On two-loop amplitudes for $t\bar{t}H$ production

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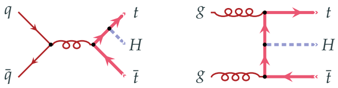
$t\bar{t}H$ production at the LHC

CRC project **B1b**, “Precision top-quark physics at the LHC”:

b) *“NNLO QCD predictions for $pp \to t\bar{t}H$ decays”.*



First observation at LHC reported in 2018. [CMS ’18, ’18, ’20, ’20, ’22; ATLAS ’18, ’20, ’23]

Current results, based on data from LHC Run 2 (2015–2018):

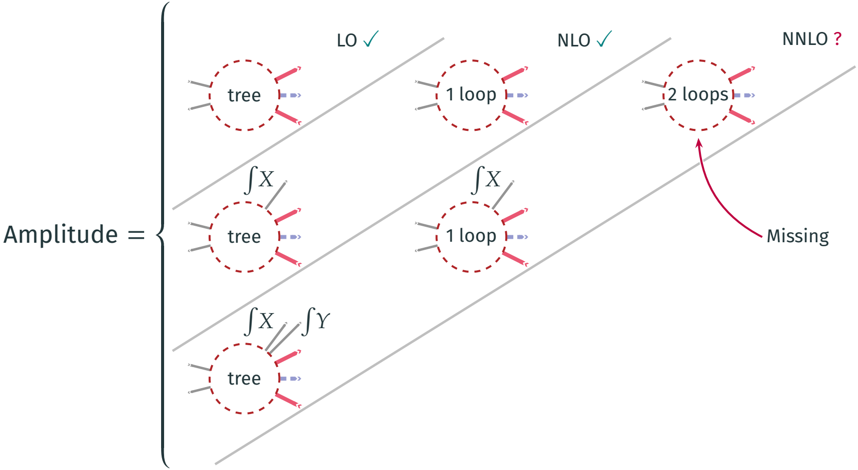
	$\sigma_{t\bar{t}H}/\sigma_{t\bar{t}H,\text{SM}}$			$\mathscr{L}$
ATLAS	0.92	$^{+0.19}_{-0.19}$(stat)	$^{+0.17}_{-0.13}$(syst)	$139\,\text{fb}^{-1}$
CMS	1.43	$^{+0.33}_{-0.31}$(stat)	$^{+0.21}_{-0.15}$(syst)	$137\,\text{fb}^{-1}$

HL-LHC will have $\mathscr{L} \sim 3000\,\text{fb}^{-1}$, reducing *statistical* uncertainty by 4x–5x.

To reduce *systematic* uncertainty: *NNLO calculation is needed*.

[HL-LHC ’19; Les Houches ’21; Snowmass ’22]

Parts of a NNLO calculation



Big missing part for NNLO: *two-loop virtual amplitudes*.

Theory results for $t\bar{t}H$ production

NLO:

- NLO QCD [Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas ’01]
  [Reina, Dawson ’01]
  [Reina, Dawson, Wackeroth ’01]
  [Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas ’02]
  [Dawson, Jackson, Orr, Reina, Wackeroth ’03]
- NLO QCD, parton shower [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli ’11]
  [Garzelli, Kardos, Papadopoulos, Trocsanyi ’11]
  [Hartanto, Jager, Reina, Wackeroth ’15]
- NLO QCD+EW [Frixione, Hirschi, Pagani, Shao, and Zaro ’14]
- NLO QCD+EW, NWA [Zhang, Ma, Zhang, Chen, Guo ’14]
  [Frixione, Hirschi, Pagani, Shao, and Zaro ’15]
- NLO QCD, off-shell [Denner, Feger ’15]
  [Stremmer, Worek ’21]
  [Denner, Lang, Pellen ’20]
  [Bevilacqua, Bi, Hartanto, Kraus, Lupattelli, Worek ’22]

Theory results for $t\bar{t}H$ production, II

NLO, contd.:

- NLO+NLL QCD [Kulesza, Motyka, Stebel, Theeuwes '15]
  [Ju, Yang '19]
- NLO+NNLL QCD [Broggio, Ferroglia, Pecjak, Signer, Yang '15]
  [Broggio, Ferroglia, Pecjak, Yang '16]
  [Kulesza, Motyka, Stebel, Theeuwes '17]
- NLO QCD+SMEFT [Maltoni, Vryonidou, Zhang '16]
- NLO QCD+EW, off-shell [Denner, Lang, Pellen, Uccirati '16]
- NLO+NNLL QCD+EW [Broggio, Ferroglia, Frederix, Pagani, Pecjak, Tsinikos '19]
- NLO QCD to $\mathcal{O}(\varepsilon^2)$ [Buccioni, Kreer, Liu, Tancredi '23]
- $t \to H$ fragmentation functions at $\mathcal{O}(y_t^2 \alpha_s)$
  [Brancaccio, Czakon, Generet, Krämer '21]

Theory results for $t\bar{t}H$ production, III

NNLO:

- NNLO QCD, flavour off-diagonal [Catani, Fabre, Grazzini, Kallweit '21]
- NNLO QCD total cross-section, soft Higgs
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]
- Two-loop QCD virtual amplitude, IR poles [Chen, Ma, Wang, Yang, Ye '22]
- Two-loop QCD master integrals, N_f-part, leading N_c
  [Cordero, Figueiredo, Kraus, Page, Reina '23]
- Two-loop QCD virtual amplitude, high-energy boosted limit
  [Wang, Xia, Yang, Ye '24]

This talk:

- *Two-loop QCD virtual amplitude, $q\bar{q}$ channel, N_f-part*
  [Agarwal, Heinrich, Jones, Kerner, Klein, Lang, V.M., Olsson '24]

The amplitude

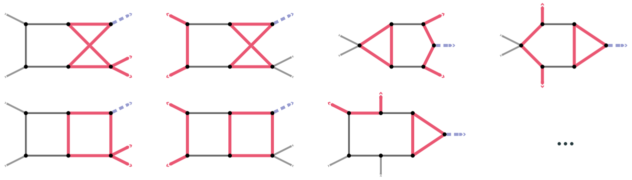
Model: QCD with a scalar H, n_l light (massless) quarks, n_h heavy (top) quarks.
Amplitude of $q\bar{q} \to t\bar{t}H$ projected onto Born, and decomposed in α_s as

$$\langle \mathrm{AMP} \,|\, \mathrm{AMP}_{\mathrm{tree}} \rangle = \mathscr{A} + \left(\frac{\alpha_s}{2\pi}\right) \mathscr{B} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathscr{C} \,.$$

As a proof-of-concept: only parts proportional to *n_l or n_h* in $\mathscr{C}$ for now.

Why is the calculation complicated?

1. IBP reduction of the amplitude to master integrals is too complicated to be computed symbolically.
   * 5 legs and 2 masses $(m_t, m_H) \Rightarrow$ 7 scales (6 scaleless variables).
2. Massive two-loop integrals contributing to $\mathscr{C}$ are not known analytically.



Calculation method

1. Generate Feynman diagrams. [QGRAF]
   - $\Rightarrow$ 249 non-zero diagrams.
2. Insert Feynman rules, apply the projector $|\,\text{AMP}_{\text{tree}}\rangle$.
3. Sum over the spinor and color tensors. [FORM; COLOR.H]
   - $\Rightarrow$ ~20000 scalar integrals;
   - $\Rightarrow$ 9 structures: $\{n_h|n_l\}\, C_A C_F N_c$, $\{n_h|n_l\}\, C_F^2 N_c$, $\{n_h|n_l\}\, d_{33}$, $\{n_h|n_l\}^2\, C_F N_c$.
4. Resolve integal symmetries, construct integral families. [FEYNSON]
   - $\Rightarrow$ 43 families (28 up to external leg permutation).
   - $\Rightarrow$ 831 master integrals in total.
5. Optimize the selection of master integrals.
   - * Criteria: quasi-finite, d-factorizing, fast to evaluate with pySECDEC.
   - * Allow increasing denominator powers up to 5 dots, and adding dimensional shifts (integrals in $d = 6 - 2\varepsilon$ and $d = 8 - 2\varepsilon$).
6. Generate IBP relations, dimensional recurrence relations. [KIRA]

...

(All of the items here use Alibrary).

Calculation method, II

7. Precompute (“trace”) the IBP solution for each family with *Rational Tracer*.
8. Precompile the *pySecDec* integration library for the amplitude pieces (each color structure is a separate weighted sum of master integrals).
9. *For each point* in the phase space:
   - 9.1 Solve IBP relations using the precomputed trace. [Ratracer; FireFly]
     - ⁎ Each variable set to a rational number.
   - 9.2 Evaluate the amplitudes as weighted sums of masters. [pySecDec]
     - ⁎ The weights are taken from the IBP solution.
   - 9.3 Apply renormalization and pole subtraction.
     [Ferroglia, Neubert, Pecjak, Yang ’09; Bärnreuther, Czakon, Fiedler ’13]
   - 9.4 Save the result.

Solving IBP with Rational Tracer

Linear equation system solving with *Ratracer*: [V.M. '22]

- Write down a system of equations (i.e. in a text file).
- Solve the system *once* using modular artihmetics with variables set to integers, and *record every arithmetic operation* into a file (a *"trace"*).
- Optimize the trace (constant propagation, dead code elimination, etc).
- Use the usual modular function reconstruction methods (i.e. FireFly) by *replaying the trace* many times (with different inputs).

$\Longrightarrow$ Around 10x faster black-box evaluation than Kira on average.

Additional trick:

- Take a trace and *expand it in ε*, save it as a new trace.
  - $\Longrightarrow$ Get the ε expansion of the IBP coefficients directly as the trace outputs.
  - $\Longrightarrow$ Eliminate ε from the list of variables.

$\Longrightarrow$ Overall 3x-4x performance gain for this calculation.

In our case (all variable set to numbers, ε eliminated via expansion):

- No function reconstruction needed! Each output is a rational number.
- Under 2 CPU minutes per point (scales well with threads).
  - Down from $\sim$1 hour on 16 cores with Kira+FireFly!

Amplitude evaluation with pySecDec

pySecDec: library for numerically evaluating Feynman integrals via *sector decomposition* and *Monte Carlo integration*. [Heinrich et al '23, '21, '18, '17]

- Takes a specification for *weighted sum of integrals* (i.e. amplitudes), produces an integration library.
  - One sum per color structure.
  - Integrals sampled adaptively to reach the requested precision of the sums.
  - The 831 masters decomposed into ∼18000 sectors (∼28000 integrals).
- Around 4x-5x speedup in version 1.6 with the new integrator "disteval".
- Integration time to get 0.3% precision for this calculation on a GPU:
  - from *5 minutes in the bulk* of the phase-space,
  - to ∞ near boundaries (e.g. high-energy region) due to growing cancellations between and inside the integrals.

Particularly large cancellations between sunrise and snow-cone integrals in the high energy region:

- The IBP coefficients become $\mathcal{O}(10^{20})$, while the amplitude is $\mathcal{O}(10^{-3})$.
- Knowing the integrals at full double precision (16 digits) is not enough!

⟹ Make pySecDec use *double-double* (32 digits) for integrals that need it.

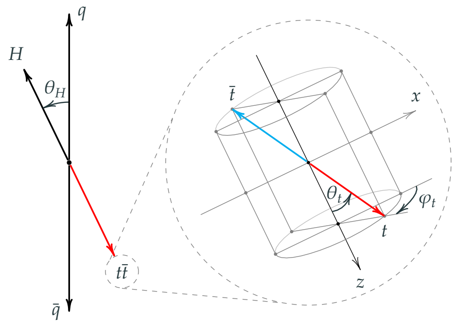
  - Around 20x performance hit on GPU, but 20-digit precision for snow cones reachable.

Phase-space parameters

To parameterize the $q\bar{q} \to t\bar{t}H$ phase-space, instead of

$$s = \left(p_q + p_{\bar{q}}\right)^2 \in \left[\,(2m_t + m_H)^2\,;\,\infty\right],$$
$$s_{t\bar{t}} = \left(p_t + p_{\bar{t}}\right)^2 \in \left[\,(2m_t)^2\,;\,\left(\sqrt{s} - m_H\right)^2 - (2m_t)^2\,\right],$$

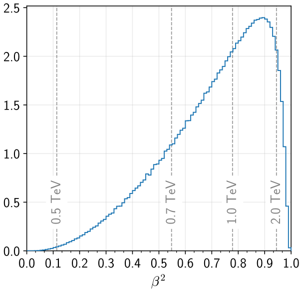
introducing:

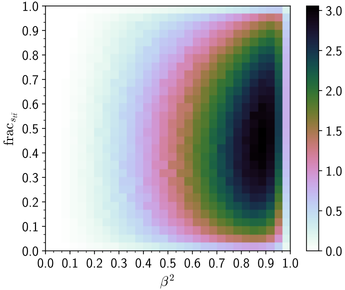


$$\beta^2 \equiv 1 - \frac{s_{min}}{s} \in [0;1],$$
$$\mathrm{frac}_{s_{t\bar{t}}} \equiv \frac{s_{t\bar{t}} - s_{t\bar{t},min}}{s_{t\bar{t},max} - s_{t\bar{t},min}} \in [0;1],$$
$$\theta_H \in [0;\pi],$$
$$\theta_t \in [0;\pi],$$
$$\varphi_t \in [0;2\pi].$$

Which parts of the phase-space are relevant?

Event density at the LHC according to the tree-level amplitude:





To cover 90% of events: $\beta^2 \in [0.34, 0.95]$, that is $\sqrt{s} \in [580\ \text{GeV}, 2.1\ \text{TeV}]$.

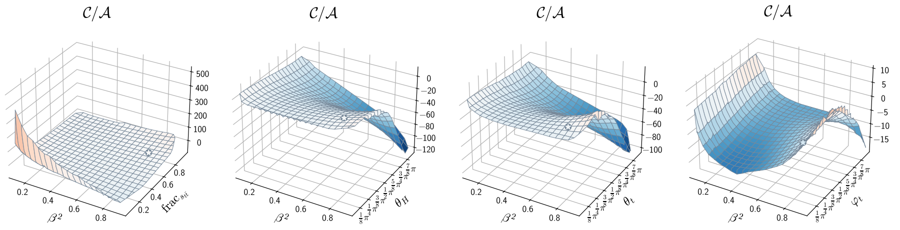
* * *

Example results as two-dimensional slices around the center point of:

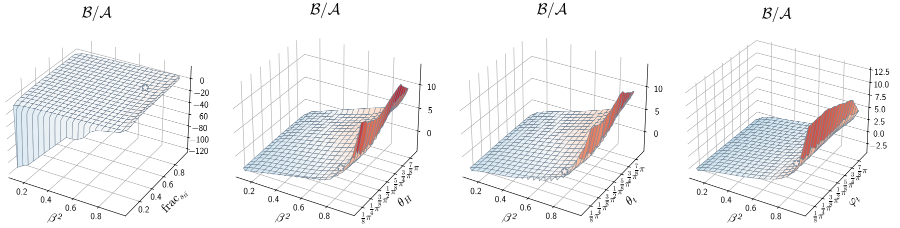
$$\beta^2 = 0.8, \qquad \mathrm{frac}_{s_{t\bar{t}}} = 0.7,$$
$$\cos\theta_H = 0.8, \qquad \cos\theta_t = 0.9, \qquad \cos\varphi_t = 0.7,$$
$$m_H^2 = 12/23\, m_t^2, \qquad \mu = s/2.$$

Resulting slices in β^2 and $\mathrm{frac}_{s_{t\bar{t}}}$, θ_H, θ_t, φ_t

N_f part of the two-loop amplitude (*our result*):

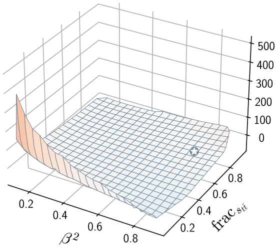


One-loop amplitude (*already known*):

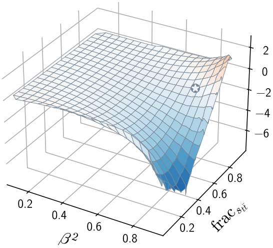


Resulting slices in β^2 and $\mathrm{frac}_{s_{t\bar{t}}}$

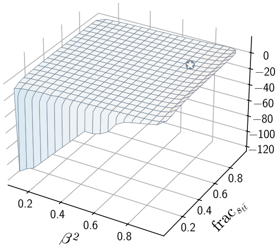
$\mathcal{C}/\mathcal{A}$



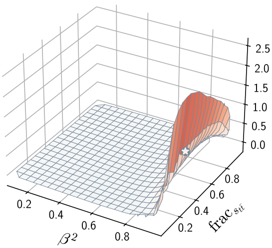
$\mathcal{C} \times (\text{phase-space density}) \times 10^3$



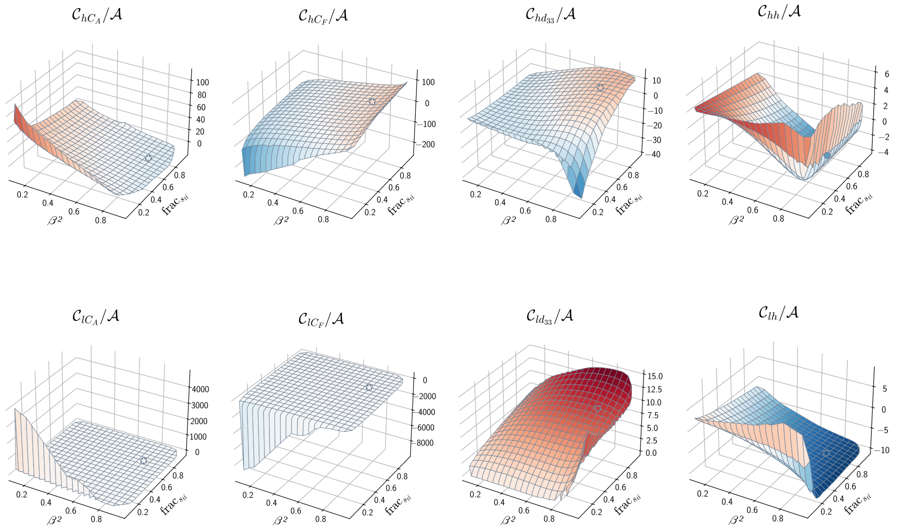
$\mathcal{B}/\mathcal{A}$



$\mathcal{B} \times (\text{phase-space density}) \times 10^3$



Resulting slices in β^2 and $\mathrm{frac}_{s_{t\bar{t}}}$ by color factor



Summary & Outlook

Done:

- N_f-part of the two-loop virtual amplitude for $q\bar{q} \to t\bar{t}H$.
- Subprojects:
  - Peformace and precision improvements in pySecDec.
  - Faster IBP solving with Ratracer.
  - Amplitude generation with Alibrary.

In progress:

- The rest of the two-loop virtual amplitude for $q\bar{q} \to t\bar{t}H$.
- Interpolation for the results.

Todo:

- Full two-loop virtual amplitude for $gg \to t\bar{t}H$.
- Phenomenological applications.

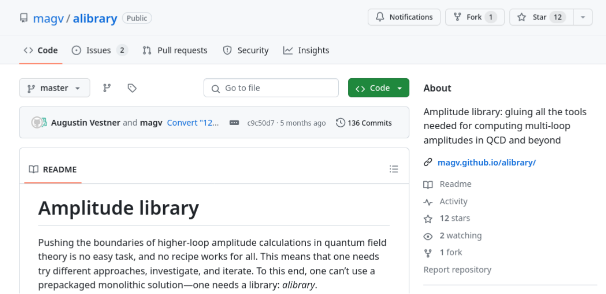
Backup slides

Amplitude library

Most of this work is powered by Alibrary (*“amplitude library”*). It provides functions and interfaces to tools for multiloop calculations in Mathematica:

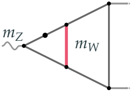
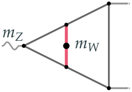
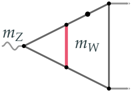
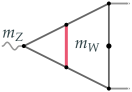
- Diagram generation and visualization (Qgraf, Graphviz, TikZ).
- Feynman rule insertion.
- Tensor trace summation (Form, Color.h).
- Integral symmetries, IBP families (Feynson).
- Export to/from IBP solvers (Kira, Fire+LiteRed).
- Export to/from pySecDec.

github.com/magv/alibrary



On the choice of master integrals

Integration time of similarly looking integrals to 10^{-3} precision:[1]

	orders	t, s		orders	t, s
	$\varepsilon^{-3} \dots \varepsilon^{0}$	27		$\varepsilon^{-2} \dots \varepsilon^{0}$	57
	$\varepsilon^{-2} \dots \varepsilon^{0}$	1230		$\varepsilon^{-2} \dots \varepsilon^{0}$	>9000

Takeaway: for best performance, test the integration speed and adjust the selection of the master integrals.

[1] pySecDec 1.5.3, NVidia A100 GPU.