

# EFT descriptions of Higgs boson pair production

Annual Meeting of the CRC TRR 257, A1b

Jannis Lang | March 11, 2024

INSTITUTE FOR THEORETICAL PHYSICS

Mainly based on collaborative research works:

[1] [JHEP 08 \(2022\) 079](#)

[Heinrich,JL,Scyboz '22]

[2] <https://arxiv.org/abs/2304.01968>

[CERN LHC Higgs WG4 note]

[3] <https://arxiv.org/abs/2310.18221>

[Di Noi,Gröber,Heinrich,JL,Vitti '23]

[4] <https://arxiv.org/abs/2311.15004>

[Heinrich,JL '23]

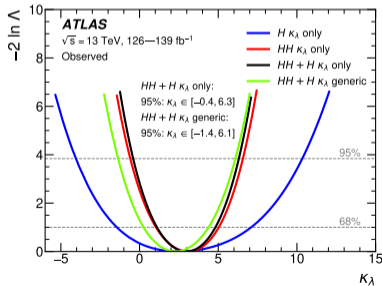
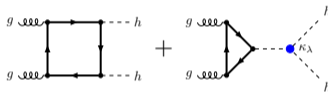
# Relevance of $hh$ production in an EFT framework

■ Impressive experimental results on Higgs couplings!

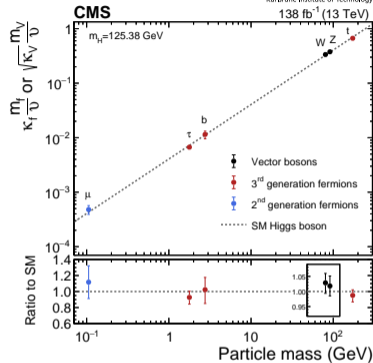
■ Is Higgs potential SM-like?

$$V_{\text{SM}} \sim \frac{m_h^2}{2} h^2 + \kappa\lambda \frac{m_h^2}{2v} h^3 + \frac{\lambda}{4} h^4$$

⇒  $\kappa\lambda$  accessible in  $hh$  production:

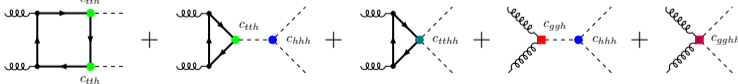


[2211.01216]



[2207.00043]

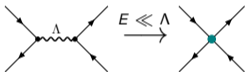
■ More consistent approach: Bottom-up EFT (assuming scale separation)



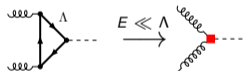
# Effective field theory basics

## Top-down perspective

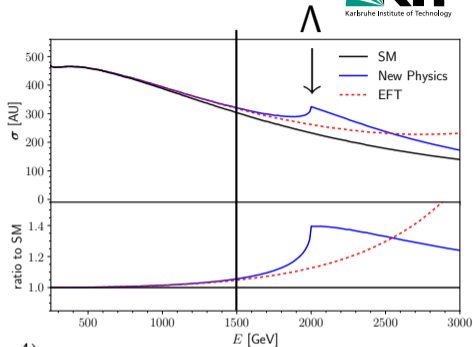
- Effective low-energy description integrating out heavy particles with mass  $M \sim \Lambda$
- Example: Fermi theory of weak interaction and heavy top limit



$$\Delta\mathcal{L} = \frac{C}{\Lambda^2} \bar{\psi}_i \gamma^\mu \psi_j \bar{\psi}_k \gamma_\mu \psi_l + \mathcal{O}(\Lambda^{-4})$$



$$\Delta\mathcal{L} = \frac{g_s^2 C'}{(16\pi^2)\Lambda^2} (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu} + \mathcal{O}(\Lambda^{-4})$$



## Bottom-up EFT: systematic parameterisation for unknown new physics above energy scale $\Lambda$

- Standard Model Effective Field Theory (SMEFT)
- Higgs Effective Field Theory (HEFT)

# Two bottom-up EFT systematics: SMEFT vs. HEFT

$d_c$  : canonical dimension  
 $L, \mathbf{L}$  : loop factor  $(16\pi^2)^{-1}$   
 $g_s$  : strong coupling

## SMEFT:

- Decoupling scenario for  $\Lambda \rightarrow \infty$ : doublet Higgs
- Expansion of contributions according to

$$\mathcal{O} \left( \Lambda^{-d_c} (g_s^2 L)^{l_{\text{QCD}}} \mathbf{L}^{l_{\text{non-QCD}}} \right)$$

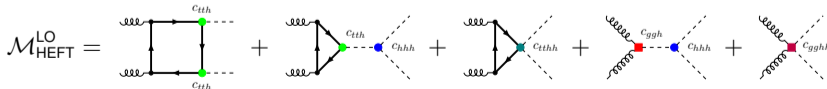
$$\Delta \mathcal{L}_{\text{SMEFT}}^{\text{lead}} = \frac{C_{H\Box}}{\Lambda^2} (\phi^\dagger \phi) \Box (\phi^\dagger \phi) + \frac{C_{HD}}{\Lambda^2} (\phi^\dagger D_\mu \phi)^* (\phi^\dagger D^\mu \phi) + \frac{C_H}{\Lambda^2} (\phi^\dagger \phi)^3$$

$$+ \frac{C_{tH}}{\Lambda^2} \left( (\phi^\dagger \phi) (\bar{Q}_L t_R \tilde{\phi}) + \text{H.c.} \right) + \frac{C_{HG}}{\Lambda^2} (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu}$$

## HEFT:

- Non-decoupling scenario: singlet Higgs
- Contributions ordered by loop expansion

$$\mathcal{L}_{\text{HEFT}}^{\text{lead}} = -m_t \left( C_{tth} \frac{h}{v} + C_{ttth} \frac{h^2}{v^2} \right) \bar{t}t - C_{hhh} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left( C_{ggh} \frac{h}{v} + C_{gggh} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a\mu\nu}$$



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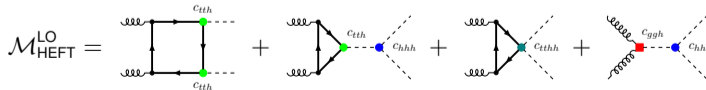
$$+ \frac{C_{tH}}{\Lambda^2} \left( (\phi^\dagger \phi) (\bar{Q}_L t_R \tilde{\phi}) + \text{H.c.} \right) + \frac{C_{HG}}{\Lambda^2} (\phi^\dagger \phi) G_{\mu\nu}^a G^{a\mu\nu}$$

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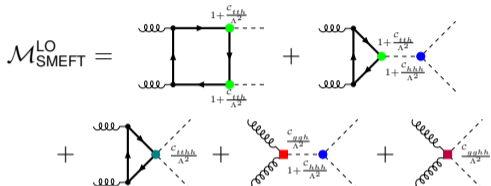
HEFT	Warsaw
$C_{hhh}$	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H; \text{kin}}$
$C_{tth}$	$1 + \frac{v^2}{\Lambda^2} C_{H; \text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{tH}$
$C_{ttth}$	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{tH} + \frac{v^2}{\Lambda^2} C_{H; \text{kin}}$
$C_{ggh}$	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s(\mu)} C_{HG}$
$C_{gggh}$	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s(\mu)} C_{HG}$



Not generally applicable:  
 $C_i \sim \mathcal{O}(1)$  possible  $\leftrightarrow \frac{E^2}{\Lambda^2} C_i \ll 1$

# SMEFT truncation

SMEFT truncation of amplitude ( $\frac{c_i}{\Lambda^2} = c_i - c_{i,sm}$ ):

$$\begin{aligned}
 \mathcal{M}_{\text{SMEFT}}^{\text{LO}} &= \text{[Diagram 1]} + \text{[Diagram 2]} \\
 &+ \text{[Diagram 3]} + \text{[Diagram 4]} + \text{[Diagram 5]} \\
 &= \underbrace{\mathcal{M}_{\text{SM}}^{\text{LO}}}_{\mathcal{O}(g_S^2 L)} + \underbrace{\mathcal{M}_{\text{dim6}}^{\text{LO}}}_{\mathcal{O}(\Lambda^{-2}(g_S^2 L))} + \underbrace{\mathcal{M}_{\text{dim6}^2}^{\text{LO}}}_{\mathcal{O}(\Lambda^{-4}(g_S^2 L))}
 \end{aligned}$$


SMEFT truncation of cross section:

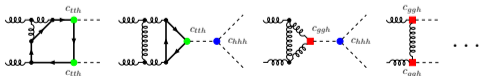
$$\sigma \simeq \begin{cases} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} & \text{(a)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} & \text{(b)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} & \text{(c)} \\ \sigma_{(\text{SM} + \text{dim6} + \text{dim6}^2) \times (\text{SM} + \text{dim6} + \text{dim6}^2)} & \text{(d)} \end{cases}$$

# SMEFT truncation

SMEFT truncation of amplitude  $\left(\frac{C_i}{\Lambda^2} = c_i - c_{i,sm}\right)$ :

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 \end{aligned}$$

⇒ Implemented at NLO QCD in the POWHEG BOX translating  $ggHH \rightarrow ggHH_{\text{SMEFT}}$ :



SMEFT truncation of cross section:

$$\sigma \simeq \begin{cases} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} & \text{(a)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} & \text{(b)} \\ \sigma_{(\text{SM} + \text{dim6}) \times (\text{SM} + \text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} & \text{(c)} \\ \sigma_{(\text{SM} + \text{dim6} + \text{dim6}^2) \times (\text{SM} + \text{dim6} + \text{dim6}^2)} & \text{(d)} \end{cases}$$

**Reals:** Modified version of 1-loop ME provider GoSam

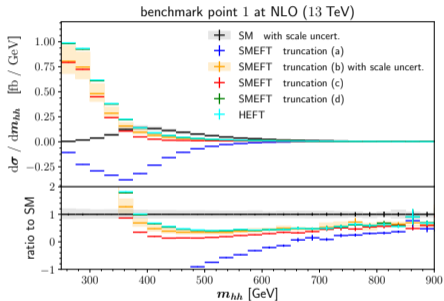
**Virtuals:** Adjust grids encoding 2-loop corrections

$$\mathcal{V}_{\text{fin}} = \sum a_i C_{hh}^{n_{hh,i}} C_{tth}^{n_{tth,i}} C_{ttth}^{n_{ttth,i}} C_{gg}^{n_{gg,i}} C_{ggh}^{n_{ggh,i}}$$

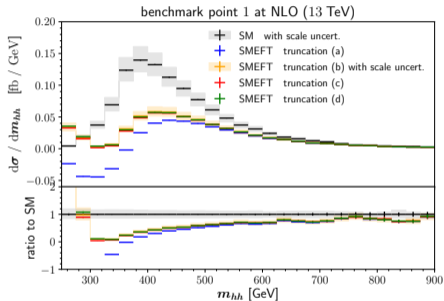
# Invariant mass distributions at NLO QCD [1]

- HEFT benchmark 1:  
enhanced low- $m_{hh}$  region

$C_{hhh}$	$C_{tth}$	$C_{ttth}$	$C_{ggh}$	$C_{gggh}$	$C_{H; kin}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$

- Truncation (a): negative cross section
- ⇒ Valid HEFT point invalid in SMEFT after naive translation

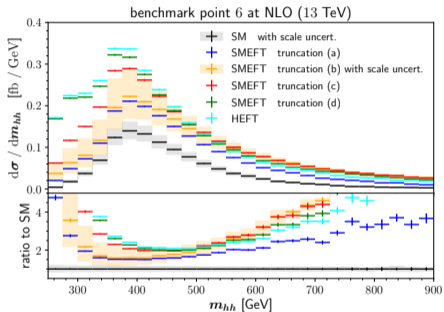
- Distributions converge for increasing  $\Lambda$
- ⇒ Consistent with measure for truncation validity



# Invariant mass distributions at NLO QCD [1]

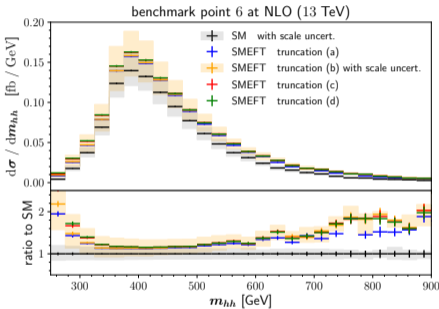
- HEFT benchmark 6:  
close-by double peaks

$C_{hhh}$	$C_{tth}$	$C_{ttth}$	$C_{ggg}$	$C_{gggh}$	$C_{H; kin}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV



$\Lambda = 1 \text{ TeV}$

- No negative cross sections
- Typical shape not recovered for SMEFT (except for (d))



$\Lambda = 2 \text{ TeV}$

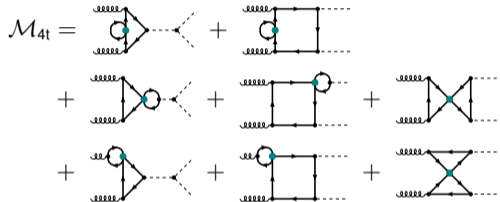
- Difference HEFT ↔ (d) mainly from  $\alpha_s(\mu)$
- Shapes converge faster for increasing  $\Lambda$

# Amplitude with $C_{tG}$ and 4-top insertion [4]

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2} \left( (\bar{Q}_L \sigma^{\mu\nu} T^a t_R \tilde{\phi}) G_{\mu\nu}^a + \text{H.c.} \right)$$



$$\begin{aligned} \mathcal{L}_{4t} = & \frac{C_{Qt}^{(1)}}{\Lambda^2} (\bar{Q}_L \gamma^\mu Q_L) \bar{t}_R \gamma_\mu t_R \\ & + \frac{C_{Qt}^{(8)}}{\Lambda^2} (\bar{Q}_L \gamma^\mu T^a Q_L) \bar{t}_R \gamma_\mu T^a t_R \\ & + \frac{C_{QQ}^{(1)}}{\Lambda^2} (\dots) + \frac{C_{QQ}^{(8)}}{\Lambda^2} (\dots) + \frac{C_{tt}}{\Lambda^2} (\dots) \end{aligned}$$



- $\mathcal{M}_{tG}, \mathcal{M}_{4t} \sim \mathcal{O}((g_s^2 L) \Lambda^{-2})$  subleading w.r.t.  $\mathcal{M}_{\text{dim6}}^{\text{LO}} \sim \mathcal{O}((g_s^2 L) \Lambda^{-2})$
- 4-top contributions factorise in 1-loop structures  $\Rightarrow$  Added analytically to ggHH\_SMEFT
- Cross check with GoSam or multi-loop framework alibrary (QGraf, FORM, Kira, pySecDec)

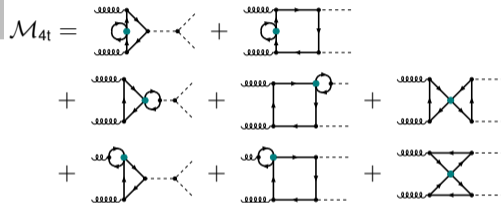
# Amplitude with $C_{tG}$ and 4-top insertion [3,4]

$$\mathcal{L}_{tG} \sim \frac{C_{tG}}{\Lambda^2} \bar{t} \sigma^{\mu\nu} T^a t \frac{h+v}{\sqrt{2}} G_{\mu\nu}^a$$



Chiral currents!  
 $\mathbb{P}_{L/R} = (1 \mp \gamma_5)/2$

$$\mathcal{L}_{4t} \sim \frac{C_{Qt}^{(1)}}{\Lambda^2} \bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R t$$



$$+ \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{t} \mathbb{P}_R \gamma^\mu \mathbb{P}_L T^a t \bar{t} \mathbb{P}_L \gamma_\mu \mathbb{P}_R T^a t$$

$$+ \frac{C_{QQ}^{(1)}}{\Lambda^2} (\dots) + \frac{C_{QQ}^{(8)}}{\Lambda^2} (\dots) + \frac{C_{tt}}{\Lambda^2} (\dots)$$

- $\mathcal{M}_{tG}, \mathcal{M}_{4t} \sim \mathcal{O}((g_s^2 L) \Lambda^{-2})$  subleading w.r.t.  $\mathcal{M}_{\text{dim6}}^{\text{LO}} \sim \mathcal{O}((g_s^2 L) \Lambda^{-2})$
- 4-top contributions factorise in 1-loop structures  $\Rightarrow$  Added analytically to ggHH\_SMEFT
- Cross check with GoSam or multi-loop framework alibrary (QGraf, FORM, Kira, pySecDec)
- Study structure of different  $\gamma_5$  scheme choices in dimensional regularisation

# 4-top contributions and the $\gamma_5$ scheme [3,4]

$\bar{\gamma}_5$  in 4-dim: (1)  $\{\bar{\gamma}_5, \bar{\gamma}^\mu\} = 0$  (2)  $\text{Tr}[\bar{\gamma}^{\mu_1} \bar{\gamma}^{\mu_2} \bar{\gamma}^{\mu_3} \bar{\gamma}^{\mu_4} \bar{\gamma}_5] = -4i \bar{\epsilon}^{\mu_1 \mu_2 \mu_3 \mu_4}$  (3)  $\text{Tr}[\Gamma_1 \Gamma_2 \bar{\gamma}_5] = \text{Tr}[\Gamma_2 \bar{\gamma}_5 \Gamma_1]$

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$\gamma_5$  in  $D$ -dim: **NDR:**  $\{\gamma_5, \gamma^\mu\} = 0$

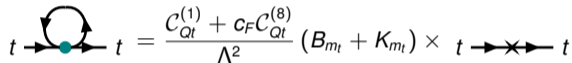
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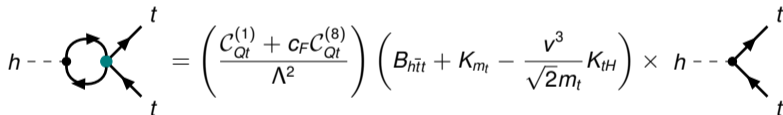
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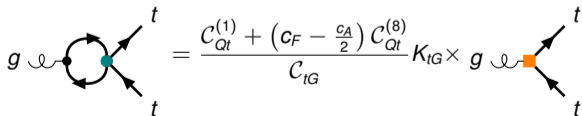
$$t \text{ loop } t = \frac{C_{Qt}^{(1)} + C_F C_{Qt}^{(8)}}{\Lambda^2} (B_{m_t} + K_{m_t}) \times t \text{ cross } t$$

$$K_{m_t} = \begin{cases} -\frac{m_t^2}{8\pi^2} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$



$$h \text{ loop } t = \left( \frac{C_{Qt}^{(1)} + C_F C_{Qt}^{(8)}}{\Lambda^2} \right) \left( B_{h\bar{t}t} + K_{m_t} - \frac{v^3}{\sqrt{2}m_t} K_{tH} \right) \times h \text{ vertex } t$$

$$K_{tH} = \begin{cases} \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$



$$g \text{ loop } t = \frac{C_{Qt}^{(1)} + (C_F - \frac{C_A}{2}) C_{Qt}^{(8)}}{C_{tG}} K_{tG} \times g \text{ vertex } t$$

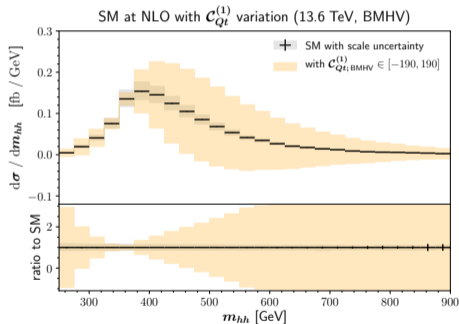
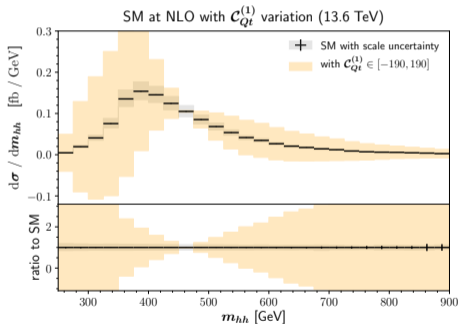
$$K_{tG} = \begin{cases} -\frac{\sqrt{2}m_t g_s}{16\pi^2 v} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$

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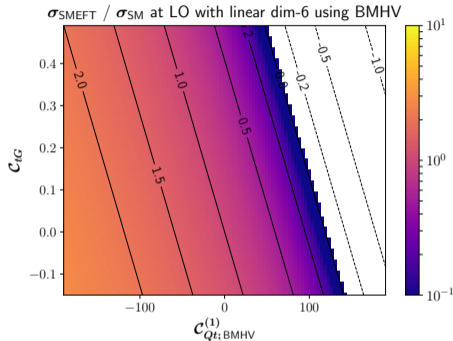
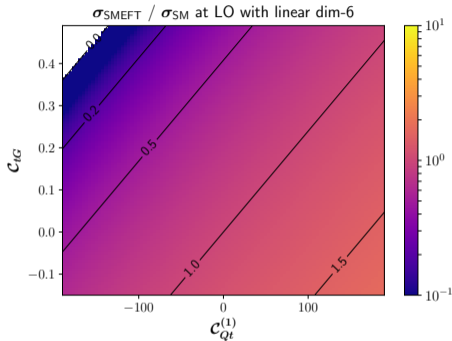
- Amplitude structure for single 4-top Wilson coefficients  $\gamma_5$  scheme dependent

Ranges from  $\mathcal{O}(\Lambda^{-2})$  marginalised fits of [2105.00006 (SMEFIT collaboration, Ethier et al.)]

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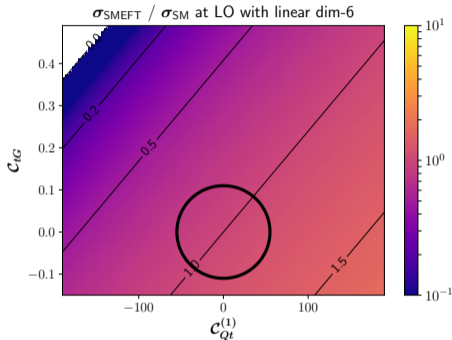
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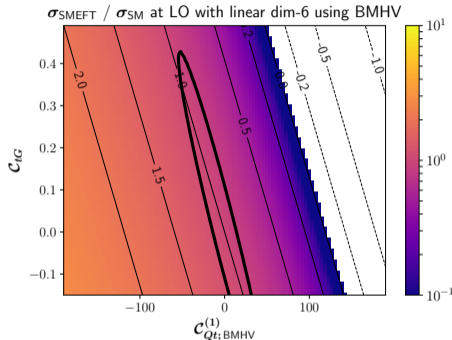
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$$C_{tH}^{\text{BMHV}} = C_{tH}^{\text{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} \left( C_{Qt}^{(1)} + C_F C_{Qt}^{(8)} \right)$$

$$C_{tG}^{\text{BMHV}} = C_{tG}^{\text{NDR}} - \frac{\sqrt{2}m_t g_s}{16\pi^2 v} \left( C_{Qt}^{(1)} + \left( C_F - \frac{C_A}{2} \right) C_{Qt}^{(8)} \right)$$

- equivalent parameterisation of new physics
- $\Rightarrow \gamma_5$  scheme independence requires multi-parameter contributions

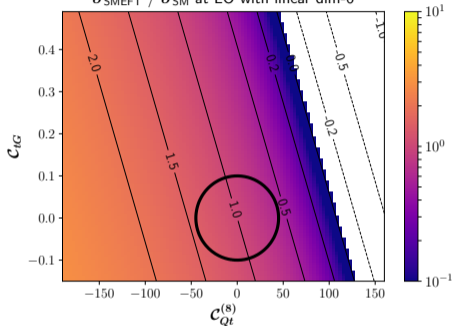
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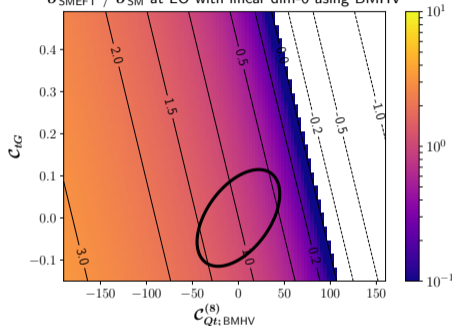
**NDR:**  $\{\gamma_5, \gamma^\mu\} = 0$

$\sigma_{\text{SMEFT}} / \sigma_{\text{SM}}$  at LO with linear dim-6



**BMHV:**  $\gamma_5 \equiv \bar{\gamma}_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$\sigma_{\text{SMEFT}} / \sigma_{\text{SM}}$  at LO with linear dim-6 using BMHV



$$C_{tH}^{\text{BMHV}} = C_{tH}^{\text{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} (C_{Qt}^{(1)} + C_F C_{Qt}^{(8)})$$

$$C_{tG}^{\text{BMHV}} = C_{tG}^{\text{NDR}} - \frac{\sqrt{2}m_t g_s}{16\pi^2 v} \left( C_{Qt}^{(1)} + \left( C_F - \frac{C_A}{2} \right) C_{Qt}^{(8)} \right)$$

■ equivalent parameterisation of new physics

⇒  $\gamma_5$  scheme independence requires multi-parameter contributions

Ranges from  $\mathcal{O}(\Lambda^{-2})$  marginalised fits of [2105.00006 (SMEFIT collaboration, Ethier et al.)]

# Summary

- State-of-the-art SMEFT predictions with ggHH\_SMEFT:  
(Public and implemented in the POWHEG BOX)

$C_H, C_{tH}, C_{HG}, C_H; \text{kin}$  @NLO QCD

$C_{tG}$  &  $C_{Qt}^{(1)}, C_{Qt}^{(8)}, C_{QQ}^{(1)}, C_{QQ}^{(8)}, C_{tt}$

**truncation options**

$\gamma_5$  schemes **NDR & BMHV**

- Naive translation from HEFT  $\rightarrow$  SMEFT can lead out of validity of  $\Lambda^{-2}$  expansion
- $\gamma_5$  scheme independence at higher orders requires inclusive selection of contributions
- $gg \rightarrow h(h)$  can potentially help to improve global fits of  $C_{Qt}^{(1)}$  and  $C_{Qt}^{(8)}$

## Future directions

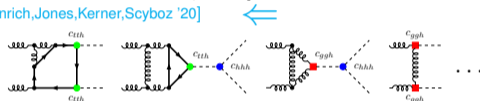
- Scale dependence of Wilson coefficients
- EW corrections required for complete subleading operator contribution
- Exhaustive study of  $\gamma_5$  schemes in EFTs

# Public implementations of EFT tools for $gg \rightarrow hh$

HTL = Heavy top limit ( $m_t \rightarrow \infty$ )

## Higgs Effective Field Theory (HEFT):

- LO and NLO QCD HTL HPAIR [Gröber,Mühlleitner,Spira,Streicher '15]
- Full  $m_t$  NLO QCD POWHEG-BOX-V2/ $ggHH$  [Buchalla,Capozi,Celis,Heinrich,Scyboz '18]  
[Heinrich,Jones,Kerner,Luisoni,Scyboz '19]  
[Heinrich,Jones,Kerner,Scyboz '20]
- **Non-public** state-of-the-art NNLO' (HTL NNLO, full  $m_t$  NLO) [de Florian,Fabre,Heinrich,Mazitelli,Scyboz '21]



## Standard Model Effective Field Theory (SMEFT):

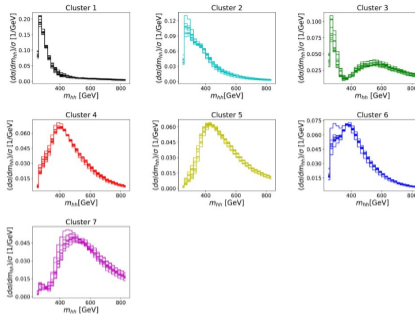
- LO and NLO QCD HTL HPAIR [Gröber,Mühlleitner,Spira,Streicher '15]
- LO (1-loop) including  $C_{tG}$  SMEFT@NLO + MG5\_aMC@NLO [Degrande,Durieux,Maltoni,Mimasu,Vryonidou,Zhang '20]
- Full  $m_t$  NLO QCD POWHEG-BOX-V2/ $ggHH\_SMEFT$  [1]
- with truncation options [4]
- +  $C_{tG}$  and 4-top



# Naive benchmark translation

Consider HEFT benchmark points with characteristic  $m_{hh}$ -distributions:

- Benchmark 1: enhanced low  $m_{hh}$  region
- Benchmark 6: close-by double peaks
- benchmark 3: enhanced low  $m_{hh}$  region and second local maximum above  $m_{hh} \simeq 2m_t$



[Capozzi, Heinrich '19] [2]

benchmark	$C_{hhh}$	$C_t$	$C_{tt}$	$C_{ggh}$	$C_{gggh}$	$C_{H; kin}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
SM	1	1	0	0	0	0	0	0	0	1 TeV
1	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
6	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV
3	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387	1 TeV

⇒ SMEFT expansion based on  $E^2 \frac{C_i}{\Lambda^2} \ll 1$  justified?

$C_{HG}$  obtained using  $\alpha_s(m_Z) = 0.118$

# Naive benchmark translation



Total cross section generated at  $\sqrt{s} = 13$  TeV [1]

benchmark	$\sigma_{\text{NLO}}$ [fb] option (b)	$\sigma_{\text{NLO}}$ [fb] option (a)	$\sigma_{\text{NLO}}$ [fb] HEFT
SM	$27.94^{+13.7\%}_{-12.8\%}$	-	-
$\Lambda = 1$ TeV			
1	$71.95^{+20.1\%}_{-15.7\%}$	<b>-57.64</b>	91.62
3	$68.69^{+9.4\%}_{-9.5\%}$	30.15	70.20
6	$70.18^{+18.8\%}_{-15.5\%}$	50.82	87.9
$\Lambda = 2$ TeV			
1	$14.53^{+12.6\%}_{-12.2\%}$	6.44	-
3	$30.80^{+14.4\%}_{-13.6\%}$	28.41	-
6	$34.80^{+16.8\%}_{-14.9\%}$	33.6	-

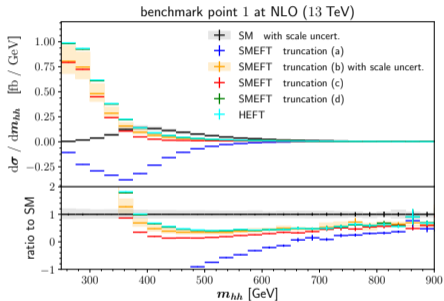
V  
V  
V  
V

0.118

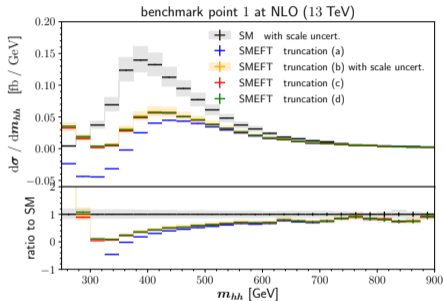
# Invariant mass distributions at NLO QCD [1]

- HEFT benchmark 1:  
enhanced low- $m_{hh}$  region

$C_{hhh}$	$C_{tth}$	$C_{ttth}$	$C_{ggh}$	$C_{gggh}$	$C_{H; kin}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$

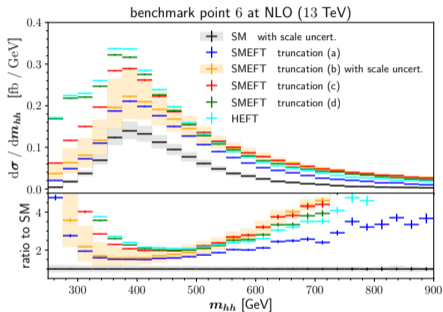
- Truncation (a): negative cross section
- ⇒ Valid HEFT point invalid in SMEFT after naive translation

- Distributions converge for increasing  $\Lambda$
- ⇒ Consistent with measure for truncation validity

# Invariant mass distributions at NLO QCD [1]

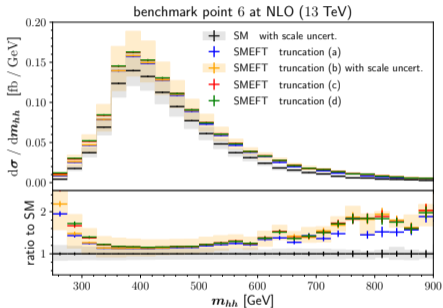
- HEFT benchmark 6:  
close-by double peaks

$C_{hhh}$	$C_{tth}$	$C_{ttth}$	$C_{ggg}$	$C_{gggh}$	$C_{H; \text{kin}}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV



$\Lambda = 1 \text{ TeV}$

- No negative cross sections
- Typical shape not recovered for SMEFT (except for (d))



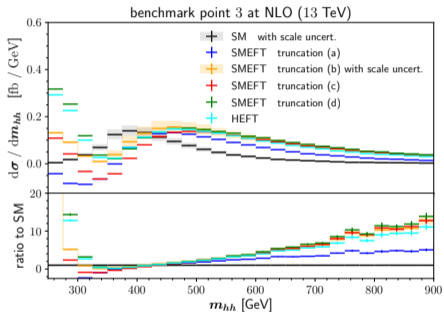
$\Lambda = 2 \text{ TeV}$

- Difference HEFT ↔ (d) mainly from  $\alpha_s(\mu)$
- Shapes converge faster for increasing  $\Lambda$

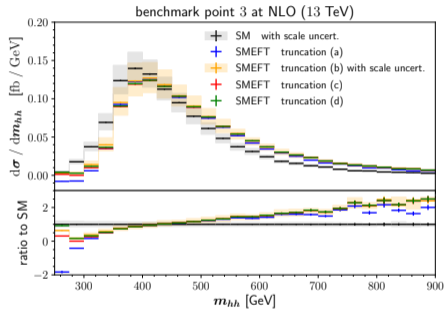
# Invariant mass distributions at NLO QCD [1]

- HEFT benchmark 3:  
enhanced low- $m_{hh}$  region  
and 2nd local maximum

$C_{hhh}$	$C_{tth}$	$C_{ttth}$	$C_{ggg}$	$C_{gggh}$	$C_{H; kin}$	$C_H$	$C_{tH}$	$C_{HG}$	$\Lambda$
-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV



$\Lambda = 1 \text{ TeV}$



$\Lambda = 2 \text{ TeV}$



# Estimating theory uncertainties

$$\Delta\sigma \sim \begin{matrix} +\Delta_{\text{scale}+} \\ -\Delta_{\text{scale}-} \end{matrix} \begin{matrix} +\Delta_{m_t \text{ scheme}+} \\ -\Delta_{m_t \text{ scheme}-} \end{matrix} \pm \Delta_{\text{num. grid}} \quad (\pm \Delta_{\text{EFT trunc.}}) \quad \pm \Delta_{\text{PDF}+\alpha_s} \quad \pm \Delta_{\text{EW}} \quad \{\pm \Delta_{\text{Decay}}\}$$

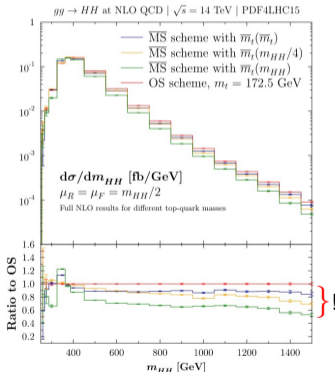
- $\Delta_{\text{scale} \pm}$ : Determined by 7-point variation of  $\mu_R, \mu_F = \{0.5, 1, 2\} \cdot \mu_0$   
 $\mathcal{O}(15\%)$  for NLO QCD SM, 15-20% for NLO QCD SMEFT truncation (b) benchmark 1 & 6
- $\Delta_{m_t \text{ scheme} \pm}$ : In principle needs determination for each point in EFT parameter space! (not yet available) [Baglio et al '18] [Baglio et al '20] [Baglio et al '20]
- $\Delta_{\text{num. grid}}$ : Numerical uncertainty of  $\mathcal{V}_{\text{fin}}$  due to grid population, not covered by Monte Carlo statistical uncertainty of POWHEG!
- $\Delta_{\text{EFT trunc.}}$ : No quantitative prescription, qualitative observation of truncation options
- $\Delta_{\text{PDF}+\alpha_s} \approx 3\%$  ( $\sqrt{s} = 13 \text{ TeV}$ ): B.I. NNLO HTL and employing PDF4LHCNNLO [twiki *hh* cross group]  
 stable for  $c_{hhh}$  variation, but might rise if tail enhanced
- $\Delta_{\text{EW}}$ : NLO EW for SM available, 10% effects w.r.t. LO QCD for threshold and tails [Bi, Huang, Ma, Yu '23]  
 unknown for EFT scenario, combines with subleading operator contributions!

# $m_t$ renormalisation scheme uncertainty

[Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '18]  
 [Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira,Streicher '20]  
 [Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira '20]

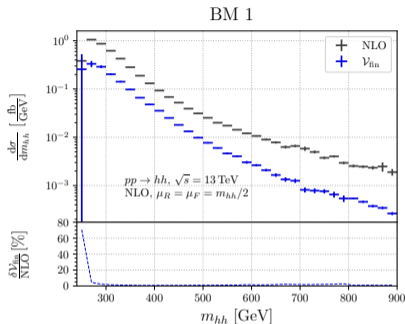
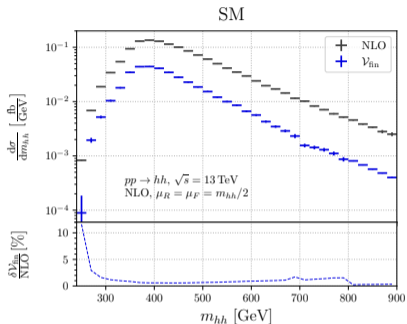
$$\bar{m}_t(m_t) = \frac{m_t}{1 + \frac{4}{3} \frac{\alpha_s(m_t)}{\pi} + K_2 \left( \frac{\alpha_s(m_t)}{\pi} \right)^2 + K_3 \left( \frac{\alpha_s(m_t)}{\pi} \right)^3}$$

- Prediction depends on  $m_t$  scheme (on-shell vs.  $\overline{MS}$  with varying scale)
- Uncertainty sensitive to choice of  $C_{hhh} = \kappa_\lambda$
- Sensitivity to variations of  $C_{tt}$  expected



$\kappa_\lambda = -10$ :	$\sigma_{tot} = 1438(1)_{-6\%}^{+10\%}$ fb,
$\kappa_\lambda = -5$ :	$\sigma_{tot} = 512.8(3)_{-7\%}^{+10\%}$ fb,
$\kappa_\lambda = -1$ :	$\sigma_{tot} = 113.66(7)_{-9\%}^{+8\%}$ fb,
$\kappa_\lambda = 0$ :	$\sigma_{tot} = 61.22(6)_{-12\%}^{+6\%}$ fb,
$\kappa_\lambda = 1$ :	$\sigma_{tot} = 27.73(7)_{-18\%}^{+4\%}$ fb,
$\kappa_\lambda = 2$ :	$\sigma_{tot} = 13.2(1)_{-23\%}^{+1\%}$ fb,
$\kappa_\lambda = 2.4$ :	$\sigma_{tot} = 12.7(1)_{-22\%}^{+4\%}$ fb,
$\kappa_\lambda = 3$ :	$\sigma_{tot} = 17.6(1)_{-15\%}^{+9\%}$ fb,
$\kappa_\lambda = 5$ :	$\sigma_{tot} = 83.2(3)_{-4\%}^{+13\%}$ fb,
$\kappa_\lambda = 10$ :	$\sigma_{tot} = 579(1)_{-4\%}^{+12\%}$ fb

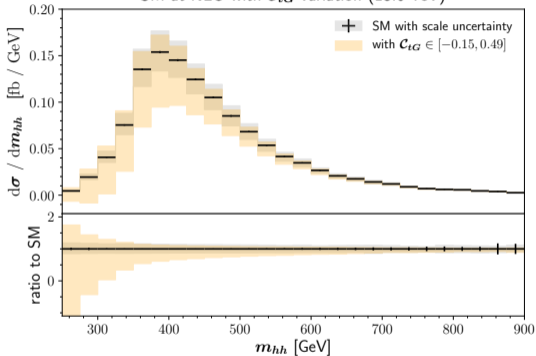
# Numerical grids uncertainty



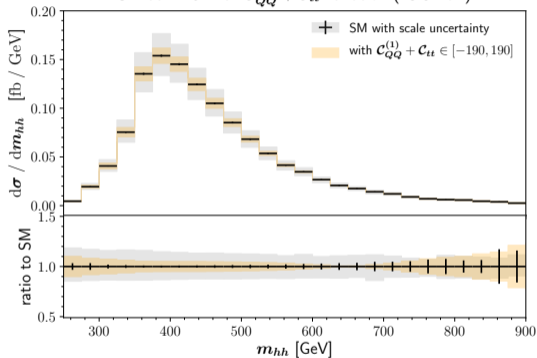
- Low (and high)  $m_{hh}$  region very sparsely populated in virtual grids, due to small contribution in SM
- ⇒  $\mathcal{O}(12\%)$  uncertainty for SM in first bin not represented by Monte Carlo statistical uncertainty in POWHEG
- ⇒ Uncertainty much worse for scenarios with enhanced low  $m_{hh}$  region

# $C_{tG}$ and irrelevant 4-top contributions [4]

SM at NLO with  $C_{tG}$  variation (13.6 TeV)



SM at NLO with  $C_{QQ}^{(1)} + C_{tt}$  variation (13.6 TeV)



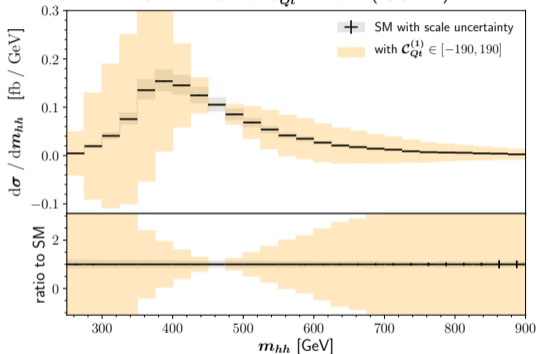
Ranges from  $\mathcal{O}(\Lambda^{-2})$  marginalised fits of [2105.00006 (SMEFIT collaboration, Ethier et al.)]

# 4-top contributions and the $\gamma_5$ scheme [3,4]

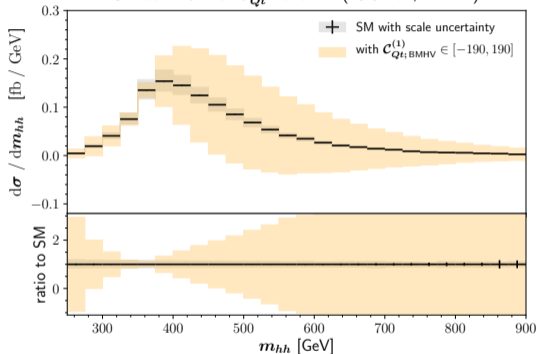
**NDR:**  $\{\gamma_5, \gamma^\mu\} = 0$

**BMHV:**  $\gamma_5 \equiv \bar{\gamma}_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

SM at NLO with  $C_{Qt}^{(1)}$  variation (13.6 TeV)



SM at NLO with  $C_{Qt}^{(1)}$  variation (13.6 TeV, BMHV)



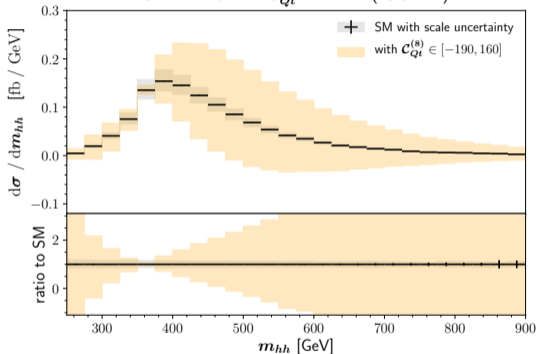
Ranges from  $\mathcal{O}(\Lambda^{-2})$  marginalised fits of [2105.00006 (SMEFIT collaboration, Ethier et al.)]

# 4-top contributions and the $\gamma_5$ scheme [3,4]

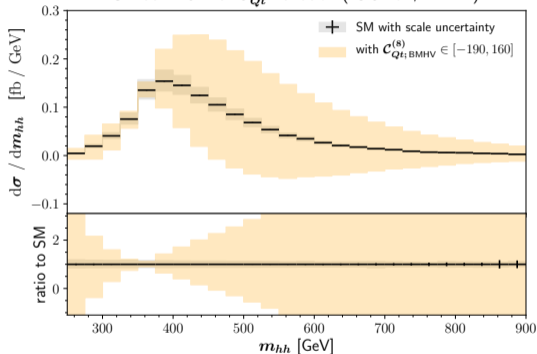
**NDR:**  $\{\gamma_5, \gamma^\mu\} = 0$

**BMHV:**  $\gamma_5 \equiv \bar{\gamma}_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

SM at NLO with  $C_{Qt}^{(8)}$  variation (13.6 TeV)



SM at NLO with  $C_{Qt}^{(8)}$  variation (13.6 TeV, BMHV)



Ranges from  $O(\Lambda^{-2})$  marginalised fits of [2105.00006 (SMEFIT collaboration, Ethier et al.)]

# $\gamma_5$ scheme translation

$$\mathcal{L}_{4t} \supset \frac{C_{Qt}^{(1)}}{\Lambda^2} (\bar{Q}_L \gamma^\mu Q_L) \bar{t}_R \gamma_\mu t_R$$

$$+ \frac{C_{Qt}^{(8)}}{\Lambda^2} (\bar{Q}_L \gamma^\mu T^a Q_L) \bar{t}_R \gamma_\mu T^a t_R$$

$$\mathcal{L}_{\psi 2\phi 2D} \supset \frac{C_{HQ}^{(1)}}{\Lambda^2} (\phi^\dagger i \overleftrightarrow{D}^\mu \phi) \bar{t}_R \gamma_\mu t_R$$

$$+ \frac{C_{Ht}}{\Lambda^2} (\phi^\dagger i \overleftrightarrow{D}^\mu \phi) (\bar{Q}_L \gamma_\mu Q_L)$$

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2} \left( (\bar{Q}_L \sigma^{\mu\nu} T^a t_R \tilde{\phi}) G_{\mu\nu}^a + \text{H.c.} \right)$$

$$\mathcal{L}_{tH} = \frac{C_{tH}}{\Lambda^2} \left( (\phi^\dagger \phi) (\bar{Q}_L t_R \tilde{\phi}) + \text{H.c.} \right)$$

$$y_t^{\text{BMHV}} = y_t^{\text{NDR}} \left( 1 - \frac{\lambda v^2}{16\pi^2} \frac{C_{Qt}^{(1)}}{\Lambda^2} + c_F C_{Qt}^{(8)} - \frac{\lambda v^2}{32\pi^2} \frac{C_{HQ}^{(1)} - C_{Ht}}{\Lambda^2} + \dots \right)$$

$$C_{tH}^{\text{BMHV}} = C_{tH}^{\text{NDR}} + \frac{y_t (y_t^2 - \lambda)}{8\pi^2} \left( C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) - \frac{y_t (y_t^2 + 3\lambda)}{48\pi^2} \left( C_{HQ}^{(1)} - C_{Ht} \right) + \dots$$

$$C_{tG}^{\text{BMHV}} = C_{tG}^{\text{NDR}} - \frac{g_s y_t}{16\pi^2} \left( C_{Qt}^{(1)} + \left( c_F - \frac{c_A}{2} \right) C_{Qt}^{(8)} \right) + \frac{g_s y_t}{48\pi^2} \left( C_{HQ}^{(1)} - C_{Ht} \right) + \dots$$

# More details about HEFT

- HEFT:**
- Non-linear theory (EW $\chi$ L)
  - Motivation as analogue to chiral pert. theory
  - BSM: can be strongly coupling New Physics
  - Light Higgs is EW gauge singlet
  - Goldstone matrix transforms non-trivially

$$D_\mu h = \partial_\mu h$$

$$U(x) \rightarrow g_L(x) U(x) g_Y^\dagger(x), \quad \text{with } U = \exp(i\sigma^a \varphi^a / v) \text{ and } g_L \in SU(2)_L, g_Y \in U(1)_Y \subset SU(2)_R$$

$$D_\mu U = \partial_\mu U + igW_\mu^i T_L^i U - ig' B_\mu U T_R^3$$

- Chiral dimension of operators  $d_\chi(\partial, \bar{\psi}\psi, g, y) = 1$
- Expansion in  $\frac{f^2}{\Lambda^2} \sim \frac{1}{16\pi^2}$  ( $\Rightarrow$  loop counting)

$$\mathcal{L}_{HEFT} \sim \mathcal{L}_{HEFT}^{LO} + \sum_{L=1} \sum_i \left( \frac{1}{16\pi^2} \right)^L c_i \mathcal{O}_i^{d_\chi=2+2L}$$



# More details about HEFT

**HEFT:** ■ Explicit form of LO Lagrangian:

$$\mathcal{L}_{HEFT}^{LO} = \mathcal{L}_4 + \frac{v^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle (1 + F_U(h)) + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) - v \left[ \bar{q}_L \left( \hat{Y}_U + F_{\hat{Y}_U^{(n)}}(h) \right) U \begin{pmatrix} u_R \\ 0 \end{pmatrix} \dots + \text{h.c.} \right]$$

with  $F_i(h) = \sum_{n=1}^{\infty} f_i^{(n)} \left( \frac{h}{v} \right)^n$

⇒ Relevant parts for  $gg \rightarrow hh$ :

$$\mathcal{L}_{HEFT} \supset \underbrace{-m_t \left( c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2} \right) \bar{t}t - c_{hhh} \frac{m_h^2}{2v} h^3}_{\subset \mathcal{L}_{HEFT}^{LO}} + \underbrace{\frac{\alpha_s}{8\pi} \left( c_{ggh} \frac{h}{v} + c_{gggh} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a\mu\nu}}_{\subset \mathcal{L}_{HEFT}^{NLO}}$$

