

Karlsruher Institut für Technologie

Non-factorizable QCD corrections in hadron collider processes

Kirill Melnikov

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Non-factorizable corrections appear in QCD processes which, at tree level, are mediated by exchanges of colorless particles. Two most famous examples are single top production and Higgs production in weak boson fusion. These processes are used by the LHC collaborations to study e.g. tbW and HVV couplings, as well as other physical quantities related to top and Higgs physics.

At NLO QCD, two classes of contributions appear. However, the non-factorizable contributions vanish at this order because of color conservation.

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At NNLO QCD, the situation changes: non-factorizable contributions do not vanish anymore. However, they are colorsuppressed relative to the factorizable ones. As the result, the NNLO QCD corrections to both single top production and to Higgs boson production in WBF were originally computed neglecting the non-factorizable contributions.

> **Non-factorizable corrections at NNLO are effectively abelian. This reduces the number of diagrams one needs to consider and leads to other simplifications.**

$$
\frac{\text{nfact}}{\text{fact}} \sim N_c^{-2} \sim 10^{-1}
$$

$$
2T^aT^b = \{T^a, T^b\} + [T^a, T^b]
$$

Expansion around the forward limit of tagging jets leads to the eikonal approximation for the loop integrand. $p_{\perp,3} \sim p_{\perp,4} \sim m_V \sim m_H \ll \sqrt{s}$

The color suppression argument does not appear to hold upon a more careful analysis because there is a peculiar dynamical enhancement of the non-factorizable corrections. This enhancement was discovered when trying to understand if it is possible to compute the required two-loop amplitude, if even approximately.

where forward (tagging) jets with large invariant mass and large rapidity separation are selected. To understand how to construct an expansion of the amplitude, we employ the kinematics of weak boson fusion

Liu, Melnikov, Penin

The main idea behind the eikonal approximation is that high-energy quarks follow straight lines and do not recoil because the exchanged gluons are soft. Rules for constructing the amplitude are:

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$$

- eikonal propagators for quark lines: $(2pk + i0)$
- 2) eikonal couplings of quarks to gluons:
- 3) no longitudinal loop momenta components in gluon and vector boson propagators:

$$
p_{\perp}^{j_1, j_2} > 25 \text{ GeV}, \quad |y_{j_1, j/2}| < 4.5
$$

$$
|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600 \text{ GeV}
$$

Kirill Melnikov **Non-factorizable corrections in weak boson fusion processes**

Sudakov, Lipatov, Gribov, Cheng, Wu, Chang, Ma

Typical WBF cuts

$$
-2ie p\mu
$$

in gluon and vector boson propagators: $k^{-2} \rightarrow -k_{\perp}^{-2}$

Dramatic simplifications occur if the eikonal approximation for integrands are used and if various diagrams, that contribute to the amplitude, are combined before attempting to integrate them over the loop momentum.

> lim $p_4 \rightarrow p_2$ [1 $\frac{1}{-2p_2k + i0} +$ 1 $2p_4k + i0$ $\Big] = -\frac{2i\pi}{s}$ *s* $\delta(\alpha)$

$$
d^4k = \frac{s}{2} \, d\alpha \, d\beta \, d^2k_\perp
$$

Kirill Melnikov **Non-factorizable corrections in weak boson fusion processes**

$$
\lim_{p_3 \to p_1} \left[\frac{1}{2p_1k + i0} - \frac{1}{-2p_3k + i0} \right] = -\frac{2i\pi}{s} \delta(\beta)
$$
\n
$$
k = \alpha p_1 + \beta p_2 + k_\perp
$$

$$
\left[\frac{1}{2p_1k+i0} + \frac{1}{-2p_3k+i0}\right] \left[\frac{1}{-2p_2k+i0} + \frac{1}{2p_4k+i0}\right]
$$

transverse momentum distribution rapidity distribution lower pane, the ratio of non-factorisable corrections to the leading order distribution Liu, Melnikov, Penin **Wang and See termin and See text for Guarroz**, Bronnum-Hansen, Melnikov, Wang

8 Kirill Melnikov **Non-factorizable corrections in weak boson fusion processes** corrections remains constant across the phase space. IN FIGURE BUILDING COLLECTIONS IN MEAN DUSULE NON-FIGURE CORRECTIONS ON THE TOP-

We find non-factorizable corrections to Higgs production in WBF to be between 0.5 and 1 percent, depending on a kinematic distribution. This result is not significantly smaller than NNLO QCD factorizable corrections and is more important than the N3LO QCD ones.

For the single top production, the exact calculation can be done using semi-numerical methods and very similar results are obtained.

Higgs production in WBF

σ \sim The top-quark transverse momentum distribution. In the upper panel, the upper panel of uppe blue corresponds to the leading order distribution where \sim

Since non-factorizable contributions may be somewhat relevant, two questions arise:

1) how reliable is the leading eikonal approximation for Higgs boson production in WBF?

2) how important are all other contributions to NNLO corrections (the emission of two gluons or a one-loop correction to the single gluon emission) in the non-factorizable case? This question applies to both single top production and to Higgs production in WBF.

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To go beyond the leading eikonal approximation, we need to parametrise the external momenta in a way that makes it clear how the forward scattering limit is approached. **b** To go beyond the leading eikonal approximation, we need to parametrise retain the next-to-eikonal accuracy. In Section 5, we have the person in Section 5, we have the infrared contain how the forward scattering limit is annmached nit is approached. We have suddenly decomposition of the four-momenta of the four-momenta of the outgoing q $approached.$ *q*(*p*1) + *q*(*p*2) ! *q*(*p*3) + *q*(*p*4) + *H*(*pH*)*.* (2.1) We perform the Sudakov decomposition of the four-momenta of the outgoing quarks and where 3 is the 10 million of the momentum conservation condition conditi

It follows from the Higgs boson on-shell condition that changes in the "longitudinal" momenta fractions are proportional to the absolute value of the transverse momentum, not the transverse momentum squared. and the absolute value of the transverse frioritentum, not the transverse mo *momentum squared. p*4 $\frac{1}{2}$ $\frac{1}{2$ write
Write
Write condition that changes in the "longitudinal" momenta fractions are proportional Ine transverse momentum, not the transverse momentum square

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10 Constants and the Monte CCD corrections to hadron collider processes and the Kirill Melnikov constants and the Kirill Melnikov constants and the Monte CCD corrections to hadron collider processes and the Monte CCD corre kinematical configurations where the transverse momenta of the tra note that we intervention that we intervention a parameter in the small number of various ratios of various ratios \sim Non-factorizable QCD c

*p*³ = ↵3*p*¹ + 3*p*² + *p*3*,*? *,* We define two auxiliary vectors *q*¹ and *q*² which describe momentum transfers from We define two auxiliary vectors *q*¹ and *q*² which describe momentum transfers from

$$
p_{3} = \alpha_{3}p_{1} + \beta_{3}p_{2} + p_{3,\perp} \t p_{4} = \alpha_{4}p_{1} + \beta_{4}p_{2} + p_{4,\perp}
$$
\n
$$
\beta_{3} = \frac{p_{3,\perp}^{2}}{s\alpha_{3}}, \quad \alpha_{4} = \frac{p_{4,\perp}^{2}}{s\beta_{4}}
$$
\n
$$
\beta_{2} = 1 - \alpha_{3} \t \delta_{4} = 1 - \beta_{4}
$$
\n
$$
q(p_{1}) + q(p_{2}) \rightarrow q(p_{3}) + q(p_{4}) + H(p_{H})
$$
\n
$$
\delta_{3}\delta_{4} \sim \frac{m_{V}^{2}}{s} \sim \frac{m_{H}^{2}}{s} \sim \frac{p_{3,\perp}^{2}}{s} \sim \frac{p_{4,\perp}^{2}}{s} \sim \lambda \ll 1
$$
\n
$$
\sigma_{3} \sim \delta_{4} \sim \sqrt{\lambda}
$$

To understand how to expand the virtual amplitude for Higgs boson production in WBF around the eikonal limit, we study the dependence of the integrand on the small parameter and identify various "integration regions". A power
counting applied to individual diagrams indicates that already at leading power, a large number of various re counting applied to individual diagrams indicates that, already at leading power, a large number of various regions contributes. konal limit, we e 1 1 1 momentum *k*¹ and write *^Jµ*⌫(*k*1*, k*¹ *^q*1) is shown on the right.

$$
k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}
$$

$$
\left(\mathcal{M}^{(a)}\sim\lambda^{-2}\,,\quad \mathcal{M}^{(b)}\sim\lambda^{-2}\,,\quad \mathcal{M}\right)
$$

*p*₃

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We now proceed in a standard way. Starting from the general expression for the integrand, we apply scaling of the external momenta and the loop momentum in a particular region to simplify the various propagators and, eventually, the integrand. rd way Starting from we would be would be would be the state of the state of the state in powers of . To understand how to do tatter the Sudakov parameters $\mathcal{L}_{\mathcal{A}}$ parametrization of the loop of Table 1: Kinematic regions relevant for one-loop non-factorizable contributions. Symmet-coupling of the vector boson *V* to the Higgs boson is given by *igVVH gµ*⌫ and that the To we now proceed in a standard way. Starting inon-trile general express coupling of the massive vector boson to quarks is vector-like, *ig^W µ*. Since we work with external momenta and the loop momentum in a particular region to To was the the non-factorizable and the subsequent was the subsequent that the subsequently the subsequent of coupling indirepted and the loop indireption in a particular region $\frac{1}{2}$ coupling of the massive vector boson to quarks is vector-like, *ig^W µ*. Since we work with \bigcup *d*1*d*3*d*⁴ *J*w proceed in a standard way. Olaning nom tr
I al momenta and the loop momentum in a par \blacksquare

$$
\mathcal{A}_{1} = \int \frac{d^{d}k_{1}}{(2\pi)^{d}} \frac{1}{d_{1}d_{3}d_{4}} J_{\mu\nu}(k_{1}, -k_{1} - q_{1}) \tilde{J}^{\mu\nu}(-k_{1}, k_{1} - q_{2})
$$

\n
$$
d_{1} = k_{1}^{2} + i0, \quad d_{3} = (k_{1} + q_{1})^{2} - m_{V}^{2} + i0, \quad d_{4} = (k_{1} - q_{2})^{2} - m_{V}^{2} +
$$

\n
$$
\rho_{i}(k) = \frac{1}{(p_{i} + k)^{2} + i0} \qquad i = 1, 2, 3, 4
$$

\n
$$
J^{\mu\nu}(k_{1}, -k_{1} - q_{1}) = \langle 3| \left[\frac{\gamma^{\nu}(\hat{p}_{1} + \hat{k}_{1})\gamma^{\mu}}{\rho_{1}(k_{1})} + \frac{\gamma^{\mu}(\hat{p}_{3} - \hat{k}_{1})\gamma^{\nu}}{\rho_{3}(-k_{1})} \right] |1]
$$

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$$
\tilde{J}^{\mu\nu}(-k_{1}, k_{1} - q_{2}) = \langle 4| \left[\frac{\gamma^{\nu}(\hat{p}_{2} + \hat{k}_{1})\gamma^{\mu}}{\rho_{2}(k_{1})} + \frac{\gamma^{\nu}(\hat{p}_{4} - \hat{k}_{1})\gamma_{\mu}}{\rho_{4}(-k_{1})} \right] |2]
$$

12 Kirill Melnikov Kirill Melnikov Non-factorizable QCD corrrections to hadron collider processes , *и*, *м*(*e*) и, *м*(*e*) и, *м*(*e*) и, *м*(*e*) и, *м*(*e*) Figure 1: The one-loop and the construction of the left of the left of the left of the left of the construction of the con

*|*2] *,*

The possibility to compute both leading and first subleading correction to the non-factorizable amplitude in WBF with a relative ease is the consequence of the fact that integrations over two longitudinal components of the loop momentum factorize and that gauge cancellations ensure that contributions of many regions are suppressed relative
The expectations based on "naive" power counting to expectations based on "naive" power counting. momentum *k*¹ and write ⇢1(*k*1) ⇡ *s*¹ + ¹ + *i*0 *,* ⇢2(*k*1) ⇡ *s*↵¹ + ¹ + *i*0 *,* ith a relative ease is the consequence of the fact that integrations over two longitudinal components of the loop observed that integrations over \mathcal{O} . In a possibility to compute both leading and first subleading correction to the non-factorizable amp **Iomentur d**₁, doctorion *is baloca* on *i*¹, *i*0*, i*⁰ $\frac{1}{2}$ is sequence of $\frac{1}{2}$. at gauge cancellations ensure that contributions of many regions are suppressed relative
charge cancellations ⇢3(*k*1) ⇡ *s*↵3¹ + ⇥3*,*¹ + *i*0 *,* ⇢4(*k*1) ⇡ *s*4↵¹ + ⇥4*,*¹ + *i*0 *.* ne possibility to compute both leading and first subleading correction to the non-factoriza *d*₁ *d*₁ *d*₁ *i d*₁ *i*₀*, d*₁ *i*₀*, d*₁ *i*₂*, d*₁ *i*² sequence or the ract that integrations over two iongitudinal con-
Sequence or the ract that integrations of many rogions

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k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}
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k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}
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$$
\Phi^{\mu\nu} = \int_{-\sigma}^{\sigma} \frac{d\beta_1}{2\pi i} \frac{\Delta_{3,1}}{s\delta_3(\beta_1 - \beta_3) + \Delta_{3,1} + i0} \langle 3| \left[\frac{\gamma^{\nu}(\hat{p}_1 + \hat{k}_{1,\perp})\gamma^{\mu}}{s\beta_1 + \Delta_1 + i0} + \frac{\gamma^{\mu}(\hat{p}_3 - \hat{k}_{1,\perp})\gamma^{\nu}}{-s\alpha_3\beta_1 + \Theta_{3,1} + i0} \right] |1]
$$

$$
\tilde{\Phi}^{\mu\nu} = \int_{-\sigma}^{\sigma} \frac{d\alpha_1}{2\pi i} \frac{\Delta_{4,1}}{-s\delta_4(\alpha_1 + \alpha_4) + \Delta_{4,1} + i0} \langle 4| \left[\frac{\gamma^{\nu}(p_2 + \hat{k}_{1,\perp})\gamma^{\mu}}{-s\alpha_1 + \Delta_1 + i0} + \frac{\gamma^{\nu}(p_4 - \hat{k}_{1,\perp})\gamma^{\mu}}{s\beta_4\alpha_1 + \Theta_{4,1} + i0} \right] |2]
$$

In region "b", gauge cancellations ensure additional suppression of one of the currents. Similar cancellations allow us to completely discard regions "c" and "d" at next-to-leading power. In region "b", gauge cancellations ensure additional suppression of one of the currents. Similar cancellations allow us allul is chisule au
allul is chisule au to completely discard regions "c" and "d" at next-to-leading power. regions "c" and "d" at ne T additional cunnmection of one of the currents *Similar cancellat* particular, the particular of the contribution of the since with the single with the single with the single with $\frac{1}{2}$ of nove to logalize nouver. \mathfrak{r} **Count Contract** and "d" at hext-to-leading power. particular, the numeration of the contribution of the contribution of the contribution of the contribution of α in region *b*) we should replace *k*¹ with *k*¹ ! 1*p*² + *k*1*,*? in *both* currents. Suppose we Indeed, using the fact that ¹ 1*/s,* ⇥3*,*1*/s*, we expand the current and obtain *^Jµ*⌫(*k*1*, q*¹ *^k*1) ⇡ *^p^µ*

$$
k_{1} = \alpha_{1}p_{1} + \beta_{1}p_{2} + k_{1,\perp}
$$
\n
$$
\Delta_{1,1} = -k_{1,\perp}^{2} + k_{2,\perp}^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{2,1} = -k_{1,\perp}^{2} + k_{2,\perp}^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{3,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{3,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{3,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{4,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^{2} - m_{V}^{2}
$$
\n
$$
\Delta_{5,1} = -\mathbf{k}_{1,\perp} + \mathbf{p}_{3,\perp}
$$
\n
$$
\Delta_{6,1} = -\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp} + \mathbf{p}_{3,\perp}
$$
\n
$$
\Delta_{7,1} = -m_{V}
$$
\n
$$
\Delta_{8,1} = -(\mathbf{k}_{1,\perp} + 2\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})
$$
\n
$$
J^{\mu\nu}(k_{1}, -q_{1} - k_{1}) \approx \nu_{1}^{2}\nu_{1}^{2}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{1}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}{2}}\sqrt{\frac{\lambda_{2}}
$$

a Kirill Melnikov **Non-factorizable QCD corrrections to hadron collider processes Non-factorizable QCD corrrections to hadron collider processes** current *^Jµ*⌫(*k*1*, q*¹ *^k*1), are to be taken at leading power in . *Non-factorizable QCD corrrections to had*

over transversal components of the loop momentum.

A similar analysis of the two-loop amplitude leads to the conclusion that contributions of momenta regions which are identical to the one-loop case need to be considered. Importantly, factorization works in a very similar manner, so that the "upper-line" and the "lower-line" currents factorize and can be treated separately. one-loop case. It relies on the fact that for soft and collinear gluons, fermion currents simplify dramatically. Consider, for example, the case where *k*¹ is Glauber and *k*² is soft. writch are identical to the one-loop case need to be considered. Importantly, rac
Comilar mannor, so that the "unnor line" and the "lewer line" currents factorize an continual tribution, so that the apper-intel and the lower-intel currents factorize and A similar analysis of the two-loop amplitude leads to the conclusion that contributions of momenta regions similar manner, so t reaus to the conclusion that contributions of monental egions
and to be considered almostantly fosterization werks in a yery contribution in the contribution of the contribution is a contribution of the contribution is a contribution of
The "lewer line" eurrente fectorize and con he treated concretely the lower-line c $\overline{0}$ one-loop case need to be considered. Importantly, factorization works in a very ered. Importantly, factorization works in a very
currents factorize and can be treated separately. the two 's scale as provided as μ pu: integrand which is valid *both* in the Glauber region and in the mixed region, we need to and both and both and both and both and the contributions of the two and the two and one of the two and one of t

and the contributions of the two and two and two and two and one of the two and one of the two and one of th e "upper-line" and the "lower-line" currents fac $\frac{1}{10}$ rize and can be treated separately.

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where M is defined in Eq. (3.36) and the function \mathcal{M}^2 reads the function \mathcal{M}^2

V

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$$
\mathcal{M}_2 = -\frac{1}{2} \frac{g_s^4}{(4\pi)^2} \left(\frac{1}{2} \{T^a, T^b\} \right)_{i_3 i_1} \left(\frac{1}{2} \{T^a, T^b\} \right)_{i_4 i_2} \mathcal{M}_0 \mathcal{C}_2
$$

$$
\boxed{\mathcal{C}_2 = 4 \int \frac{d^{d-2} \mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}} \frac{d^{d-2} \mathbf{k}_{2,\perp}}{\pi (2\pi)^{1-2\epsilon}} \frac{(\mathbf{p}_{3,\perp}^2 + m_V^2)(\mathbf{p}_{4,\perp}^2 + m_V^2)}{\Delta_1 \Delta_2 \Delta_{3,12} \Delta_{4,12}} \left[1 - \delta_3 \left(\frac{m_V^2}{\mathbf{p}_{3,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{3,12}}\right) - \delta_4 \left(\frac{m_V^2}{\mathbf{p}_{4,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{4,12}}\right)\right]}
$$

 \overline{a}

kⁱ = ↵*ip*¹ + *ip*² + *ki,*?*, i* = 1*,* 2 *.* (4.6)

$$
d\hat{\sigma}_{nf}^{NNLO} = \frac{N_c^2 - 1}{4N_c^2} \alpha_s^2 \mathcal{C}_{nf} d\hat{\sigma}^{LO} \qquad \qquad \boxed{\begin{array}{c} c_1 = -\frac{1}{\epsilon} + \mathcal{C}_{1,0} + \epsilon \mathcal{C}_{1,1} & \mathcal{C}_2 = \frac{1}{\epsilon^2} - \frac{2}{\epsilon} \mathcal{C}_{1,0} + \mathcal{C}_{2,0} \\ \mathcal{C}_{nf} = \mathcal{C}_{1,0}^2 - 2 \mathcal{C}_{1,1} - \mathcal{C}_{2,0} \end{array}}
$$

17 Kirill Melnikov Kirill Melnikov Non-factorizable QCD corrrections to hadron collider processes and the function zapie world confections to hadron confection order processes in the state of the confection against numerical processes in the state of the confection against numerical processes in the state of the state results are sure to the factorizable OOD agreement to be already agriculture the accuracy of t of our result in a realistic setting, we compare the one-loop amplitude including leading lea Non-factorizable QCD corrrections to hadron collider processes

The cross section is obtained by computing the sum of the square of the one-loop amplitude and the interference of the two-loop amplitude with the Born amplitude. In this combination, the infra-red divergences cancel out and the finite remainder is obtained. The finite remainder can be computed analytically, by applying standard methods of multi-loop computations, albeit at $d = 2$. and the surface support that an all that we have a mother than the second strong second strong second than the second strong The cross section is obtained by computing the sum of the square of the one-loop amplitude and the computed with we contained to obtained. The infire fortained oan be compated
methods of multi-loop computations albeit at d = 2 the lot include of intenting approximate its expanding the set of \Box . plitude v 1e finite re
utations, a ✏ *C* α **C** α + α + ²)*,* The dross section is obtained by computing the sum of the square of the dife-loop are
interference of the two-loop amplitude with the Born amplitude the this combination divergences cancel out and the finite remainder is obtained. The finite remainder can be computed t analytically, by applying standard in Eurous on multi-loop computations, albeit at $u - z$ Interference of the two-loop amplitude with the Born amplitude. In this combination, the infra-red area of the two-loop amplitude with the Born amplitude. In this combination, the infra-red compu generated using Julius envitude the this combination the infra-red $\frac{1}{200}$ butlem is obtained by seperiting the ourse of the previous of the previous experituals and the efference of the two-loop amplitude with the Born amplitude. In this combination, the infra-red \cdot th divergences cancel out and the finite remainder is obtained. The finite remainder can be computed tically, by applying standard methods of multi-loop computations, albeit at $d = 2$. the two 's scale as ^p , then ↵¹² and ¹² also scale as ^p The cross section is obtained by computing the sum of the square of the one-loop interference of the two analytically, by applying standard methods of multi $-|c|$) op computations, albeit at **region is obtained by computing the cross section is obtained by computing the 11 J I** Interference of the two-loop amplitude with the same as in the same as in the one-loop case and we have an Building on the experience with the one-loop calculation reported in the previous calculation reported in the p
Surface of the previous calculation reported in the previous calculation of the previous calculation of the pr

$$
d\hat{\sigma}_{nf}^{NNLO} = \frac{N_c^2 - 1}{4N_c^2} \alpha_s^2 C_{nf} d\hat{\sigma}^{LO}
$$

$$
C_1 = -\frac{1}{\epsilon} + C_{1,0} + \epsilon
$$

$$
C_{nf} = C_{1,0}^2
$$

$$
j[a_1,a_2,a_3,a_4]=\frac{(m_V^2)^{2\epsilon}}{\pi^{d-2}\Gamma(1+\epsilon)^2}\int\frac{{\rm d}\mathbf{k}_{1,\perp}^{d-2}{\rm d}\mathbf{k}_{2,\perp}^{d-2}}{\Delta_1^{a_1}\Delta_2^{a_2}\Delta_{3,12}^{a_3}\Delta_{4,12}^{a_4}}\overset{\frac{p_{3,\perp}}{2\epsilon}}{\int_{{f_1}}^{{f_2}}{\Delta_1^{a_2}\Delta_{4,12}^{a_3}\Delta_{4,12}^{a_4}}}
$$

$$
\Delta_i = -\mathbf{k}_{i,\perp}^2
$$
\n
$$
\Delta_{3,i} = -(\mathbf{k}_{i,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2
$$
\n
$$
\Delta_{4,i} = -(\mathbf{k}_{i,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2
$$

The next-to-leading terms change the leading eikonal contribution to the non-factorizable corrections by thirty percent, depending on the observable. Hence, the eikonal expansion is not perfect but, most likely, it provides a reasonable apportently on the observable. Therice, the enterial expansion is

$$
d\hat{\sigma}_{\rm nf}^{\rm NNLO} = \frac{N_c^2 - 1}{4N_c^2} \alpha_s^2 \mathcal{C}_{\rm nf} \ d\hat{\sigma}^{\rm LO} \qquad \begin{array}{c} p_{\perp}^{j_1, j_2} > 25 \text{ GeV}, \quad |y_{j_1, j}| \\ |y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1, j} \end{array}
$$

Other contributions that need to be considered for computing physical quantities (such as the cross section) are the double-real emission contribution and the real-virtual contribution. An important problem that one faces, when dealing with these terms, is to extract and remove singularities that arise upon integration over energies and angles of the emitted gluon (soft and collinear singularities).

A general solution to this problem requires the development of intricate infra-red subtraction schemes. However, in case of non-factorizable corrections, this problem simplifies because 1) there are no collinear singularities (emissions and absorptions must occur on different lines) and 2) the non-factorizable corrections are, effectively, abelian.

Real-virtual contribution to NNLO corrections Double-real contribution to NNLO corrections

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Iterative subtraction of soft singularities from the double-real and real-virtual contributions as well as the use of Catani's formula for divergences of double-virtual corrections leads to the following compact result for complete NNLO contribution to differential cross section. The computer is readered to the reporting compact require. non-factorizable one-loop amplitudes for the processes for the processes of the processes of the processes of
The processes of the proce **hd real-virtual contrik**

$$
d\sigma_{\rm mlo}^{\rm nf} = \frac{T_R^2 (N_c^2 - 1)}{2s} \left[\langle F_{\rm LM}^{\rm nf}(1, 2, 3, 4 | 5, 6) \rangle \right. \n+ \langle F_{\rm LV}^{\rm rf}(1, 2, 3, 4 | 5) \rangle + \langle F_{\rm LVV}^{\rm rf}(1, 2, 3, 4) \rangle \right] \n= \frac{T_R^2 (N_c^2 - 1)}{2s} \left[\langle \left[I - S_6 \right] F_{\rm LM}^{\rm nf}(1, 2, 3, 4 | 5, 6) \rangle \right. \n- 2 \frac{\tilde{\alpha}_s}{2\pi} \langle \left[I - S_5 \right] \mathcal{W}(E_5; 1, ..., 4) F_{\rm LM}^{\rm nf}(1, 2, 3, 4 | 5) \rangle \n+ 2 \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 \langle \mathcal{W}(E_{\rm max}; 1, ..., 4)^2 F_{\rm LM}^{\rm nf}(1, 2, 3, 4) \rangle \n+ \langle \left[I - S_5 \right] F_{\rm LV, fin}^{\rm nf}(1, 2, 3, 4 | 5) \rangle \n- 2 \frac{\tilde{\alpha}_s}{2\pi} \langle \mathcal{W}(E_{\rm max}; 1, ..., 4) F_{\rm LV, fin}^{\rm nf}(1, 2, 3, 4) \rangle \n+ \langle F_{\rm LVV, fin}^{\rm rf}(1, 2, 3, 4) \rangle \right].
$$

$$
\mathcal{W}(E; 1, 2, 3, 4) \equiv \kappa_{qQ} \left[\left(\frac{2E}{\mu} \right)^{-2\epsilon} K_{\text{nf}}(\epsilon) - I_1(\epsilon) \right]
$$

$$
= \kappa_{qQ} \left[-2 \ln \left(\frac{2E}{\mu} \right) \ln \left(\frac{p_1 \cdot p_4 \ p_3 \cdot p_2}{p_1 \cdot p_2 \ p_3 \cdot p_4} \right) + \sum_{\substack{i \in \{1, 3\} \\ j \in \{2, 4\}}} \lambda_{ij} \left(\frac{1}{2} \ln^2(\eta_{ij}) + \text{Li}_2(1 - \eta_{ij}) \right) \right] + \mathcal{O}(\epsilon)
$$

Campanario, Figi, Plätzer

and Melnikov a

The non-factorizable contributions are dominated by the virtual corrections. This is true for both the single top and the Higgs production in WBF, but this feature becomes extreme (a factor of a 10⁵ difference) in the latter case (the consequence of how WBF events are selected). p
Put the Higgs production in WRF thut this feature $\frac{1}{2}$ single top and the mggs production in vvDF, $\frac{1}{2}$

22 Kirill Melnikov Kirill Melnikov Non-factorizable QCD corrrections to hadron collider processes ↵³ ⁴ *s* ↵³ ⁴ culture (41) Non-factorizable QCD corrrections, to hadron collider proces In comparison, virtual corrections do not vanish in the forward region. In factor is a shown in Ref. [8], they are shown in Ref. [8], they Non-factorizable QCD corrrections to hadron collider processes put at the controller in this section present present in the present of the control of

To understand this suppression, consider emission of soft gluon, compare the change of the cross section caused by the emission of two soft gluons with the double-virtual corrections. section caused by the emission of two soft gluons with the double-virtual *pi,*? *· p*¹ = *pi,*? *· p*² = 0 *,* (39) po difference and dapproducin, donorate announcement of during the chipero and different of and or re gradi ✓ ↵˜*s* ith *N*₂ *C*_{*M*} 2*, 2 a*_{*, 2} <i>d*₂ *d*₂ </sub> *re* the change of t

> 3*,*? nf lim p_5, p_6 \rightarrow 0 $|\mathcal{M}(1_q, 2_Q, 3_q, 4_Q)|$

$$
\lim_{p_5, p_6 \to 0} |\mathcal{M}(1_q, 2_Q, 3_q, 4_Q)|_{\text{nf}}^2 = (N_c^2 - 1) \text{Eik}_{\text{nf}}(p_5) \text{Eik}_{\text{nf}}(p_6) \mathcal{A}_0^2(1, 2, 3, 4)
$$
\n
$$
\text{Eik}(p) = \sum_{i \in [1,3], j \in [2,4]} \lambda_{ij} \frac{p_i p_j}{(p_i p)(p_j p)}
$$
\n
$$
L(1, 2, 3, 4) = \ln \left(\frac{p_1 \cdot p_4 p_3 \cdot p_2}{p_1 \cdot p_2 p_3 \cdot p_4} \right) \approx \frac{2 \vec{p}_{3, \perp} \cdot \vec{p}_{4, \perp}}{s} \sim 10^{-2}
$$
\n
$$
\sigma_{RR} \sim \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 N_c^2 \langle L^2(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) \right) \approx \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 10^{-4} \sigma_{\text{LO}}
$$
\n
$$
\sigma_{VV} \sim \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 N_c^2 \langle \chi_{\text{nf}}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) \right) \approx \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 10 \sigma_{\text{LO}}
$$
\n
$$
\sigma_{VV} \sim \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 N_c^2 \langle \chi_{\text{nf}}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) \right) \approx \left(\frac{\tilde{\alpha}_s}{2\pi} \right)^2 10 \sigma_{\text{LO}}
$$

$$
\lim_{p_b, p_b \to 0} |\mathcal{M}(1_q, 2_Q, 3_q, 4_Q)|_{\text{nf}}^2 = (N_c^2 - 1)\text{Eik}_{\text{nf}}(p_5)\text{Eik}_{\text{nf}}(p_0) \mathcal{A}_0^2(1, 2, 3, 4)
$$
\n
$$
\text{Eik}(p) = \sum_{i \in [1,3]: j \in [2,4]} \lambda_{ij} \frac{p_i p_j}{(p_i p)(p_j p)}
$$
\n
$$
L(1, 2, 3, 4) = \ln \left(\frac{p_1 \cdot p_4 p_3 \cdot p_2}{p_1 \cdot p_2 p_3 \cdot p_4}\right) \approx \frac{2\bar{p}_{3,\perp} \cdot \vec{p}_{4,\perp}}{s} \sim 10^{-2}
$$
\n
$$
\sigma_{RR} \sim \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 N_c^2 \langle L^2(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) \times \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 10^{-4} \sigma_{\text{LO}} \approx \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 10 \sigma_{\text{LO}} \approx \frac{\sigma_{RR}}{\sigma_{VV}} \sim 10^{-5}
$$

$$
\lim_{\rho_{B} \to 0} |\mathcal{M}(1_{q}, 2_{Q}, 3_{q}, 4_{Q})|_{\text{nf}}^{2} = (N_{c}^{2} - 1)\text{Eik}_{\text{nf}}(p_{5})\text{Eik}_{\text{nf}}(p_{6}) \mathcal{A}_{0}^{2}(1, 2, 3, 4)
$$
\n
$$
\text{Eik}(p) = \sum_{i \in [1,3]; j \in [2,4]} \lambda_{ij} \frac{p_{i}p_{j}}{(p_{i}p)(p_{j}p)}
$$
\n
$$
L(1, 2, 3, 4) = \ln \left(\frac{p_{1} \cdot p_{4} \cdot p_{3} \cdot p_{2}}{p_{1} \cdot p_{2} \cdot p_{3} \cdot p_{4}}\right) \approx \frac{2\vec{p}_{3,1} \cdot \vec{p}_{4,1}}{s} \sim 10^{-2}
$$
\n
$$
\text{Var } \sim \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} N_{c}^{2} \langle L^{2}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) + \sum_{i \in [1,3]} \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} 10^{-4} \sigma_{\text{LO}}
$$
\n
$$
V \sim \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} N_{c}^{2} \langle \chi_{\text{nf}}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) + \sum_{i \in [1,3]} \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} 10^{-4} \sigma_{\text{LO}}
$$
\n
$$
V \sim \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} N_{c}^{2} \langle \chi_{\text{nf}}(1, 2, 3, 4) F_{\text{LM}}^{\text{nf}}(1, 2, 3, 4) \rangle \approx \left(\frac{\tilde{\alpha}_{s}}{2\pi}\right)^{2} 10^{-4} \sigma_{\text{LO}}
$$

$$
Eik(p) = \sum_{i \in [1,3]; j \in [2,4]} \lambda_{ij} \frac{p_i p_j}{(p_i p)(p_j p)}
$$

on *^C*nf but, since we are interested in computing *^O*(0↵³

 \hat{f} = $\gamma_{\rm nf}$ = $t_1 + \nu_1$) $(1 - \nu_2)$
 $C_1(\nu_1)$ $C_1(\nu_2)$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $t + C_{\text{ref}}^{(1)}$ + $\mathcal{O}(\alpha_s^2 \beta_0^2)$ $C_{\text{nf}}^{(1)} = C_1^{(0)} C_1^{(1)} - 3C_1^{(2)}$ $\frac{\nu_1}{\nu_1}$ (1 $\frac{\nu_2}{\nu_2}$) $C_1(\nu_{12})$ $C_1(\nu_{13})$ *,* $(\nu_1)\Gamma(1-\nu_2)$
 $(\nu_1)(1-\nu_2)$ $\Gamma(1-\nu_{12})$ is the transverse momentum, we obtain the following representation for the f (0) \overline{a} ิ์
า1 $\begin{pmatrix} 1 & 1 \end{pmatrix}$ $C^{(0)}C^{(1)} = 3C^{(2)}$ $\frac{C_1}{\Gamma(1-\nu_{12})}C_1(\nu_{12})$ $C_1($ $C_{\rm nf}^{(1)}$ $C_1(\nu) = \frac{1}{\nu}$ ν $+\sum$ *i*=1 $C_1^{(i)} \nu^i$ $\binom{2}{5}$ $\binom{2}{0}$ ↵*s*⁰ \int_0^{∞} $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ *µ*2*e*5*/*³ $\alpha^{(1)}$ $\alpha^{(0)}\alpha^{(1)}$ $\alpha^{(2)}$ $\alpha^{(2)}$ $\bigcup_{i=1}^n$ $\binom{2}{0}$ $C_{\rm nf}^{(0)} =$ $\left(C_1^{(0)}\right)$ \setminus^2 $-2C_1^{(1)}$ \overline{C} $q(1) =$ *m*² *V* $\frac{1}{1}$ $\frac{\prime}{i=1}$ $=$ ϵ ⁺ *^O*(↵² $\frac{1}{2}$ $C_{\rm nf}^{(1)} = C_1^{(0)} C_1^{(1)} - 3C_1^{(2)} + 2\zeta_3$

 \overline{a}

The dependence of not-factorizable corrections on the renormalization scale is quite strong; one can try to accommodate effects of the running coupling constant into the calculation by employing Brodsky-Lepage-Mackenzie philosophy. **112** corrections when the corrections of the contributions of the corrections of the contributions of the correction of the $\frac{1}{2}$ The dependence of pet $f(x)$, we have experience on the x below, we experience of the required of the re into one of the two gluon propagators, see Figure 2 for an example. that is not infrared divergent and, as a consequence, in Eq. (2.6) we can replace the consequence of \setminus **space-times on the renormalization scale is question of the second dimension of the second dimension** If **y** ig coupling constant into the calculation by employing Brousky-Lepage-
C2. The rank-2 tensor in the limit of vanishing and vanishing and vanishing coupling consumers and the calculation |
|ization ation sca $\frac{1}{2}$ **T**² *Z*
20rmacione en the reperpendization ecolo is quite etrepe: ene can trute that *C*nf is not infrared divergent and, as a consequence, in Eq. (2.6) we can replace the culation by employin *,* The dependence of not-ractorizate corrections on the renormalization scale is quite strong, one can try to accommodate effects of the running coupling constant into the calculation by employing Brodsky-Lepage-*^O*(0↵³ *^s*) corrections. One should sum over *N^f* flavors running in the bubble. that we will use the calculation of the calculation \mathcal{C} 1 the renormalization scale is quite strong; one can trant into the calculation by employing Brodsky-Length \sim dependence of pat $\left\{\right\}\right\}$ bmmodate effects of the running coupling constant into the calculation by employing Brods
kenzie, philosophy TUE UT HUL-TACTURISCHE CUITECHUITS UIT *ration* integration over the one-loop case provided that we replace \mathcal{C}^{max} work able corrections on the renormalization scale is quite strong; one can try to *^C*2(⌫1*,* ⌫2) = ⌫¹² $(1 - \frac{1}{2})$ inter funning coupling constant into the calculation by employing Brodsky-Lepage- $\overline{}$

^s) corrections to Higgs boson

and Melnikov Mon-factorizable QCD corrrections to hadron collider processes Non-factorizable QCD corrrections for hadron collider processes \overline{O} \overline{O} \overline{O} \overline{O} Mon-factorizable QCD corrrections to hadron collider processes ¹ + 2⇣3*.* (2.19) ⁵Using Eq. (2.11) and the fact that *^F*21(2 + ⌫*,* ⌫*,* ¹*, x*)= 1+ *^O*(⌫), it is easy to check that the first term

 $-4,12$

$$
C_{1}(\nu) = -2 \int \frac{d^{2} \mathbf{k}_{1}}{2\pi} \frac{\Delta_{3} \Delta_{4} m_{V}^{2\nu}}{\Delta_{1}^{1+\nu} \Delta_{3,1} \Delta_{4,1}} \qquad C_{2}(\nu_{1},\nu_{2}) = 4 \int \frac{d^{2} \mathbf{k}_{1}}{2\pi} \frac{d^{2} \mathbf{k}_{2}}{\Delta_{1}^{1+\nu_{1}} \Delta_{2}^{1+\nu_{2}} \Delta_{3,1} \Delta_{4,12}}
$$

\n
$$
C_{2}(\nu_{1},\nu_{2}) = \frac{\nu_{12}}{\nu_{1}\nu_{2}} \frac{\Gamma(1+\nu_{12})}{\Gamma(1+\nu_{1})\Gamma(1+\nu_{2})} \frac{\Gamma(1-\nu_{1})\Gamma(1-\nu_{2})}{\Gamma(1-\nu_{12})} C_{1}(\nu_{12}) \qquad C_{1}(\nu) = \frac{1}{\nu} + \sum_{i=1} C_{1}^{(i)} \nu^{i}
$$

\n
$$
C_{2}^{(0)} - \frac{\nu_{12}}{\nu_{1}\nu_{2}} \frac{\Gamma(1+\nu_{12})}{\Gamma(1+\nu_{1})\Gamma(1+\nu_{2})} \frac{\Gamma(1-\nu_{12})}{\Gamma(1-\nu_{12})} C_{1}(\nu_{12}) \qquad C_{1}(\nu) = \frac{1}{\nu} + \sum_{i=1} C_{1}^{(i)} \nu^{i}
$$

$$
C_{\rm nf} = C_{\rm nf}^{(0)} + \frac{\alpha_s \beta_0}{\pi} \left(C_{\rm nf}^{(0)} \ln \left(\frac{\mu^2 e^{5/3}}{m_V^2} \right) + C_{\rm nf}^{(1)} \right) + \mathcal{O}(\alpha_s^2 \beta_0^2) \qquad C_{\rm nf}^{(1)} = C_{\rm nf}^{(1)}
$$

$$
\begin{array}{|c|c|} \hline \textbf{1} & C_{\rm nf} = 4\int\frac{\mathrm{d}^2\mathbf{k}_1}{(2\pi)}\frac{\mathrm{d}^2\mathbf{k}_2}{(2\pi)}\frac{\Delta_3\Delta_4}{\tilde{\Delta}_1\tilde{\Delta}_2}\left(\frac{\Delta_3\Delta_4}{\Delta_{3,1}\Delta_{4,1}\Delta_{3,2}\Delta_{4,2}}-\frac{1}{\Delta_{3,12}\Delta_{4,12}}\right) \\ & \hline \textbf{1} & \Delta_i = \mathbf{k}_i^2, \quad \Delta_{3,i} = (\mathbf{k}_i-\mathbf{p}_3)^2+m_V^2, \quad \Delta_{4,i} = (\mathbf{k}_i+\mathbf{p}_4)^2+m_V^2, \quad i=1,2,12 \\ & \Delta_i = \Delta_i\,\left(1+\frac{\beta_0\alpha_s}{2\pi}\ln\frac{\mathbf{k}_i^2}{\mu^2c^{5/3}}\right) \\ & \hline \Delta_i = \Delta_i\,\left(1+\frac{\beta_0\alpha_s}{2\pi}\ln\frac{\mathbf{k}_i^2}{\mu^2c^{5/3}}\right) \\ & C_1(\nu) = -2\int\frac{\mathrm{d}^2\mathbf{k}_1}{2\pi}\,\frac{\Delta_3\,\Delta_4\,m_V^{2\nu}}{\Delta_1^{1+\nu}\Delta_{3,1}\Delta_{4,1}}\qquad \qquad C_2(\nu_1,\nu_2) = 4\int\frac{\mathrm{d}^2\mathbf{k}_1}{2\pi}\frac{\mathrm{d}^2\mathbf{k}_2}{\Delta_1^{1+\nu_1}\Delta_2^{1+\nu_2}\Delta_{3,12}\Delta_{4,12}} \end{array}
$$

$$
C_{\rm nf} = 4 \int \frac{d^2 \mathbf{k}_1}{(2\pi)} \frac{d^2 \mathbf{k}_2}{(2\pi)} \frac{\Delta_3 \Delta_4}{\tilde{\Delta}_1 \tilde{\Delta}_2} \left(\frac{\Delta_3 \Delta_4}{\Delta_{3,1} \Delta_{4,1} \Delta_{3,2} \Delta_{4,2}} - \frac{1}{\Delta_{3,12} \Delta_{4,12}} \right)
$$

$$
\Delta_i = \mathbf{k}_i^2, \quad \Delta_{3,i} = (\mathbf{k}_i - \mathbf{p}_3)^2 + m_V^2, \quad \Delta_{4,i} = (\mathbf{k}_i + \mathbf{p}_4)^2 + m_V^2, \quad i = 1, 2, 12
$$

$$
\tilde{\Delta}_i = \Delta_i \left(1 + \frac{\beta_0 \alpha_s}{2\pi} \ln \frac{\mathbf{k}_i^2}{\mu^2 e^{5/3}} \right)
$$

$$
C_2(\nu_1, \nu_2) = 4 \int \frac{d^2 \mathbf{k}_1}{2\pi} \frac{d^2 \mathbf{k}_2}{2\pi} \frac{\Delta_3 \Delta_4 m_V^{2(\nu_1 + \nu_2)}}{\Lambda^{1 + \nu_1} \Lambda^{1 + \nu_2} \Delta_{3,12} \Delta_{4,12}}
$$

 \blacksquare

$$
\begin{array}{c}\n663 \\
683 \\
-112\n\end{array}
$$

↵*s*⁰

Bronum-Hansen, Long, Melnikov Llopoop Lope Molpikou expand the quantity of the qua $\overline{\mathsf{C}}$ *µ*2*e*5*/*³ ikov

BLM corrections can be computed analytically; the scale dependence stabilizes.

$$
C_{1}^{(0)} = \int_{0}^{1} dt \frac{\Delta_{x}\Delta_{y}}{r_{12}^{2}} \left[\ln r_{2} - 2\ln r_{12} + \frac{r_{2} - r_{1}}{r_{2}} \right],
$$

\n
$$
C_{1}^{(1)} = \int_{0}^{1} dt \frac{\Delta_{x}\Delta_{4}}{r_{12}^{2}} \left[\frac{1}{2} \ln^{2} r_{12} - \ln r_{12} \left(\frac{r_{2} - r_{1}}{r_{2}} + \ln \frac{r_{2}}{r_{12}} \right) \right],
$$

\n
$$
+ 2\ln \frac{r_{2}}{r_{12}} + \frac{\pi^{2}}{6} - \ln^{2} \left(\frac{r_{1}}{r_{12}} \right) \right],
$$

\n
$$
C_{1}^{(2)} = \int_{0}^{1} dt \frac{\Delta_{x}\Delta_{4}}{r_{12}^{2}} \left[-\frac{1}{6} \ln^{3} r_{12} + \frac{1}{2} \ln^{2} r_{12} \left(\frac{r_{2} - r_{1}}{r_{2}} + \ln \frac{r_{2}}{r_{12}} \right) \right]
$$

\n
$$
+ \frac{\pi^{2} r_{2} - r_{1}}{6} + \ln^{2} \left(\frac{r_{2}}{r_{12}} \right) \ln \frac{r_{1}}{r_{12}} - \ln r_{12} \left(\frac{\pi^{2}}{6} + 2 \ln \frac{r_{2}}{r_{12}} - \ln^{2} \left(\frac{r_{1}}{r_{12}} \right) \right)
$$

\n
$$
- \frac{r_{2} - r_{1}}{r_{2}} \ln^{2} \left(\frac{r_{1}}{r_{12}} \right) - \ln \frac{r_{2}}{r_{12}} \left(\frac{\pi^{2}}{6} - \ln^{2} \left(\frac{r_{1}}{r_{12}} \right) \right) + 2 \text{Li}_{3} \left(\frac{r_{2}}{r_{12}} \right) - 2 \zeta_{3}
$$

\n
$$
\frac{\sum_{i=1}^{3} \frac{\ln 3}{4} \pi_{i}}{\sum_{i=1}^{3} \frac{\ln 3 \pi_{i}}{\ln 2 \pi_{i}} \left(\frac{r_{1
$$

$$
\boxed{\sigma_{\rm nf}^{\rm LO} = -2.97^{+0.69}_{+0.52}~{\rm fb}, \quad \sigma_{\rm nf, \beta_0}^{\rm NLO} = -3.20}
$$

$$
r_1 = \frac{\mathbf{p}_3^2}{m_V^2} + \frac{\mathbf{p}_4^2}{m_V^2} + \frac{\mathbf{p}_H^2}{m_V^2} t(1-t) \qquad r_2 = 1 - \frac{\mathbf{p}_H^2}{m_V^2} t(1-t) \qquad r_{12} = r_1 + r_2
$$

$$
\overline{\sigma_{\mathrm{nf}\,,\beta_{0}}^{\mathrm{NLO}} = -3.20_{+0.14}^{-0.01} \mathrm{\;fb}}
$$

Non-factorizable corrections have the following properties:

3) they can reach a percent level in kinematic distributions and they are strongly kinematic-dependent; they can be

- they start contributing at next-to-next-to-leading order for the first time;
-
- studied independently of real-emission contributions since they are infra-red finite.
- (eikonal expansion), that can also be extended to provide the next-to-leading power accuracy;
- contributions are very much suppressed.
- corrections and treating them in the spirit of BLM scale-setting procedure.

4) for Higgs production in WBF, the non-factorizable corrections can be studied using expansion around the forward limit

5) the non-factorizable corrections are (very) strongly dominated by the double-virtual contributions; the real-emission

6) the scale-dependence of non-factorizable contributions can be strongly reduced by computing O(nf) three-loop

We discussed the non-factorizable corrections to process of the weak boson fusion type (Higgs production in WBF, single top production). Our interest to understand these effect is related to an impressive progress in computing factorizable corrections, e.g. where N3LO QCD corrections to inclusive WBF have been computed.

2) they are colour-suppressed but dynamically enhanced; the enhancement is related to the Coulomb (Glauber) phase;