QCD and electroweak corrections to Higgs Boson Pair Production

CRC meeting 2024, 11-12 March 2024

Matthias Steinhauser | TTP KIT

Team

A3b: Precision predictions for Higgs boson properties as a probe for New Physics

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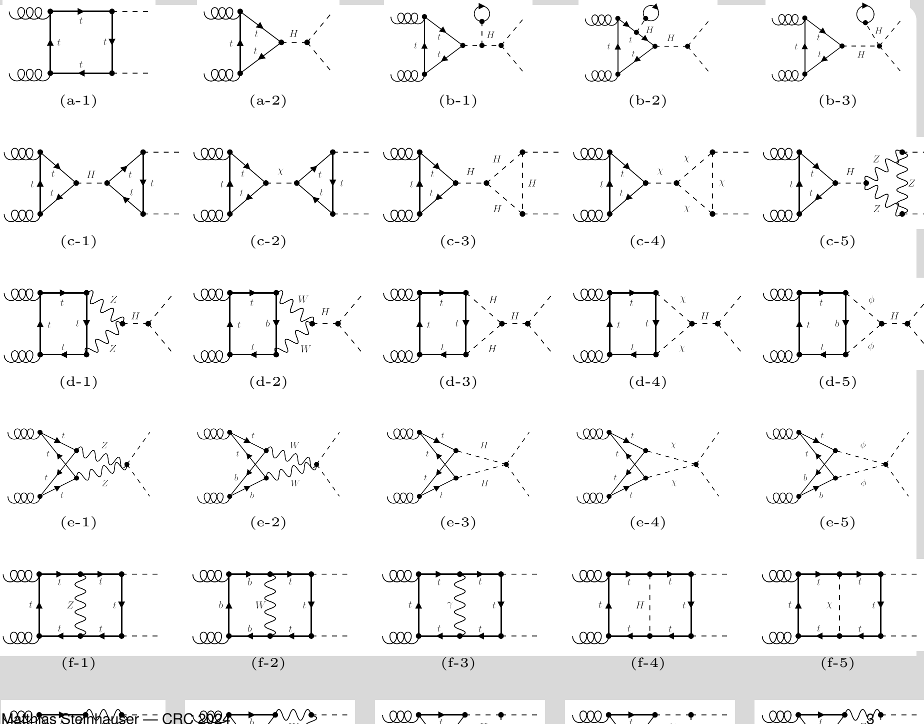
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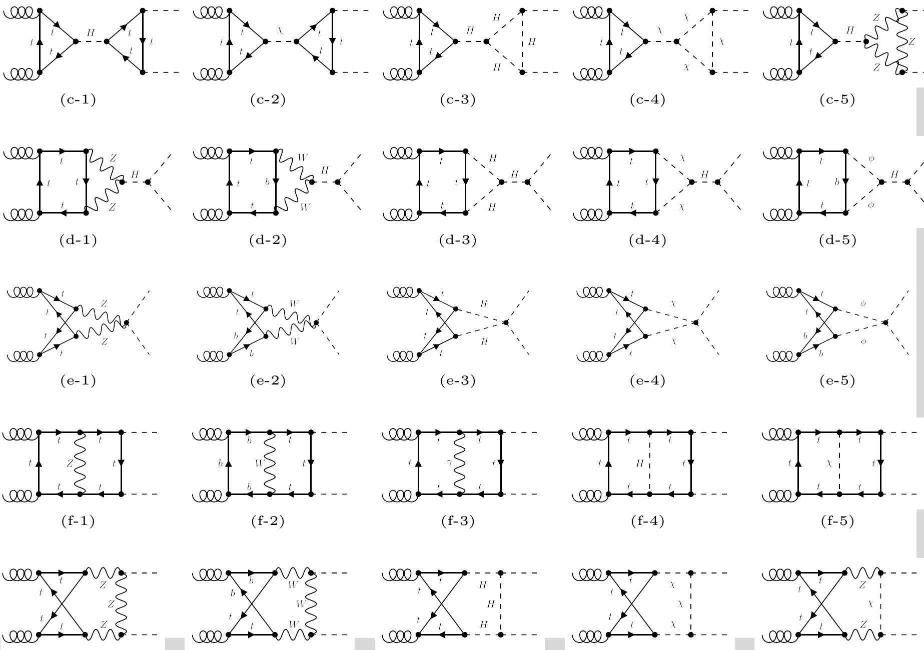

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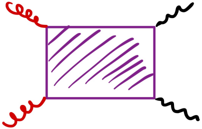
(f-1) (f-2) (f-3) (f-4) (f-5)

(g-1) (g-2) (g-3) (g-4) (g-5)

Analytic expansions



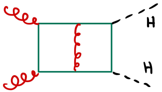
(s, t, m_t, m_Z, m_H)

- Numeric calculations [Heinrich,...,Mühlleitner,...]
- Analytic calculations [Duhr,...,Tancredi,...]
- Analytic expansions:
  - large-mass expansion: $m_t^2 \gg s, t, \ldots$
    `exp` [Harlander,Seidensticker,Steinhauser'98]
    "simple": vacuum integrals and massless integrals
  - high energy: $m_t^2 \ll s, t, \ldots$
    involved asymptotic expansion
    complicated MIs
  - $t \to 0$
    (often) Taylor expansion

High energy expansion

[Davies,Mishima,Steinhauser,Wellmann'18,. . . ,Davies,Mishima,Schönwald,Steinhauser,Zhang'22]

- Taylor expansion in m_H
- IBP reduction (s, t, m_t)
- differential equations in m_t^2/s; ansatz for $m_t^2 \ll s, t$
- BCs depend on s and t (can be quite complicated)
- deep expansion: $(m_t^2)^{16} \ldots (m_t^2)^{50} \ldots$



Padé improvement

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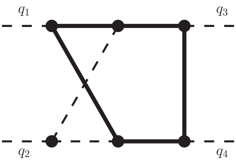
$$\sum_{k=0}^{N} c_k (m_t^2)^k = \frac{a_0 + \cdots + a_r (m_t^2)^r}{1 + b_1 + \cdots + b_s (m_t^2)^s} = [r/s](m_t^2) \qquad r + s = N$$

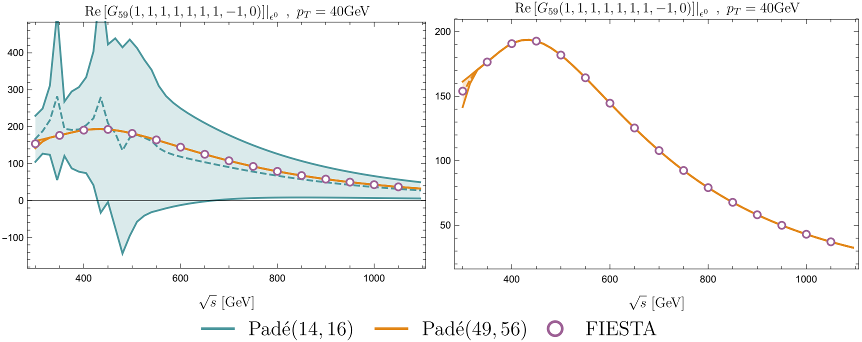
➪ Central value and corresponding uncertainty
for each phase-space point $(\sqrt{s}, p_T)$

$$p_T^2 = (tu - m_H^4)/s, s + t + u = 2m_H^2$$

High energy expansion ⊕ PA

- expansion up to $N_{\max} = (m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$

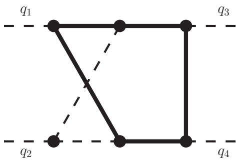


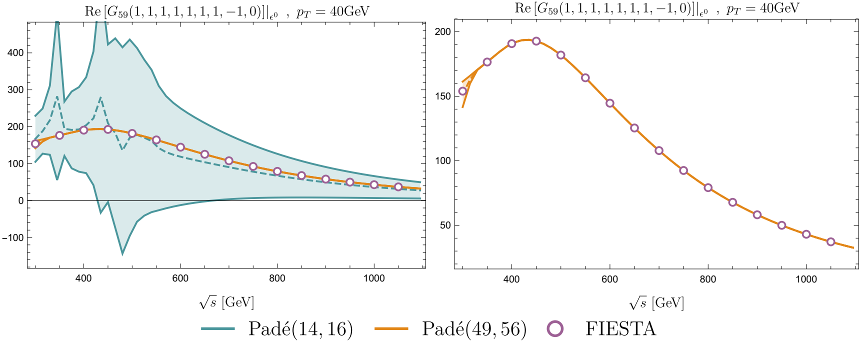


High energy expansion $\oplus$ PA

- expansion up to $N_{\max} = (m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$

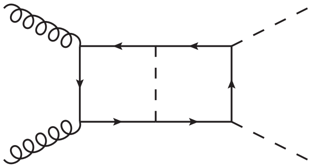
PA is a precision tool





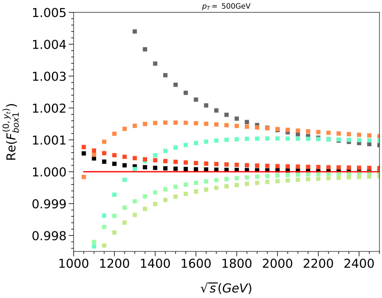
High energy expansion for $gg \to HH$

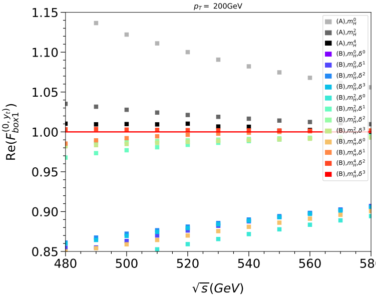
. . . electroweak corrections: H exchange



- A: $s, t \gg m_t^2 \gg (m_H^{\rm int})^2, (m_H^{\rm ext})^2$
- B: $s, t \gg m_t^2 \approx (m_H^{\rm int})^2 \gg (m_H^{\rm ext})^2$
- expand in

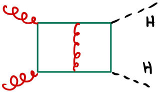
  $\{\delta = 1 - \frac{m_H^{\rm int}}{m_t}, \frac{m_H^{\rm ext}}{m_t}\}$ or $\{\frac{m_H^{\rm int}}{m_t}, \frac{m_H^{\rm ext}}{m_t}\}$





[Davies,Mishima,Schönwald,Steinhauser,Zhang'22]

$t \to 0$ expansion



[Bonciani,Degrassi,Giardino,Gröber'18]
[Bellafronte,Degrassi,Giardino,Gröber,Vitti'22; ...]
[Davies,Mishima,Schönwald,Steinhauser'23]

- forward scattering kinematics
- Taylor expansion
- same differential equations as for high-energy expansion $(\{s, t, m_t^2\})$
- construct for each MI expansion in t
- BC at $t = 0$: $f(s/m_t^2)$
- compute $f(s/m_t^2)$ with "expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

"Expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

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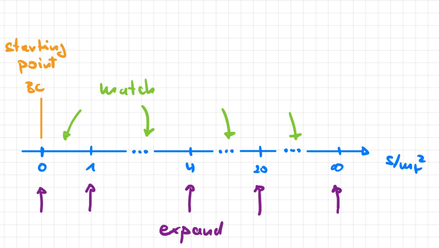
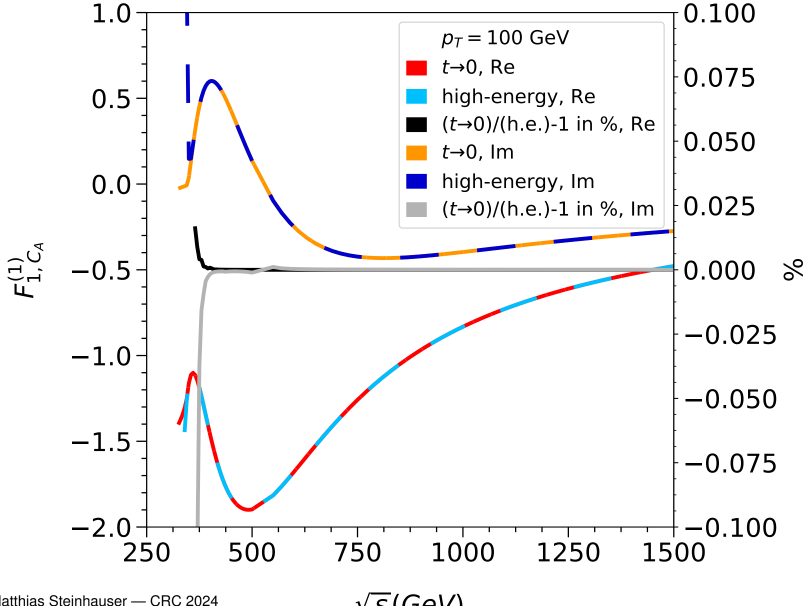


- semi-analytic results for $f(s/m_t^2)$
- differential equation for MIs in s/m_t^2
- (Power-log) ansatz for MIs
  ➪ insert in differential equation
  ➪ linear equations
- BCs for $s/m_t^2 \to 0$ ("simple")
- move step-by-step to $s/m_t^2 \to \infty$
- thresholds are properly taken into account by the ansatz

Expansion of (unknown) function $f(s/m_t^2)$ around properly chosen s/m_t^2 values with precise numerical coefficients

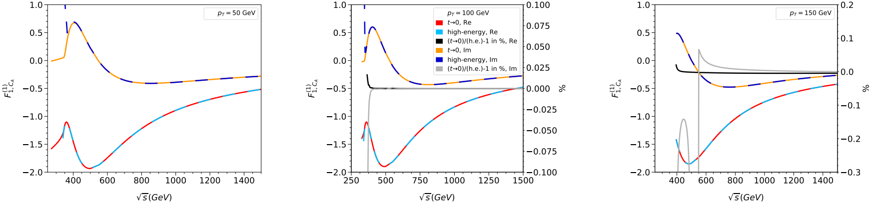
Similar approaches: [Blümlein,Czakon,Laporta,Lee,Liu,Smirnov,...]

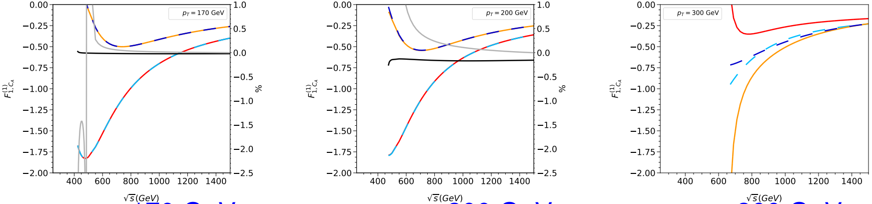
Combine: $t \to 0$ and h.e. at 2 loops (QCD)



Combine: $t \to 0$ and h.e. at 2 loops (QCD)

$p_T = 50$ GeV $p_T = 100$ GeV $p_T = 150$ GeV





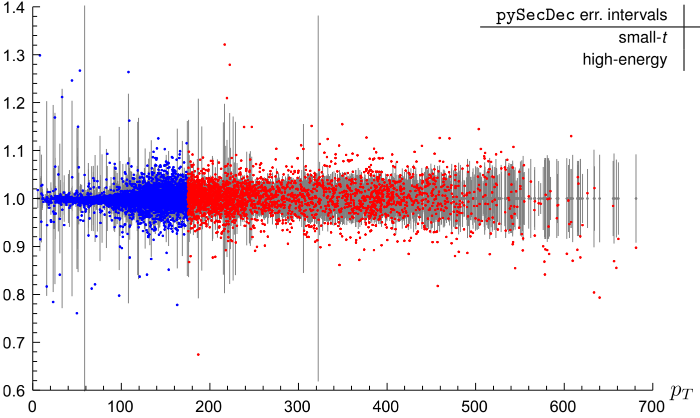
$p_T = 170$ GeV $p_T = 200$ GeV $p_T = 300$ GeV

[Davies,Mishima,Schönwald,Steinhauser'23]

$\mathcal{V}_{\rm fin}$: virtual NLO QCD corrections

Comparison to "pySecDec"

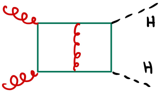
pySecDec err. intervals	1σ	2σ	3σ
small-t	0.57	0.85	0.92
high-energy	0.65	0.94	0.99



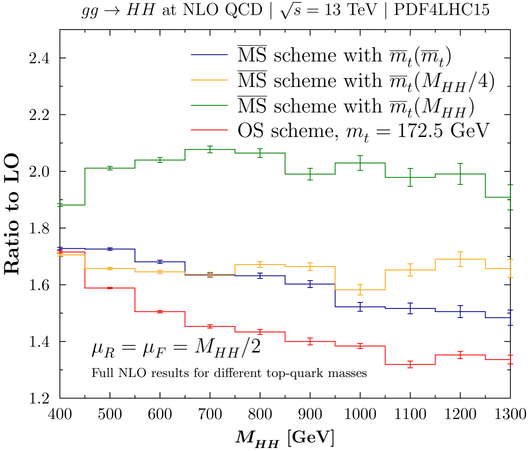
https://github.com/mppmu/hhgrid

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

$gg \to HH$ at NLO

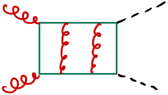


[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]



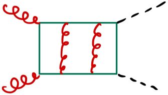
$gg \to HH$ **at 3 loops (NNLO)**

large-mass expansion: DONE [Davies,Steinhauser'19]
high-energy: NO

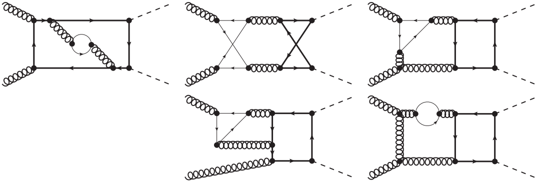
$t \to 0$: YES, if we can do the reduction for  $\{s, t, m_t^2\}$

➪ currently not possible

But: invert order:

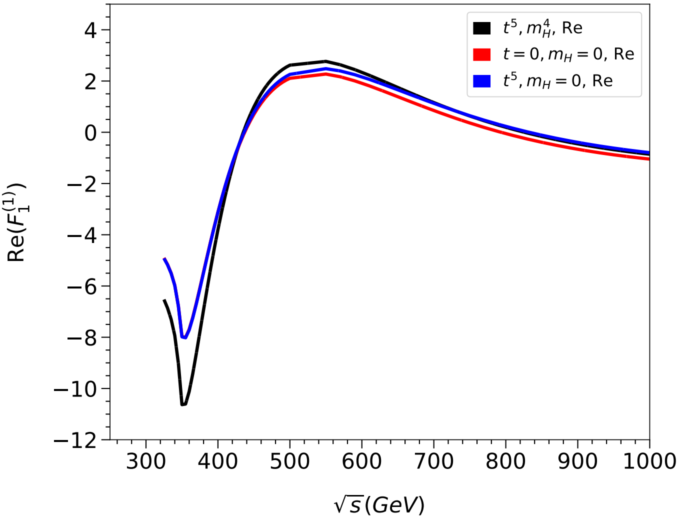
1. expand in t ➪ no t dependence
2. reduce:  $\{s, m_t^2\}$ (still challenging)

Fermionic corrections



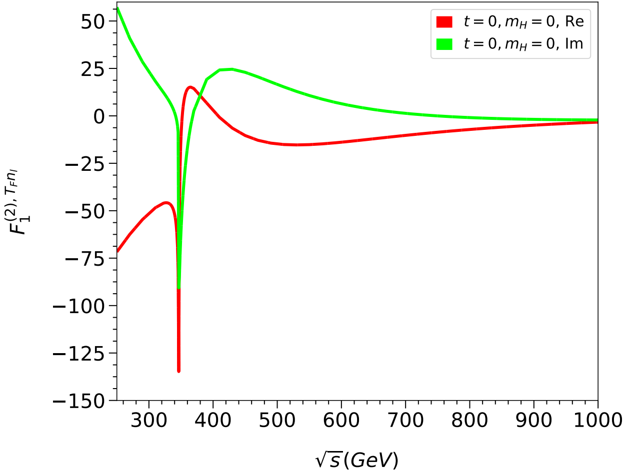
- $t = 0$, $m_H = 0$
- 31 integral families — tapir: [Gerlach,Herren,Lang]
- 176 MIs — kira: [Klappert,Lange,Maierhöfer,Usovitsch'20]
- useful: `LiteRed` [Lee], `LIMIT` [Herren], `Feynson` [Magerya]
- reduction: about 1 week for most complicated family

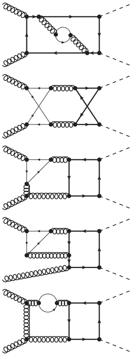
2-loop results for $t = 0$



3-loop n_l for $t = 0, m_H = 0$

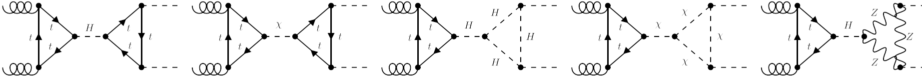
[Davies,Schönwald,Steinhauser'23]



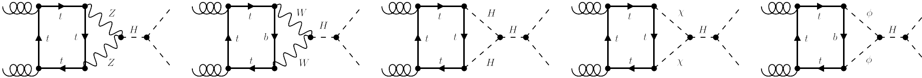


At threshold: NLO: $v \log v, v^2 \log v, v^3 \log^2 v, \ldots$

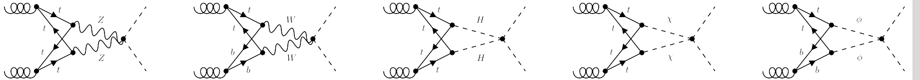
$v = \sqrt{1 - 4m_t^2/s}$ NNLO: $v \log^2 v, v^2 \log^2 v, v^3 \log^3 v, \ldots$



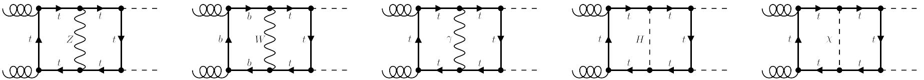
(c-1) (c-2) (c-3) (c-4) (c-5)



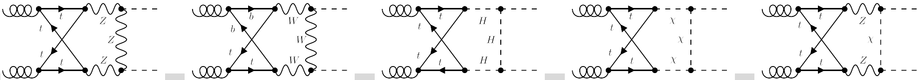
(d-1) (d-2) (d-3) (d-4) (d-5)



(e-1) (e-2) (e-3) (e-4) (e-5)



(f-1) (f-2) (f-3) (f-4) (f-5)



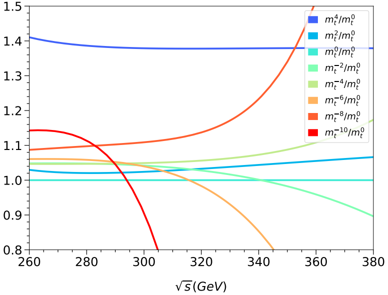
(g-1) (g-2) (g-3) (g-4) (g-5)

Full electroweak corrections to $gg \to HH$ in large-m_t limit

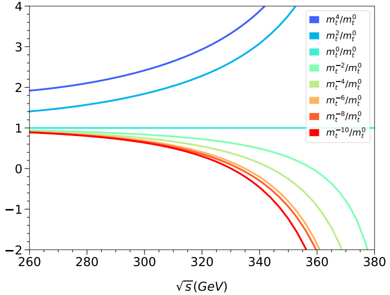
[Davies,Mishima,Schönwald,Steinhauser,Zhang'23]

- $m_t \gg m_H, m_Z, m_W$, check that ξ_Z, ξ_W drop out
- expansion up to $1/m_t^{10}$
- on-shell renormalization (exact in m_t, m_H, m_Z, m_W)

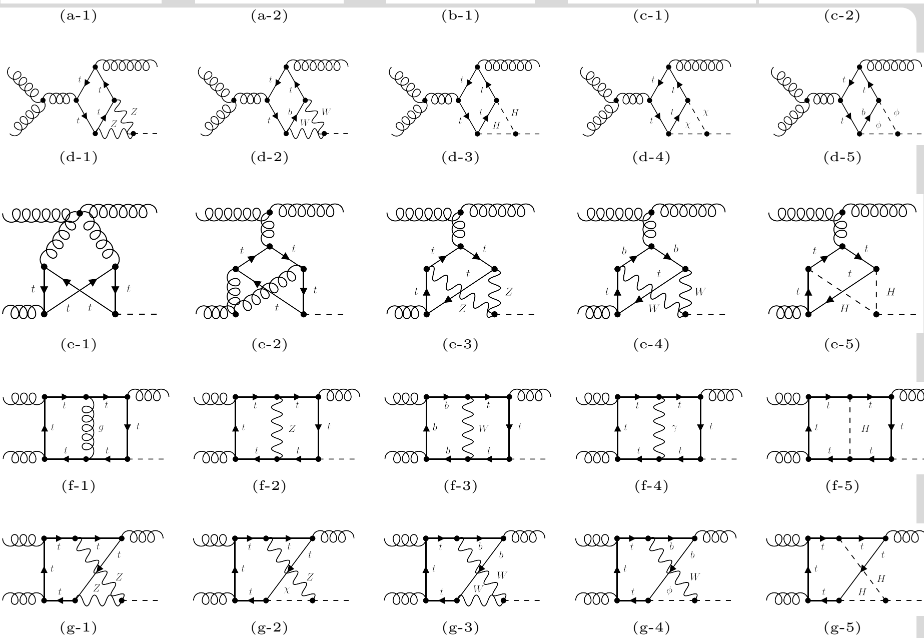
NLO: ratio to m_t^0



no $\sqrt{s} = m_t + m_W$ cut



- leading Yukawa correction [Mühlleitner,Schlenk,Spira'22]
- full numerical calculation [Bi,Huang,Huang,Ma,Yu'23]
  $\mathrm{d}\sigma/\mathrm{d}M_{HH}$: $+15\% \ldots -10\%$


(a-1) (a-2) (b-1) (c-1) (c-2)
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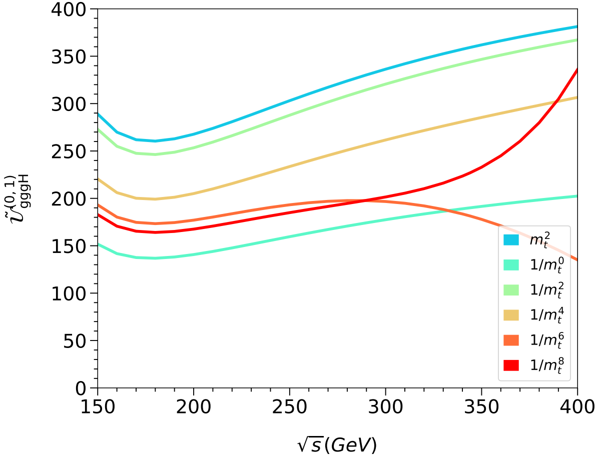
(f-1) (f-2) (f-3) (f-4) (f-5)
(g-1) (g-2) (g-3) (g-4) (g-5)

Full electroweak corrections to $gg \to Hg$ in large-m_t limit

[Davies,Mishima,Schönwald,Steinhauser,Zhang'23]

NLO $|\mathcal{M}|^2$



Conclusions

- Combine expansions (large-m_t, high-energy, $t \to 0$)
- "Expand and match"
- QCD: 3-loop $gg \to HH$, $gg \to ZH$, $gg \to ZZ$, ...
  (with massive m_t, m_H, m_Z) in reach
  semi-analytic $t \to 0$ expansion
- Electroweak corrections:
  expand (in addition) in mass differences, e.g. $1 - m_H/m_t$
- Semi-analytic expressions ➪ fast and flexible