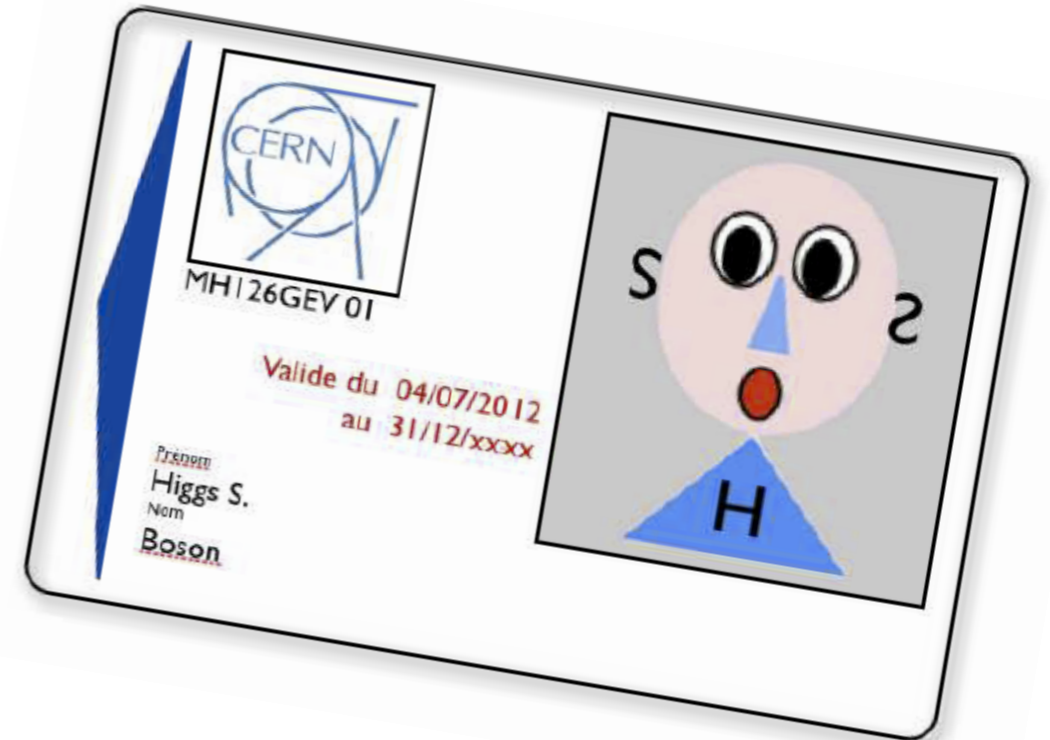


*Recent Progress in Precision Higgs  
Pair and NMSSM Mass Calculations*

Margarete Mühlleitner (KIT)  
Annual Meeting CRC TRR 257  
11 March 2024

# Outline

- Higgs Mass Predictions in the CP-Violating High-Scale NMSSM
- $\mathcal{O}(\alpha_t^2)$  corrections to the trilinear Higgs self-coupling in the CP-violating NMSSM
- NLO QCD Corrections w/ full top-mass dependence to 2HDM hH and AA production
- Top-Yukawa induced EW corrections to SM HH



Involved ITP members in A3a and A3b projects:

Thomas Biekötter, Lisa Biermann, Christoph Borschensky,  
Sauro Carlotti, Pedro Gabriel, Stefan Liebler, Dat Nguyen

# Higgs Mass Predictions in the $CP$ -Violating High-Scale NMSSM



# Higgs Mass in Supersymmetry

## Higgs boson mass:

- \* SM: fundamental parameter, not predicted by the theory
- \* Supersymmetry: calculable from input parameters; quantum corrections  $\Delta m^2_H$  are important!

$$\begin{aligned} \text{MSSM: } m_H^2 &\approx M_Z^2 \cos^2 2\beta + \Delta m_H^2 \leftarrow (85 \text{ GeV})^2! \\ \text{NMSSM: } m_H^2 &\approx M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \Delta m_H^2 \leftarrow (55 \text{ GeV})^2 \end{aligned}$$

## NMSSM:

- \* less important loop corrections compared to the MSSM
- \* solves little hierarchy problem

[Kim, Nilles, '84]

## Comparison of calculated mass value (at high precision) w/ experimental data:

- \* indirect constraint on (N)MSSM parameter space

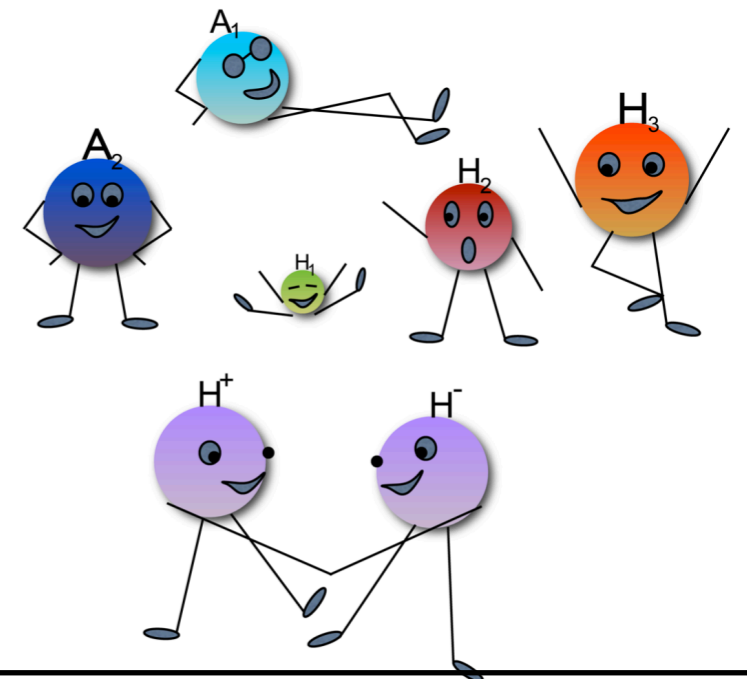
# SUSY Higgs Masses

- ♦ **Supersymmetry:** requires at least 2 complex Higgs doublets
- ♦ **Minimal Supersymmetric extension (MSSM):** 2 complex Higgs doublets

5 Higgs bosons:  $h, H, A, H^+, H^-$   
4 neutralinos:  $\tilde{\chi}_i^0$  ( $i = 1, \dots, 4$ )

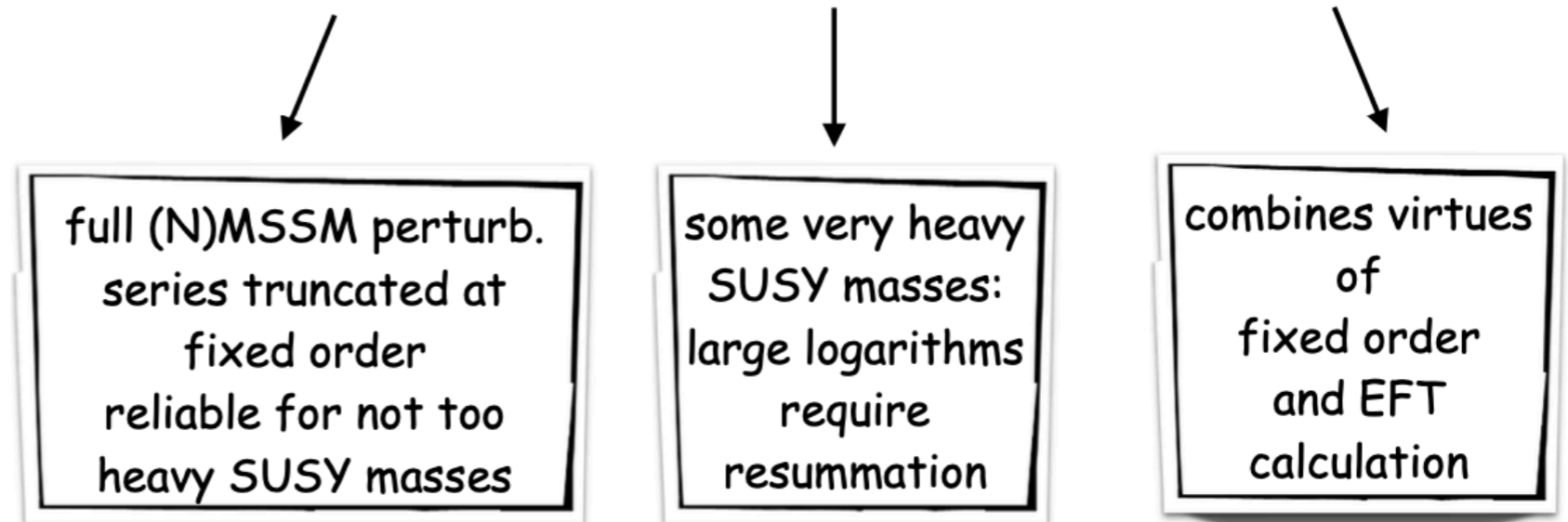
- ♦ **Next-to-MSSM (NMSSM):** 2 complex Higgs doublets plus complex singlet field
- ♦ **Enlarged Higgs and neutralino sector:**

7 Higgs bosons:  $H_1, H_2, H_3, A_1, A_2, H^+, H^-$   
5 neutralinos:  $\tilde{\chi}_i^0$  ( $i = 1, \dots, 5$ )



# Spectrum Calculations

❖ **Methods for Higgs mass calculations: fixed-order (FO) - effective field theory (EFT) - hybrid**



❖ **Fixed-order calculation and large mass scales:**

FO contains terms  $\sim y_x \ln(M_{Sx}^2/M_x^2)$  with  $y_x$  Yukawa coupling,  $M_x$  ( $M_{Sx}$ ) mass of (SUSY) particle  
most important contribution from top/stop sector  $\leadsto$  large hierarchy  $\leadsto$  large logs!  $\leadsto$  resummation needed for reliable results

❖ **EFT calculation:** SUSY couplings matched to corresponding couplings in EFT theory such that physics

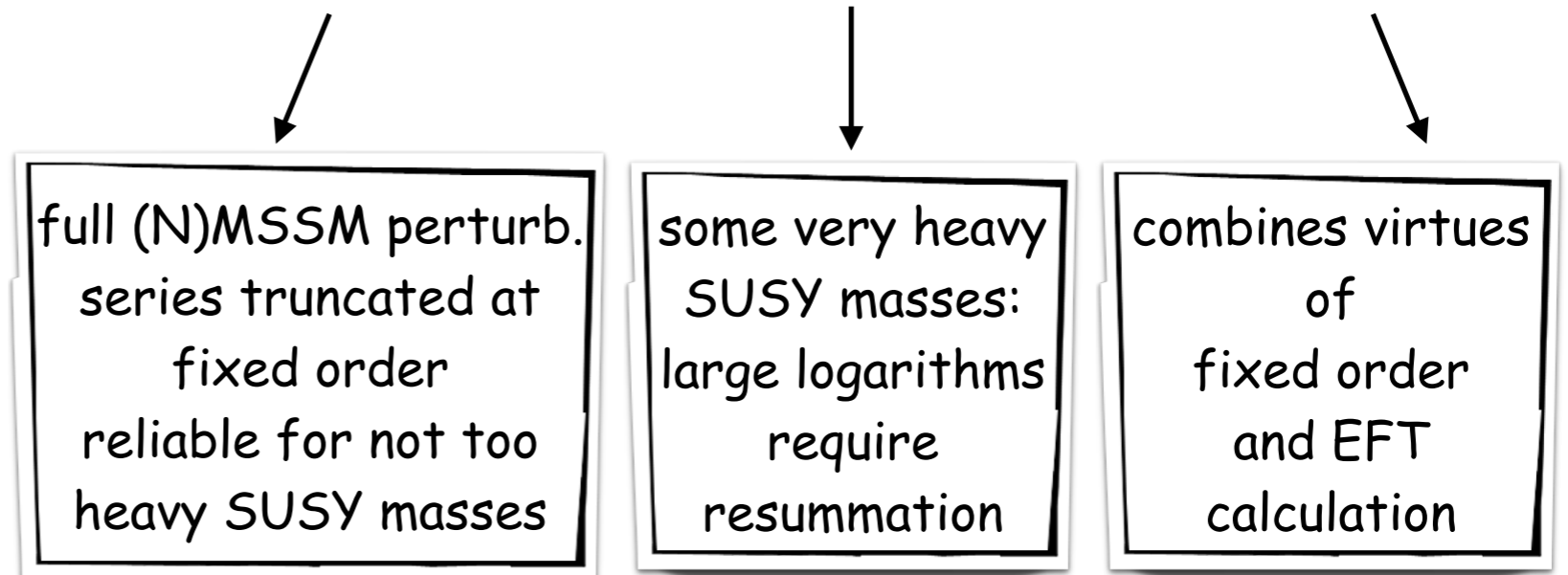
at **matching scale  $\mu_R$**  is the same; only light SM-like particles & heavy BSM particles: **SM = EFT  $\Rightarrow$**

**$\lambda^{SM}(\mu_R) = \lambda^{BSM}(\mu_R)$**  [receives only BSM contributions]; we have terms like  $y_x \ln(M_{Sx}^2/M_x^2)$ , respectively

$y_x (\ln(M_{Sx}^2/\mu_R^2) + \ln(\mu_R^2/M_x^2))$ . With  $\mu_R = M_{Sx} \Rightarrow$   **$y_x \ln(\mu_R^2/M_x^2) \Leftarrow$  resummed via RGEs for  $y_x$**

# Spectrum Calculations

❖ Methods for Higgs mass calculations: fixed-order (FO) - effective field theory (EFT) - hybrid



❖ Status MSSM spectrum calculations:

FO: up to 2-loop in on-shell (OS) and DR scheme, partial 3-loop in DR scheme

EFT: up to N<sup>2</sup>LL (included in calculators), N<sup>3</sup>LL

Hybrid: FeynHiggs, FlexibleEFTHiggs, N<sup>3</sup>LO+N<sup>3</sup>LL QCD corrections [Harlander, Klappert, Voigt, '19]

❖ Status NMSSM spectrum calculations:

FO: up to 2-loop in mixed OS-DR scheme and in DR-scheme

EFT: matching to quartic coupling in NMSSM w/ all BSM particles at TeV scale

e.g. [Gabelmann, MM, Staub, '18, '19][Bagnaschi et al, '22]

Hybrid: FlexibleEFTHiggs, SARAH+SPheno

# NMSSM Spectrum Calculators

- FlexibleSUSY [Athron,Bach,Harries,Kotlarski,Kwasnitza,Park,Stöckinger,Voigt,Ziebell]: DR, FO & hybrid, through FlexibleEFTHiggs
- NMSSMCALC:[Baglio,Borschensky,Dao,Gabelmann,Gröber,Krause,Le,MM,Rzehak,Spira,Streicher,Walz]: FO, real & complex NMSSM, DR and mixed OS-DR
- NMSSMTools [Ellwanger,Gunion,Hugonie]: FO, DR scheme
- SOFTSUSY [Allanach,Athron,Bednyakov,Tunstall,Voig,RuizdeAustri,Williams]: FO, DR scheme
- SPheno [Porod,Staub]: FO, DR scheme, quartic and pole mass matching

## Remarks:

- comparison of codes in DR scheme: [Staub,Athron,Ellwanger,Gröber,MM,Slavich,Voigt,'15]  
FlexibleSUSY,NMSSMCALC,NMSSMTools, SOFTSUSY,SPheno
- comparison of codes in mixed OS-DR scheme: [Drechsel,Gröber,Heinemeyer,MM,Rzehak,Weiglein,'16]  
FeynHiggs, NMSSMCALC
- solution of Goldstone boson catastrophe [Braathen,Goodsell,'16], [Braathen,Goodsell,Staub,'17]
- advances in FeynHiggs: [Drechsel,Galeta,Heinemeyer,Hollik,Liebler,Moortgat-Pick,Paßehr,Weiglein]  
real&complex NMSSM, GNMSSM: 1-loop in, 2-loop&resummation of HO log-effects only in MSSM limit, no public code yet
- OS masses CP-violating NMSSM, consistent description production/decay [Domingo,Drechsel,Paßehr]



# Matching Conditions

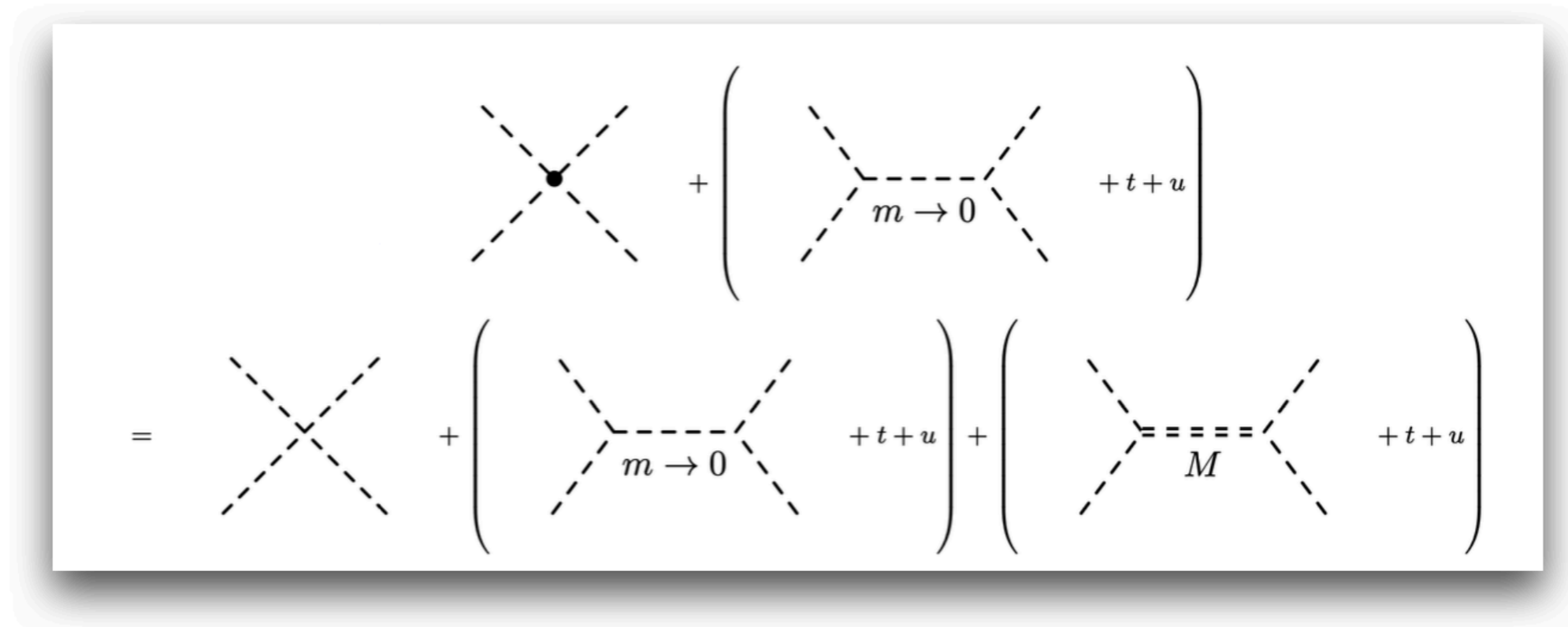
Quartic Coupling Matching (unbroken EW symmetry;  $v_u, v_d \rightarrow 0$ ,  $\tan\beta = v_u/v_d = \text{const.}$ ,  $v_s \neq 0$ ):

[Bagnaschi et al., '22] for real NMSSM  
our work: complex NMSSM

$$\lambda_H^{\text{SM}, \overline{\text{MS}}}(Q_{\text{match}}) = \lambda_H^{\text{NMSSM}, \overline{\text{MS}}}(Q_{\text{match}})$$

effective quartic coupling after subtracting the SM contributions:

$$\lambda_{\text{NMSSM}}^{\overline{\text{DR}}}(Q_{\text{match}}) = \lambda_{\text{NMSSM}}^{\text{tree}} + \Delta\lambda_{\text{NMSSM}}^{1l} + \Delta\lambda_{\text{MSSM}}^{2l}$$



$m$  ( $M$ ) light  
(heavy) mass  
scale

----- light scalars    ===== heavy scalars

# Matching Conditions

Quartic Coupling Matching (unbroken EW symmetry:  $v_u, v_d \rightarrow 0$ ,  $\tan\beta = v_u/v_d = \text{const.}$ ,  $v_S \neq 0$ ):

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$$\begin{aligned} \lambda_{\text{NMSSM}}^{\text{tree}} = & \underbrace{\frac{1}{4}(g_1^2 + g_2^2) \cos^2 2\beta}_{\text{MSSM D-terms}} + \underbrace{\frac{1}{2}|\lambda| \sin^2 2\beta}_{\text{NMSSM F-terms}} \\ & - \underbrace{\frac{1}{24|\kappa|^2 M_S^2 (3M_S^2 + M_{A_S}^2)} \left( 3|\kappa|^2 M_{H^\pm}^2 - 3|\kappa|^2 M_{H^\pm}^2 \cos 4\beta \right.}_{\text{s/t/u-channel } S} \\ & \left. + (3M_S^2 + M_{A_S}^2) (|\kappa||\lambda| \cos \varphi_y \sin 2\beta - 2|\lambda|^2) \right)^2 \\ & - \underbrace{\frac{3}{8M_{A_S}^2} |\lambda|^2 (3M_S^2 + M_{A_S}^2) \sin^2 2\beta \sin^2 \varphi_y}_{\text{s/t/u-channel } A_S} \end{aligned}$$

# Matching Conditions

Quartic Coupling Matching (unbroken EW symmetry:  $v_u, v_d \rightarrow 0$ ,  $\tan\beta = v_u/v_d = \text{const.}$ ,  $v_s \neq 0$ ):

$$\lambda_H^{\text{SM}, \overline{\text{MS}}}(Q_{\text{match}}) = \lambda_H^{\text{NMSSM}, \overline{\text{MS}}}(Q_{\text{match}})$$

[Bagnaschi et al., '22] for real NMSSM  
our work: complex NMSSM

effective quartic coupling after subtracting the SM contributions:

$$\lambda_{\text{NMSSM}}^{\overline{\text{DR}}}(Q_{\text{match}}) = \lambda_{\text{NMSSM}}^{\text{tree}} + \Delta\lambda_{\text{NMSSM}}^{1l} + \Delta\lambda_{\text{MSSM}}^{2l}$$

Remark: shift due to NMSSM calc. done in  $\overline{\text{DR}}$  and discarded SM contribution done in  $\overline{\text{MS}}$  taken into account

## Loop corrected NMSSM masses and couplings from NMSSMCALC

NMSSMCALC

Calculator of One-Loop and  
 $O(\alpha_t \alpha_s + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$  Two-Loop  
Higgs Mass Corrections  
and of Higgs Decay Widths  
in the CP-conserving and the CP-violating NMSSM

Computation of the Loop-Corrected Effective Higgs Self-Couplings  
and the Loop-Corrected Higgs-to-Higgs Decays  
up to  $O(\alpha_t \alpha_s + \alpha_t^2)$

Computation of the muon anomalous magnetic moment and the electric dipole moment

**New: Computation of the  $q$  parameter up to  $O(\alpha + \alpha_t \alpha_s + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ ; W-mass prediction in the SM, at 1-loop NMSSM, 2-loop QCD NMSSM, 2-loop EW NMSSM**

[Baglio, Borschensky, Dao, Gabelmann, Gröber, Krause, Le, MM, Rzehak, Spira, Streicher, Walz]

The program package NMSSMCALC calculates the one-loop and  $O(\alpha_t \alpha_s + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$  corrected Higgs boson masses and the Higgs decay widths and branching ratios within the CP-conserving and the CP-violating NMSSM.

The decay calculator is based on an extension of the program HDECAY 6.10 now.

The effective loop-corrected trilinear Higgs self-couplings and loop-corrected Higgs-to-Higgs decays are provided up to  $O(\alpha_t \alpha_s + \alpha_t^2)$ .

The program also provides the options to calculate the electron and muon anomalous magnetic moments and, in the CP-violating case, the electric dipole moments.

The program provides the  $q$  parameter up to  $O(\alpha + \alpha_t \alpha_s + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ ; the W-mass prediction in the SM, at 1-loop NMSSM, 2-loop QCD NMSSM, 2-loop EW NMSSM.

# Matching Conditions

**Pole Mass Matching/„Hybrid“ (broken EW symmetry,  $v \ll M_{\text{SUSY}}$ ):**

e.g. [Athron et al, '16]

$$M_{h,\text{SM}}^2 \stackrel{!}{=} M_{h,\text{NMSSM}}^2$$

$$M_{h,X}^2 = m_{h,X}^2 - \hat{\Sigma}_{h,X}(M_{h,X}^2) \quad \text{with} \quad X = \text{SM, NMSSM}$$

$m_{h,\text{SM}}$  and  $m_{h,\text{NMSSM}}$  denote the running  $\overline{\text{MS}}$  and  $\overline{\text{DR}}$  masses of the SM(-like) Higgs states

**Tree Level:**  $m_{h,\text{SM}}^2 = 2\lambda_{\text{SM}}^{\text{eff.}} v_{\text{SM}}^2 \stackrel{!}{=} m_{h,\text{NMSSM}}^2 \quad \leadsto \quad \lambda_{\text{SM}}^{\text{eff.}} = \frac{m_{h,\text{NMSSM}}^2}{2v_{\text{NMSSM}}^2}$

**use**  $v_{\text{SM}}^2 = v_{\text{NMSSM}}^2 + \delta v^2 = v_{\text{NMSSM}}^2 \left(1 + \frac{\delta v^2}{v^2}\right)$  **with**  $\frac{\delta v^2}{v^2} = \left[\hat{\Sigma}'_{h,\text{NMSSM}}(0) - \hat{\Sigma}'_{h,\text{SM}}(0)\right] + \mathcal{O}(v^4/M_{\text{SUSY}}^4)$

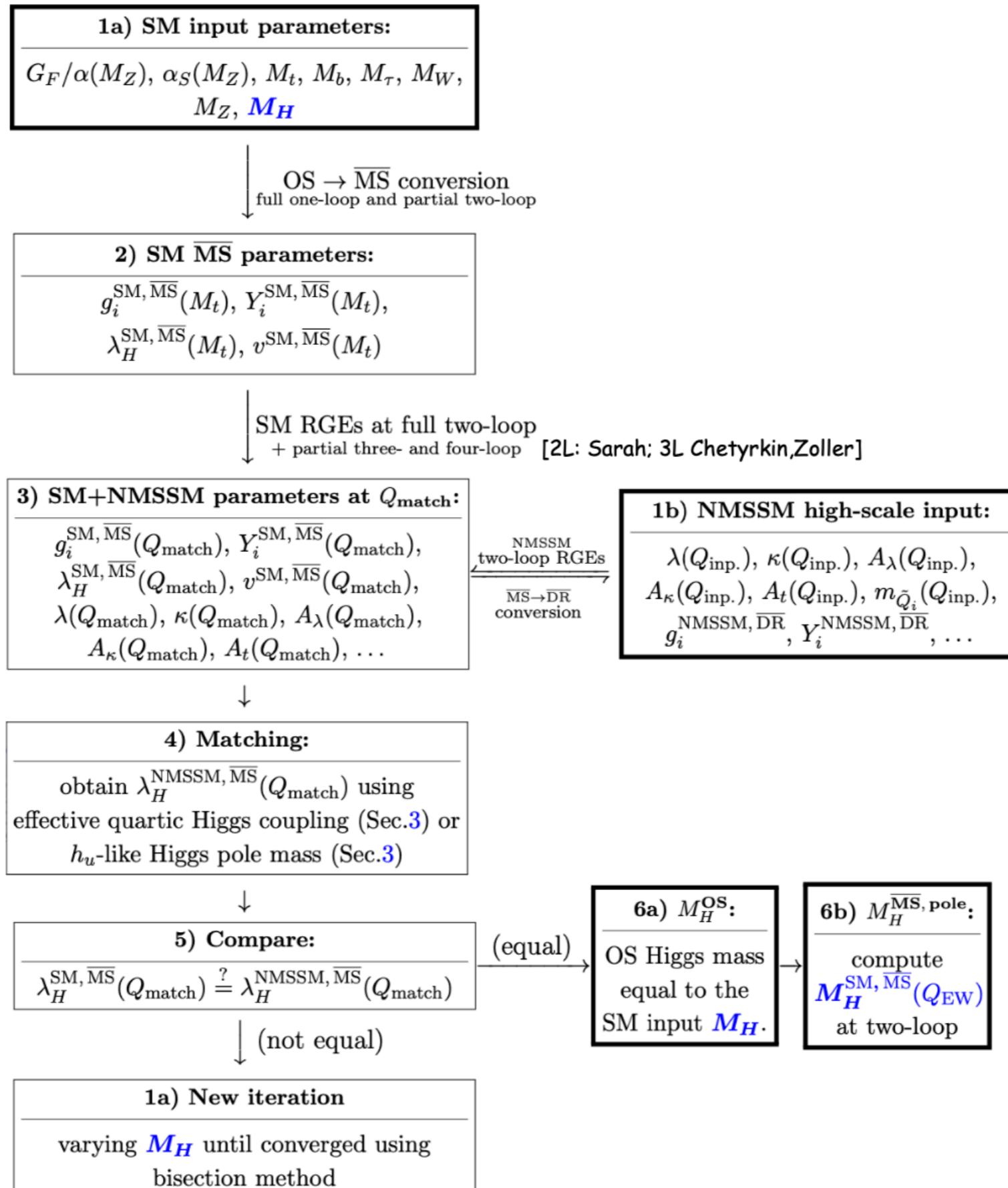
**One-Loop Level:**  $\lambda_{\text{SM}}^{\text{eff.}} = \frac{1}{2v_{\text{SM}}^2} \left[ m_{h,\text{NMSSM}}^2 - \hat{\Sigma}_{h,\text{NMSSM}}(m_{h,\text{NMSSM}}^2) + \hat{\Sigma}_{h,\text{SM}}(m_{h,\text{SM}}^2) \right]$

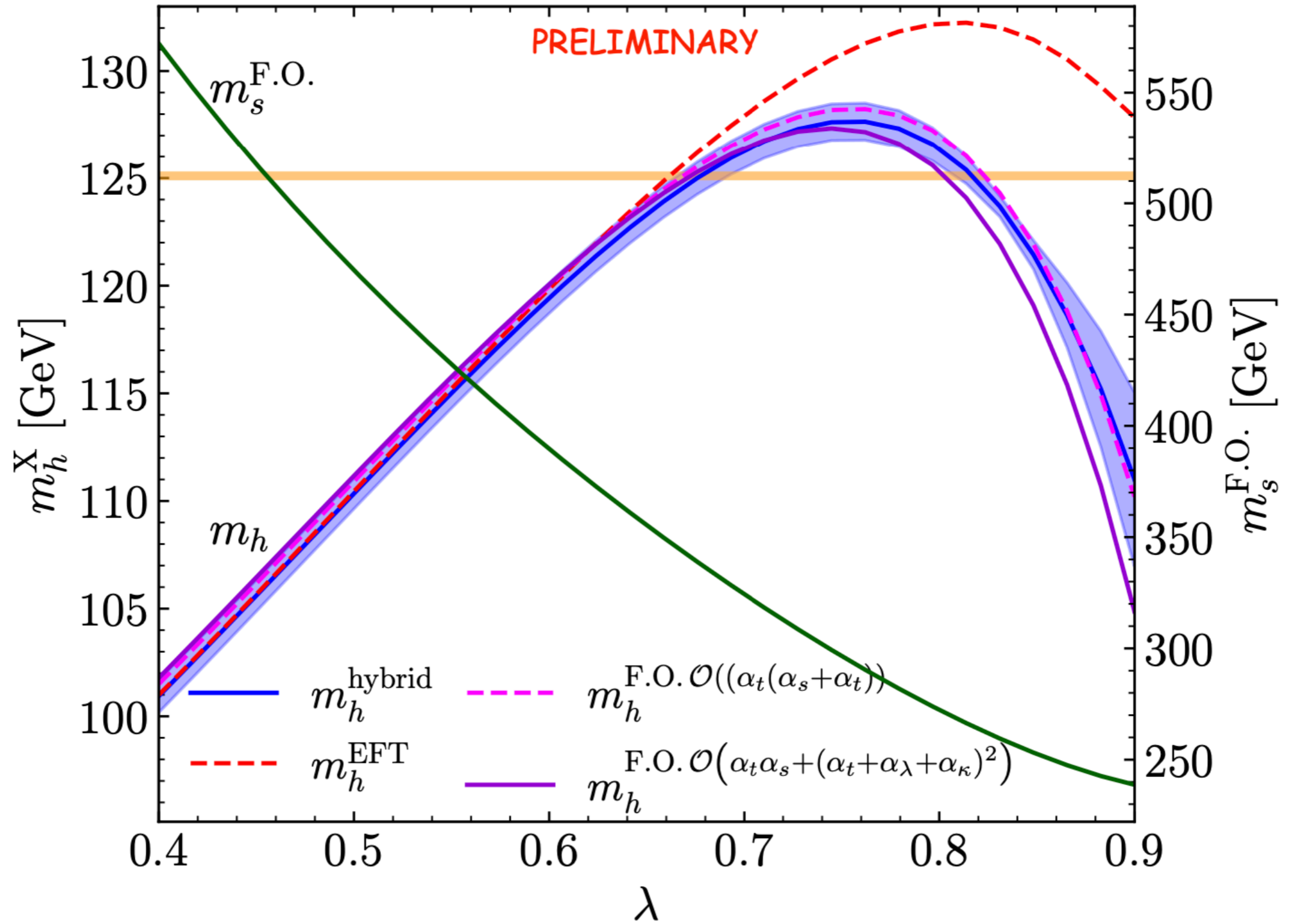
**with**  $\hat{\Sigma}_{h,X}(m_{h,X}^2) = \hat{\Sigma}_{h,X}(0) + m_{h,X}^2 \hat{\Sigma}'_{h,X}(0) + \mathcal{O}(m_{h,X}^4)$  **and**  $v_{\text{SM}} \rightarrow v_{\text{NMSSM}}$  **so that**

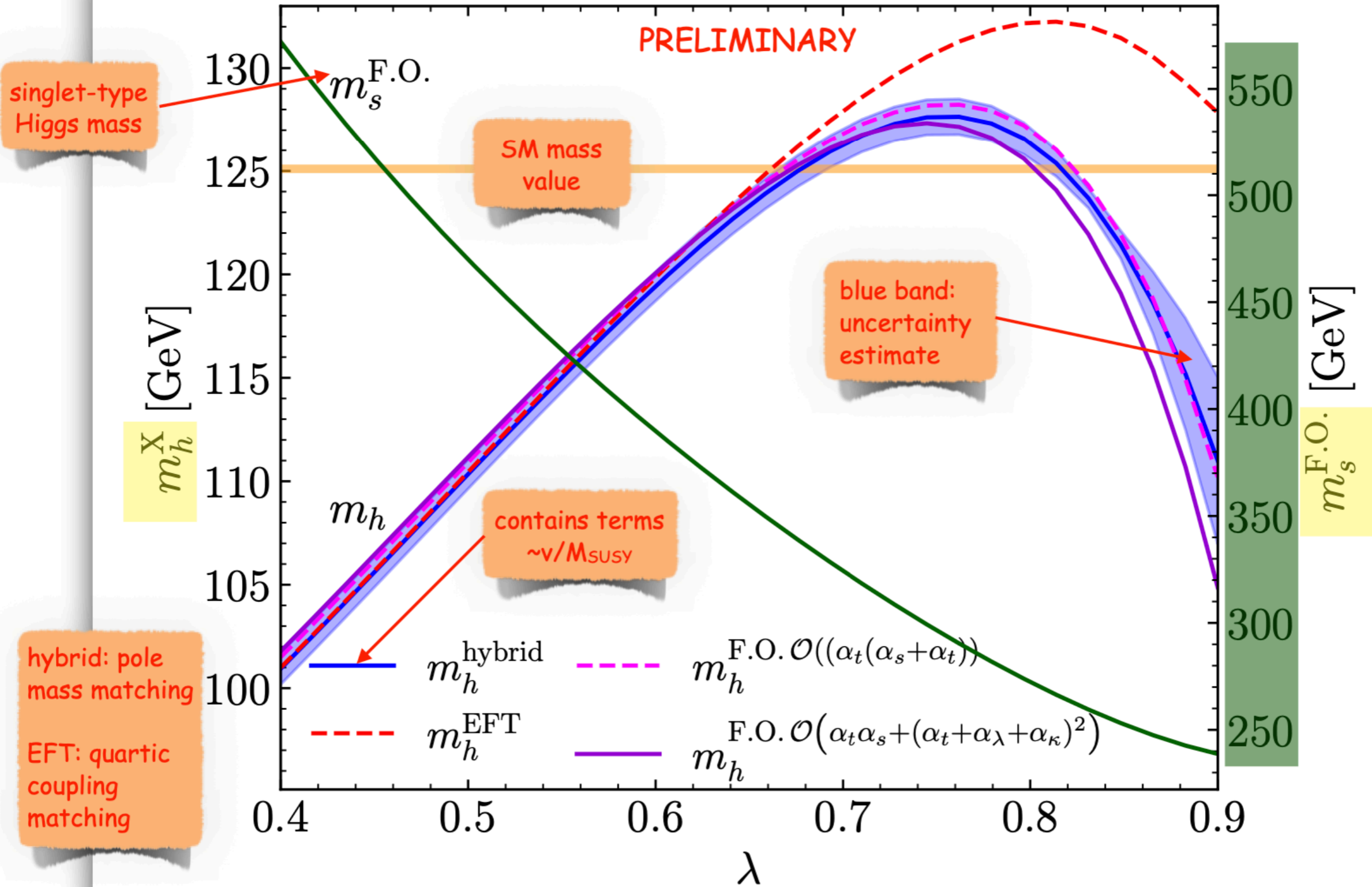
Leading terms in expansion in  $v/M_{\text{SUSY}}$

$$\lambda_{\text{SM}}^{\text{eff.}} = \frac{1}{2v_{\text{NMSSM}}^2} \left[ m_{h,\text{NMSSM}}^2 - \Delta\hat{\Sigma}_h - 2m_{h,\text{NMSSM}}^2 \Delta\hat{\Sigma}'_h \right] \quad \text{with} \quad \Delta\hat{\Sigma}_h^{(l)} \equiv \Sigma_{h,\text{NMSSM}}^{(l)}(0) - \hat{\Sigma}_{h,\text{SM}}^{(l)}(0)$$

# Schematic Procedure implemented in NMSSMCALC







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# Uncertainty Estimate

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## Blue uncertainty band:

### SM uncertainties:

- scheme uncertainty using either  $G_F$  or  $\alpha_{\text{QED}}(m_Z)$  as input (estimates missing 2-loop EW corrections in the relation between Lagrangian  $\overline{\text{MS}}$  and physical OS parameters)
- scheme and scale uncertainty:  $M_H^{\text{OS}} - M_H^{\overline{\text{MS}},\text{pole}}(\mu_{\text{ren}})$  (estimates missing corrections in the relation  $\lambda^{\text{SM},\overline{\text{MS}}}$  and  $M_h^{\text{SM},\text{OS}}$ )
- Estimate missing corrections in the relation between  $m_t^{\overline{\text{MS}}}(M_t)$  and  $M_t^{\text{OS}}$  by in/excluding corrections of  $O(\alpha_s^3)$  & higher [mr by Kniehl, Pikelner, Veretin; SMDR by Martin, Robertson]

### SUSY uncertainties:

- scale uncertainty by varying  $Q_{\text{match}}$ : estimates missing 2-loop corrections in the matching condition
- for the quartic coupling matching: difference between the quartic-coupling and pole-mass matching as an estimate of the  $v/M_{\text{SUSY}}$  terms that are not included in the quartic-coupling matching



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## Information on Used Benchmark Point

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	$\tan \beta$	$\lambda$	$\kappa$	$M_1$	$M_2$	$M_3$	$A_0$	$A_\lambda$	$A_\kappa$	$\mu_{eff.}$	$m_{\tilde{Q}_L}$	$m_{\tilde{t}_R}$
BP1	1.27	0.73	0.62	0.14	1.18	2.3	-0.39	0.06	-1.44	0.49	1.79	1.51

All parameters with mass dimension are given in units of TeV. All soft SUSY breaking trilinear couplings are set equal to  $A_0$ , all soft SUSY breaking left-handed doublet and Right-handed singlet masses are set equal to  $m_{\tilde{Q}_L}$  and  $m_{\tilde{t}_R}$ , respectively.

$\mathcal{O}(\alpha_t^2)$  corrections to the trilinear Higgs self-coupling in the  $CP$ -violating NMSSM





# Trilinear Higgs Self-Couplings

♦ SM Higgs potential in physical gauge:

$$V(H) = \frac{1}{2} M_H^2 H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

Higgs mass :  $M_H = \sqrt{2\lambda} v$

trilinear Higgs self-coupling :  $\lambda_{HHH} = 3M_H^2 / M_Z^2$  

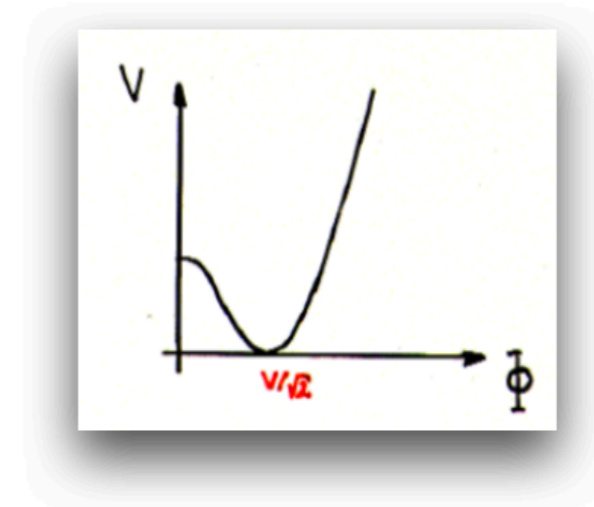
quadrilinear Higgs self-coupling :  $\lambda_{HHHH} = 3M_H^2 / M_Z^4$  

(units  $\lambda_0 = 33.8 \text{ GeV} / \lambda^2$ )

♦ Masses  $M_{ij} = (\partial^2 V_H / \phi_i \phi_j) |_{\phi=0}$  and Higgs self-couplings  $\lambda_{ijk} = (\partial^3 V_H / \phi_i \phi_j \phi_k) |_{\phi=0}$  related through Higgs potential  $V_H \Rightarrow$  catch up in precision w/ masses

♦ Importance of the trilinear Higgs self-coupling:

- determines shape of the Higgs potential
- sensitive to beyond-Standard Model physics
- important input for Higgs pair production
- important input for Higgs-to-Higgs decays
- important input for electroweak phase transitions



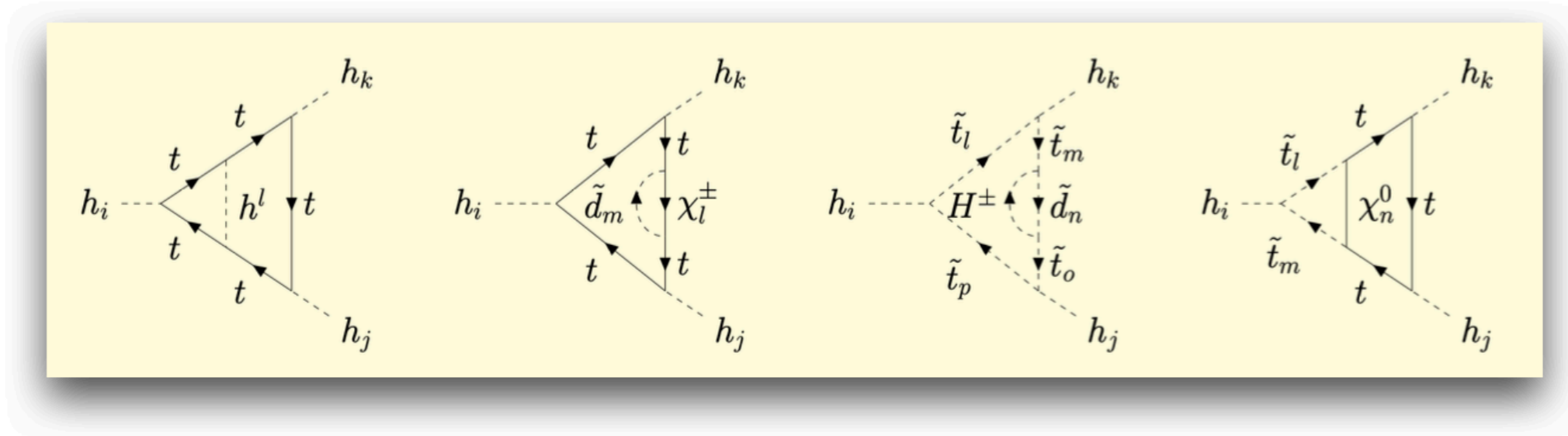
♦ Previous work: full 1-loop [Dao,MM,Streicher,Walz,'13]

2-loop at  $\mathcal{O}(\alpha_t \alpha_s)$  [Dao,MM,Ziesche,'15]

Present work: 2-loop  $\mathcal{O}(\alpha_t^2)$  [Borschensky,Dao,Gabelmann,MM,Rzehak,'22]

# Trilinear Higgs Self-Couplings at 2L $\mathcal{O}(\alpha_t^2)$

- ✦ New corrections at  $\mathcal{O}(\alpha_t^2)$ : all 2-loop diagrams with top/stops and at most one Higgs/Higgsino field, e.g.



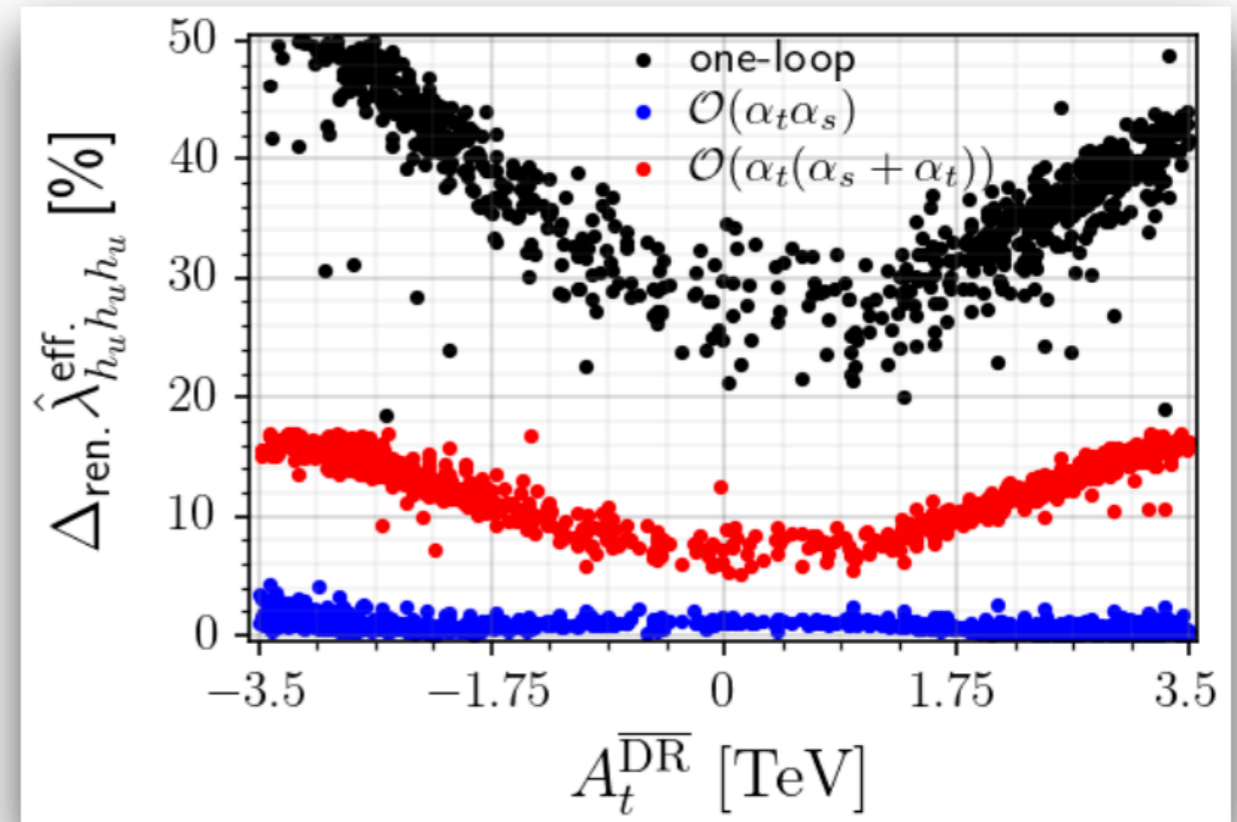
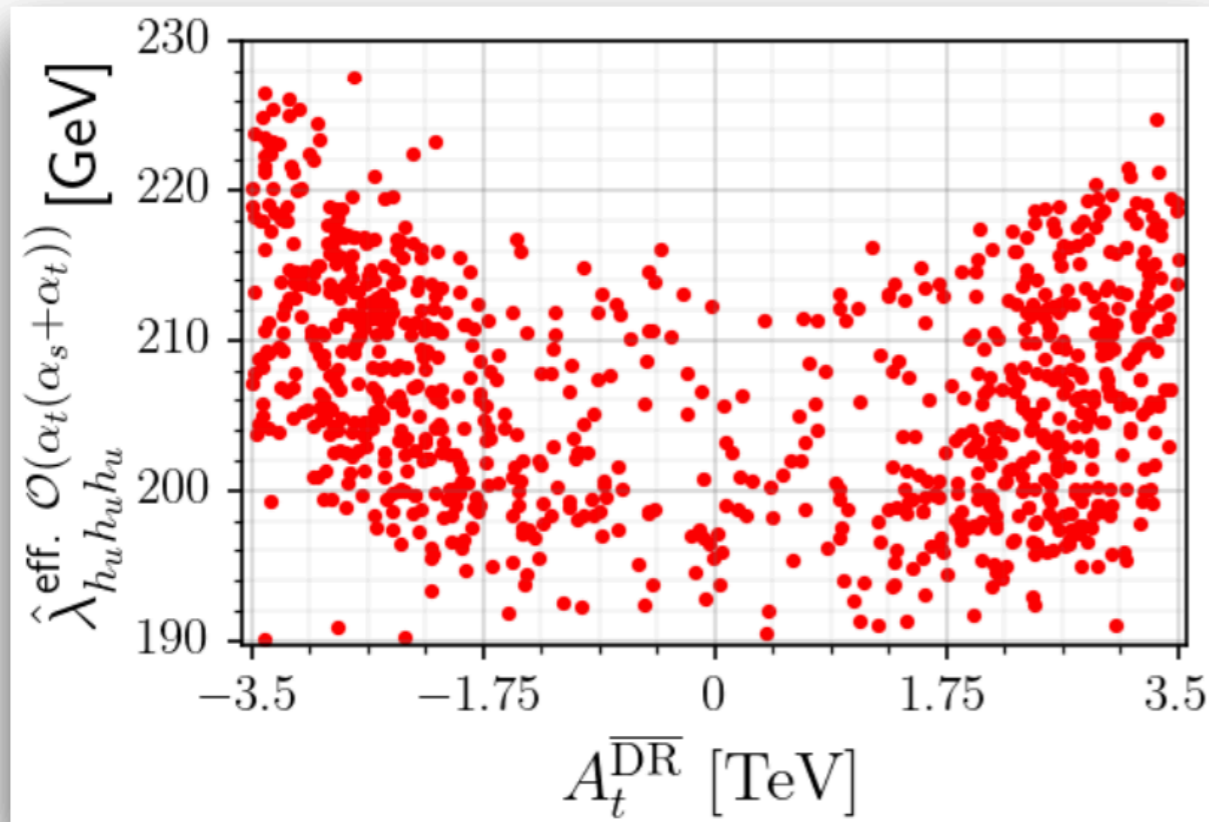
proportional to top mass  $m_t$  and soft SUSY-breaking trilinear stop mass parameter  $A_t$

- ✦ Approximations:
  - gaugeless limit  $g_1, g_2 \rightarrow 0$  (keeping  $\tan\theta_W = g_2/g_1$  fixed)
  - vanishing external momenta  $\rightarrow$  **effective coupling**

# Loop Corrected Trilinear Higgs Self-Couplings at $\mathcal{O}(\alpha_t(\alpha_s+\alpha_t))$

Corrections to  $h_u$ -like Higgs ( $\hat{=}$  SM-like Higgs)

[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]



$\hat{\lambda}_{abc}^{\text{eff}}$  : renormalized loop-corrected Higgs self-coupling at vanishing external momentum

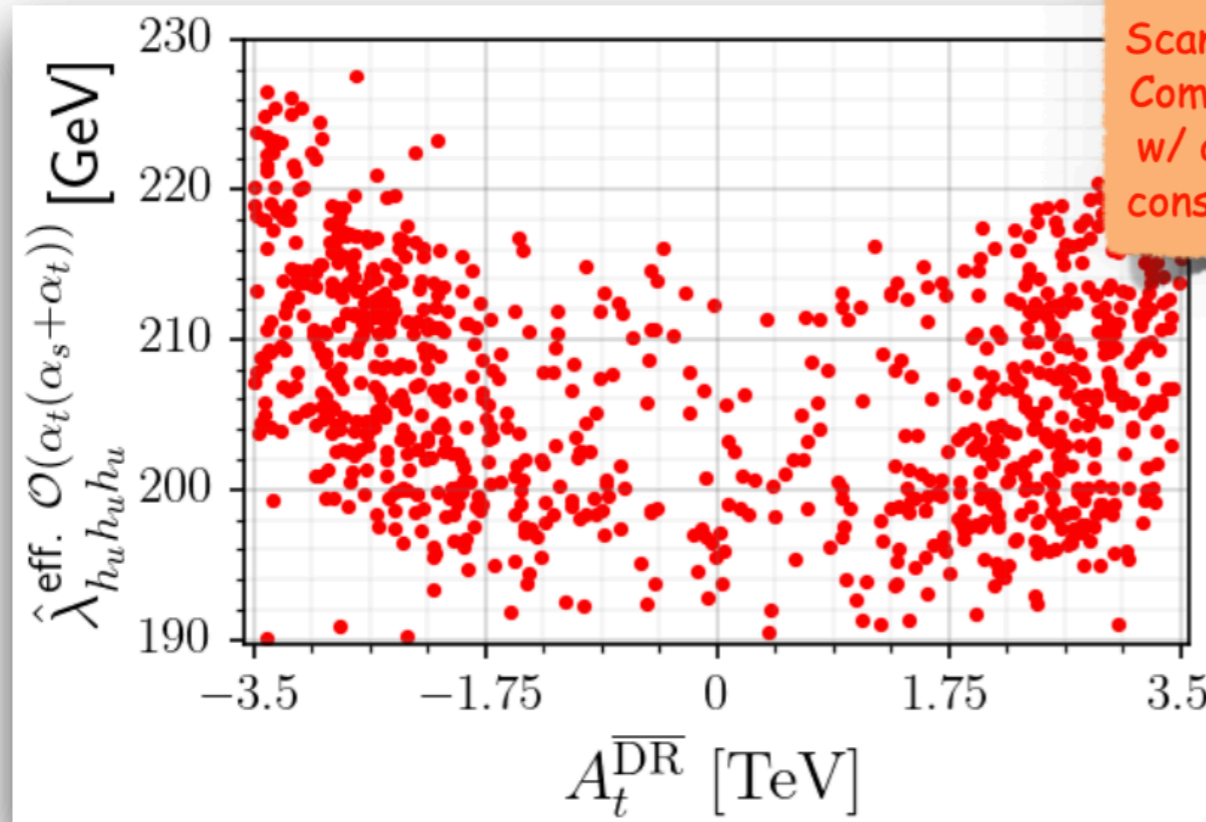
Estimate of theor. uncertainty via renorm. scheme dependence:  $\Delta_{\text{ren}} = \frac{|\lambda^{m_t(\overline{\text{DR}})} - \lambda^{m_t(\text{OS})}|}{\lambda^{m_t(\overline{\text{DR}})}}$

Results comply w/ SM value  $\lambda_{HHH}^{\text{SM}} = \frac{3M_H^2}{v} = 191 \text{ GeV}$  within theoretical uncertainty

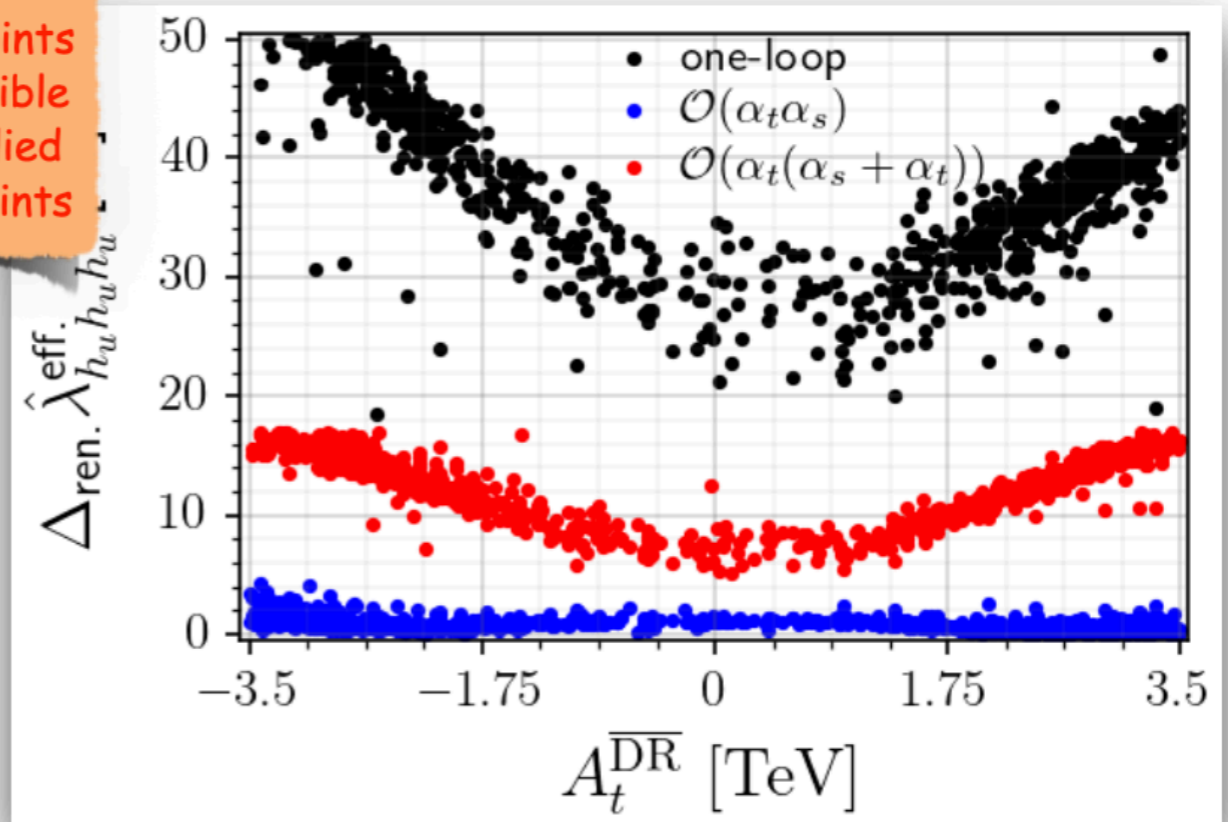
# Loop Corrected Trilinear Higgs Self-Couplings at $\mathcal{O}(\alpha_t(\alpha_s+\alpha_t))$

Corrections to  $h_u$ -like Higgs ( $\hat{=}$  SM-like Higgs)

[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]



Parameter Scan points Compatible w/ applied constraints



$\hat{\lambda}_{abc}^{\text{eff}}$  : renormalized loop-corrected Higgs self-coupling at vanishing external momentum

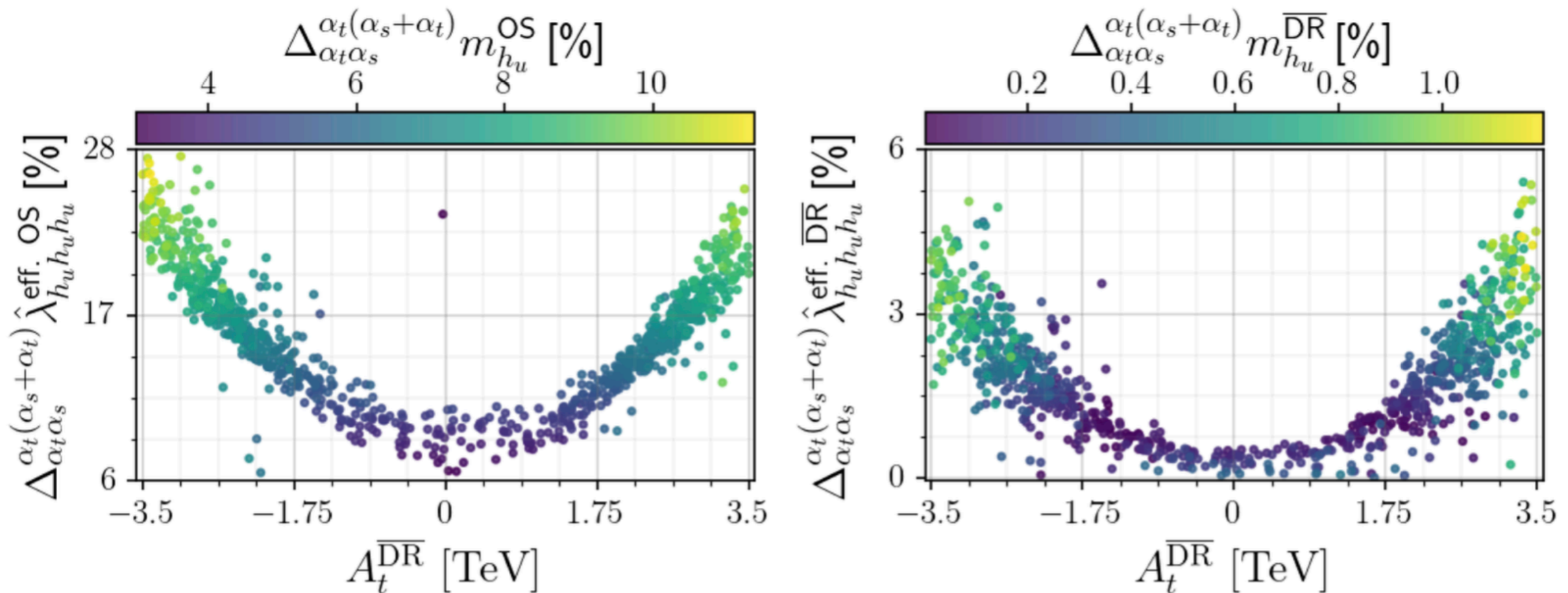
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Results comply w/ SM value  $\lambda_{HHH}^{\text{SM}} = \frac{3M_H^2}{v} = 191 \text{ GeV}$  within theoretical uncertainty

# Size of the Corrections at $\mathcal{O}(\alpha_t(\alpha_s+\alpha_t))$

## Corrections to $h_u$ -like Higgs ( $\hat{=}$ SM-like Higgs)

[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]



$$\Delta_{\alpha_i}^{\alpha_{i+1}} = \frac{|\lambda^{\alpha_{i+1}} - \lambda^{\alpha_i}|}{\lambda^{\alpha_i}}$$

- Correlation with size of mass corrections
- Smaller corrections in the DRbar than in the OS scheme due to partial resummation of higher-order terms

# Impact on Higgs Pair Production

## Benchmark Point BP10:

[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]

**Parameter Point BP10:** All complex phases are set to zero and the remaining input parameters are given by

$$\begin{aligned}
 |\lambda| &= 0.65, \quad |\kappa| = 0.65, \quad \text{Re}(A_\kappa) = -432 \text{ GeV}, \quad |\mu_{\text{eff}}| = 225 \text{ GeV}, \quad \tan \beta = 2.6, \\
 M_{H^\pm} &= 611 \text{ GeV}, \quad m_{\tilde{Q}_3} = 1304 \text{ GeV}, \quad m_{\tilde{t}_R} = 1576 \text{ GeV}, \quad m_{\tilde{X} \neq \tilde{Q}_3, \tilde{t}_R} = 3 \text{ TeV}, \\
 A_t &= 46 \text{ GeV}, \quad A_b = -1790 \text{ GeV}, \quad A_\tau = -93 \text{ GeV}, \quad A_c = 267 \text{ GeV}, \\
 A_s &= -618 \text{ GeV}, \quad A_\mu = 1851 \text{ GeV}, \quad A_u = -59 \text{ GeV}, \quad A_d = -175 \text{ GeV}, \\
 A_e &= 1600 \text{ GeV}, \quad |M_1| = 810 \text{ GeV}, \quad |M_2| = 642 \text{ GeV}, \quad M_3 = 2 \text{ TeV}. \quad (38)
 \end{aligned}$$

	$h_1 [h_u]$	$h_2 [h_s]$	$h_3 [h_d]$	$a_1 [a_s]$	$a_2 [a_d]$
tree-level	97.21	307.80	626.13	556.71	617.22
one-loop	131.46 (114.81)	299.65 (299.28)	625.96 (625.52)	543.58 (543.69)	615.82 (616.01)
two-loop $\mathcal{O}(\alpha_t \alpha_s)$	118.90 (120.36)	299.40 (299.38)	625.78 (625.58)	543.73 (543.60)	615.90 (615.96)
two-loop $\mathcal{O}(\alpha_t(\alpha_s + \alpha_t))$	123.53 (120.14)	299.44 (299.38)	625.89 (625.57)	543.73 (543.60)	615.90 (615.96)
two-loop $\mathcal{O}(\alpha_{\lambda\kappa}^2)$	122.36 (119.97)	300.27 (299.90)	625.94 (625.65)	543.34 (543.47)	615.91 (616.01)



# Impact on Higgs Pair Production

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[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]

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$$\begin{aligned}
 |\lambda| &= 0.65, \quad |\kappa| = 0.65, \quad \text{Re}(A_\kappa) = -432 \text{ GeV}, \quad |\mu_{\text{eff}}| = 225 \text{ GeV}, \quad \tan \beta = 2.6, \\
 M_{H^\pm} &= 611 \text{ GeV}, \quad m_{\tilde{Q}_3} = 1304 \text{ GeV}, \quad m_{\tilde{t}_R} = 1576 \text{ GeV}, \quad m_{\tilde{X} \neq \tilde{Q}_3, \tilde{t}_R} = 3 \text{ TeV}, \\
 A_t &= 46 \text{ GeV}, \quad A_b = -1790 \text{ GeV}, \quad A_\tau = -93 \text{ GeV}, \quad A_c = 267 \text{ GeV}, \\
 A_s &= -618 \text{ GeV}, \quad A_\mu = 1851 \text{ GeV}, \quad A_u = -59 \text{ GeV}, \quad A_d = -175 \text{ GeV}, \\
 A_e &= 1600 \text{ GeV}, \quad |M_1| = 810 \text{ GeV}, \quad |M_2| = 642 \text{ GeV}, \quad M_3 = 2 \text{ TeV}. \quad (38)
 \end{aligned}$$

	$h_1 [h_u]$	$h_2 [h_s]$	$h_3 [h_d]$	$a_1 [a_s]$	$a_2 [a_d]$
tree-level	97.21	307.80	626.13	556.71	617.22
one-loop	131.46	299.65	625.96	543.58	615.82
	(114.81)	(299.28)	(625.52)	(543.69)	(616.01)
one-loop $\mathcal{O}(\alpha_t \alpha_s)$	118.90	299.40	625.78	543.73	615.90
	(120.36)	(299.38)	(625.58)	(543.60)	(615.96)
one-loop $\mathcal{O}(\alpha_t(\alpha_s + \alpha_t))$	123.53	299.44	625.89	543.73	615.90
	(120.14)	(299.38)	(625.57)	(543.60)	(615.96)
two-loop $\mathcal{O}(\alpha_{\lambda\kappa}^2)$	122.36	300.27	625.94	543.34	615.91
	(119.97)	(299.90)	(625.65)	(543.47)	(616.01)

di-Higgs cxn dominated by resonant  $h_2$  production w/  $h_2 \rightarrow h_u h_u$

# Impact on Higgs Pair Production

[Borschensky, Dao, Gabelmann, MM, Rzehak, '22]

loop order  
mass tril.cplg.



'1L1L'	$\sigma^{\text{OS}}$ [fb]	$\sigma^{\overline{\text{DR}}}$ [fb]	$\kappa_{H_1 H_1 H_1}^{\text{OS}}$	$\kappa_{H_1 H_1 H_1}^{\overline{\text{DR}}}$	$\kappa_{H_2 H_1 H_1}^{\text{OS}}$	$\kappa_{H_2 H_1 H_1}^{\overline{\text{DR}}}$	$\Delta_{\text{ren}}\sigma$
'inp'	63.72	62.14	0.54	0.71	-0.25	-0.30	2.5%
'proc'	76.83	61.48	1.01	1.04	-0.30	-0.31	25%
'at2at2'	$\sigma^{\text{OS}}$ [fb]	$\sigma^{\overline{\text{DR}}}$ [fb]	$\kappa_{H_1 H_1 H_1}^{\text{OS}}$	$\kappa_{H_1 H_1 H_1}^{\overline{\text{DR}}}$	$\kappa_{H_2 H_1 H_1}^{\text{OS}}$	$\kappa_{H_2 H_1 H_1}^{\overline{\text{DR}}}$	$\Delta_{\text{ren}}\sigma$
'inp'	68.98	61.25	0.61	0.65	-0.27	-0.28	12.6%
'proc'	71.69	62.57	1.03	1.02	-0.30	-0.31	14.6%



- 'inp': loop-corrected masses and mixing angles (->Yukawa & trilinear couplings) in tree-level-like formula: HO corrections to input parameters
- 'proc': additionally including loop-corrected trilinear Higgs self-coupling -> HO corrections to observable included (though only partially)
- 'inp': scheme dependence of input parameters uncanceled by scheme dependence of process-dependent corrections (at the same loop order)
- 'proc': remaining large uncertainty (14.6%): remaining missing EW corrections might be important

NLO QCD Corrections to 2HDM  $hH$  and  $AA$  production



# Ultimate Test of the Higgs Mechanism

$$V(H) = \frac{1}{2} M_H^2 H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

- Higgs mass :  $M_H = \sqrt{2\lambda} v$
- trilinear Higgs self-coupling :  $\lambda_{HHH} = 3M_H^2/M_Z^2$  
- quadrilinear Higgs self-coupling :  $\lambda_{HHHH} = 3M_H^2/M_Z^4$  
- (units  $\lambda_0 = 33.8 \text{ GeV}/\lambda^2$ )

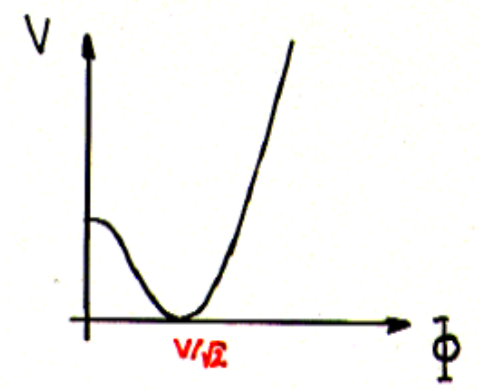
- (a) trilinear coupling : via Higgs pair production
- (b) quadrilinear coupling : via triple Higgs production

measurement of the Higgs self-couplings and reconstruction of the Higgs potential }  $\Rightarrow$  establish the scalar sector of the Higgs mechanism experimentally

$$V(\Phi) = \lambda \left( \Phi^\dagger \Phi - \frac{v}{2} \right)^2$$

$v = 246 \text{ GeV}$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \sim$$



Slides from LCWS, Fermilab, 10/2000  
 [Djouadi, Kilian, MM, Zerwas, 03/99, 04/99]

Dominant process at the LHC:  $gg \rightarrow HH$

# Higher-Order Corrections to Higgs Pair Production in Gluon Fusion

- ♦ 2-loop QCD corrections:  $\approx 70%$  [HTL,  $\mu=M_{HH}/2$ ] [Dawson,Dittmaier,Spira]
- ♦ 2-loop QCD corrections:  $\sigma = \sigma_0 + \sigma_1/m_t^2 + \dots + \sigma_4/m_t^8$   
[refinement: full LO at differential level] [Grigo,Hoff,Melnikov,Steinhauser]
- ♦ Mass effects @ NLO in real corrections:  $\sim -10%$   
[Frederix,Frixione,Hirschi,Maltoni,Mattelaer,Torrielli,Vryonidou,Zaro]
- ♦ NLO QCD w/ full top mass dependence:  $\sim 15%$  mass effects on top of LO, 20-30% for distributions [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke], [Baglio,Campanario,Glaus,MM,Ronca,Spira,Streicher]  
Combined uncertainties [Baglio,Campanario,Glaus,MM,Ronca,Spira]
- ♦ NNLO QCD corrections:  $\sim 20%$  [HTL] [de Florian,Mazzitelli; Grigo,Melnikov,Steinhauser]
- ♦ Light fermion three-loop corrections [Davies,Schönwald,Steinhauser]
- ♦ N<sup>3</sup>LO QCD corrections:  $\sim 5%$  [HTL] [Chen,Li,Shao,Wang]
- ♦ NNLO Monte Carlo: inclusion of full top-mass effects @ NLO [partly at NNLO]  
[Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli]
- ♦ NLO: matching to parton showers [Heinrich,Jones,Kerner,Luisoni,Vryonidou]

# Higher-Order Corrections to Higgs Pair Production in Gluon Fusion

## ♦ New expansion/extrapolation methods:

(i)  $1/m_+^2$  expansion + conformal mapping + Padé approximants

[Gröber, Maier, Rauh]

(ii)  $p_T^2$  expansion

[Bonciani, Degassi, Giardino, Gröber]

## ♦ NLO: small mass expansion [ $Q^2 \gg m_+^2$ ]

[Davies, Mishima, Steinhauser, Wellmann]

## ♦ Combination of full NLO and small mass expansion

[Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann]

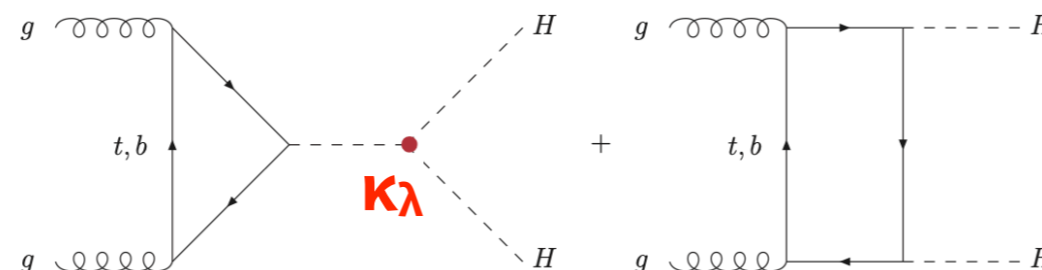
Complete list, see e.g. twiki of LHC Higgs Working Subgroup HH and recent reviews

-> recommendations for cross sections to be used given for

- different c.m. energies

- different coupling modifiers  $\kappa_\lambda$

-> uncertainties on di-Higgs cross sections



# Higher-Order Corrections in the CP-Conserving 2HDM

[Lee,'73], [Branco eal,'11]

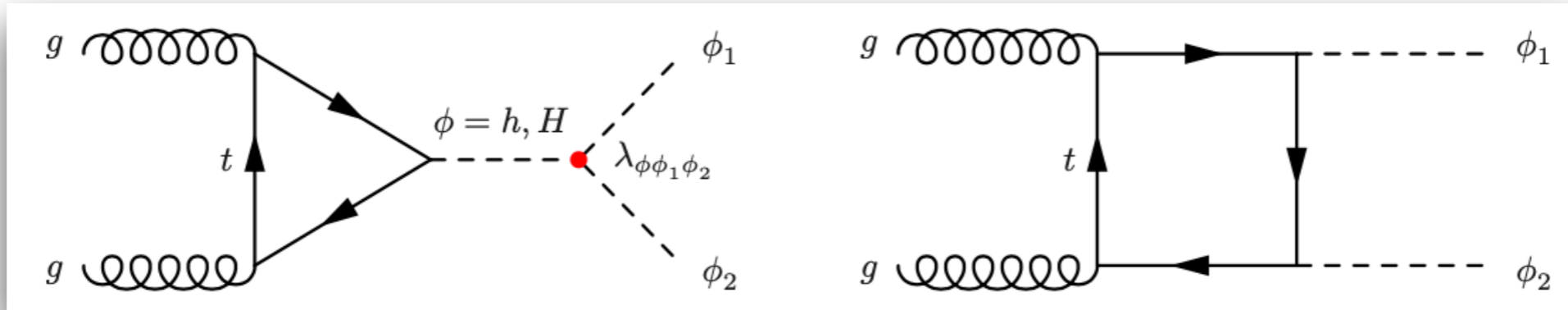
♦ 2HDM Higgs potential w/ softly broken  $\mathbb{Z}_2$  symmetry:

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] .$$

♦ Higgs spectrum after EWSB: 2 CP-even  $h, H$  with  $m_h < m_H$ ,  
1 CP-odd  $A$ ,  
charged Higgs pair  $H^\pm$

# Gluon Fusion into $\phi_1\phi_2$ with $\phi_1\phi_2=hH, AA$

♦ Contributing diagrams at leading order:



♦ 2HDM type 1 benchmark point (compatible w/ theor. & exp. constraints):

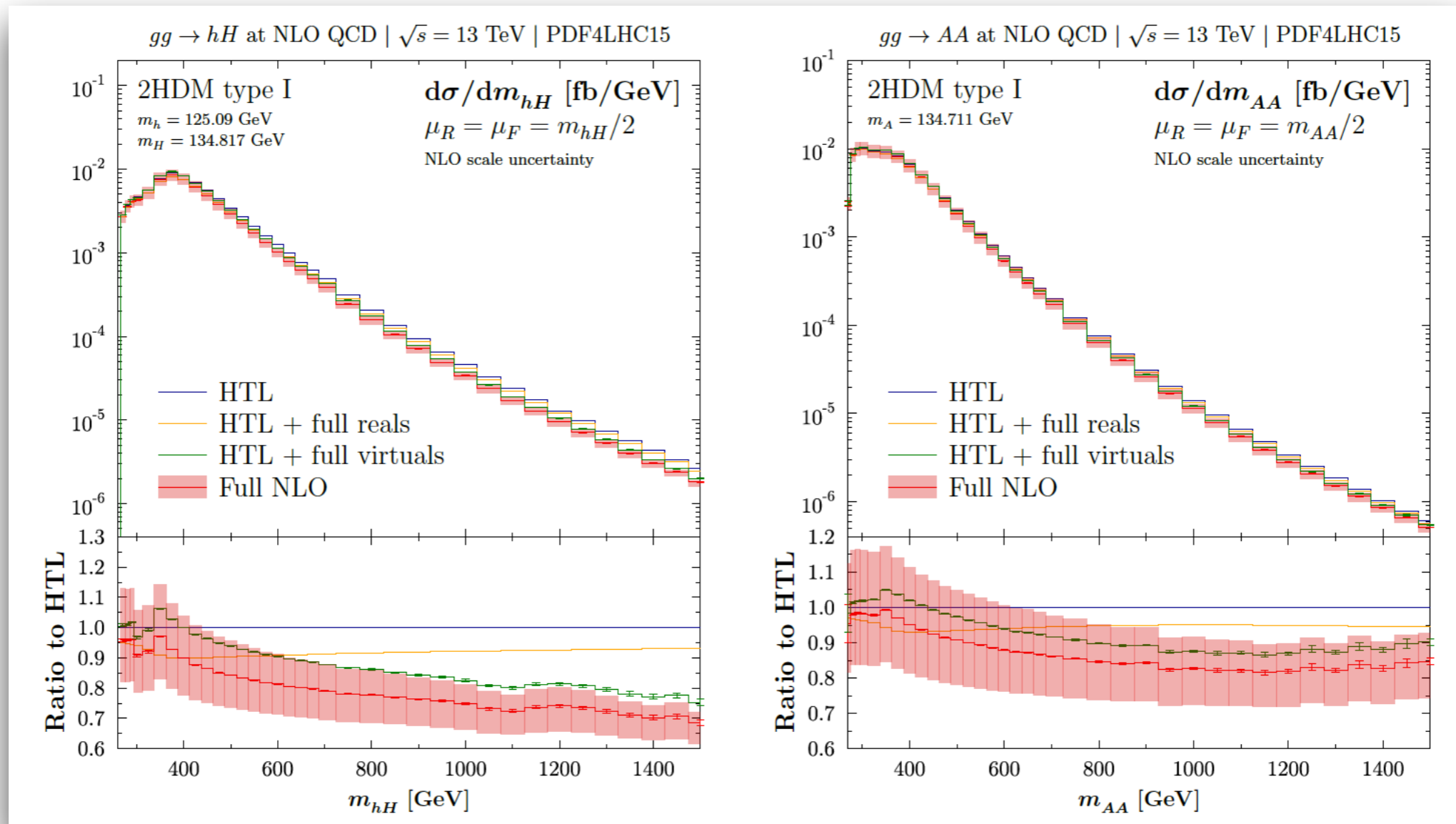
[taken from Abouabid et al.,'22]

$$\begin{aligned} m_h &= 125.09 \text{ GeV}, & m_H &= 134.817 \text{ GeV}, \\ m_A &= 134.711 \text{ GeV}, & m_{H^\pm} &= 161.5 \text{ GeV}, \\ m_{12}^2 &= 4305 \text{ GeV}^2, & \alpha &= -0.102, \\ \tan \beta &= 3.759, & v &= 246.22 \text{ GeV}. \end{aligned}$$



# NLO Top Mass Effects in Invariant Mass Distributions

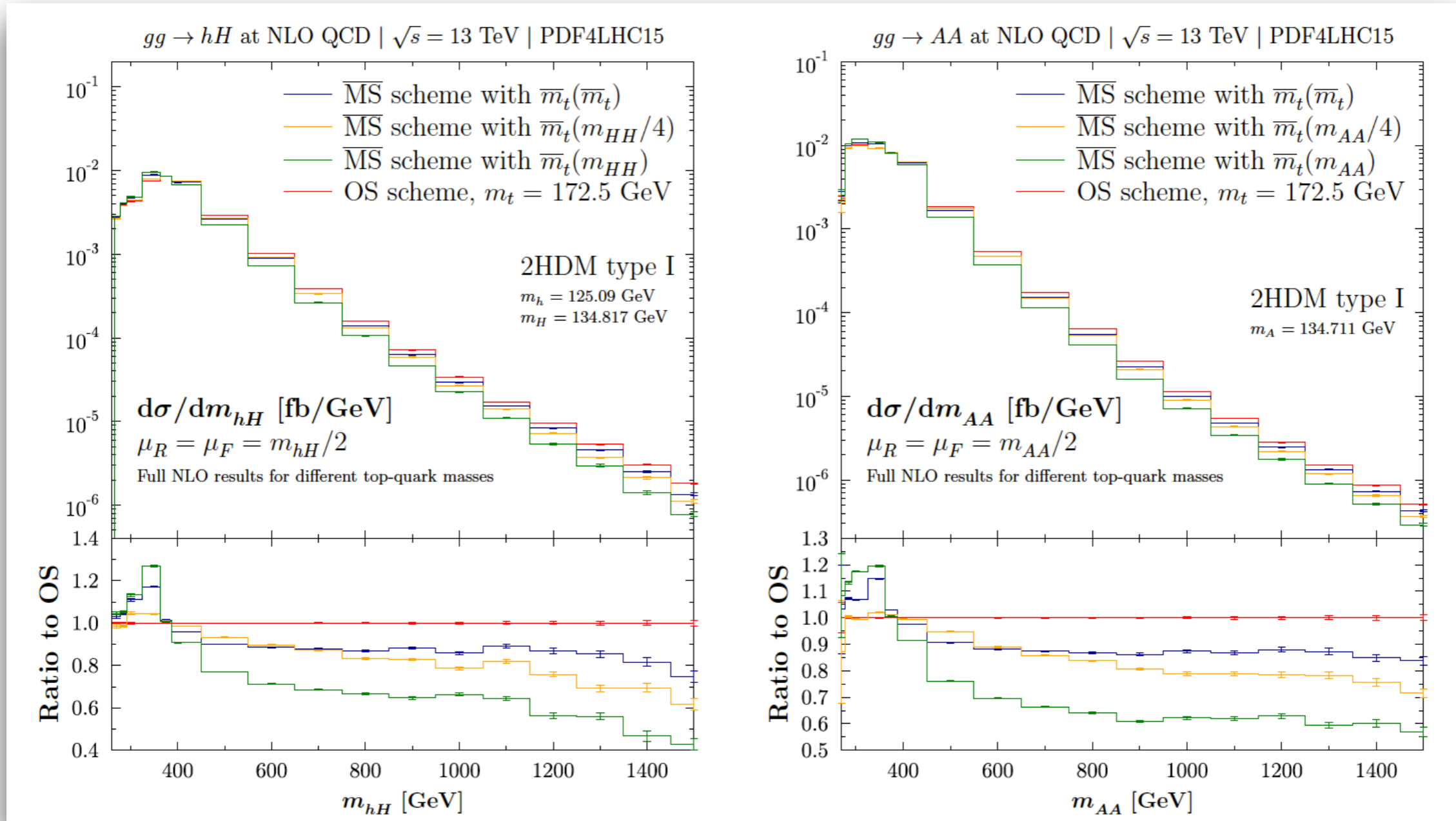
[Baglio,Campanario,Glaus,MM,Ronca,Spira,'23]



- Mass effects in distributions: -30% (-15%) at Q~1.5 TeV for hH (AA)
- increases w/ c.m. energy (results provided for 14, 27, 100 TeV)
- Mass effects on total cxn: -12% (-5%) at 13 TeV (increases w/ c.m. energy)

# Top Quark Scale and Scheme Uncertainties

[Baglio,Campanario,Glaus,MM,Ronca,Spira,'23]



# Top Quark Scale and Scheme Uncertainties in Total Cross Section

[Baglio,Campanario,Glaus,MM,Ronca,Spira,'23]

$$\begin{aligned} 13 \text{ TeV} : \quad & \sigma_{gg \rightarrow hH} = 1.592(1)^{+6\%}_{-11\%} \text{ fb}, \\ 14 \text{ TeV} : \quad & \sigma_{gg \rightarrow hH} = 1.876(1)^{+6\%}_{-11\%} \text{ fb}, \\ 27 \text{ TeV} : \quad & \sigma_{gg \rightarrow hH} = 7.036(4)^{+5\%}_{-12\%} \text{ fb}, \\ 100 \text{ TeV} : \quad & \sigma_{gg \rightarrow hH} = 60.49(4)^{+4\%}_{-14\%} \text{ fb}, \end{aligned}$$

$$\begin{aligned} 13 \text{ TeV} : \quad & \sigma_{gg \rightarrow AA} = 1.643(1)^{+9\%}_{-7\%} \text{ fb}, \\ 14 \text{ TeV} : \quad & \sigma_{gg \rightarrow AA} = 1.927(1)^{+9\%}_{-8\%} \text{ fb}, \\ 27 \text{ TeV} : \quad & \sigma_{gg \rightarrow AA} = 7.012(4)^{+8\%}_{-8\%} \text{ fb}, \\ 100 \text{ TeV} : \quad & \sigma_{gg \rightarrow AA} = 58.12(3)^{+7\%}_{-9\%} \text{ fb}. \end{aligned}$$

Top-Yukawa induced EW corrections to SM HH



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# Electroweak Corrections to SM Higgs Pair Production

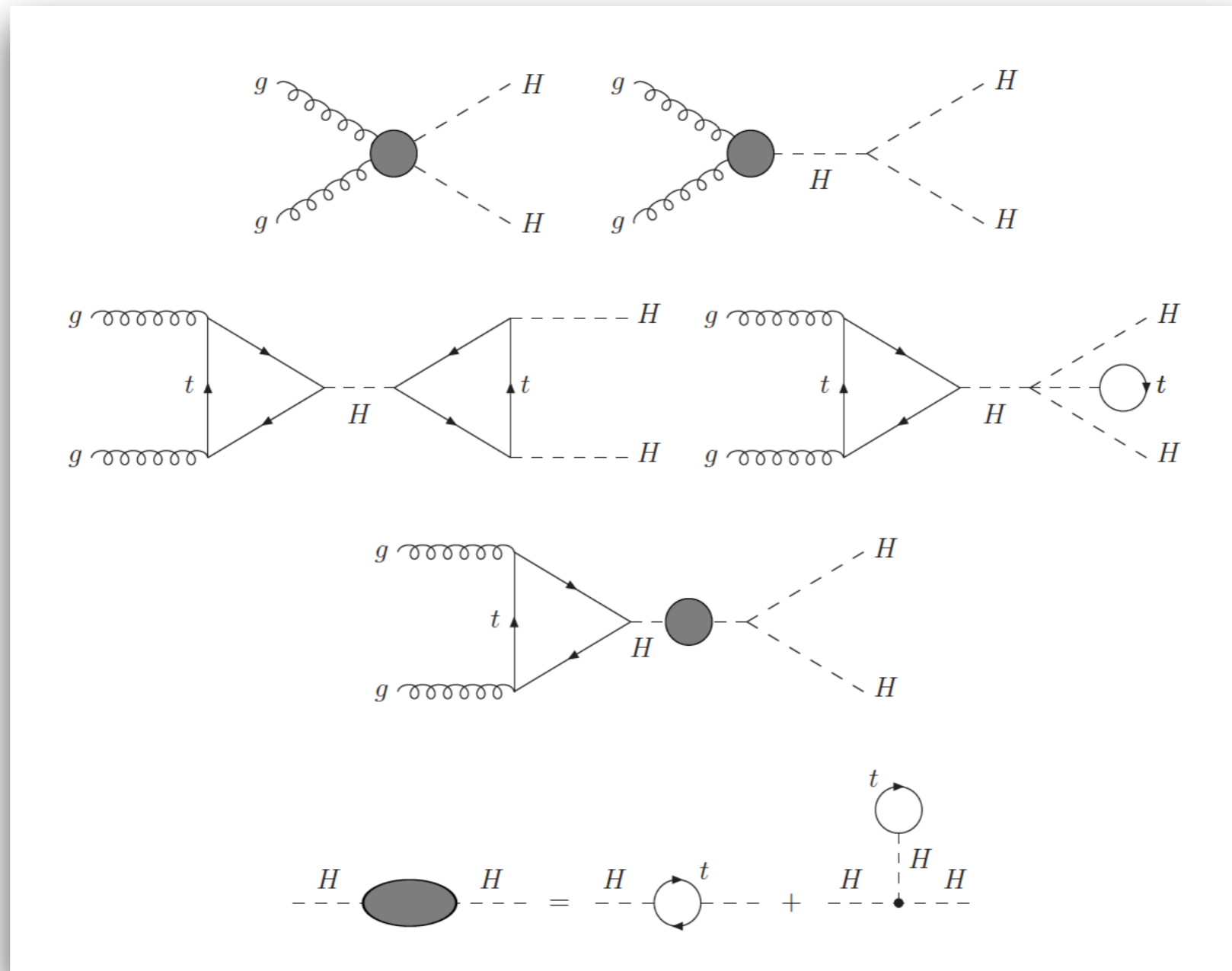
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See also:

- Next-to-leading order electroweak correction to  $gg \rightarrow HH$  and  $gg \rightarrow gH$  in the large  $m_t$ -limit  
[Davies, Schönwald, Steinhauser, Zhang, '23]
- Higgs boson contribution to the leading two-loop Yukawa corrections to  $gg \rightarrow HH$   
[Davies, Mishima, Schönwald, Steinhauser, Zhang, '22]
- Complete NLO EW corrections [Bi, Huang, Huang, Ma, Yu, '23]
- NLO Yukawa and self-coupling corrections to  $gg \rightarrow HH$  (DPG Spring Conference, '24)  
[Heinrich, Jones, Kerner, Stone, Vestner]

# Top-Yukawa-Induced Corrections to Higgs Pair Production

- ♦ Part of the electroweak corrections to Higgs pair production
- ♦ Full top-mass dependence in the triple Higgs vertex and self-energy corrections  
HTL in radiative corrections to the effective  $ggH$  and  $ggHH$  vertices (b-loops neglected)

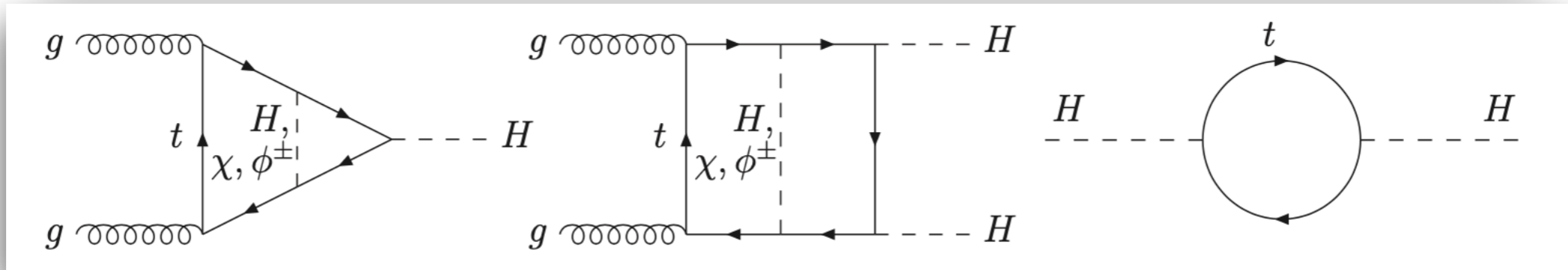


# Effective Lagrangians

- Effective  $ggH$  and  $ggHH$  vertices (top-Yukawa induced EW corrections in HTL):

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G_{\mu\nu}^a \left\{ (1 + \delta_1) \frac{H}{v} + (1 + \eta_1) \frac{H^2}{2v^2} + \mathcal{O}(H^3) \right\}$$

$$\delta_1 = \frac{x_t}{2} + \mathcal{O}(x_t^2) \quad \eta_1 = 4x_t + \mathcal{O}(x_t^2) \quad x_t = \frac{m_t^2}{(4\pi)^2 v^2}$$



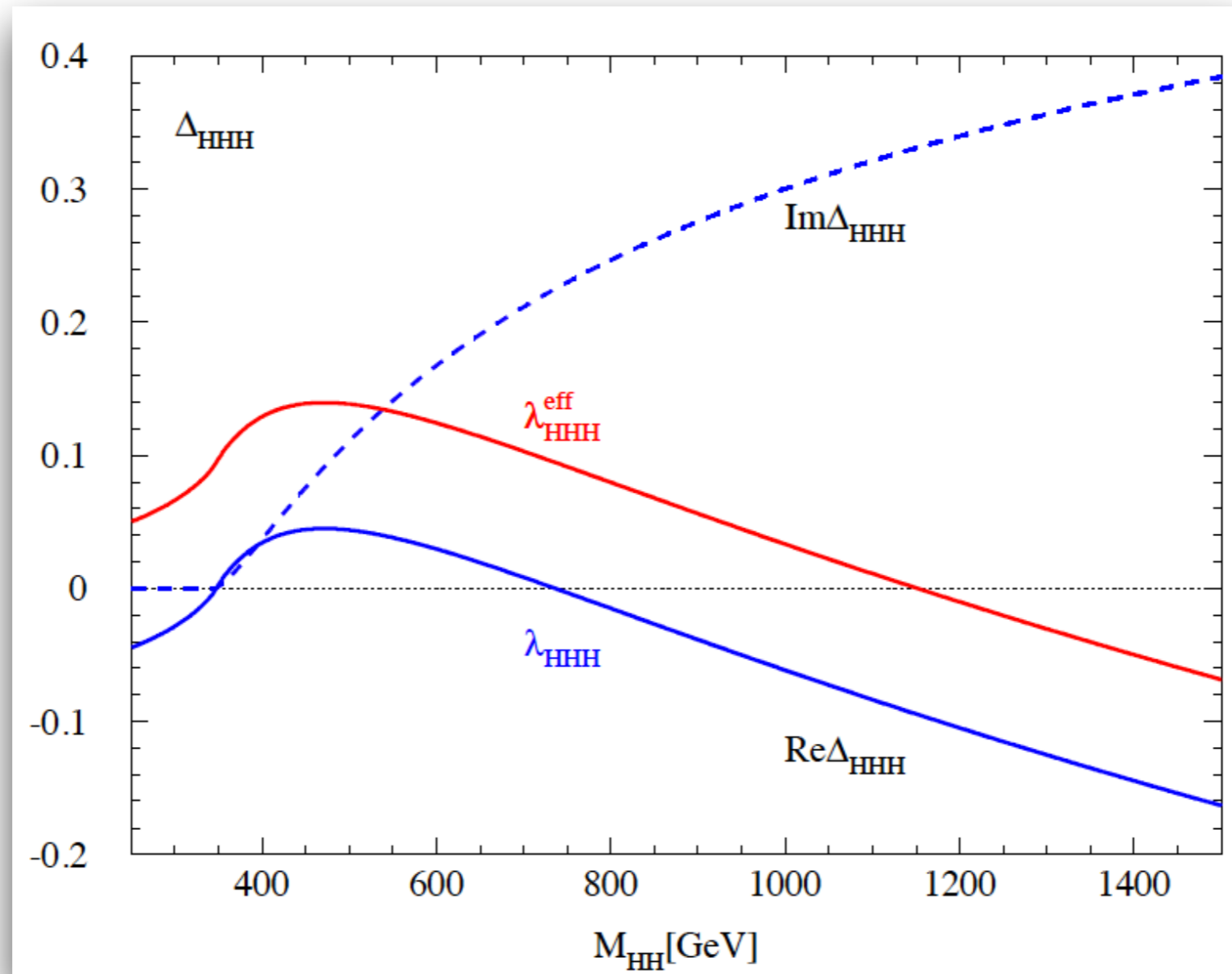
- Effective Higgs self-couplings: from effective Higgs potential

$$\lambda_{HHH}^{eff} = 3 \frac{M_H^2}{v} - \frac{3m_t^4}{\pi^2 v^3} \approx 0.91 \times 3 \frac{M_H^2}{v} \quad \Delta\lambda_{HHH} = -\frac{3m_t^4}{\pi^2 v^3}$$

$$\lambda_{HHHH}^{eff} = 3 \frac{M_H^2}{v^2} + \Delta\lambda_{HHHH} \quad \Delta\lambda_{HHHH} = -\frac{12m_t^4}{\pi^2 v^4}$$

# Relative Top-Yukawa-Induced EW Correction Factor $\Delta_{HHH}$

[MM,Schlenk,Spira,'22]

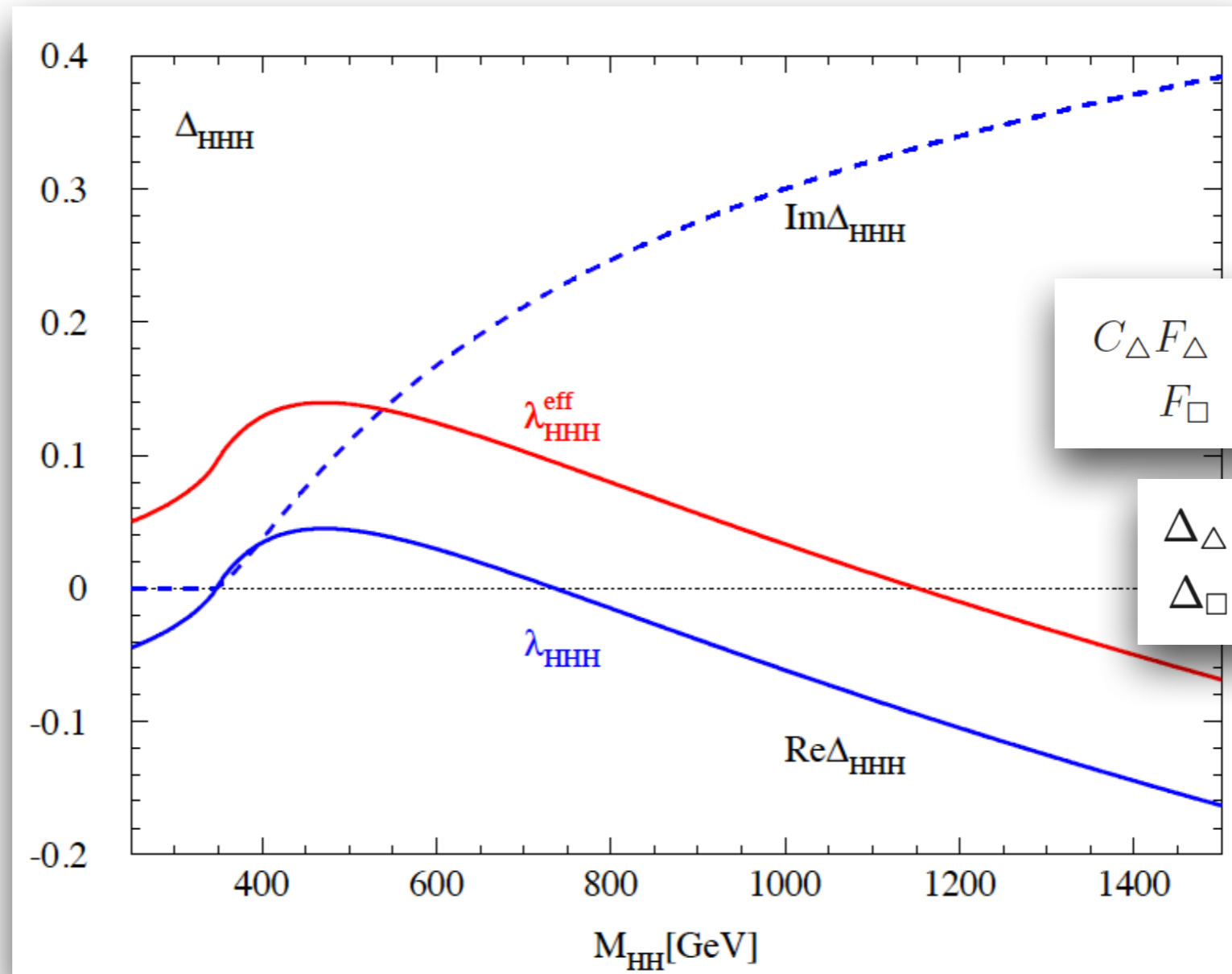


Effective trilinear coupling does not capture the bulk of the EW corrections



# Relative Top-Yukawa-Induced EW Correction Factor $\Delta_{HHH}$

[MM,Schlenk,Spira,'22]



$$C_{\Delta}F_{\Delta} \rightarrow C_{\Delta}F_{\Delta}(1 + \Delta_{\Delta})$$

$$F_{\square} \rightarrow F_{\square}(1 + \Delta_{\square}),$$

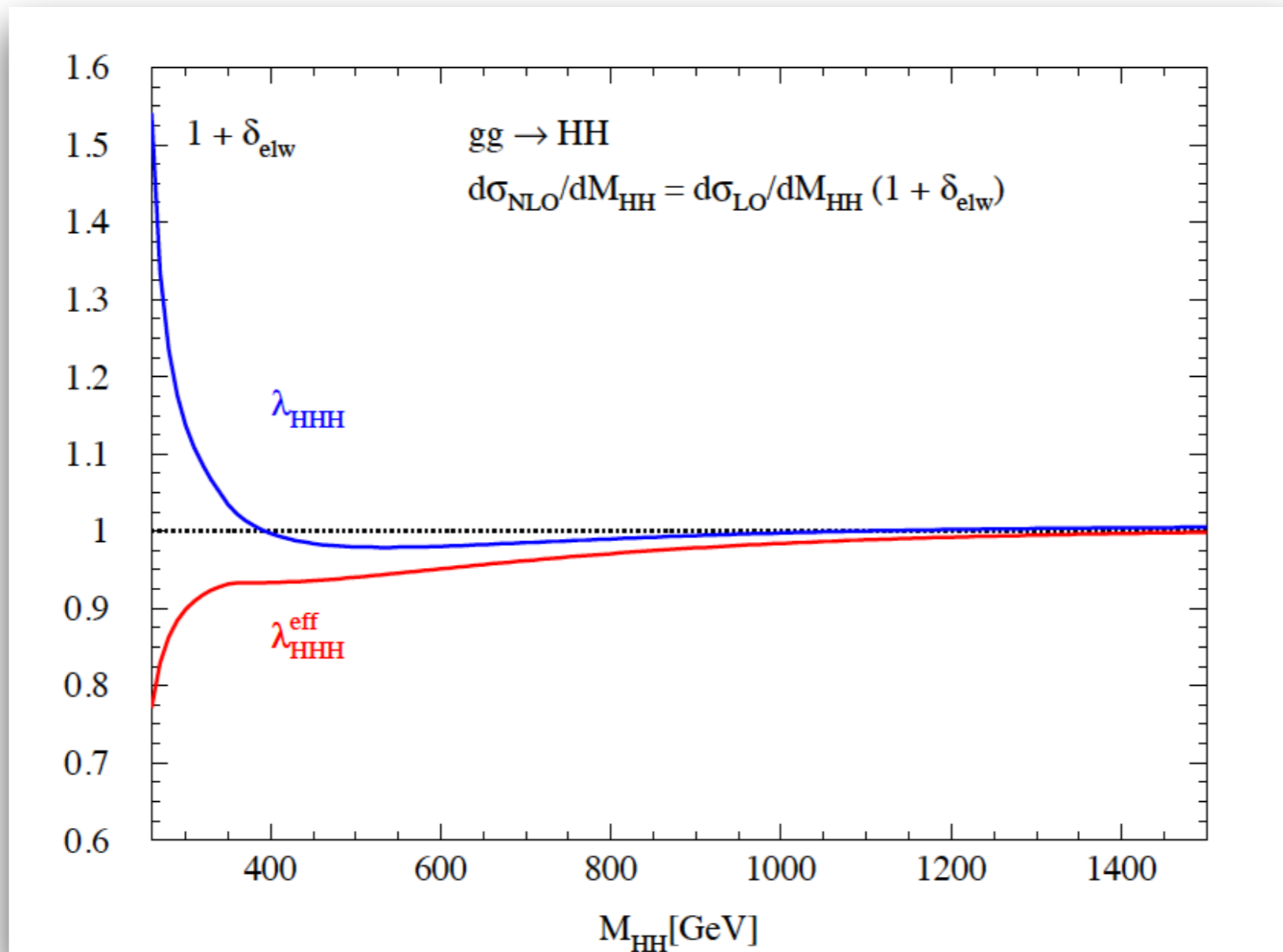
$$\Delta_{\Delta} = \delta_1 + \Delta_{HHH}$$

$$\Delta_{\square} = \eta_1,$$

Effective trilinear coupling does not capture the bulk of the EW corrections

# Relative Top-Yukawa-Induced EW Correction to differential HH prod

[MM,Schlenk,Spira,'22]



- Large enhancement near threshold because of vanishing LO matrix element
- Suppression is lifted by mismatch of EW corrections to triangle and box diagrams

# Effect of Top-Yukawa-Induced EW Corrections on Total Cxn

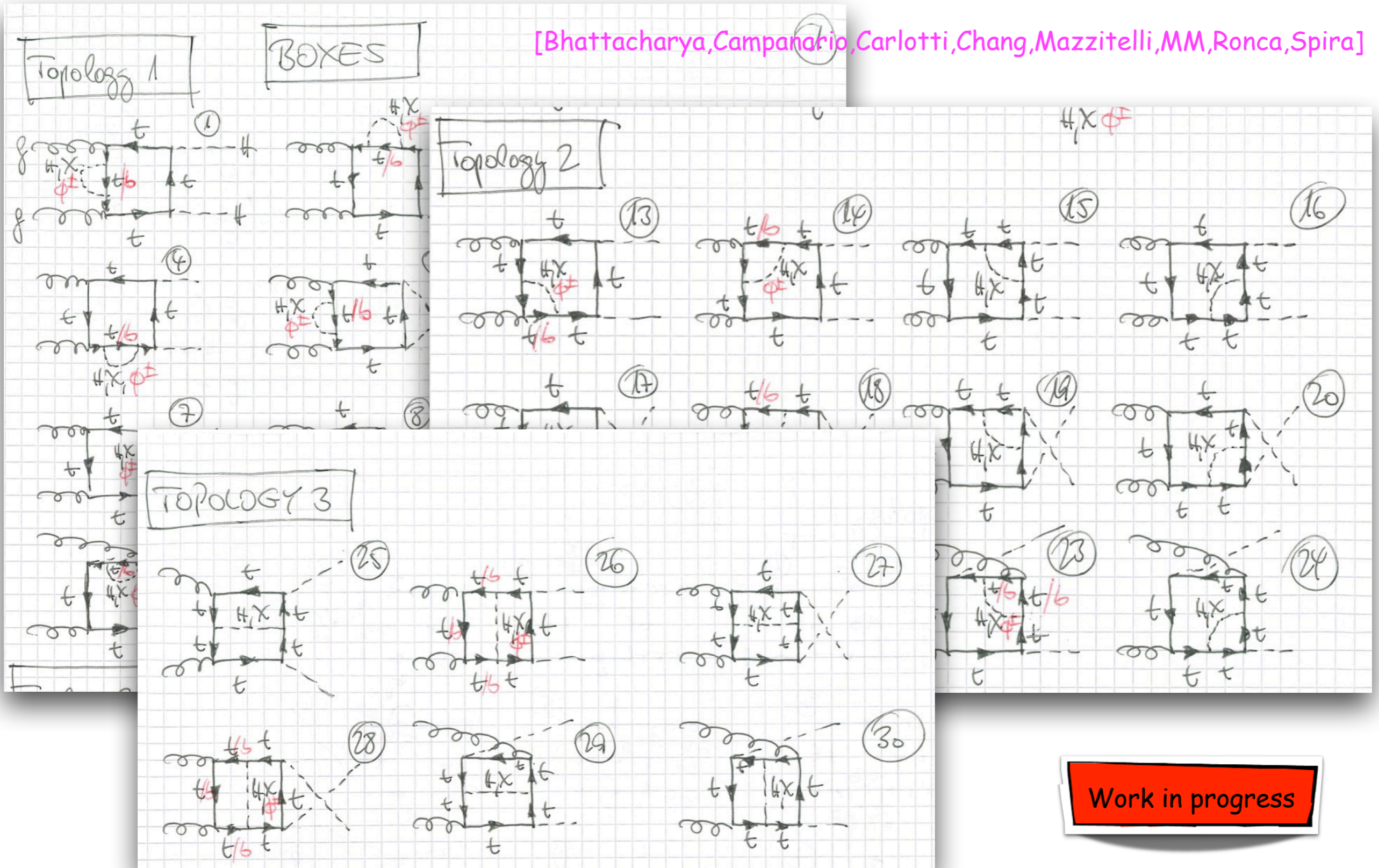
† Effect of top-Yukawa-induced EW correction on total integrated hadronic cross section:

$$\begin{aligned}\sigma &= K_{elw} \times \sigma_{LO} \\ K_{elw} &\approx 1.002 \quad (\lambda_{HHH}) \\ K_{elw}^{eff} &\approx 0.938 \quad (\lambda_{HHH}^{eff})\end{aligned}$$

- Corrections induce an effect of about 0.2%
- Bulk of corrections cannot be absorbed in the effective trilinear Higgs coupling (leads to an artificial increase of the relative EW corrections)
- ~> Inclusion of complete EW corrections is mandatory

# Top-Yukawa-Induced EW Corrections (w/ $m_{\text{top}}$ & $m_{\text{bottom}}$ dependence, gaugeless limit)

[Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, MM, Ronca, Spira]



# Top-Yukawa-Induced EW Corrections (w/ $m_{\text{top}}$ & $m_{\text{bottom}}$ dependence, gaugeless limit)

[Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, MM, Ronca, Spira]

Topology 1

BOXES

- diagrammatic approach, no reduction to master integrals
- UV divergences extracted using endpoint subtraction
- numerical instabilities: integration by parts, Richardson extrapolation

Topology 2

Topology 3


Work in progress

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## Further Work within A3a and A3b

---

- A global view of the EDM landscape [Degenkolb,Elmer,Modak,MM,Plehn,'24]
- Impact of new experimental data on the C2HDM: the strong interdependence between LHC Higgs data and the electron EDM [Biekötter,Fontes,MM,Romao,Santos,Silva,'24]
- Dark colored scalars impact on single and di-Higgs production at the LHC [Gabriel,MM,Neacsu,Santos,'23]
- Intermediate charge-breaking phases and symmetry non-restoration in the 2HDM [Aoki,Biermann,Borschensky,Ivanov,MM,Shibuya,'23]
- The  $\mathcal{O}(\alpha_t+\alpha_\lambda+\alpha_\kappa)^2$  correction to the  $\varrho$  parameter and its effect on the W boson mass [Dao,Gabelmann,MM,'23]
- Charged Higgs-boson decays into quarks [Chang,Kirk,MM,Spira,'23]
- Leptonic anomalous magnetic and electric dipole moments in the CP-violating NMSSM with and without inverse seesaw mechanism [Dao,Le,MM,'22]
- Pseudoscalar Higgs production at NLO SUSY QCD [Bagnaschi,Fritz,Liebler,MM,Nguyen,Spira,'22]
- Electroweak phase transition in a dark sector with CP violation [Biermann,MM,Müller,'22]

A top-down view of a gold-colored metal chocolate mold tray filled with various chocolates. The chocolates are in different shapes: round, teardrop, and square. Some are plain dark chocolate, while others have white fillings or decorative patterns. The lighting is warm, highlighting the metallic sheen of the mold and the smooth texture of the chocolate.

*Thank you for  
your attention!*

---

## Considered Constraints in the NMSSM

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- SM-like Higgs mass  $m_h \in [122, 129] \text{ GeV}$   
at  $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2 + \alpha_t \alpha_s)$  in the default mixed  $\overline{\text{DR}}$ -OS scheme  
with OS renormalisation in the top/stop and charged Higgs boson sectors
  - Compatibility w/ Higgs data and BSM Higgs searches
  - $\lambda, \kappa$  required to be below 0.7 (ensure roughly perturbativity below the GUT scale)
  - Neglected points with
    - (i)  $m_{\chi_i^{(\pm)}}, m_{h_i} > 1 \text{ TeV}, m_{\tilde{t}_2} > 2 \text{ TeV}$
    - (ii)  $m_{h_i} - m_{h_j} < 0.1 \text{ GeV}, m_{\chi_i^{(\pm)}} - m_{\chi_j^{(\pm)}} < 0.1 \text{ GeV}$
    - (iii)  $m_{\chi_1^\pm} < 94 \text{ GeV}, m_{\tilde{t}_1} < 1 \text{ TeV} .$
-



# LO Cross Section

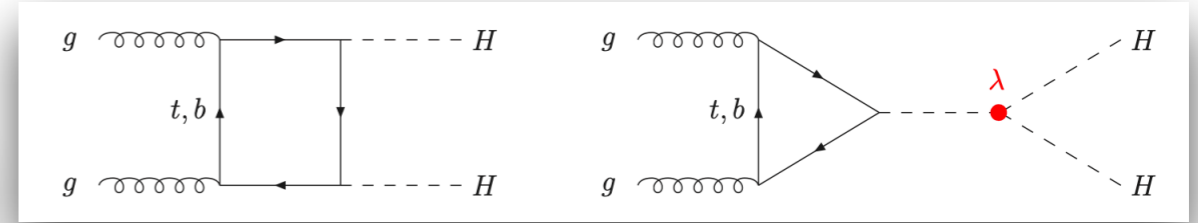
♦ The LO cross section: 2 form factors

$$\mathcal{M}(g^a g^b \rightarrow HH) = -i \frac{G_F \alpha_s(\mu_R) Q^2}{2\sqrt{2}\pi} \mathcal{A}^{\mu\nu} \epsilon_{1\mu} \epsilon_{2\nu} \delta_{ab}$$

with  $\mathcal{A}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$ ,

$$F_1 = C_\Delta F_\Delta + F_\square, \quad F_2 = G_\square$$

$$C_\Delta = \frac{\lambda_{H^3\nu}}{Q^2 - M_H^2 + iM_H\Gamma_H}$$



Heavy top limit:

$$F_\Delta \rightarrow \frac{2}{3}, \quad F_\square \rightarrow -\frac{2}{3}, \quad G_\square \rightarrow 0$$

Hadronic cross section

$$\sigma_{LO} = \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{LO}(Q^2 = \tau s)$$

Partonic cross section

$$\hat{\sigma}_{LO} = \frac{G_F^2 \alpha_s^2(\mu_R)}{512(2\pi)^3} \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} \left[ |F_1|^2 + |F_2|^2 \right]$$

$$\hat{t}_\pm = -\frac{1}{2} \left[ Q^2 - 2M_H^2 \mp Q^2 \sqrt{1 - 4\frac{M_H^2}{Q^2}} \right]$$

Gluon luminosity

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x, \mu_F) g\left(\frac{\tau}{x}, \mu_F\right)$$

# NLO Cross Section

♦ The NLO cross section:

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$C$  and  $d_{ij}$  depend on  $Q^2$

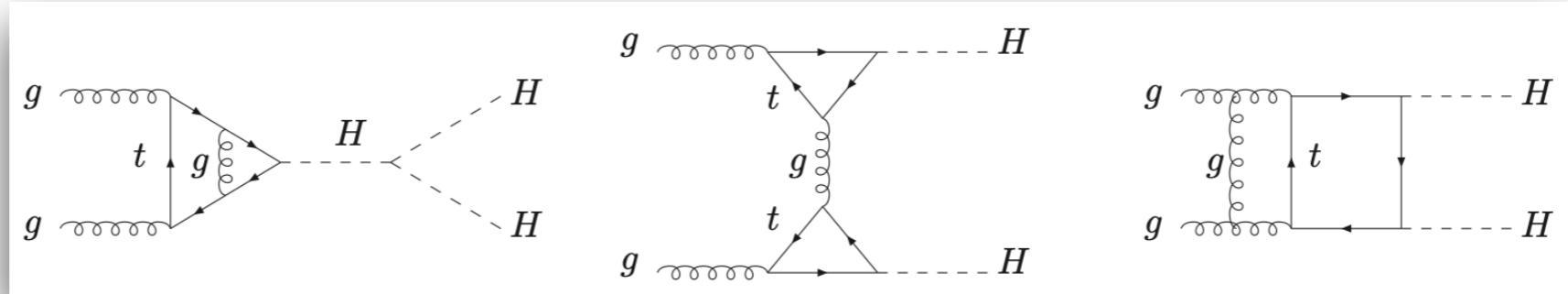
$$\begin{aligned} \sigma_{\text{LO}} &= \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\ \Delta\sigma_{\text{virt}} &= \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) C \\ \Delta\sigma_{gg} &= \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{\mu_F^2}{\tau s} \right. \\ &\quad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left( \frac{\log(1-z)}{1-z} \right)_+ \right\} \\ \Delta\sigma_{gq} &= \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{\mu_F^2}{\tau s(1-z)^2} + d_{gq}(z) \right\} \\ \Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu_R)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) d_{q\bar{q}}(z) \end{aligned}$$

♦ HTL:

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta\Delta}, \quad d_{gg} \rightarrow -\frac{11}{2}(1-z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1-z)^3$$

# Virtual Corrections

- ♦ **Contributing diagrams:** 47 generic box diagrams, 8 triangle diagrams (← single Higgs), 1 PR (←  $H \rightarrow Z\gamma$ )



- ♦ **Full diagram w/o tensor reduction** → 6-dim. Feynman integral (for 2 form factors)
- ♦ **UV singularities:** → endpoint subtractions

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- ♦ **IR singularities:** IR subtraction (based on structure of integr. and rel. to HTL)
- ♦ **Thresholds:**  $Q^2 \geq 0, 4m_t^2 \rightarrow$  IBP → reduction of power of denominator

$$[m_t^2 \rightarrow m_t^2(1 - ih)]$$

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a+b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a+bx)^2}$$

## Further Computational Details

- ♦ **Renormalization:**  $\alpha_S$ : MSbar, 5 flavors,  $m_t$ : on-shell
- ♦ **Phase space integration**  $\rightarrow$  7-dim. integrals for  $d\sigma/dQ^2$
- ♦ **Subtraction of HTL**  $\rightarrow$  IR-finite mass effects [adding back HTL results  $\leftarrow$  HPAIR]
- ♦ **Extrapolation to NWA ( $h \rightarrow 0$ ):** Richardson extrapolation

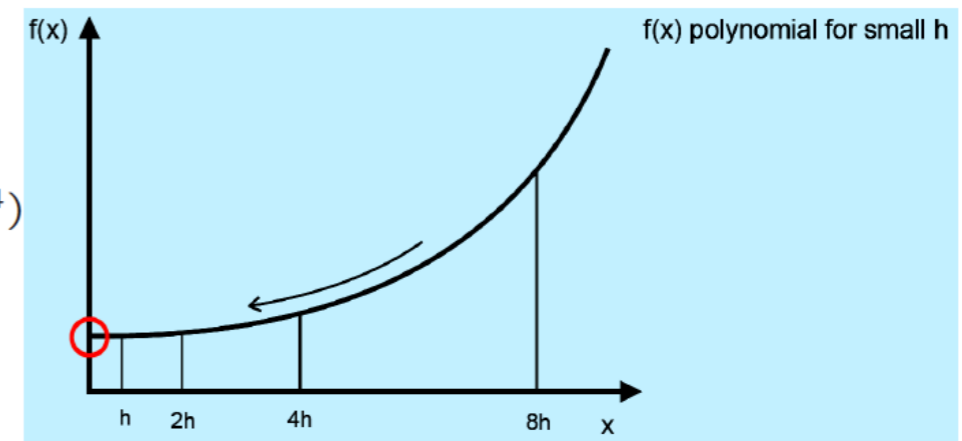
$$M_2 = 2f(h) - f(2h) = f(0) + \mathcal{O}(h^2)$$

$$M_4 = \{8f(h) - 6f(2h) + f(4h)\}/3 = f(0) + \mathcal{O}(h^3)$$

$$M_8 = \{64f(h) - 56f(2h) + 14f(4h) - f(8h)\}/21 = f(0) + \mathcal{O}(h^4)$$

etc.

$$[h \geq 0.025]$$



# Real Corrections

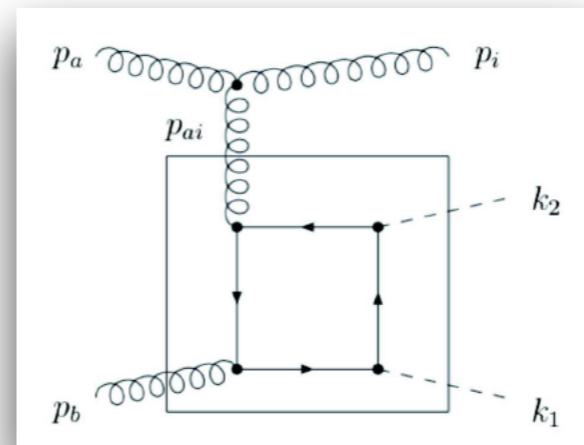
- ♦ **Full matrix element:** generated with FeynArts and FormCalc
- ♦ Matrix elements in HTL involving full LO sub-matrix elements subtracted  
→ IR-, COLL-finite [adding back HTL results ← HPAIR]

$$\sum \overline{|\mathcal{M}_{gg}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{24\pi^2 \alpha_s}{Q^4 \pi} \left\{ \frac{s^4 + t^4 + u^4 + Q^8}{stu} - 4 \frac{\epsilon}{1-\epsilon} Q^2 \right\}$$

$$\sum \overline{|\mathcal{M}_{gq}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{32\pi^2 \alpha_s}{3Q^4 \pi} \left\{ \frac{s^2 + u^2}{-t} + \epsilon \frac{(s+u)^2}{t} \right\}$$

$$\sum \overline{|\mathcal{M}_{q\bar{q}}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{256\pi^2 \alpha_s}{9Q^4 \pi} (1-\epsilon) \left\{ \frac{t^2 + u^2}{s} - \epsilon \frac{(t+u)^2}{s} \right\}$$

- ♦ **PDFs:** MSbar, 5 flavors



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## Results

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	PDF4LHC15	MMHT2014
$\sigma_{LO}$	19.80 fb	23.75 fb
$\sigma_{NLO}^{HTL}$	38.66 fb	39.34 fb
$\sigma_{NLO}$	32.78(7) fb	33.33(7) fb

# Conversion from Pole to MSbar Mass

$$F_i = F_{i,LO} + \Delta F_i = F_{i,LO} + \Delta F_{1,HTL} + \Delta F_{1,Mass}$$

Pole mass:  $F_{1,LO} = 4 m_b^2 / \hat{s}$

Conversion pole mass  $\rightarrow$  MS mass at  $\mu_b$ :  $m_t \rightarrow \bar{m}_t(\mu_b) \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{4}{3} + \log \frac{\mu_b^2}{\bar{m}_t^2} \right) \right]$

$$F_{1,LO} + \Delta F_{1,HTL}$$

$$\rightarrow \frac{4 \bar{m}_t^2(\mu_b)}{\hat{s}} \left[ 1 + 2 \cdot \frac{\alpha_s}{\pi} \cdot \frac{4}{3} + 2 \frac{\alpha_s}{\pi} \log \frac{\mu_b^2}{\bar{m}_t^2} \right]$$

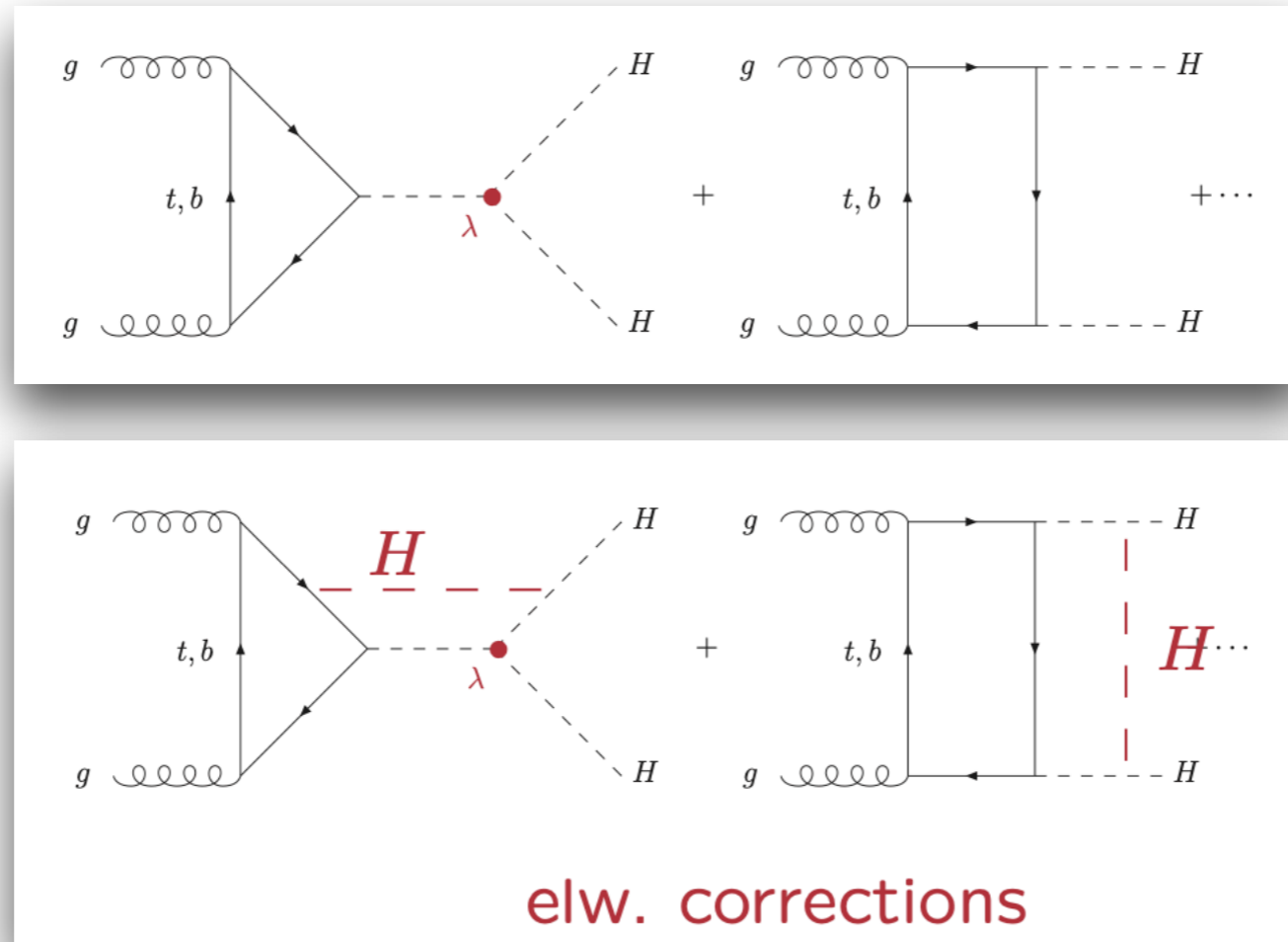
$$+ \frac{4 \bar{m}_t^2(\mu_b)}{\hat{s}} \cdot 2 \cdot \frac{\alpha_s}{\pi} \log \frac{\mu_b^2}{\bar{m}_t^2}$$

$$= \frac{4 \bar{m}_t^2(\mu_b)}{\hat{s}} \left[ 1 + 2 \frac{\alpha_s}{\pi} \left[ \log \frac{\mu_b^2}{\bar{m}_t^2} + \frac{4}{3} \right] \right]$$

# Scales for $y_+$

♦ Different scales for  $y_+$  in triangle (Q) and box ( $M_H$ ) diagrams?

→ has to hold to all orders



⇒ Same scale in all diagrams



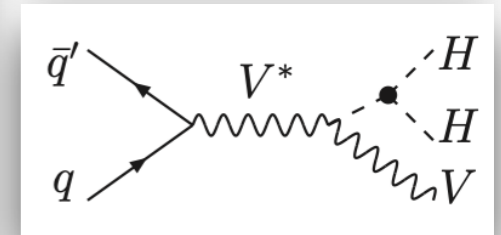
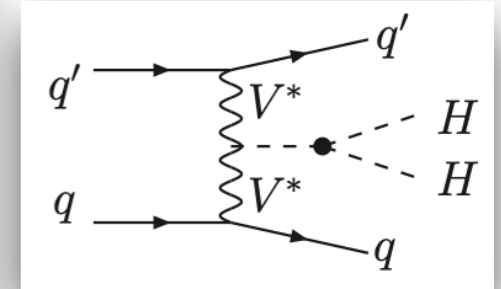
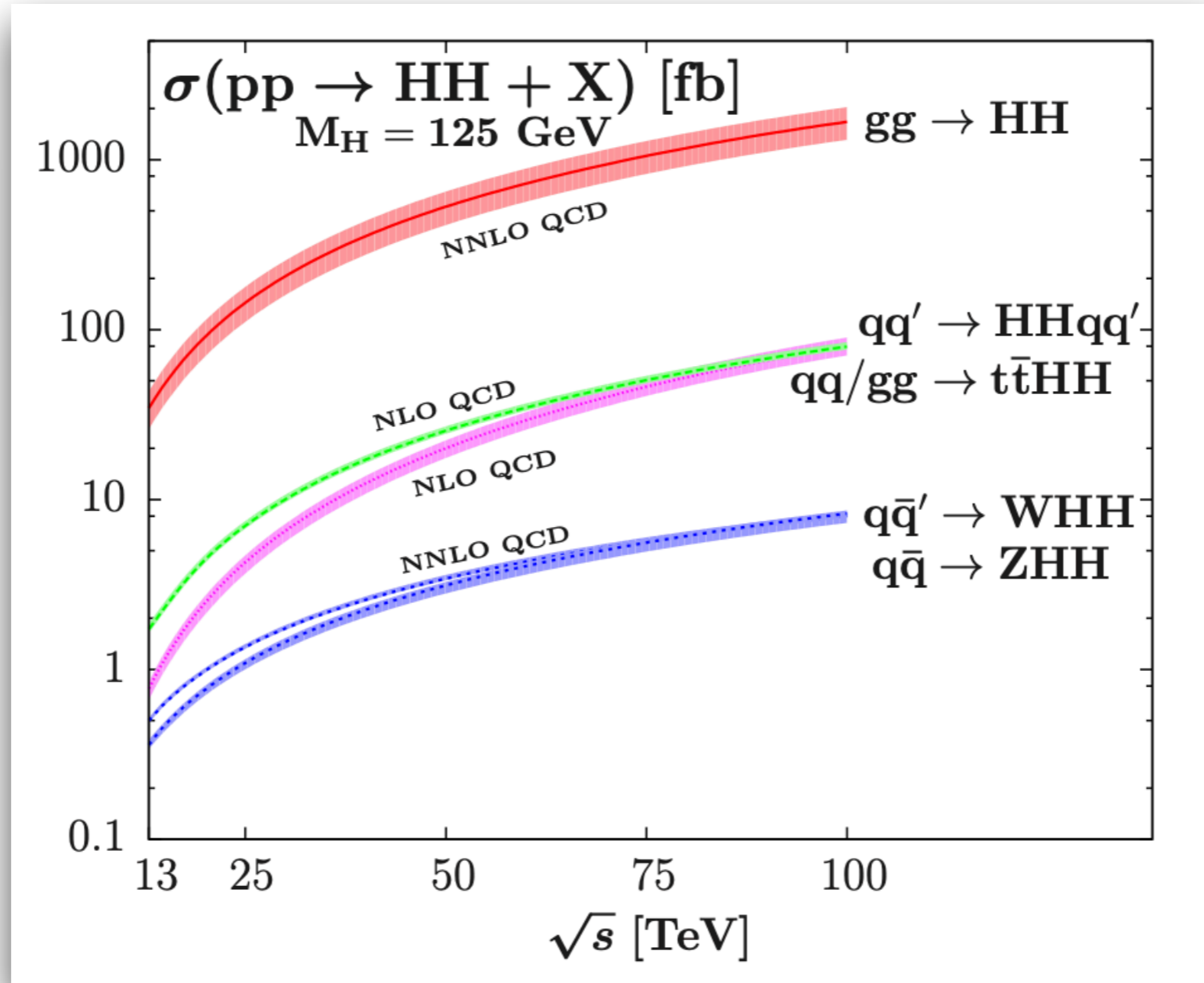
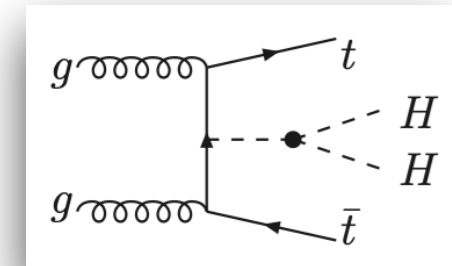
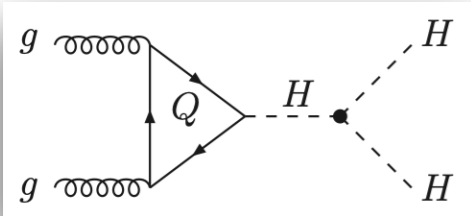
# LO Uncertainties

## † Scale and scheme uncertainties at LO

$$\begin{aligned}\frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=300 \text{ GeV}} &= 0.01656^{+62\%}_{-2.4\%} \text{ fb/GeV} \\ \frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=400 \text{ GeV}} &= 0.09391^{+0\%}_{-20\%} \text{ fb/GeV} \\ \frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=600 \text{ GeV}} &= 0.02132^{+0\%}_{-48\%} \text{ fb/GeV} \\ \frac{d\sigma(gg \rightarrow HH)}{dQ} \Big|_{Q=1200 \text{ GeV}} &= 0.0003223^{+0\%}_{-56\%} \text{ fb/GeV}\end{aligned}$$

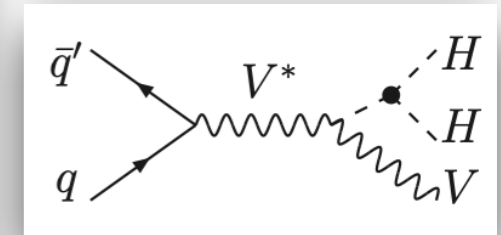
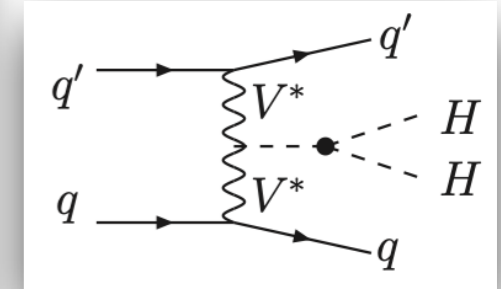
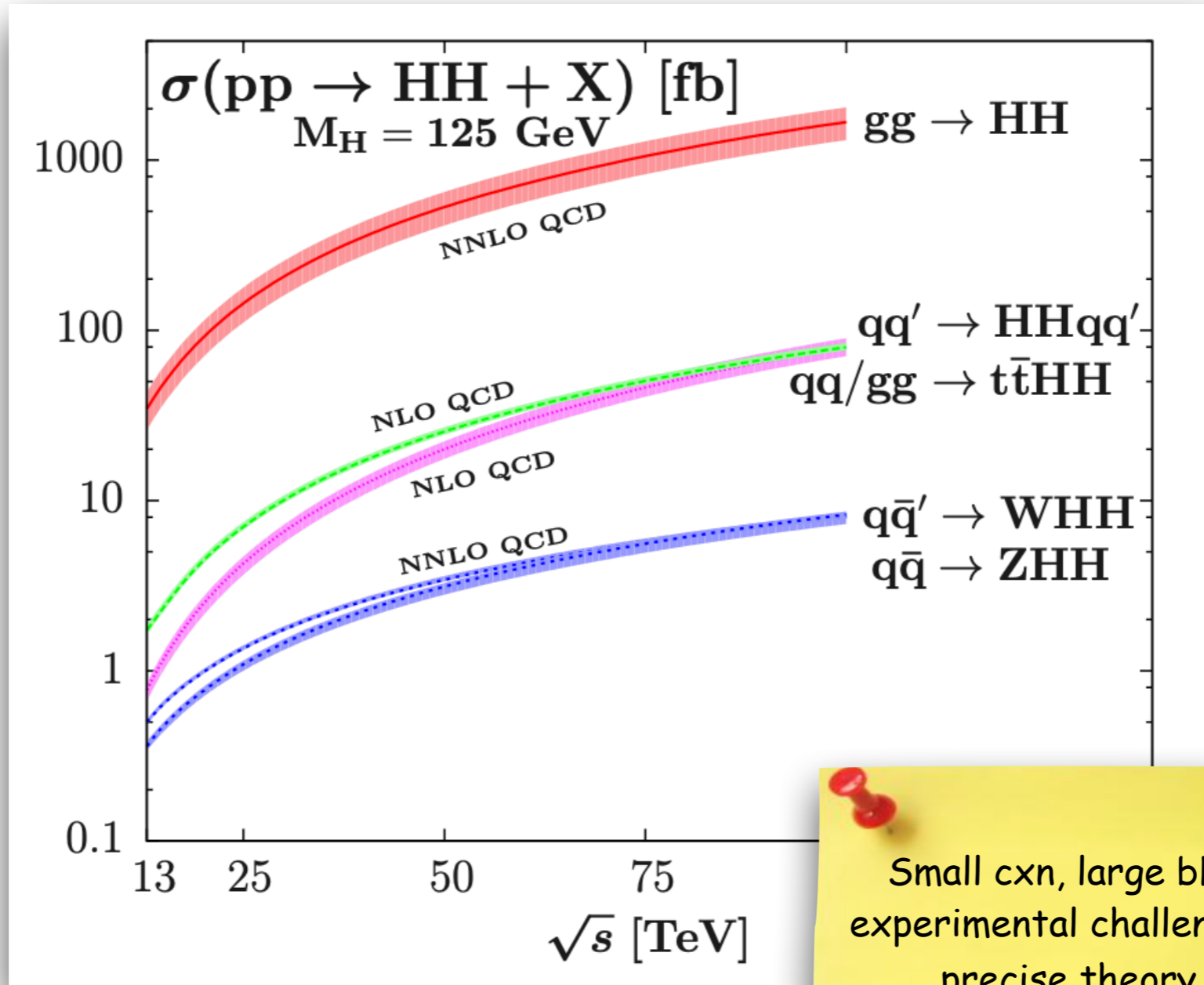
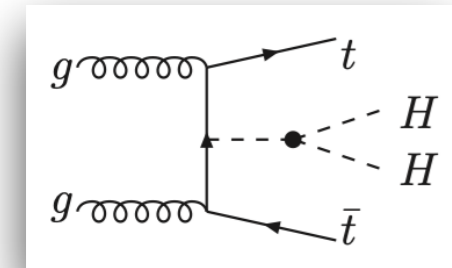
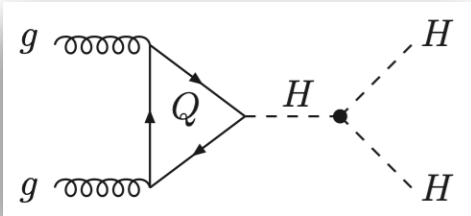
# Double Higgs Production Processes

[Baglio, Djouadi, Quévilion, '15]



# Double Higgs Production Processes

[Baglio, Djouadi, Quévilion, '15]



Small c<sub>xn</sub>, large bkg:  
 experimental challenge ~  
 precise theory  
 predictions required