

Constructing and Transforming EFT Bases with AutoEFT

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Particle Physics Phenomenology after the Higgs Discovery

Outline

Effective Field Theories

AutoEFT v1

Basis Transformations

Lagrangian

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_0 + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \dots$$

Operators \mathcal{O}

- composed out of the fields of \mathcal{L}_0
- invariant under symmetries of \mathcal{L}_0
- effective couplings \mathcal{C} (Wilson coefficients)

Operator basis

- *complete and independent* set of operators

Redundancies

1. Equations of motion: $\mathcal{O} = \mathcal{O}' + \frac{\delta \mathcal{S}_0}{\delta \Phi_i} \mathcal{O}'' \sim \mathcal{O}'$
2. Integration by parts: $\mathcal{O} = \mathcal{O}' + \partial \mathcal{O}'' \sim \mathcal{O}'$
3. Permutation symmetries: $\mathcal{O} = \pi_\Phi \circ \mathcal{O}' \sim \mathcal{O}'$
4. Fierz/Schouten identities: $g_{\mu\nu} \sigma_{\alpha\dot{\alpha}}^\mu \sigma_{\beta\dot{\beta}}^\nu = 2\epsilon_{\alpha\beta} \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$

Brief History of SMEFT

5

1979

[Weinberg]

Baryon- and Lepton-Nonconserving Processes

Steven Weinberg

Phys. Rev. Lett. **43**, 1566 – Published 19 November 1979

$$f_{abmn} \overline{\ell_{i a L}} \ell_{j b L} \phi_k^{(m)} \phi_\ell^{(n)} \epsilon_{ik} \epsilon_{j\ell}.$$

Only a single *complex* term at $d = 5$

Brief History of SMEFT



[Weinberg] [Buchmüller, Wyler]

Effective lagrangian analysis of new interactions and flavour conservation

W. Buchmüller, D. Wyler

The effective Lagrangian contains 1 dimension-five operator, which violates lepton number, and 80 baryon- and lepton-number conserving dimension-six operators.

22 terms redundant, 1 missing

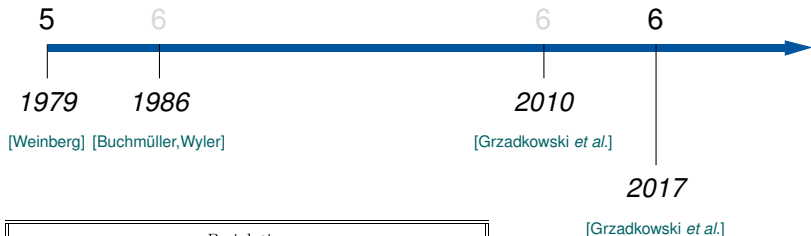
Brief History of SMEFT



<i>B</i> -violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^\gamma)^T C l_t^k]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^\gamma)^T C l_t^n]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I \varepsilon)_{jk}(\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^\gamma)^T C l_t^n]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$

59 + 5 terms ($n_f = 1$)

Brief History of SMEFT



<i>B</i> -violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^{\gamma j})^T C l_t^k]$
Q_{quq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C u_r^\beta] [(u_s^{\gamma k})^T C e_t]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jn} \varepsilon_{km} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$
Q_{dqu}	

59 + 4 terms ($n_f = 1$)

Brief History of SMEFT



$\mathcal{O}_{BQQ^i HH^i D}^{(1\sim 8)}$	$\begin{aligned} & B_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha i}) D^\mu (H^i H), \quad i B_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha i}) \left(H^i \overleftrightarrow{D}^\mu H \right), \\ & \tilde{B}_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha i}) D^\mu (H^i H), \quad i \tilde{B}_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha i}) \left(H^i \overleftrightarrow{D}^\mu H \right), \\ & B_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha j}) D^\mu (H^i H_j), \quad i B_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha j}) \left(H^i \overleftrightarrow{D}^\mu H_j \right), \\ & \tilde{B}_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha j}) D^\mu (H^i H_j), \quad i \tilde{B}_{\mu\nu} (Q_{p\alpha i} \sigma^\nu Q_r^{\dagger\alpha j}) \left(H^i \overleftrightarrow{D}^\mu H_j \right) \end{aligned}$
$\mathcal{O}_{G u_c^i u_c^i HH^i D}^{(1\sim 4)}$	$\begin{aligned} & G_{\mu\nu}^A \left(u_{c p}^a \sigma^\nu (\lambda^A)_a^b u_{c r b}^\dagger \right) D^\mu (H^i H), \quad i G_{\mu\nu}^A \left(u_{c p}^a \sigma^\nu (\lambda^A)_a^b u_{c r b}^\dagger \right) \left(H^i \overleftrightarrow{D}^\mu H \right), \\ & \tilde{G}_{\mu\nu}^A \left(u_{c p}^a \sigma^\nu (\lambda^A)_a^b u_{c r b}^\dagger \right) D^\mu (H^i H), \quad i \tilde{G}_{\mu\nu}^A \left(u_{c p}^a \sigma^\nu (\lambda^A)_a^b u_{c r b}^\dagger \right) \left(H^i \overleftrightarrow{D}^\mu H \right) \end{aligned}$
$\mathcal{O}_{W u_c^i u_c^i HH^i D}^{(1\sim 4)}$	$\begin{aligned} & W_{\mu\nu}^I \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) D^\mu (H^i \tau^I H), \quad i W_{\mu\nu}^I \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) \left(H^i \tau^I \overleftrightarrow{D}^\mu H \right), \\ & \tilde{W}_{\mu\nu}^I \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) D^\mu (H^i \tau^I H), \quad i \tilde{W}_{\mu\nu}^I \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) \left(H^i \tau^I \overleftrightarrow{D}^\mu H \right) \end{aligned}$
$\mathcal{O}_{B u_c^i u_c^i HH^i D}^{(1\sim 4)}$	$\begin{aligned} & B_{\mu\nu} \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) D^\mu (H^i H), \quad i B_{\mu\nu} \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) \left(H^i \overleftrightarrow{D}^\mu H \right), \\ & \tilde{B}_{\mu\nu} \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) D^\mu (H^i H), \quad i \tilde{B}_{\mu\nu} \left(u_{c p}^a \sigma^\nu u_{c r a}^\dagger \right) \left(H^i \overleftrightarrow{D}^\mu H \right) \end{aligned}$

[Grzadkowski et al.]

44.807 operators ($n_f = 3$) also by [Murphy; 2020]

Brief History of SMEFT



```
1 # 'ILL_3QL.yml' generated by AutoEFT 1.0.0
2 version: 1.0.0
3 type:
4 - {LL: 1, QL: 3}
5 - complex
6 generations: {LL: 3, QL: 3}
7 n_terms: 3
8 n_operators: 57
9 invariants:
10 Lorentz:
11   0(Lorentz,1): +eps(1_1,3_1)*eps(2_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
12   0(Lorentz,2): +eps(1_1,2_1)*eps(3_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
13 SU3:
14   0(SU3,1): +eps(2_1,3_1,4_1) * LL*QL(2_1)*QL(3_1)*QL(4_1)
15 SU2:
16   0(SU2,1): +eps(1_1,3_1)*eps(2_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
17   0(SU2,2): +eps(1_1,2_1)*eps(3_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
18 permutation_symmetries:
19 - vector: Lorentz * SU3 * SU2
20 - symmetry: {LL: [1], QL: [1, 1, 1]}
21 n_terms: 1
22 n_operators: 3
```

75.577.476 operators ($n_f = 3$)

Outline

Effective Field Theories

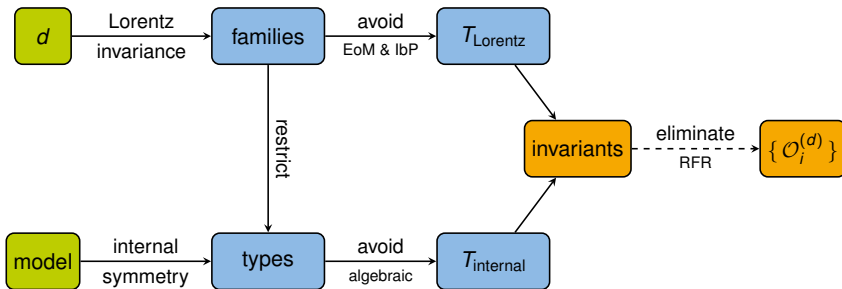
AutoEFT v1

Basis Transformations

Automated Construction



[Harlander,MS; 2023]



implemented in Python using SageMath & FORM

Custom Models — SMEFT

```
name: SMEFT
```

```
# arbitrary SU(N) and  
# U(1) groups
```

```
symmetries:
```

```
  sun_groups:
```

```
    SU3:
```

```
      N: 3
```

```
    SU2:
```

```
      N: 2
```

```
  u1_groups:
```

```
    U1: {}
```

```
fields:
```

```
  GL:
```

```
    representations:
```

```
      Lorentz: -1
```

```
      SU3: [2,1]
```

```
  uR:
```

```
    representations:
```

```
      Lorentz: 1/2
```

```
      SU3: [1]
```

```
      U1: 2/3
```

```
    generations: 3
```

```
  ...
```

Minimal Flavor Violation

Approximate flavor symmetry of the SM

$$U(3)^5 \sim U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_L \times U(3)_e$$

only broken by the Yukawa couplings Y_u, Y_d, Y_e

Field representations $u(N) \cong su(N) \times u(1)$

$$Q \sim \square_{SU(3)_Q} \otimes 1_{U(1)_Q}$$

$$u \sim \square_{SU(3)_u} \otimes 1_{U(1)_u} \quad d \sim \square_{SU(3)_d} \otimes 1_{U(1)_d}$$

$$L \sim \square_{SU(3)_L} \otimes 1_{U(1)_L} \quad e \sim \square_{SU(3)_e} \otimes 1_{U(1)_e}$$

Custom Models — MFV

```
# AutoEFT model file
name: MFV-SMEFT

symmetries:
  sun_groups:
    q3: {N: 3}
    u3: {N: 3}
    d3: {N: 3}
    l3: {N: 3}
    e3: {N: 3}
  u1_groups:
    q1: {}
    u1: {}
    d1: {}
    l1: {}
    e1: {}
```

```
fields:

  QL:
    representations: {q3: [1], q1: 1}
  uR:
    representations: {u3: [1], u1: 1}
  dR:
    representations: {d3: [1], d1: 1}
  LL:
    representations: {l3: [1], l1: 1}
  eR:
    representations: {e3: [1], e1: 1}
```

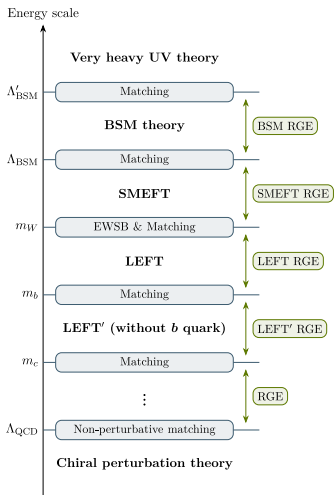
Outline

Effective Field Theories

AutoEFT v1

Basis Transformations

EFT Phenomenology



[Isidori, Wilsch, Wyler; 2023]

- loop calculations introduce $\tilde{\mathcal{O}} \notin \{\mathcal{O}_i\}$
- $\tilde{\mathcal{O}} = \sum_i a_i \mathcal{O}_i$
- different bases are more convenient at each step
- $\tilde{\mathcal{C}}_i = \sum_j b_{ij} \mathcal{C}_j$

Warsaw basis v1

$$Q_{qqq}^{(1)} = \epsilon^{\alpha\beta\gamma} \epsilon_{jk} \epsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$$

$$Q_{qqq}^{(3)} = \epsilon^{\alpha\beta\gamma} (\tau^l \epsilon_{jk}) (\tau^l \epsilon_{mn}) [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$$

AutoEFT basis $\{ \mathcal{O}_{[3]}, \mathcal{O}_{[2,1]}, \mathcal{O}_{[1,1,1]} \}$

$$Q_{qqq}^{(1)} = \frac{1}{3} \mathcal{O}_{[2,1]} - \frac{1}{3} \mathcal{O}_{[3]}$$

$$Q_{qqq}^{(3)} = -\mathcal{O}_{[1,1,1]} - \frac{1}{3} \mathcal{O}_{[2,1]} - \frac{2}{3} \mathcal{O}'_{[2,1]}$$

Repeated Fields

Basis change

$$\begin{aligned} Q_{qqq} &= -\frac{1}{2}(Q_{qqq}^{(1)} + Q_{qqq}^{(3)}) \\ &= \frac{1}{2}\mathcal{O}_{[1,1,1]} + \frac{1}{6}\mathcal{O}_{[3]} + \frac{1}{3}\mathcal{O}'_{[2,1]} \end{aligned}$$

$$\begin{aligned} \tilde{Q}_{qqq} &= -\frac{1}{2}(Q_{qqq}^{(1)} - Q_{qqq}^{(3)}) \\ &= -\frac{1}{2}\mathcal{O}_{[1,1,1]} + \frac{1}{6}\mathcal{O}_{[3]} - \frac{1}{3}(\mathcal{O}_{[2,1]} + \mathcal{O}'_{[2,1]}) \end{aligned}$$

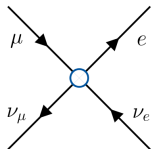
Permutation Symmetry

$$\tilde{Q}_{qqq} = (12) \circ Q_{qqq}$$

Backup

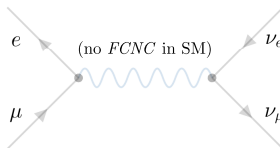
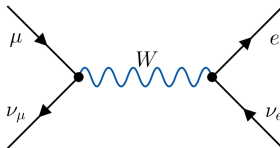
New Physics in Effective Field Theories

Fermi Theory (EFT)



possible realisations

Standard Model



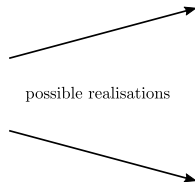
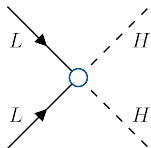
$\ll m_W$

$\sim m_W$

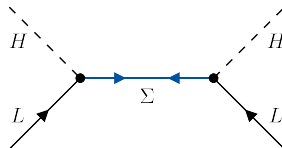
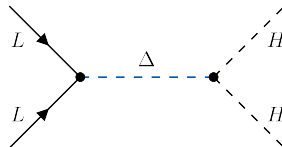
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New Physics in Effective Field Theories

SMEFT



High Energy Theory ?



Redundancies

Fierz identities

$$\begin{aligned}g_{\mu\nu}\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\beta\dot{\beta}}^{\nu} &= 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \\ \epsilon^{\alpha\beta}\delta_{\kappa}^{\gamma} + \epsilon^{\beta\gamma}\delta_{\kappa}^{\alpha} + \epsilon^{\gamma\alpha}\delta_{\kappa}^{\beta} &= 0 \\ \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta_{\dot{\gamma}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta_{\dot{\alpha}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta_{\dot{\beta}}^{\dot{\kappa}} &= 0\end{aligned}$$

Field strength tensor

$$[D_{\mu}, D_{\nu}] = -iF_{\mu\nu}$$

Redundancies

Equations of motion

$$D^2\phi + J_\phi = 0$$

$$i\cancel{D}\psi + J_\psi = 0$$

$$D_\mu F^{\mu\nu} + J_A^\nu = 0$$

Integration by parts

$$D_\mu(XY) \sim 0 \implies X D_\mu Y \sim -D_\mu X Y$$

Building Blocks

Lorentz group: $SL(2, \mathbb{C}) \simeq SU(2)_I \times SU(2)_R$

$$\phi \in (0, 0)$$

$$\psi_\alpha \in (1/2, 0) \quad \psi^{\dot{\alpha}} \in (0, 1/2)$$

$$F_{L\alpha\beta} = \frac{i}{2} F_{\mu\nu} \sigma_{\alpha\beta}^{\mu\nu} \in (1, 0) \quad F_R^{\dot{\alpha}\dot{\beta}} = -\frac{i}{2} F^{\mu\nu} \bar{\sigma}_{\mu\nu}^{\dot{\alpha}\dot{\beta}} \in (0, 1)$$

$$D_\alpha^{\dot{\alpha}} = D_\mu \sigma^{\mu\dot{\alpha}}_\alpha \in (1/2, 1/2)$$

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$G_{abc} = \epsilon_{acd} (\lambda^A)^d_b G^A$$

$$W_{ij} = \epsilon_{jk} (\tau^I)^k_i W^I$$

Explicit index notation

$$\begin{aligned}\epsilon^{abc} (L^i Q_{j,a}) (Q_{i,b} Q_c^j) &= \epsilon^{abc} L_\alpha^i Q_{j,a}^\alpha Q_{i,b}^\beta Q_{c,\beta}^j \\ &= \underbrace{\epsilon^{abc}}_{T_{\text{SU}(3)}} \underbrace{\epsilon^{ik} e^{jl}}_{T_{\text{SU}(2)}} \underbrace{\epsilon_{\alpha\beta\gamma\delta}}_{T_{\text{Lorentz}}} \underbrace{L_i^\alpha Q_{j,a}^\beta Q_{k,b}^\gamma Q_{l,c}^\delta}_{\prod_n \Phi_n}\end{aligned}$$

operators of same *type* only differ in choice of T

e.g.: $T_{\text{SU}(2)} = \epsilon^{ij} \epsilon^{kl}$

$\Rightarrow T_{\text{SU}(3)}, T_{\text{SU}(2)}$ & T_{Lorentz} govern the symmetry

General operator

$$\mathcal{O} = T_{\text{SU}(3)}^{\{g\}} T_{\text{SU}(2)}^{\{h\}} T_{\text{Lorentz}}^{\{l\}} \times \prod_{i=1}^N (D^{n_i} \Phi_i)_{\{g\}, \{h\}, \{l\}}$$

Invariant tensors

$$T_{\text{SU}(3)} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)_a^b, \epsilon_{abc}, \epsilon^{abc} \rangle$$

$$T_{\text{SU}(2)} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)_i^j, \epsilon_{ij}, \epsilon^{ij} \rangle$$

$$T_{\text{Lorentz}} \in \langle \sigma_{\alpha\beta}^{\mu\nu}, \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu}, \sigma_{\alpha\dot{\alpha}}^{\mu}, \bar{\sigma}^{\mu\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

General operator

$$\mathcal{O} = T_{\text{SU}(3)}^{\{g\}} T_{\text{SU}(2)}^{\{h\}} T_{\text{Lorentz}}^{\{l\}} \times \prod_{i=1}^N (D^{n_i} \Phi_i)_{\{g\}, \{h\}, \{l\}}$$

Invariant tensors

$$T_{\text{SU}(3)} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)_a^b, \epsilon_{abc}, \epsilon^{abc} \rangle$$

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$$T_{\text{Lorentz}} \in \langle \sigma_{\alpha\beta}^{\mu\nu}, \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu}, \sigma_{\alpha\dot{\alpha}}^{\mu}, \bar{\sigma}^{\mu\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

Irreducible Representations

Lorentz group: $SL(2, \mathbb{C}) \simeq SU(2)_l \times SU(2)_r$

$$\phi \in (0, 0) \quad \psi_L \in (1/2, 0) \quad F_L \in (1, 0)$$

Equations of motion

$$\partial^2 \phi \in \underbrace{(0, 0)}_{\sim \phi} \oplus \underbrace{(1, 0) \oplus (0, 1)}_{\sim F} \oplus (1, 1)$$

$$\partial \psi_L \in \underbrace{(0, 1/2)}_{\sim \psi_R} \oplus (1, 1/2)$$

$$\partial F_L \in \underbrace{(1/2, 1/2)}_{\sim A} \oplus (3/2, 1/2)$$

Irreducible Representations

Lorentz group: $SL(2, \mathbb{C}) \simeq SU(2)_l \times SU(2)_r$

$$\phi \in (0, 0) \quad \psi_L \in (1/2, 0) \quad F_L \in (1, 0)$$

Equations of motion

$$\partial^2 \phi \in \underbrace{(0, 0)}_{\sim \phi} \oplus \underbrace{(1, 0) \oplus (0, 1)}_{\sim F} \oplus (1, 1)$$

$$\partial \psi_L \in \underbrace{(0, 1/2)}_{\sim \psi_R} \oplus (1, 1/2)$$

$$\partial F_L \in \underbrace{(1/2, 1/2)}_{\sim A} \oplus (3/2, 1/2)$$

\Rightarrow **only keep symmetric combinations**

Equations of Motion

$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\dots\alpha^{(r-h)}\dot{\alpha}^{(1)}\dots\dot{\alpha}^{(r+h)}} \equiv \begin{cases} D_{\alpha^{(1)}\dot{\alpha}^{(1)}} \dots D_{\alpha^{(r+h)}\dot{\alpha}^{(r+h)}} \Phi_{\alpha^{(r+h+1)}\dots\alpha^{(r-h)}} , & h < 0 \\ D_{\alpha^{(1)}\dot{\alpha}^{(1)}} \dots D_{\alpha^{(r-h)}\dot{\alpha}^{(r-h)}} \Phi_{\dot{\alpha}^{(r-h+1)}\dots\dot{\alpha}^{(r+h)}} , & h > 0 \end{cases}$$

$$D_{[\alpha\dot{\alpha}} D_{\beta]\dot{\beta}} \phi = -\epsilon_{\alpha\beta} \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} D^2 \phi + \frac{i}{2} \epsilon_{\alpha\beta} \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu} [D_\mu, D_\nu] \phi$$

$$D_{[\alpha\dot{\alpha}} \psi_{\beta]} = -\epsilon_{\alpha\beta} (\not{D}\psi)_{\dot{\alpha}}$$

$$D_{[\alpha\dot{\alpha}} F_{L\beta]\gamma} = 2\epsilon_{\alpha\beta} \sigma_{\gamma\dot{\alpha}}^\nu D^\mu F_{\mu\nu}$$

$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\dots\alpha^{(r-h)}\dot{\alpha}^{(1)}\dots\dot{\alpha}^{(r+h)}} \equiv (D^{r-|h|}\Phi)_{\alpha^{r-h}}^{\dot{\alpha}^{r+h}} \in \left(\frac{r-h}{2}, \frac{r+h}{2} \right)$$

Irreducible Representations

Integration by parts

$$-D\phi_1 \phi_2 \phi_3 D\phi_4 = \phi_1 D\phi_2 \phi_3 D\phi_4 + \phi_1 \phi_2 D\phi_3 D\phi_4$$

Irreducible representation of $SU(N)$

$$- \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} = - \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}$$

Irreducible Representations

Integration by parts

$$-D\phi_1 \phi_2 \phi_3 D\phi_4 = \phi_1 D\phi_2 \phi_3 D\phi_4 + \phi_1 \phi_2 D\phi_3 D\phi_4$$

Irreducible representation of $SU(N)$

$$- \begin{array}{|c|c|} \hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} = - \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}$$

\Rightarrow only keep **Semi-Standard Young Tableaux**


Lorentz Structure as $SU(N)$ State

Young diagrams for $N = 4$ fields

$$\epsilon \sim \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \quad \tilde{\epsilon} \sim \begin{array}{|c|} \hline \overline{\square} \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

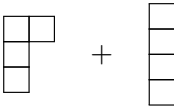


$$T_{\text{Lorentz}} = (\tilde{\epsilon}_{\dot{\alpha}k\dot{\alpha}l})^{\otimes \tilde{n}} (\epsilon^{\alpha i \alpha j})^{\otimes n} \quad n \underset{\sim}{=} \tilde{n} = 1$$



primary

+



total derivative

Irreducible Representations

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$SU(3) : \quad \mathbf{3} \sim \square \quad \bar{\mathbf{3}} \sim \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \quad \mathbf{8} \sim \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \end{array}$$

$$SU(2) : \quad \mathbf{2} \sim \square \quad \mathbf{3} \sim \square \square$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \quad Q_j \sim \boxed{j} \quad Q_k \sim \boxed{k} \quad Q_l \sim \boxed{l}$$

$$\boxed{i} \rightarrow \boxed{i \ j} \rightarrow \begin{array}{|c|c|} \hline i & j \\ \hline k & \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline i & j \\ \hline k & l \\ \hline \end{array} \sim \epsilon^{ik} \epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \rightarrow \begin{array}{|c|} \hline i \\ \hline j \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline i & k \\ \hline j & \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline i & k \\ \hline j & l \\ \hline \end{array} \sim \epsilon^{ij} \epsilon^{kl} L_i Q_j Q_k Q_l$$

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Invariant contractions

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\Rightarrow tensors according to **Littlewood-Richardson rule**

Permutation Symmetries

LQ^3

2 Lorentz \times 2 SU(2) invariant contractions:

$$\begin{aligned}\mathcal{O}_1 &= \mathcal{O}_1^{\text{Lorentz}} \otimes \mathcal{O}_1^{\text{SU}(2)} & \mathcal{O}_2 &= \mathcal{O}_1^{\text{Lorentz}} \otimes \mathcal{O}_2^{\text{SU}(2)} \\ \mathcal{O}_3 &= \mathcal{O}_2^{\text{Lorentz}} \otimes \mathcal{O}_1^{\text{SU}(2)} & \mathcal{O}_4 &= \mathcal{O}_2^{\text{Lorentz}} \otimes \mathcal{O}_2^{\text{SU}(2)}\end{aligned}$$

Change of basis

$$\begin{pmatrix} \mathcal{O}_{[3]} \\ \mathcal{O}_{[2,1],1} \\ \mathcal{O}_{[2,1],2} \\ \mathcal{O}_{[1,1,1]} \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & 2 \\ -1 & 2 & 2 & -1 \\ -1 & -1 & -1 & 2 \\ 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \end{pmatrix}$$

Permutation Symmetries

Adding flavor: $LQ^{f_1} Q^{f_2} Q^{f_3}$

$$\begin{aligned} & \kappa_{[3]}^{f_1 f_2 f_3} \mathcal{O}_{[3]}^{f_1 f_2 f_3} + (\kappa_{[2,1]}^{f_1 f_2 f_3} + \hat{\kappa}_{[2,1]}^{f_1 f_3 f_2}) \mathcal{O}_{[2,1],1}^{f_1 f_2 f_3} + \kappa_{[1,1,1]}^{f_1 f_2 f_3} \mathcal{O}_{[1,1,1]}^{f_1 f_2 f_3} \\ &= \kappa^{f_1 f_2 f_3} (\mathcal{O}_{[3]}^{f_1 f_2 f_3} + \mathcal{O}_{[2,1],1}^{f_1 f_2 f_3} + \mathcal{O}_{[1,1,1]}^{f_1 f_2 f_3}) \equiv \kappa^{f_1 f_2 f_3} \mathcal{O}_{LQ^3}^{f_1 f_2 f_3} \end{aligned}$$

Single independent term remains

$$\mathcal{O}_{LQ^3} = \mathcal{O}_1 + 2\mathcal{O}_3 + \mathcal{O}_4$$

→ $3 \cdot (10 + 8 + 1) = 57$ independent components of $\kappa^{f_1 f_2 f_3}$

Field Definitions

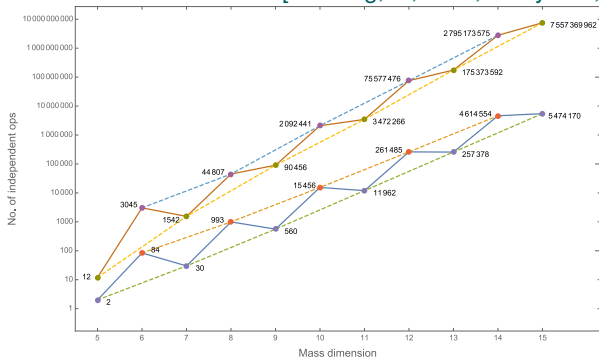
$$q_L = \begin{pmatrix} Q \\ 0 \end{pmatrix} \quad u_R = \begin{pmatrix} 0 \\ u_C^\dagger \end{pmatrix} \quad d_R = \begin{pmatrix} 0 \\ d_C^\dagger \end{pmatrix} \quad l_L = \begin{pmatrix} L \\ 0 \end{pmatrix} \quad e_R = \begin{pmatrix} 0 \\ e_C^\dagger \end{pmatrix}$$

$$F_{L/R} = \frac{1}{2} (F \pm i\tilde{F}) \quad C_{L/R} = \frac{1}{2} (C \pm i\tilde{C})$$

Hilbert Series

$$H(p, \phi_1, \dots, \phi_n) = \int d\mu_{\text{conf.}} d\mu_{\text{gauge}} \sum_{i=1}^{\infty} p^i \chi_{[i;0]}^* \prod_j \text{PE}[\phi_j \chi_j(q, \alpha, \beta)]$$

[Henning, Lu, Melia, Murayama; 2017]



Gravity

$$C_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - (g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}) + \frac{1}{3}g_{\mu[\rho}g_{\sigma]\nu}R$$

$$C_{L\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma} \sigma_{\alpha\beta}^{\mu\nu} \sigma_{\gamma\delta}^{\rho\sigma} \in (2, 0)$$

[Marinissen,Rahn,Waalewijn; 2020]

Dimension	5	6	7	8	9	10	11	12	13	14	15
One generation	2	94	30	1096	580	17797	12936	314650	291702	5812440	6518462
Three generations	12	3055	1542	45816	91284	2160964	3567228	79514441	182542620	2995340275	8023911776

TABLE III. Number of operators in the GRSMEFT of a given dimension with 1 or 3 generations.

Collaborative Research Center TRR 257



Particle Physics Phenomenology after the Higgs Discovery



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