Status of b-hadron lifetimes and B-mixing

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Particle Physics Phenomenology after the Higgs Discovery

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based on review ArXiv:2402.04224

b-hadron lifetimes

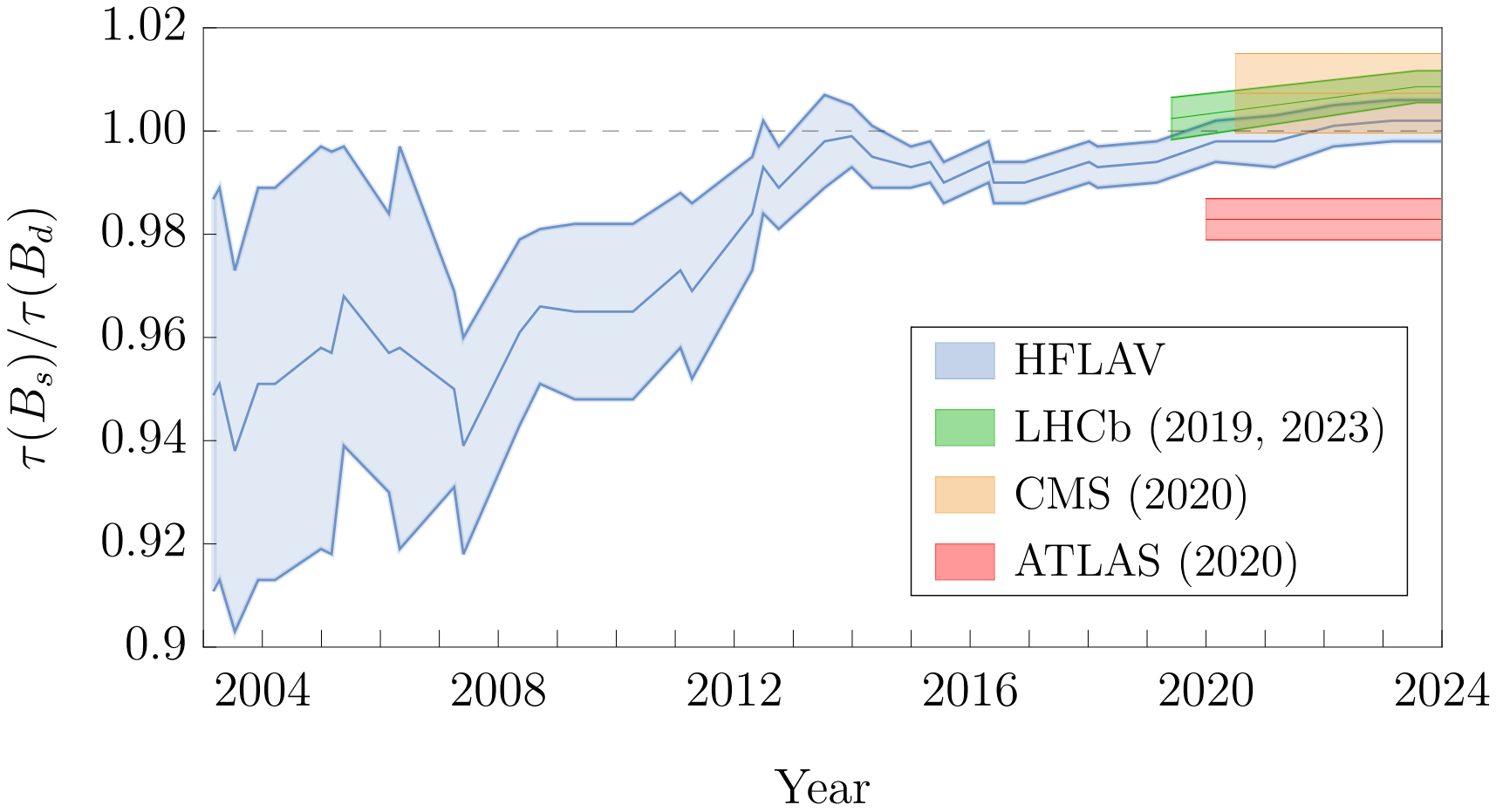
b-hadron lifetimes: experimental values

- Lifetimes of b hadrons are measured precisely at the experiment
- HFLAV and PDG

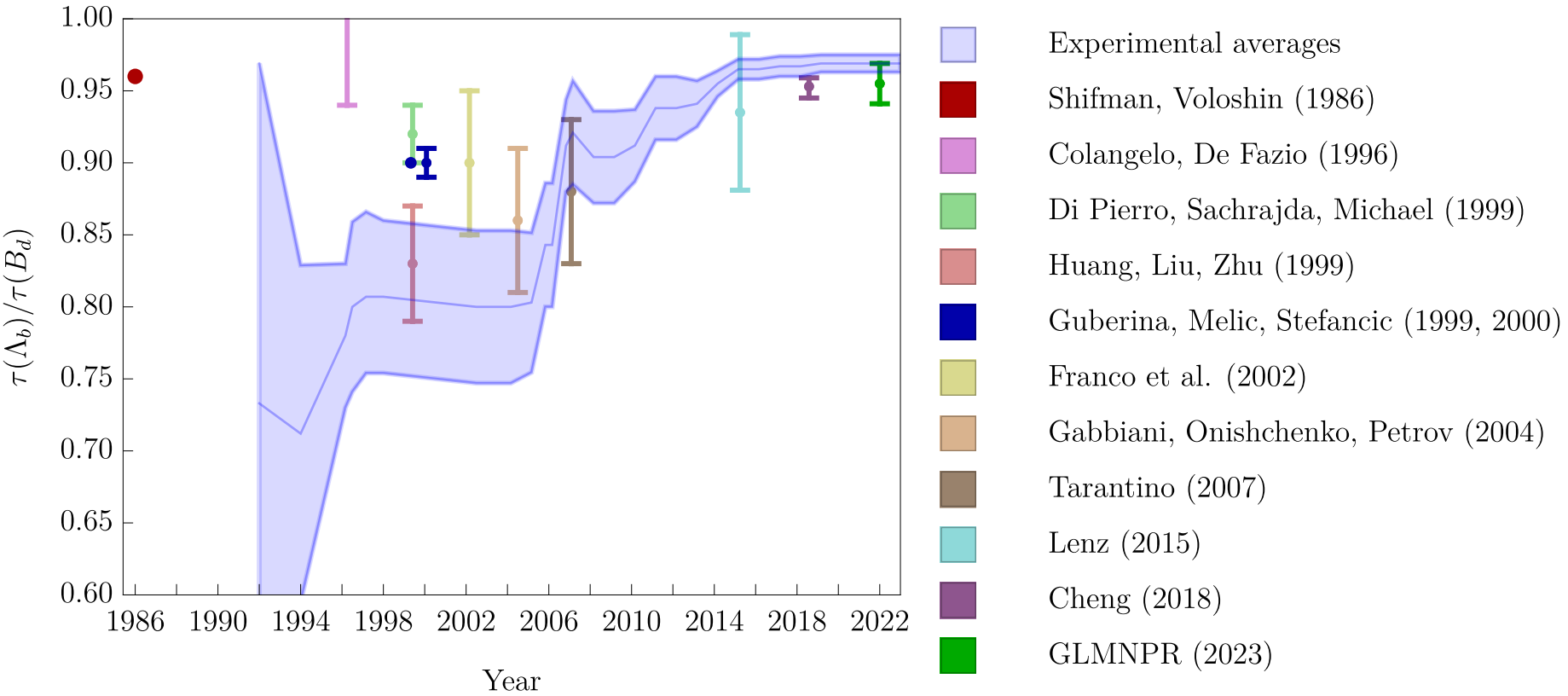
	B^+	B_d^0	B_s^0	B_c^+
τ [ps]	1.638 ± 0.004	1.519 ± 0.004	1.521 ± 0.005	0.510 ± 0.009
$\tau(X)/\tau(B_d^0)$	1.076 ± 0.004	1	1.002 ± 0.004	0.336 ± 0.006
	Λ_b^0	Ξ_b^0	Ξ_b^-	Ω_b^-
τ [ps]	1.471 ± 0.009	1.480 ± 0.030	1.572 ± 0.040	$1.64^{+0.18}_{-0.17}$
$\tau(X)/\tau(B_d^0)$	0.968 ± 0.006	0.974 ± 0.020	1.035 ± 0.027	$1.08^{+0.12}_{-0.11}$

History of $\tau(B_s^0)/\tau(B_d^0)$

[ArXiv: 2402.04224]



History of $\tau(\Lambda_b^0)/\tau(B_d^0)$



[ArXiv: 2301.07698]

b-hadron lifetimes: theory

- Total width of a hadron $\mathcal{B}$ is given by

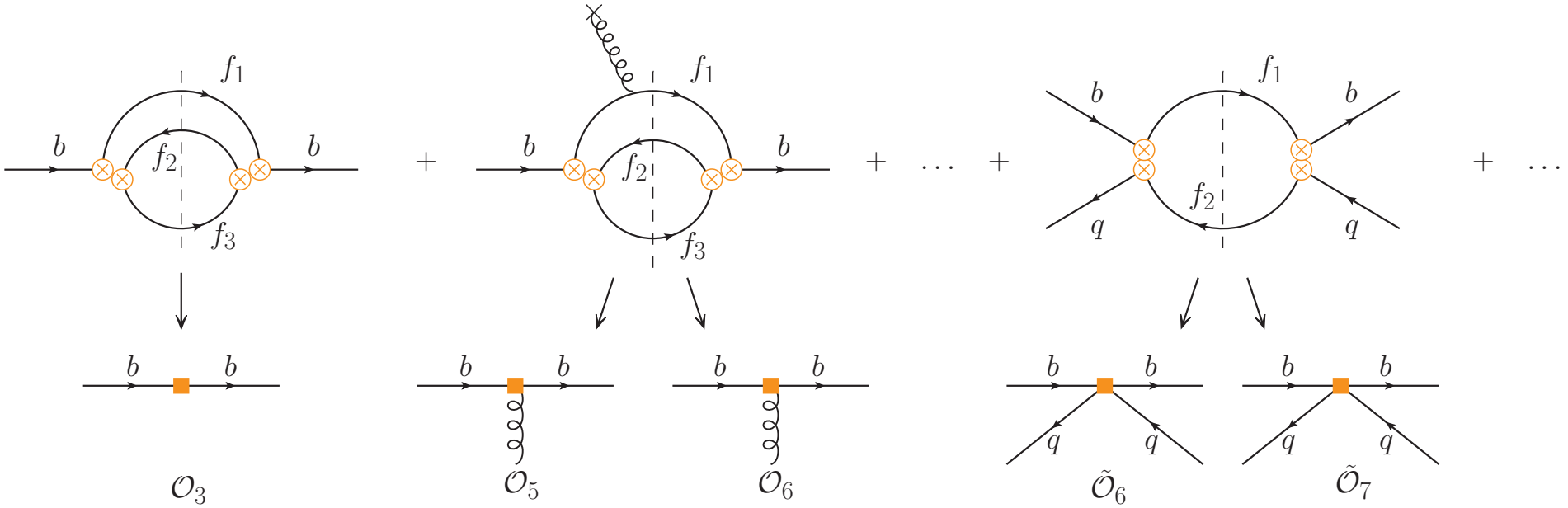
$$\Gamma(\mathcal{B}) = \frac{1}{2m_{\mathcal{B}}} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_{\mathcal{B}} - p_X) \; |\langle X(p_X)|\mathcal{H}_{\rm eff}|\mathcal{B}(p_{\mathcal{B}})\rangle|^2$$

$$\text{Optical Theorem} \quad = \frac{1}{2m_{\mathcal{B}}} \, {\rm Im} \, \langle \mathcal{B}(p_{\mathcal{B}})| \, i \int d^4x \, T \left\{ \mathcal{H}_{\rm eff}(x), \mathcal{H}_{\rm eff}(0) \right\} |\mathcal{B}(p_{\mathcal{B}})\rangle$$

- Computed using heavy quark expansion (HQE) in powers of $\Lambda/m_b \ll 1$
- Ratio of lifetimes $\dfrac{\tau(\mathcal{B}_1)}{\tau(\mathcal{B}_2)} = \dfrac{\Gamma_b + \delta\Gamma_{\mathcal{B}_2}}{\Gamma_b + \delta\Gamma_{\mathcal{B}_1}} = 1 + (\delta\Gamma_{\mathcal{B}_2} - \delta\Gamma_{\mathcal{B}_1})\,\tau(\mathcal{B}_1)$
- May be sensitive to New Physics contributions

$$\frac{\tau(\mathcal{B}_1)}{\tau(\mathcal{B}_2)} = \frac{\Gamma_b + \delta\Gamma_{\mathcal{B}_2}}{\Gamma_b + \delta\Gamma_{\mathcal{B}_1}} = 1 + \left(\delta\Gamma^{\rm SM}_{\mathcal{B}_2} - \delta\Gamma^{\rm SM}_{\mathcal{B}_1}\right)\tau(\mathcal{B}_1) + \left(\delta\Gamma^{\rm NP}_{\mathcal{B}_2} - \delta\Gamma^{\rm NP}_{\mathcal{B}_1}\right)\tau(\mathcal{B}_1)$$

HQE: diagrams



$$\Gamma(\mathcal{B}) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \ldots$$

Status of short-distance coefficients

Determined recently within CRC In progress/planned by CRC

K - Karlsruhe S - Siegen

	Semi-leptonic				Non-leptonic		
	LO	NLO	N^2LO	N^3LO	LO	NLO	N^2LO
Γ_3	✓	✓	✓	✓[1] [C1a]	✓	✓	K [C1b][5]
Γ_5	✓	✓			✓	✓[2] [C1b]	
Γ_6	✓	✓[3] [C1a]			✓[4] [C1b]	S [C1b]	
Γ_7	✓				S [C1b]		
Γ_8	✓						
$\tilde{\Gamma}_6$	✓	✓	K [C1b]		✓	✓	K [C1b]
$\tilde{\Gamma}_7$	✓	K [C1b]			✓	K [C1b]	

[1] [Fael, Schönwald, Steinhauser (2011.13654)], [Czakon, Czarnecki, Dowling (2104.05804)], [Fael, Usovitsch (2310.03685)]

[2] [Mannel, Moreno, Pivovarov (for $m_c = 0$) (2304.08964)] (including m_c effects in progress)

[3] [Mannel, Pivovarov (1907.09187)], [Mannel, Moreno, Pivovarov (2112.03875)], [Moreno (2207.14245, 2402.13805)]

[4] [Lenz, Piscopo, AR (2004.09527)], [Mannel, Moreno, Pivovarov (2004.09485)]

[5] almost finished [Egner, Fael, Schönwald, Steinhauser]

Status of non-perturbative matrix elements

In progress/planned by CRC

S - Siegen A - Aachen

$\langle Q_5 \rangle_{B_d}$	QCD sum rule ✓
	Fit of inclusive data ✓
	Lattice QCD ✓
$\langle Q_5 \rangle_{B_s}$	Spectroscopy relations ✓
$\langle Q_5 \rangle_{\mathcal{B}}$	Spectroscopy relations ✓
$\langle Q_6 \rangle_{B_d}$	EOM relation ✓
	Fit of inclusive data ✓
$\langle Q_6 \rangle_{B_s}$	EOM relation ✓
	Sum rule ✓
$\langle Q_6 \rangle_{\mathcal{B}}$	EOM relation ✓

$\langle \tilde{Q}_6 \rangle_{B_d}$	HQET sum rule ✓
	Lattice QCD [C1c] S A[1]
$\langle \tilde{Q}_6 \rangle_{B_s}$	HQET sum rule ✓
	Lattice QCD [C1c] S A[1]
$\langle \tilde{Q}_6 \rangle_{\Lambda_b}$	QCD sum rule ✓
$\langle \tilde{Q}_6 \rangle_{\mathcal{B}}$	NRCQM ✓
$\langle \tilde{Q}_7 \rangle_{B_{d,s}}$	VIA ✓
	QCD sum rule [C1c] S

$$\mathcal{B} = \{\Lambda_b, \Xi_b^0, \Xi_b^-, \Omega_b\}$$

[1] [see talk by Oliver Witzel]

B-meson lifetimes and ratios

[Lenz, Piscopo, AR (2208.02643)]

Scenario A

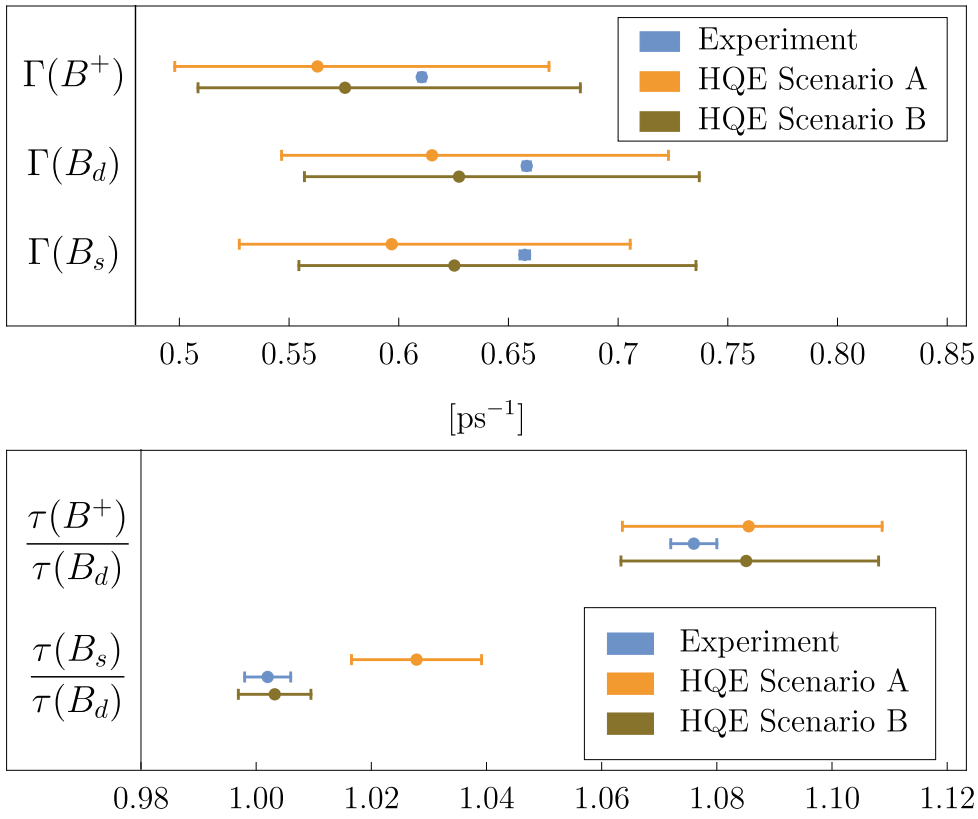
larger ρ_D^3, larger ~~$SU(3)_F$~~

(ρ_D^3 from fit of inclusive semileptonic data by [Bordone, Capdevila, Gambino, 2107.00604])

Scenario B

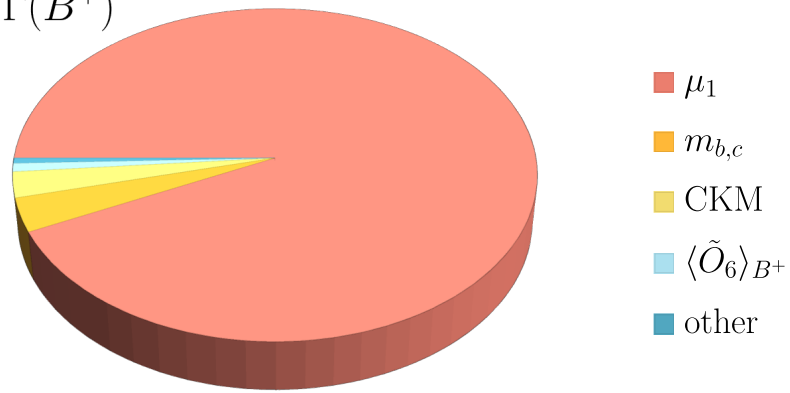
smaller ρ_D^3, smaller ~~$SU(3)_F$~~

(ρ_D^3 from fit of inclusive semileptonic data by [Bernlochner et al., 2205.10274])

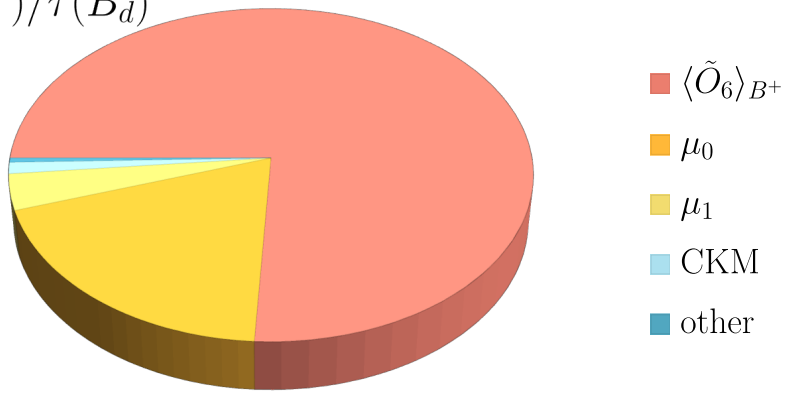


Theory uncertainties in B-meson lifetimes

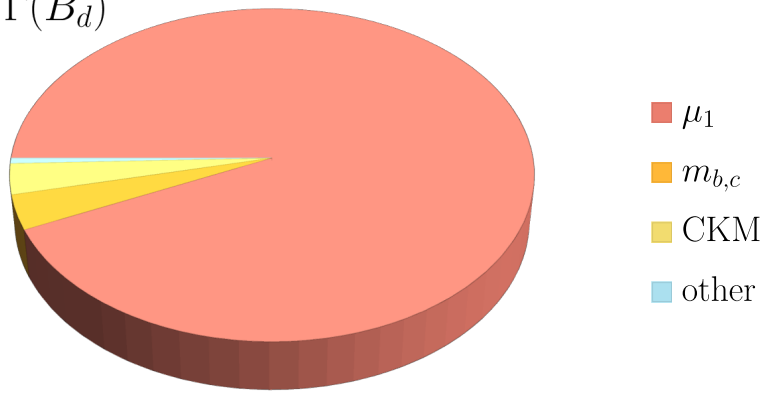
$\Gamma(B^+)$



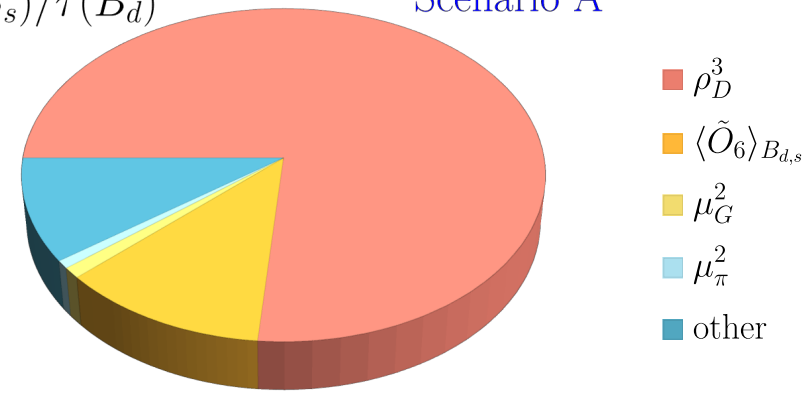
$\tau(B^+)/\tau(B_d)$



$\Gamma(B_d)$

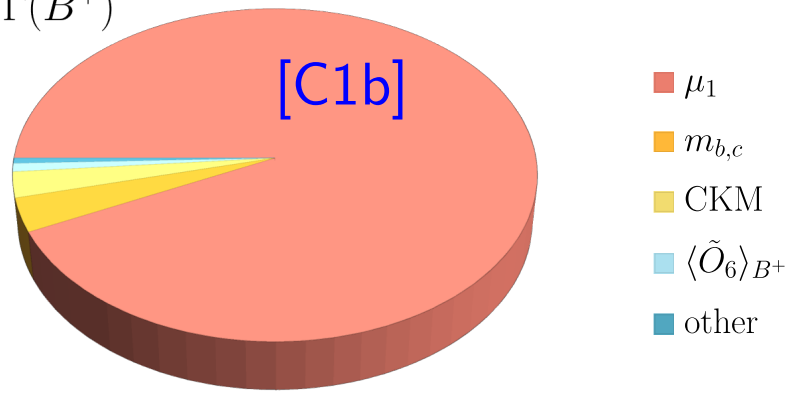


$\tau(B_s)/\tau(B_d)$ Scenario A

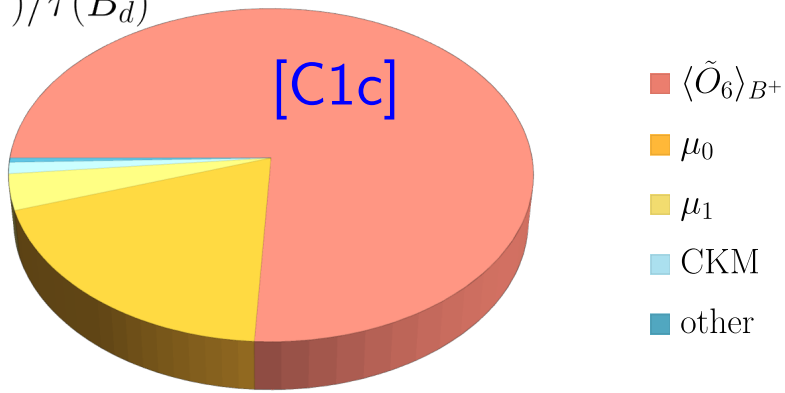


Theory uncertainties in B-meson lifetimes

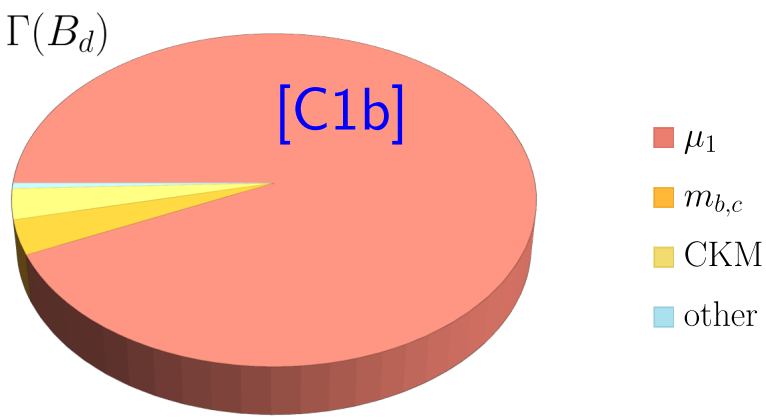
$\Gamma(B^+)$



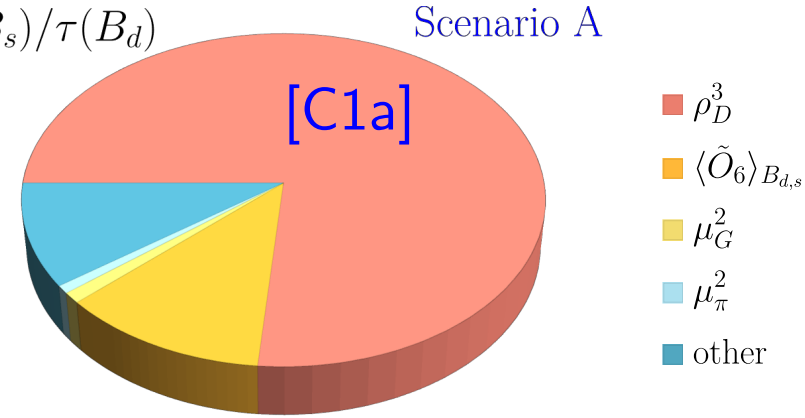
$\tau(B^+)/\tau(B_d)$



$\Gamma(B_d)$

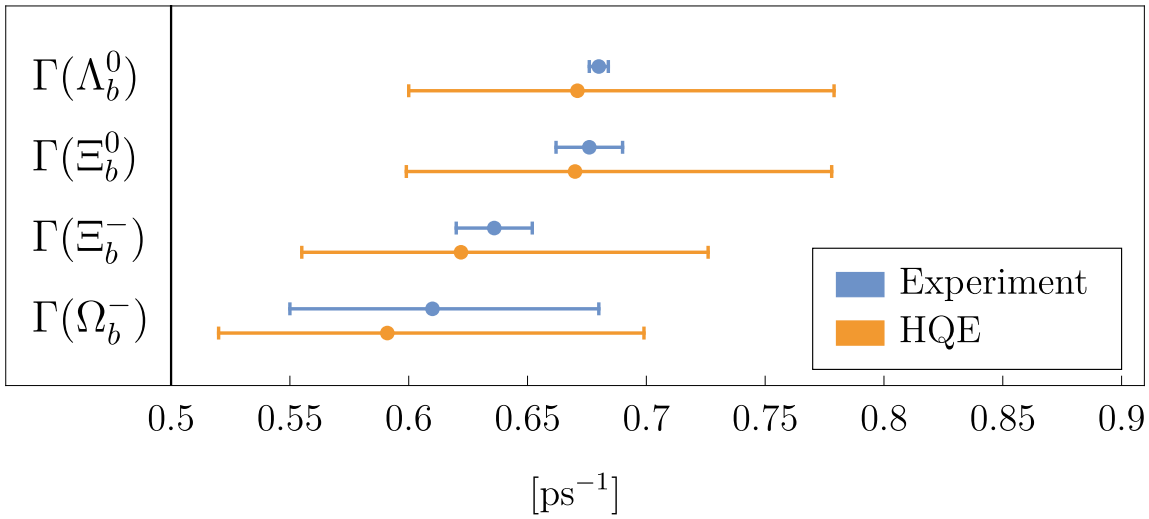


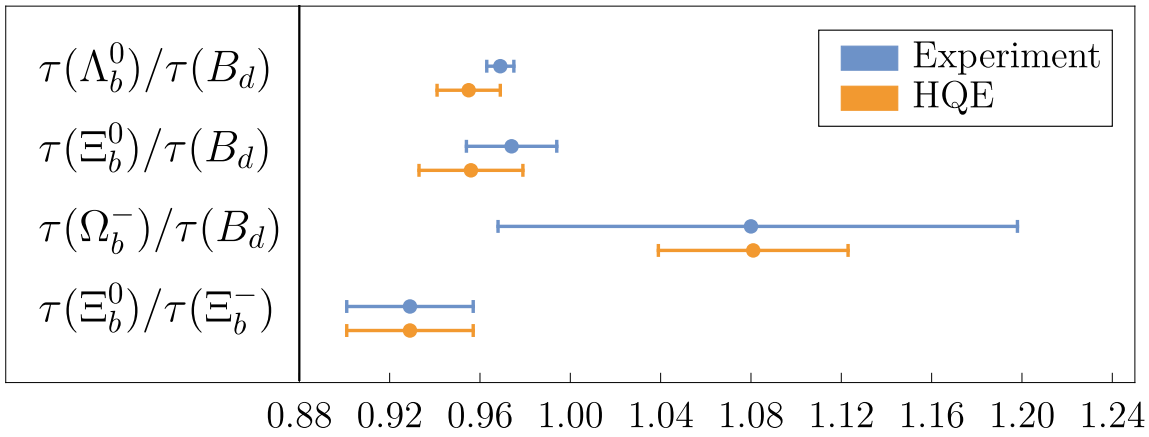
$\tau(B_s)/\tau(B_d)$ Scenario A



b-baryon lifetimes and ratios

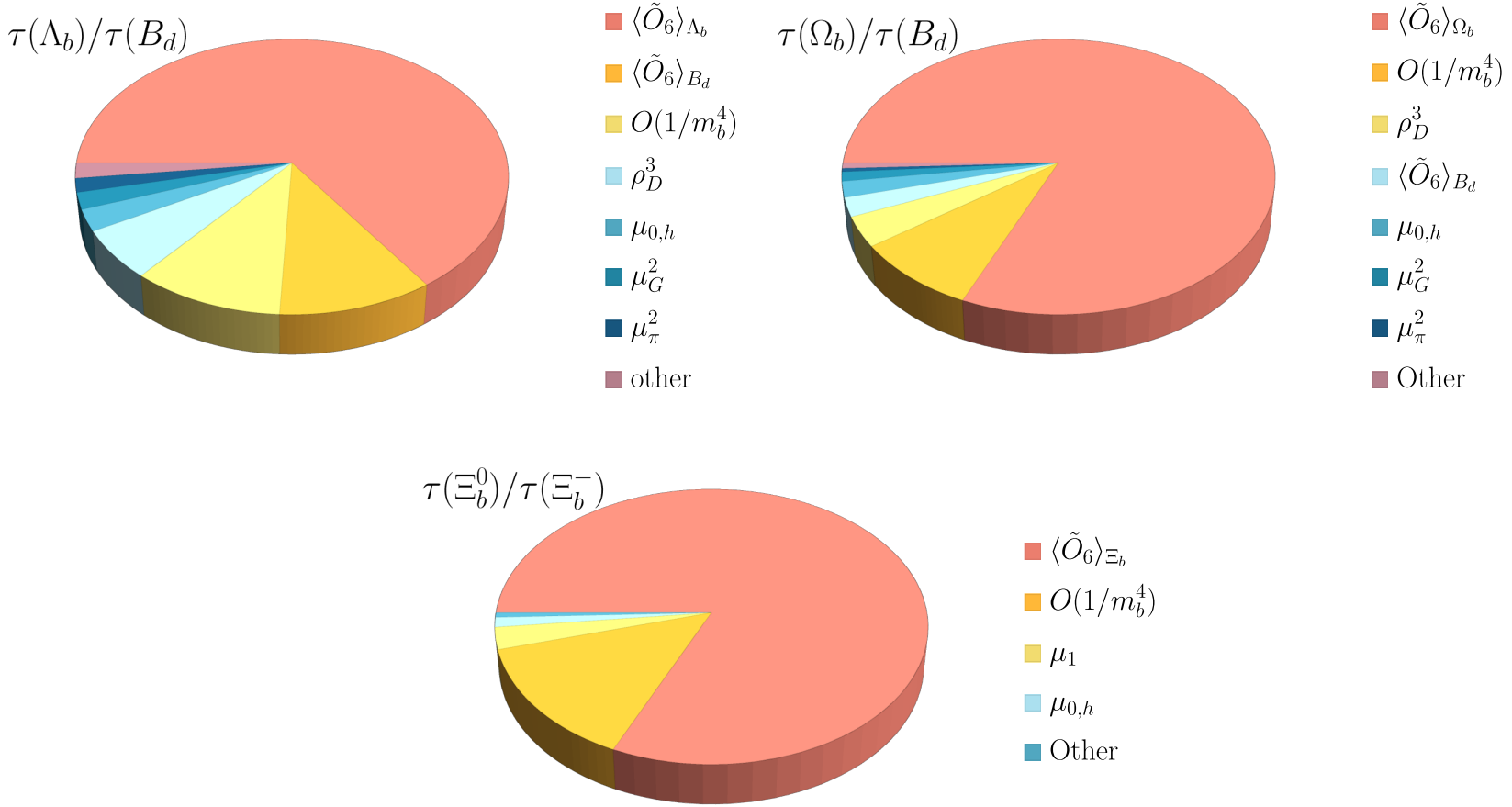
[Gratrex, Lenz, Melic, Nisandzic, Piscopo, AR (2301.07698)]





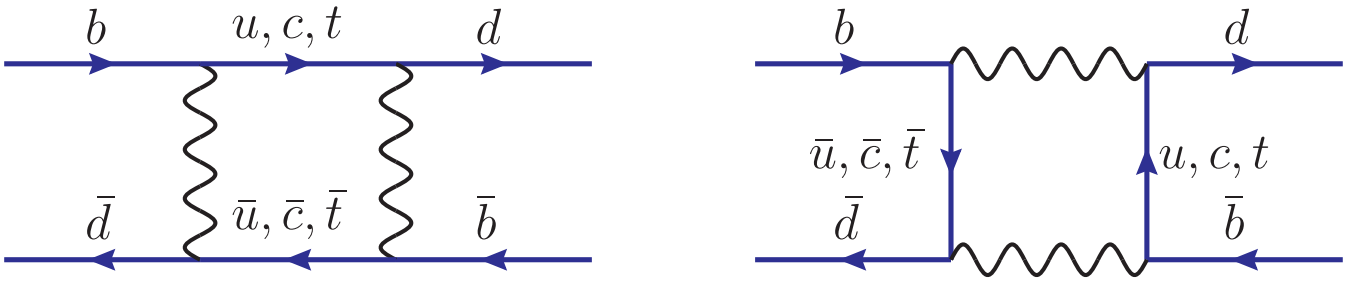
Theory uncertainties in b-baryon lifetimes

- Composition of uncertainties in total decay width of b-baryons similar to B-meson case



$B^0 - \bar{B}^0$ mixing

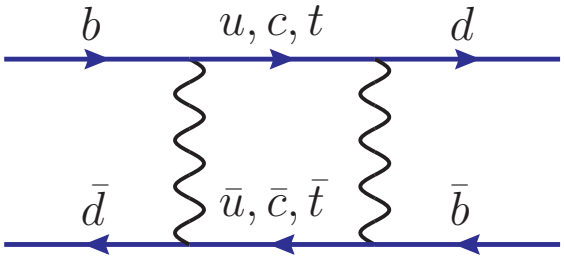
B-mixing: introduction and experimental values

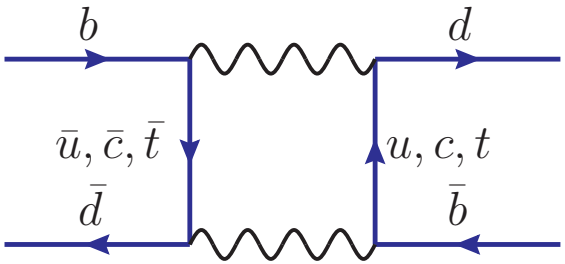


- Mass difference $\Delta M_q = M_{B_{q,(H)}} - M_{B_{q,(L)}} \approx 2|M_{12}^q|$ $\quad q = d, s$
- Decay rate difference $\Delta\Gamma_q = \Gamma_{B_{q,(L)}} - \Gamma_{B_{q,(H)}} \approx 2|\Gamma_{12}^q| \cos\phi_{12}^q$
- Flavour-specific CP asymmetries $a_{\rm fs}^q \approx \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi_{12}^q$ $\quad \phi_{12}^q = \arg(-M_{12}^q/\Gamma_{12}^q)$
- HFLAV values

$$\Delta M_d = 0.5065(19)\,\text{ps}^{-1} \qquad \Delta M_s = 17.765(6)\,\text{ps}^{-1}$$
$$\Delta\Gamma_d/\Gamma_d = 0.001(10) \qquad \Delta\Gamma_s = 0.083(5)\,\text{ps}^{-1}$$
$$a_{\rm sl}^d = -21(14)\cdot 10^{-4} \qquad a_{\rm sl}^s = -60(280)\cdot 10^{-5}$$

B-mixing: theory



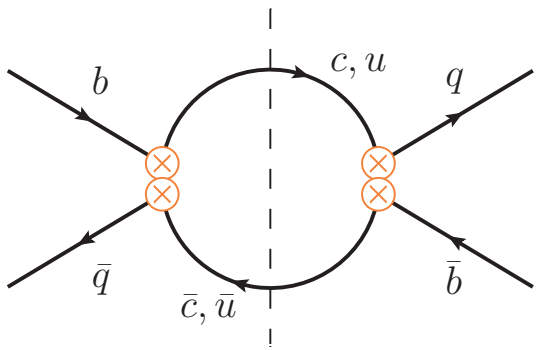


- Off-shell part of the box diagrams

$$M_{12}^q \sim |V_{tq}^* V_{tb}|^2 S_0(x_t)\, \hat{\eta}_B \underbrace{f_{B_q}^2 B_1^q}_{\sim \langle \tilde{\mathcal{O}}_6 \rangle_{B_q}}$$

- On-shell part of the box diagrams (within HQE)

$$\Gamma_{12}^q = 16\pi^2 \left(\tilde{\Gamma}_6^q \frac{\langle \tilde{\mathcal{O}}_6 \rangle_{B_q}}{m_b^3} + \tilde{\Gamma}_7^q \frac{\langle \tilde{\mathcal{O}}_7 \rangle_{B_q}}{m_b^4} + \ldots \right)$$



Theory status of B-mixing

In progress/planned by CRC

K - Karlsruhe S - Siegen A - Aachen

	LO	NLO	NNLO
$\tilde{\Gamma}_6$	✓	✓	✓[1] [C1b]
$\tilde{\Gamma}_7$	✓	K [C1b]	

[1] [Asatrian, Hovhannisyan, Nierste, Yeghiazaryan (1709.02160)]

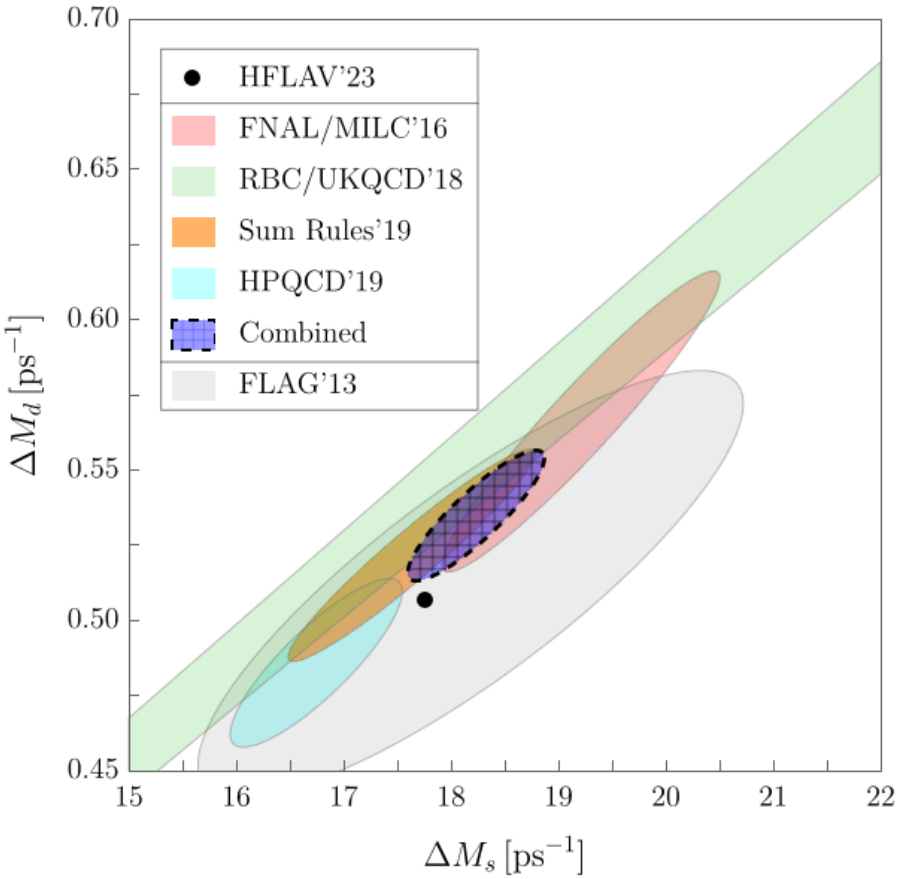
[Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, Yeghiazaryan (2006.13227)]

[Gerlach, Nierste, Shtabovenko, Steinhauser (2106.05979, 2202.12305, 2205.07907)]

m_c^2/m_b^2 corrections in progress (needed for $a_{\rm fs}^q$ at NNLO)

$\langle \tilde{Q}_6 \rangle_{B_d}$	HQET sum rule ✓
	Lattice QCD ✓ [C1c] S A
$\langle \tilde{Q}_6 \rangle_{B_s}$	HQET sum rule ✓
	Lattice QCD ✓ [C1c] S A
$\langle \tilde{Q}_7 \rangle_{B_d}$	VIA ✓
	HQET sum rule [C1c] S
$\langle \tilde{Q}_7 \rangle_{B_s}$	VIA ✓
	HQET sum rule [C1c] S

ΔM_q, $\Delta\Gamma_q$ and $a_{\rm fs}^q$



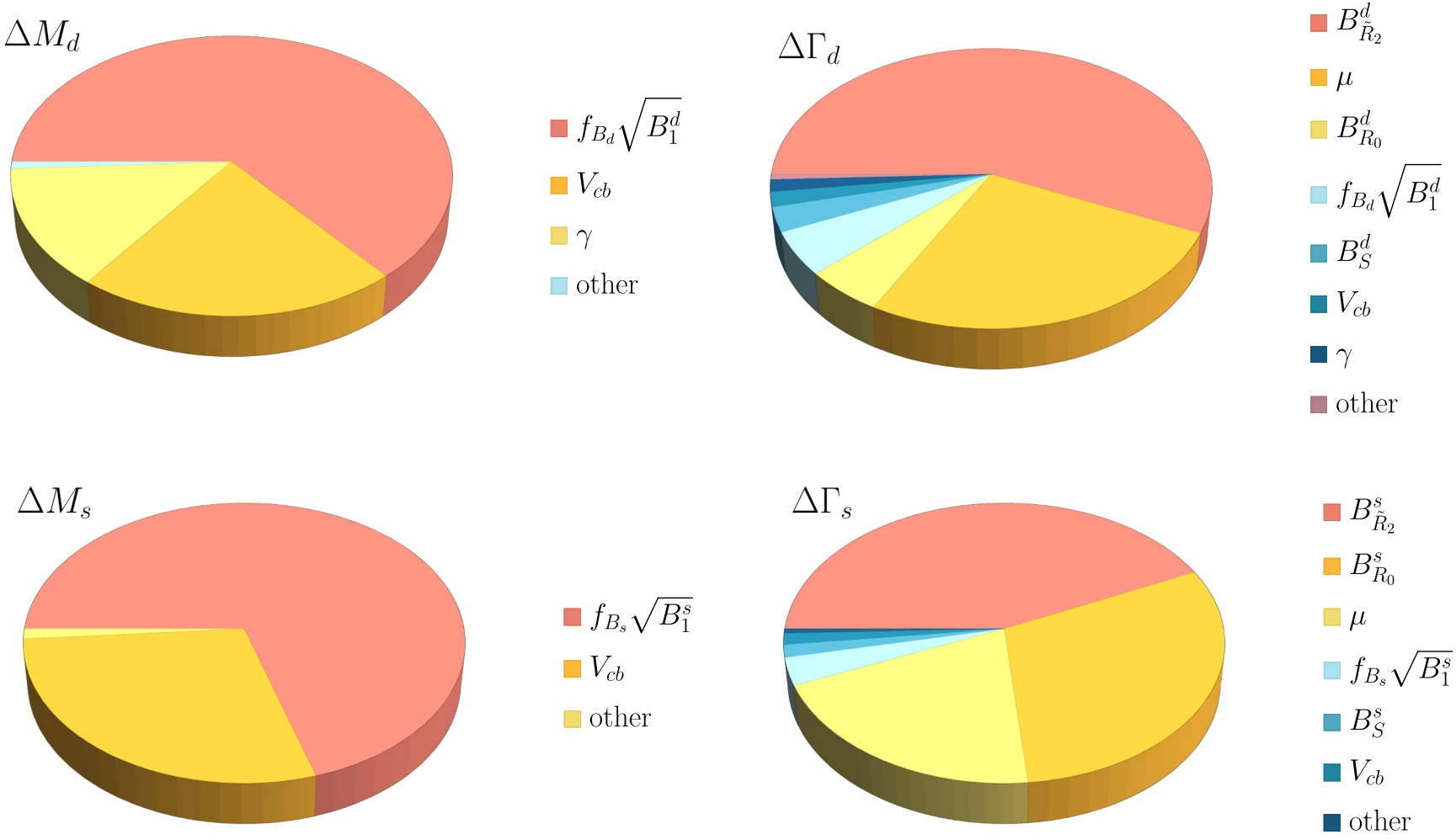
[ArXiv: 2402.04224]

$$\Delta\Gamma_d = (2.7 \pm 0.4) \cdot 10^{-3}\,\mathrm{ps}^{-1}$$
$$\Delta\Gamma_s = (9.1 \pm 1.5) \cdot 10^{-2}\,\mathrm{ps}^{-1}$$

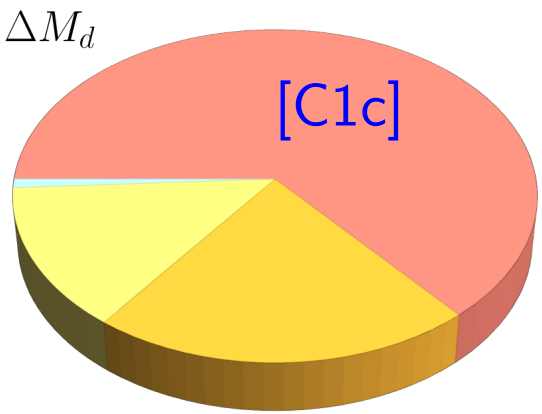
$$\frac{\Delta\Gamma_d}{\Delta M_d} = (50.5 \pm 6.8) \cdot 10^{-4}$$
$$\frac{\Delta\Gamma_s}{\Delta M_s} = (49.9 \pm 7.9) \cdot 10^{-4}$$

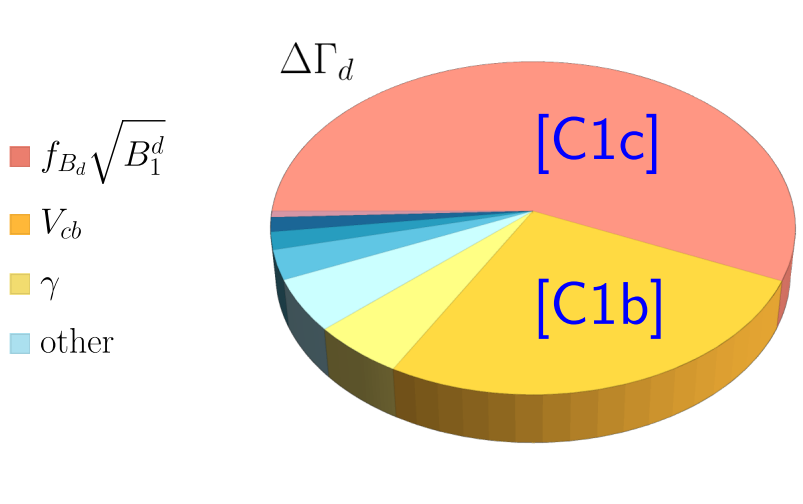
$$a_{sl}^d = -(5.1 \pm 0.5) \cdot 10^{-4}$$
$$a_{sl}^s = +(2.2 \pm 0.2) \cdot 10^{-5}$$

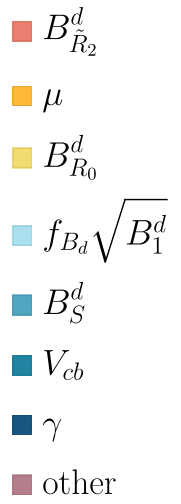
Theory uncertainties in ΔM_q and $\Delta\Gamma_q$

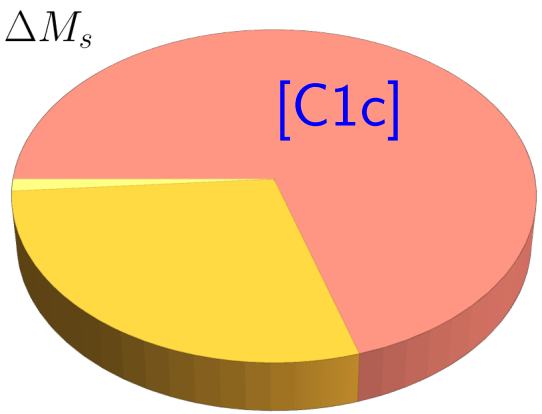


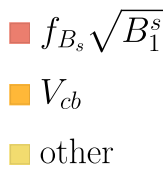
Theory uncertainties in ΔM_q and $\Delta\Gamma_q$

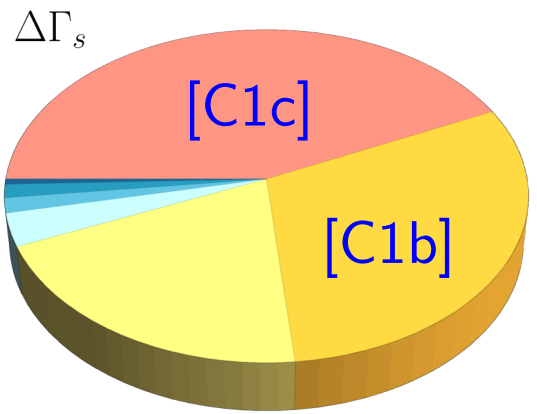


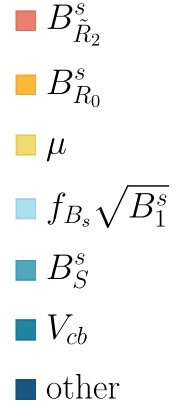




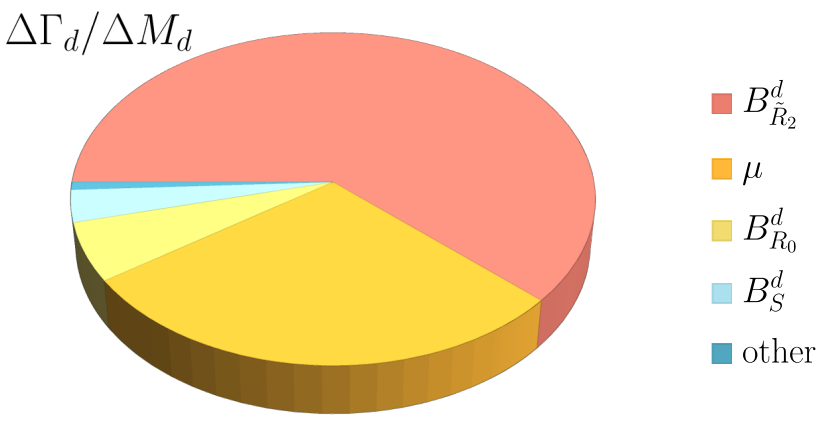


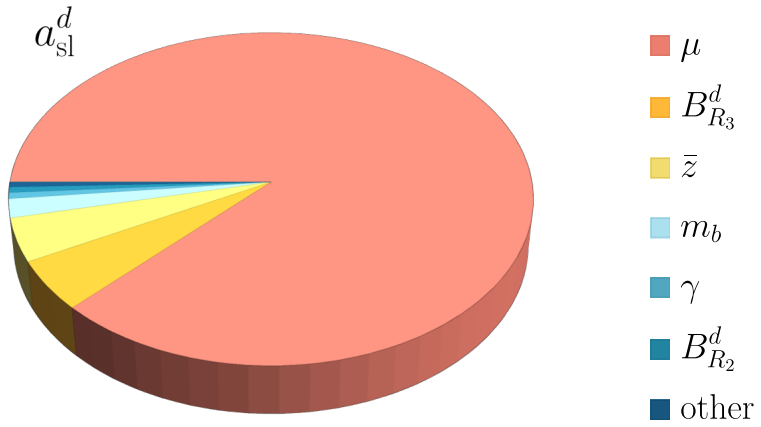


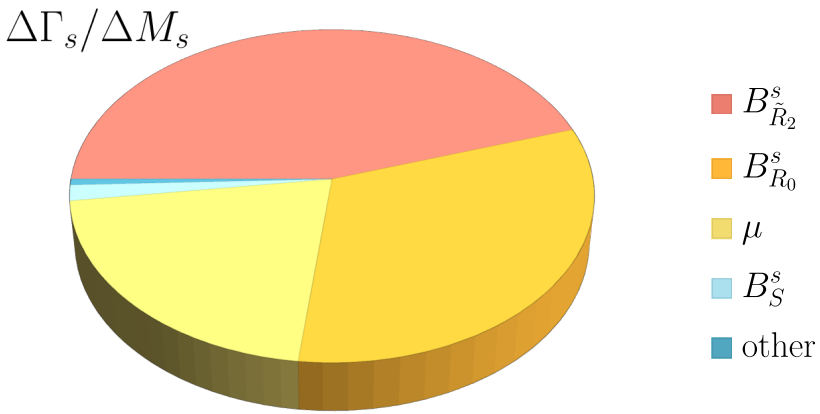


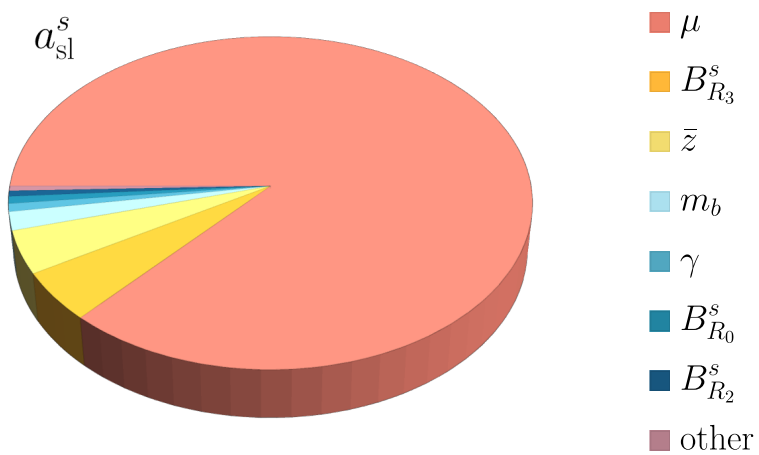


Theory uncertainties in $\Delta\Gamma_q/\Delta M_q$ and $a_{\rm sl}^q$

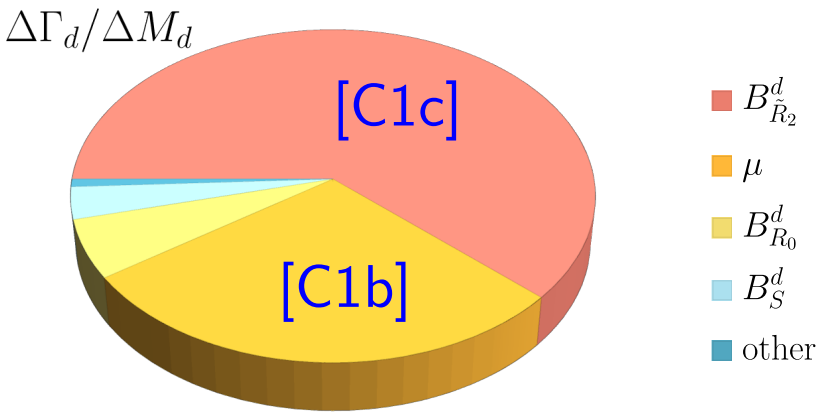


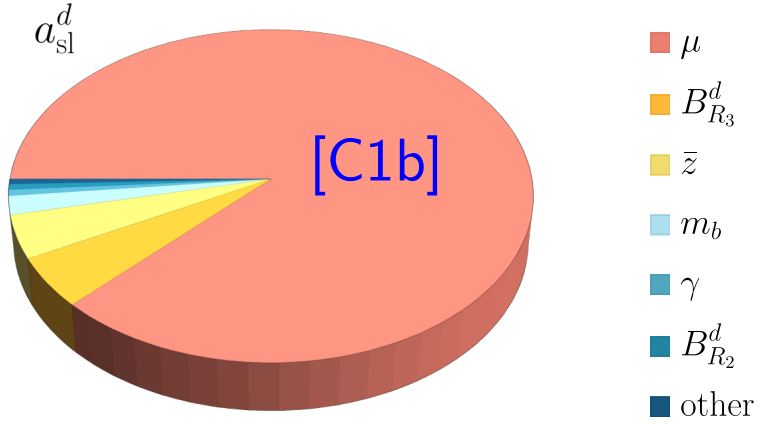


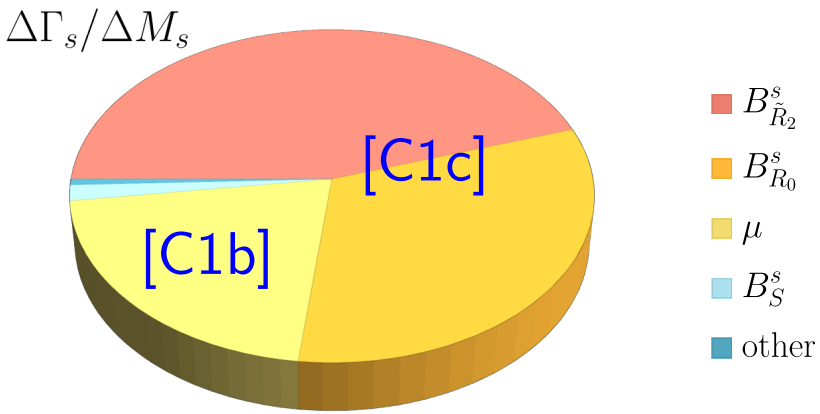


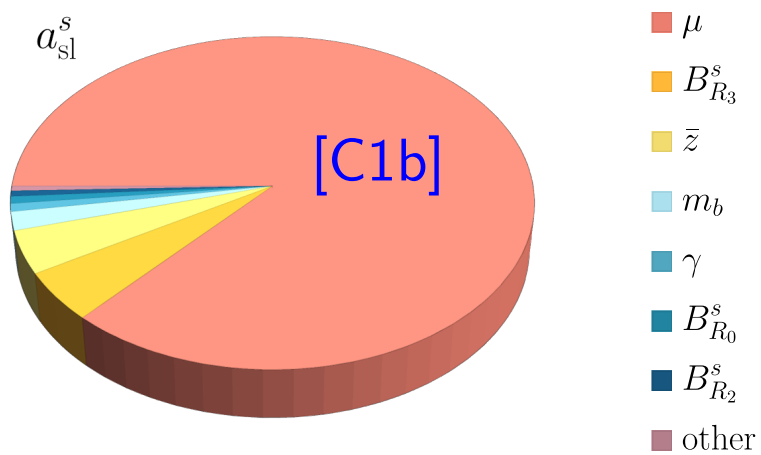


Theory uncertainties in $\Delta\Gamma_q/\Delta M_q$ and $a_{\rm sl}^q$









Conclusion and outlook

Outlook: b-hadron lifetimes

- HQE predictions for b-hadron lifetimes in good agreement with data
- Further improvements (within CRC)

	Non-leptonic		
	LO	NLO	N^{2}LO
Γ_3	✓	✓	🕒 [1]
Γ_5	✓	✓ 🕒 [2]	
Γ_6	✓	🕒	
Γ_7	🕒		
$\tilde{\Gamma}_6$	✓	✓	🕒
$\tilde{\Gamma}_7$	✓	🕒	

[C1b] Siegen, Karlsruhe

	B-mesons	b-baryons
$\langle \mathcal{O}_6 \rangle$	ρ_D^3 from fit ?	
$\langle \tilde{\mathcal{O}}_6 \rangle$	LQCD 🕒	LQCD ?
$\langle \tilde{\mathcal{O}}_7 \rangle$	HQET SR 🕒	

[C1c] Siegen, Aachen

🕒 in progress/planned (by CRC)

[1] almost finished [Egner, Fael, Schönwald, Steinhauser]

[2] including m_c effects in progress

Outlook: B-mixing

- Predictions for B-mixing observables also consistent with current data
- Further improvements (within CRC)

	LO	NLO	NNLO
$\tilde{\Gamma}_6$	✓	✓	✓ 🕒 [1]
$\tilde{\Gamma}_7$	✓	🕒	

[C1b] Karlsruhe

$\langle \tilde{Q}_6 \rangle_{B_d}$	✓ Lattice QCD 🕒
$\langle \tilde{Q}_6 \rangle_{B_s}$	✓ Lattice QCD 🕒
$\langle \tilde{Q}_7 \rangle_{B_d}$	HQET sum rule 🕒
$\langle \tilde{Q}_7 \rangle_{B_s}$	HQET sum rule 🕒

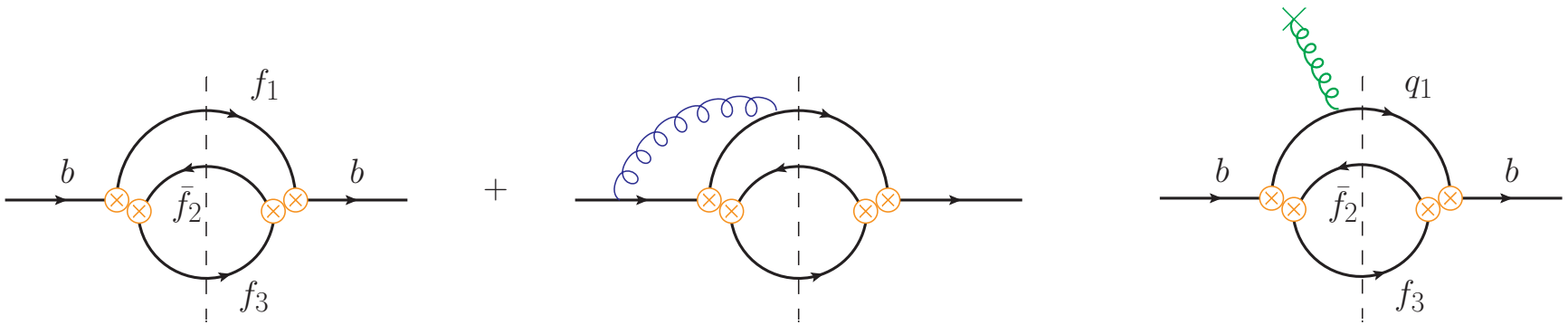
[C1c] Siegen, Aachen

🕒 in progress/planned (by CRC)

[1] m_c^2/m_b^2 corrections in progress (needed for $a_{\rm fs}^q$ at NNLO)

Backup

Two-quark contribution

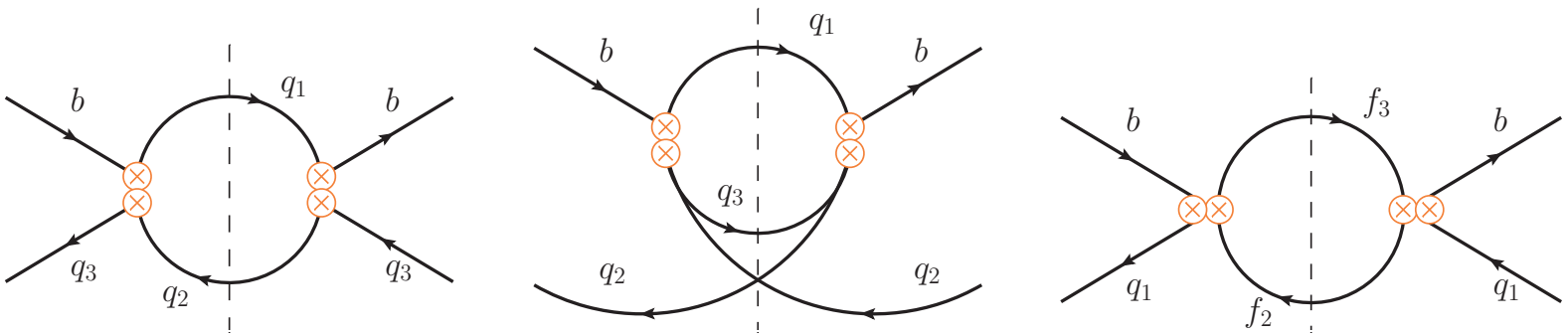


- Perturbatively calculable short-distance coefficients

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \dots$$

  - ⋆ Universal for all heavy hadrons containing a b-quark
- Matrix elements of two-quark operators $\langle \mathcal{O}_5 \rangle$, $\langle \mathcal{O}_6 \rangle$, ...
  - ⋆ Depend on b-hadron spectator quark(s)

Four-quark contribution



- Perturbatively calculable short-distance coefficients

$$\tilde{\Gamma}_i = \tilde{\Gamma}_i^{(0)} + \frac{\alpha_s}{4\pi}\tilde{\Gamma}_i^{(1)} + \dots$$

  - ⋆ Dependent on b-hadron spectator quark(s)
- Matrix elements of four-quark operators $\langle\tilde{\mathcal{O}}_6\rangle$, $\langle\tilde{\mathcal{O}}_7\rangle$, ...
  - ⋆ Depend on b-hadron spectator quark(s)

Definition of non-perturbative parameters

- Matrix elements of dimension-5 two-quark operators

$$
\begin{aligned}
2m_B\,\mu_\pi^2(\mathcal{B}) &= -\langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iD^\mu)b_v|\mathcal{B}(p_\mathcal{B})\rangle \\
2m_\mathcal{B}\,\mu_G^2(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|\mathcal{B}(p_\mathcal{B})\rangle
\end{aligned}
$$

- Matrix elements of dimension-6 two-quark operators

$$
\begin{aligned}
2m_\mathcal{B}\,\rho_D^3(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu)b_v|\mathcal{B}(p_\mathcal{B})\rangle \\
2m_\mathcal{B}\,\rho_{LS}^3(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})\,b_v|\mathcal{B}(p_\mathcal{B})\rangle
\end{aligned}
$$

- Dimension-6 four-quark operators

$$\mathcal{O}_1^q = (\bar{h}_v^i\gamma_\mu(1-\gamma_5)q^i)(\bar{q}^j\gamma^\mu(1-\gamma_5)h_v^j) \qquad \mathcal{O}_2^q = (\bar{h}_v^i(1-\gamma_5)q^i)(\bar{q}^j(1+\gamma_5)h_v^j)$$

$$\mathcal{O}_3^q = (\bar{h}_v^i\gamma_\mu(1-\gamma_5)q^j)(\bar{q}^j\gamma^\mu(1-\gamma_5)h_v^i) \qquad \mathcal{O}_4^q = (\bar{h}_v^i(1-\gamma_5)q^j)(\bar{q}^j(1+\gamma_5)h_v^i)$$

- Matrix elements of dimension-6 four-quark operators for B-mesons

$$\langle B_q|\mathcal{O}_n^q|B_q\rangle = f_{B_q}^2 M_{B_q}^2\, B_n(B_q)$$

Definition of non-perturbative parameters

- Matrix elements of dimension-6 four-quark operators for b-baryons in non-relativistic constituent quark model (NRCQM)

$$\frac{\langle \Lambda_b|\mathcal{O}_1^q|\Lambda_b\rangle}{2M_{\Lambda_b}} = -y_{\tilde{q}}\,\frac{4}{3}\,\frac{M_{\Sigma_b^*}-M_{\Sigma_b}}{M_{B^*}-M_B}\,|\Psi^B(0)|^2$$

$$\frac{\langle \Xi_b^0|\mathcal{O}_1^u|\Xi_b^0\rangle}{2M_{\Xi_b}} = \frac{\langle \Xi_b^-|\mathcal{O}_1^d|\Xi_b^-\rangle}{2M_{\Xi_b}} = -y_{\tilde{q}}\,\frac{4}{3}\,\frac{M_{\Xi_b^*}-M_{\Xi_b'}}{M_{B^*}-M_B}\,|\Psi^B(0)|^2$$

$$\frac{\langle \Xi_b^-|\mathcal{O}_1^s|\Xi_b^-\rangle}{2M_{\Xi_b}} = \frac{\langle \Xi_b^0|\mathcal{O}_1^s|\Xi_b^0\rangle}{2M_{\Xi_b}} = -y_s\,\frac{4}{3}\,\frac{M_{\Xi_b^*}-M_{\Xi_b'}}{M_{B_s^*}-M_{B_s}}\,|\Psi^{B_s}(0)|^2$$

$$\frac{\langle \Omega_b^-|\mathcal{O}_1^s|\Omega_b^-\rangle}{2M_{\Omega_b}} = -y_s\,6\,\frac{4}{3}\,\frac{M_{\Omega_b^*}-M_{\Omega_b}}{M_{B_s^*}-M_{B_s}}\,|\Psi^{B_s}(0)|^2$$

$$|\Psi^{B_q}(0)|^2 = \frac{F_{B_q}^2(\mu_0)}{12} \qquad y_q = \frac{m_b^b\, m_q^b}{m_b^m\, m_q^m} \qquad \langle\mathcal{B}|\mathcal{O}_{2,3,4}^q|\mathcal{B}\rangle \sim \langle\mathcal{B}|\mathcal{O}_1^q|\mathcal{B}\rangle$$