PRECISION CALCULATIONS AT LEADING AND NEXT-TO-LEADING POWER

[GUIDO BELL]

Universität Siegen

P H

CPPS

Overview

Project B2a: Automated calculations in SCET

- NNLO soft, jet and beam functions
- high-precision resummations
- phenomenological applications

leading power

⇒ automation

Project B1e: Power corrections in collider processes

- structure of non-perturbative corrections
- resummations at next-to-leading power
- power corrections to slicing variables

next-to-leading power

⇒ new concepts

Publications

Project B2a: Automated calculations in SCET

2312.06496	N3LL resummation of one-jettiness for Z-boson plus jet production at hadron colliders	Alioli, GB, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, Rahn
2312.11626	The NNLO soft function for N-jettiness in hadronic collisions	GB, Dehnadi, Mohrmann, Rahn
2312.14089	FeynCalc 10: Do multiloop integrals dream of computer codes?	Shtabovenko, Mertig, Orellana
to appear	The NNLO gluon beam function for jet-veto resummation	GB, Brune, Das, Wald

Project B1e: Power corrections in collider processes

2302.02729	Linear power corrections to single top production processes at the LHC	Makarov, Melnikov, Nason, Ozcelik
2307.02286	Subleading effects in soft-gluon emission at one-loop in massless QCD	Czakon, Eschment, Schellenberger
2308.05526	Linear power corrections to top quark pair production in hadron collisions	Makarov, Melnikov, Nason, Ozcelik
2309.08410	Soft-overlap contribution to $B_c \to \eta_c$ form factors: diagrammatic resummation of double logarithms	GB, Böer, Feldmann, Horstmann, Shtabovenko
2311.03990	Effects of Renormalon Scheme and Perturbative Scale Choices on Determinations of the Strong Coupling from e^+e^- Event Shapes	GB, Lee, Makris, Talbert, Yan

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N-jettiness

The NNLO soft function for N-jettiness in hadronic collisions

Guido Bell[a], Bahman Dehnadi[b], Tobias Mohrmann[a] and Rudi Rahn[c]

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[b] *Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany*

[c] *Department of Physics and Astronomy, University of Manchester, Manchester, M13 9PL, United Kingdom*

Abstract

We compute the N-jettiness soft function in hadronic collisions to next-to-next-to-leading order (NNLO) in the strong-coupling expansion. Our calculation is based on an extension of the `SoftSERVE` framework to soft functions that involve an arbitrary number of lightlike Wilson lines. We present numerical results for 1-jettiness and 2-jettiness, and illustrate that our formalism carries over to a generic number of jets by calculating a few benchmark points for 3-jettiness. We also perform a detailed analytic study of the asymptotic behaviour of the soft-function coefficients at the edges of phase space, when one of the jets becomes collinear to another jet or beam direction, and comment on previous calculations of the N-jettiness soft function.

2312.11626

N^3LL resummation of one-jettiness for Z-boson plus jet production at hadron colliders

Simone Alioli,[1] Guido Bell,[2] Georgios Billis,[1] Alessandro Broggio,[3] Bahman Dehnadi,[4] Matthew A. Lim,[5] Giulia Marinelli,[1,4] Riccardo Nagar,[1] Davide Napoletano,[1] and Rudi Rahn[6]

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[4] *Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany*

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(Dated: December 12, 2023)

We present the resummation of one-jettiness for the colour-singlet plus jet production process $pp \to (\gamma^*/Z \to \ell^+\ell^-) + \text{jet}$ at hadron colliders up to the fourth logarithmic order (N^3LL). This is the first resummation at this order for processes involving three coloured partons at the Born level. We match our resummation formula to the corresponding fixed-order predictions, extending the validity of our results to regions of the phase space where further hard emissions are present. This result paves the way for the construction of next-to-next-to-leading order simulations for colour-singlet plus jet production matched to parton showers in the GENEVA framework.

I. INTRODUCTION

The study of the production of a colour singlet system at large recoil is of crucial importance for the physics programme at the Large Hadron Collider. In particular, theoretical predictions for γ^*/Z+jet production are needed at higher precision to match the accuracy reached by experimental measurements of the Z boson transverse momentum (q_T) spectrum. Combining next-to-next-to-leading order (NNLO) predictions for γ^*/Z+jet [1–6] with q_T resummation [7–14] provides an accurate description of this distribution over the whole kinematic range and can be used to extract α_s [15] and as a background for new physics searches.

The one-jettiness variable is a suitable event shape for colour singlet (L) + jet production which does not suffer

We define the one-jettiness resolution variable as [16]

$$\mathcal{T}_1 = \sum_k \min\left\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\right\}, \qquad (1)$$

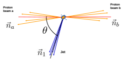
with $q_{a,b} = x_{a,b} E_{\rm cm}\, n_{a,b}/2 = E_{a,b}\, n_{a,b}$ and $q_J = E_J\, n_J$, where E_J is the jet energy. The beam directions are $n_{a,b} = (1,0,0,\pm 1)$ while the massless jet direction is $n_J = (1, \vec{n}_J)$. In eq. (1) the sum runs over the four-momenta p_k of all partons which are part of the hadronic final state. We use a geometric measure where $Q_i = 2\rho_i E_i$ with $i = a,b,J$ is proportional to the energy of the beam or jet momenta. This particular choice is preferable because it is independent of the total jet energy, but makes the one-jettiness definition frame dependent. Results in frames that differ by a longitudinal boost can be obtained by making different choices for

2312.06496

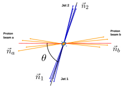
N-jettiness

Definition

$$\mathcal{T}_N = \sum_i \min\left\{ n_a \cdot k_i,\ n_b \cdot k_i,\ n_1 \cdot k_i, \ldots,\ n_N \cdot k_i \right\}$$



$\mathcal{T}_1 \simeq 0$



$\mathcal{T}_1 = \mathcal{O}(1)$

$\mathcal{T}_2 \simeq 0$

Motivation

- slicing variable for higher-order calculations
- jet resolution variable in `Geneva` MC framework
- jet substructure studies

Factorisation

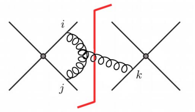
[Stewart, Tackmann, Waalewijn 10]

$$\frac{d\sigma}{d\mathcal{T}_N} = \sum_{i,j,\{k_n\}} B_i \otimes B_j \otimes \prod_{n=1}^{N} J_{k_n} \otimes \mathrm{tr}\left[H_{ij\to\{k_n\}} * S_{ij\to\{k_n\}} \right] + \mathcal{O}(\mathcal{T}_N)$$

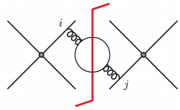
$\Rightarrow$ need NNLO soft function for generic number of jets N

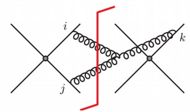
Calculation

$$S(\tau,\mu) = \sum_{i\in X} \langle 0|\underbrace{(S_{n_1} S_{n_2} \dots S_{n_N})^\dagger}_{\text{soft Wilson lines}} |X\rangle\langle X| S_{n_1} S_{n_2} \dots S_{n_N} |0\rangle \underbrace{\mathcal{M}(\tau;\{k_i\})}_{\text{N-jettiness measure}}$$



$$S^{(2,\mathrm{RV})}(\varepsilon) = C_A \sum_{i\neq j} \mathbf{T}_i\cdot\mathbf{T}_j\, S_{ij}^{(2,\mathrm{Re})}(\varepsilon) + \sum_{i\neq j\neq k} (\lambda_{ij}-\lambda_{ip}-\lambda_{jp})\, f_{ABC}\, \mathbf{T}_i^A\, \mathbf{T}_j^B\, \mathbf{T}_k^C\, S_{ijk}^{(2,\mathrm{Im})}(\varepsilon)$$



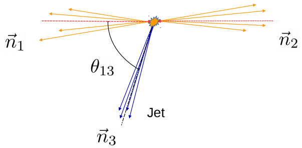


$$S^{(2,RR)}(\varepsilon) = T_F n_f \sum_{i\neq j} \mathbf{T}_i\cdot\mathbf{T}_j\, S_{ij}^{(2,q\bar{q})}(\varepsilon) + C_A \sum_{i\neq j} \mathbf{T}_i\cdot\mathbf{T}_j\, S_{ij}^{(2,gg)}(\varepsilon) + \frac{1}{4}\sum_{i\neq j}\sum_{k\neq l} \{\mathbf{T}_i\cdot\mathbf{T}_j\,,\,\mathbf{T}_k\cdot\mathbf{T}_l\} S_{ij}^{(1)}(\varepsilon)\, S_{kl}^{(1)}(\varepsilon)$$

$\Rightarrow$ extent `SoftSERVE` strategy to non-back-to-back Wilson lines

[GB, Rahn, Talbert 18]

1-jettiness

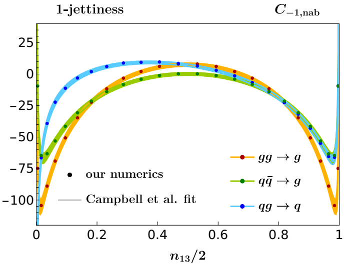


- one kinematic variable

  $n_{13} \equiv n_1 \cdot n_3 = 1 - \cos\theta_{13}$

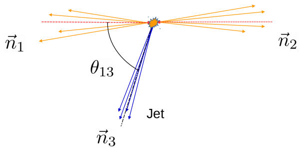
$\Rightarrow$ scan 24 configurations

Two-loop coefficient in distribution space



- very good agreement with previous calculations [Boughezal, Liu, Petriello 15; Campbell, Ellis, Mondini, Williams 17]
- can one understand the divergent behaviour at the endpoints?

1-jettiness



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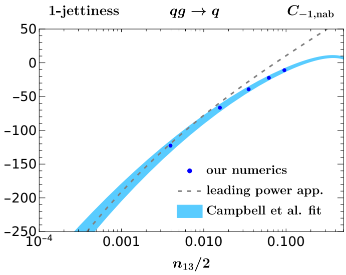
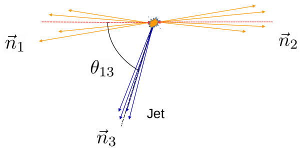
Two-loop coefficient in distribution space



- analytic method-of-regions analysis

  to derive the leading-power asymptotics

1-jettiness



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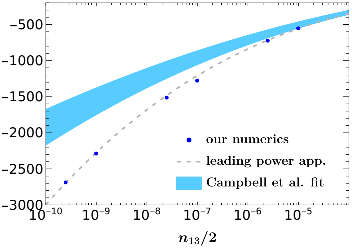
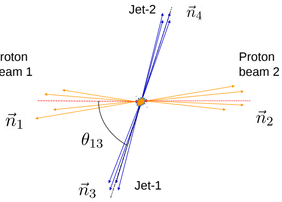


- analytic method-of-regions analysis

  to derive the leading-power asymptotics
- numerics stable in deep endpoint region
- relevant for soft functions that are

  defined in highly boosted frame

2-jettiness

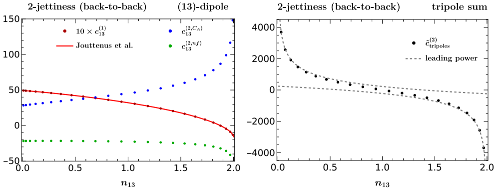


- three kinematic variables

  $\theta_{13}, \theta_{14}, \varphi_4 \quad \rightarrow \quad n_{13}, n_{14}, n_{34}$

$\Rightarrow$ scan 28.776 configurations

Two-loop coefficient in Laplace space

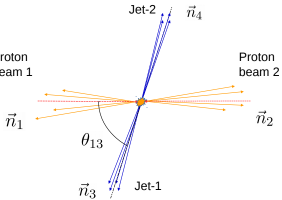


- tripoles differ from [Jin, Liu 19]
- results recently confirmed

  by independent calculation

  [Agarwal, Melnikov, Pedron 24]

2-jettiness



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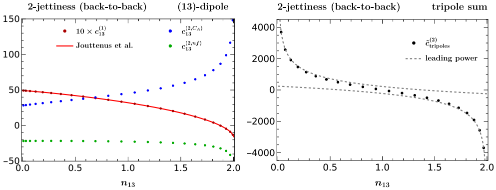

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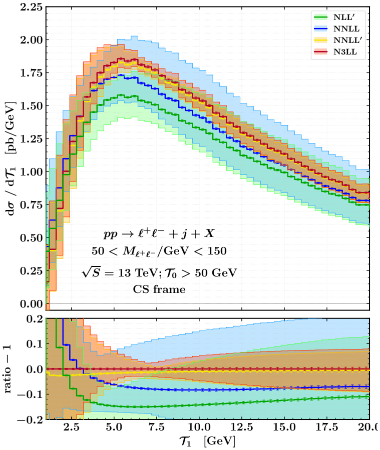
  [Agarwal, Melnikov, Pedron 24]

3-jettiness involves 10 dipoles, 60 tripoles and $\sim 45 \cdot 10^6$ configurations . . .

Z+jet production

First N3LL resummation of a 1-jet observable



- N3LL matched to NLO prediction for $\gamma^*/Z + 2$ jets production
- enables the construction of NNLO+PS generators for processes with one jet within the `Geneva` framework
- similar implementation in `MiNNLO-PS` generator in progress

[Ebert, Rottoli, Wiesemann, Zanderighi, Zanoli 24]

e^+e^- event shapes

Effects of Renormalon Scheme and Perturbative Scale Choices on Determinations of the Strong Coupling from e^+e^- Event Shapes

Guido Bell,[1,*] Christopher Lee,[2,†] Yiannis Makris,[2,3] Jim Talbert,[4,2,‡] and Bin Yan[2,5,§]

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[2] *Theoretical Division, Los Alamos National Laboratory, P.O. Box 1663, MS B283, Los Alamos, NM 87545, USA*

[3] *INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy*

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We study the role of renormalon cancellation schemes and perturbative scale choices in extractions of the strong coupling constant $\alpha_s(m_Z)$ and the leading non-perturbative shift parameter Ω_1 from resummed predictions of the e^+e^- event shape thrust. We calculate the thrust distribution to N^{3}LL$'$ resummed accuracy in Soft-Collinear Effective Theory (SCET) matched to the fixed-order $\mathcal{O}(\alpha_s^2)$ prediction, and perform a new high-statistics computation of the $\mathcal{O}(\alpha_s^3)$ matching in EERAD3, although we do not include the latter in our final α_s fits due to some observed systematics that require further investigation. We are primarily interested in testing the phenomenological impact sourced from varying amongst three renormalon cancellation schemes and two sets of perturbative scale profile choices. We then perform a global fit to available data spanning center-of-mass energies between 35-207 GeV in each scenario. Relevant subsets of our results are consistent with prior SCET-based extractions of $\alpha_s(m_Z)$, but we are also led to a number of novel observations. Notably, we find that the combined effect of altering the renormalon cancellation scheme and profile parameters can lead to few-percent-level impacts on the extracted values in the $\alpha_s - \Omega_1$ plane, indicating a potentially important systematic theory uncertainty that should be accounted for. We also observe that fits performed over windows dominated by dijet events are typically of a higher quality than those that extend into the far tails of the distributions, possibly motivating future fits focused more heavily in this region. Finally, we discuss how different estimates of the three-loop soft matching coefficient $c_{\tilde{S}}^3$ can also lead to measurable changes in the fitted $\{\alpha_s, \Omega_1\}$ values.

2311.03990

α_s determination

Event-shape fits tend to give low value of α_s

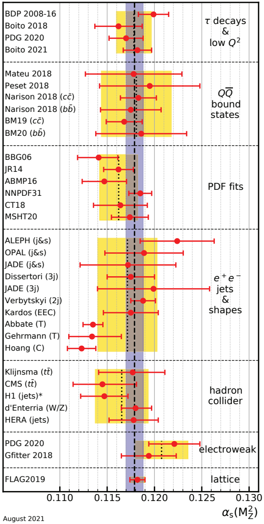
$$\alpha_s(M_Z) = \begin{cases} 0.1179 \pm 0.0009 & \text{PDG world average} \\ 0.1135 \pm 0.0011 & \text{Thrust} \\ 0.1123 \pm 0.0015 & \text{C-parameter} \end{cases}$$

Recent focus: Non-pert. effects from 3-jet configurations

- C-parameter in symmetric 3-jet limit [Luisoni, Monni, Salam 20]
- general renormalon analysis [Caola, Ferrario Ravasio, Limatola, Melnikov, Nason 21; + Ozcelik 22]
- implementation in α_s fit [Nason, Zanderighi 23]

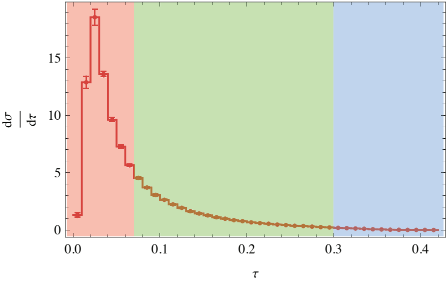
Our goal: scrutinise if the systematic uncertainties

of the 2-jet predictions are under control



Perturbative treatment

Thrust distribution



peak region

- very sensitive to non-perturbative effects

tail region

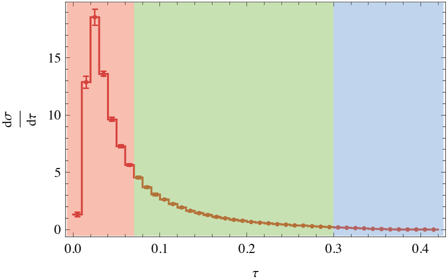
- resummation of Sudakov logarithms

far-tail region

- fixed-order QCD, but few events

Perturbative treatment

Thrust distribution



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tail region

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far-tail region

- fixed-order QCD, but few events

N^3LL' resummation using SCET technology

- computation of missing 3-loop soft constant on-going [Baranowski, Delto, Melnikov, Wang 22; + Pikelner 24; Chen, Feng, Jia, Liu 22]

Matched to $\mathcal{O}(\alpha_s^2)$ fixed-order prediction

- high-statistics runs reveal instabilities in `EERAD3` [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 14]

Non-perturbative treatment

Gapped shape function

$$S(k,\mu_S) = \int dk' \; \underbrace{S_{PT}(k-k',\mu_S)}_{\text{perturbative soft function}} \; \underbrace{f_{\rm mod}(k'-2\overline{\Delta})}_{\text{shape-function model}}$$

- gap parameter $\overline{\Delta}$ models minimal soft momentum of hadronic final state

⇒ S_{PT} and $\overline{\Delta}$ suffer from renormalon ambiguities in the $\overline{\text{MS}}$ scheme [Hoang, Stewart 07]

Non-perturbative treatment

Gapped shape function

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- gap parameter $\overline{\Delta}$ models minimal soft momentum of hadronic final state

$\Rightarrow$ S_{PT} and $\overline{\Delta}$ suffer from renormalon ambiguities in the $\overline{\rm MS}$ scheme [Hoang, Stewart 07]

Renormalon subtraction

$$\overline{\Delta} = \underbrace{\Delta(\mu_\delta,\mu_R)}_{\text{renormalon free}} + \underbrace{\delta(\mu_\delta,\mu_R)}_{\text{cancels renormalon ambiguity of } S_{PT}}$$

$\Rightarrow$ class of schemes that is free of leading renormalon [Bachu, Hoang, Mateu, Pathak, Stewart 20]

$$\frac{d^n}{d(\ln\nu)^n} \ln\left[\widetilde{S}_{PT}(\nu,\mu_\delta)\, e^{-2\nu\delta(\mu_\delta,\mu_R)}\right]_{\nu=\xi/\mu_R} = 0$$

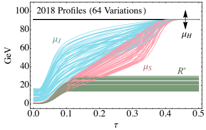
Scheme choices

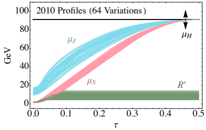
Two renormalon schemes

R Scheme: $\{n, \xi, \mu_\delta, \mu_R\} = \{1, e^{-\gamma_E}, \mu_S, R\}$ used in previous α_s fits

R* Scheme: $\{n, \xi, \mu_\delta, \mu_R\} = \{1, e^{-\gamma_E}, R^\star, R^\star\}$ new scheme

Two perturbative scale choices





2018 scales more conservative than those used in previous fits

Scheme choices

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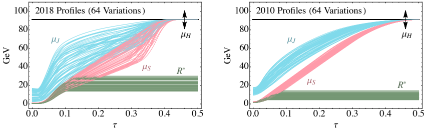
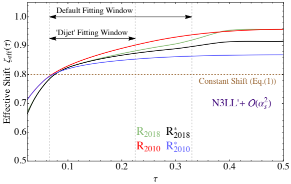
Two perturbative scale choices



2018 scales more conservative

than those used in previous fits

Effective shift of perturbative distribution

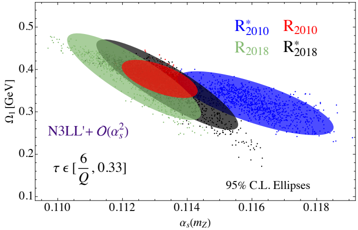


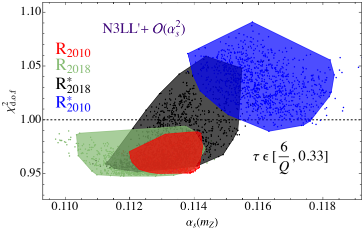
corresponds to $\lesssim$ 10% modification

of leading 2-jet power correction

Results

χ^2 fit to global thrust data with $Q \in [35, 207]$ GeV





- siginificant scheme dependence
- spread of $\{\alpha_s, \Omega_1\}$ values much larger than R_{2010} ellipse would suggest
- we also find that fits which focus more on dijet events show better fit quality

$\Rightarrow$ sign of additional systematic theory uncertainties?

Next-to-leading power

Soft-overlap contribution to $B_c \to \eta_c$ form factors: diagrammatic resummation of double logarithms

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Using diagrammatic resummation techniques, we investigate the double-logarithmic series of the "soft-overlap" contribution to $B_c \to \eta_c$ transition form factors at large hadronic recoil, assuming the scale hierarchy $m_b \gg m_c \gg \Lambda_{\rm QCD}$. In this case, the hadronic bound states can be treated in the non-relativistic approximation and the relevant hadronic matrix elements can be computed perturbatively. This setup defines one of the simplest examples to study the problem of endpoint singularities appearing in the factorization of exclusive B-decay amplitudes. We find that the leading double logarithms arise from a peculiar interplay of soft-quark "endpoint logarithms" from ladder diagrams with energy-ordered spectator-quark propagators, as well as

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FeynCalc 10: Do multiloop integrals dream of computer codes?

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Abstract

In this work we report on a new version of FeynCalc, a Mathematica package widely used in the particle physics community for manipulating quantum field theoretical expressions and calculating Feynman diagrams. Highlights of the new version include greatly improved capabilities for doing multiloop calculations, including topology identification and minimization, optimized tensor reduction, rewriting of scalar products in terms of inverse denominators, detection of equivalent or scaleless loop integrals, derivation of Symanzik polynomials, Feynman parametric as well as graph representation for master integrals and initial support for handling differential equations and iterated integrals. In addition to that, the new release also features completely rewritten routines for color algebra simplifications, inclusion of symmetry relations between arguments of

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Next-to-leading power

Significant interest in extending SCET technology to subleading power

- threshold resummation [Beneke et al 18-20]
- bottom-induced $H \to \gamma\gamma$ and $gg \to H$ [Neubert et al 19-22]
- thrust distribution [Stewart et al 19; Beneke et al 22]
- electron-muon backward scattering [GB, Böer, Feldmann 22]
- leptonic B decays [Feldmann, Gubernari, Huber, Seitz 22; Cornella, König, Neubert 22]
- resolved contribution in $B \to X_s\gamma$ [Hurth, Szafron 23]

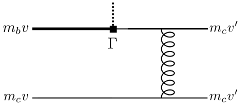
Key problem

- naive factorisation may lead to endpoint-divergent convolutions $\int_0^1 \mathrm{d}z\; h(z)\, j(z) = \infty$

$\Rightarrow$ will resort to diagramatic techniques in the following

$B_c \to \eta_c$ form factors

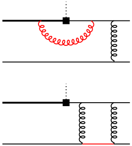
Heavy-to-light transition in non-relativistic approximation $(m_b \gg m_c \gg \Lambda_{\rm QCD})$



$$F(\gamma) = \frac{1}{2E_\eta} \langle \eta_c(p_\eta) | \, \bar{c}\Gamma b \, | B_c(p_B) \rangle$$

Double logarithmic enhancement at large recoil $\gamma \equiv v \cdot v' = \mathcal{O}(m_b/m_c)$

$$F(\gamma) \propto 1 + \frac{\alpha_s}{4\pi} \Big\{ \underbrace{- C_F}_{\text{soft gluons}} + \underbrace{\frac{9}{5} C_F - \frac{1}{5} C_A}_{\text{soft quarks}} \Big\} \ln^2(2\gamma) + \mathcal{O}(\alpha_s^2)$$

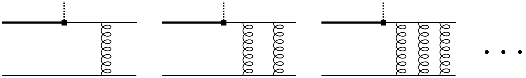


$\Rightarrow$ non-trivial interplay of soft-gluon and soft-quark corrections

What is the all-order structure of the double logarithmic series?

Soft-quark corrections

In light-cone gauge all soft-quark corrections arise from energy-ordered ladder diagrams



Structure familiar from electron-muon backward scattering [GB, Böer, Feldmann 22]

$$f_m(\ell_+, \ell_-) = 1 + \underbrace{\frac{\alpha_s C_F}{2\pi} \int_{\ell_-}^{p_{2-}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{\ell_+} \frac{dk_+}{k_+}}_{\text{rung of a ladder}} f_m(k_+, k_-)$$

- more complicated Dirac structure leads to mixing effects
- soft-gluon corrections modify each rung in the ladder

Integral equations

Double-logarithmic series is governed by coupled integral equations

$$F(\gamma) \simeq \xi_0 \, \exp\left\{-\frac{\alpha_s C_F}{4\pi} \ln^2(2\gamma)\right\} \left(24\, f(m_c, m_c) - 4\right)$$

$$f(\ell_+, \ell_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{\ell_-}^{p_{2-}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{\ell_+} \frac{dk_+}{k_+} \, e^{-S(k_+,k_-)} \left(f(k_+, k_-) - \frac{C_A - 2C_F}{4C_F} \underbrace{f_m(k_+, k_-)}_{\text{mixing}} + \frac{C_A}{4C_F} \right)$$

$$f_m(\ell_+, \ell_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{\ell_-}^{p_{2-}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{\ell_+} \frac{dk_+}{k_+} \, \underbrace{e^{-S(k_+,k_-)}}_{\text{soft gluons}} \, f_m(k_+, k_-)$$

- analytic solution unknown
- iterative solution up to $\mathcal{O}(\alpha_s^{80})$
- asymptotic behaviour for $\alpha_s \ln^2(2\gamma) \to \infty$
- recover bottom-induced $H \to \gamma\gamma$ and electron-muon backward scattering in certain limits

Fixed-order checks

Does this reproduce the correct 2-loop and 3-loop logarithms?

- reconstruct logarithms with method-of-regions techniques

$\Rightarrow$ requires computation of purely hard-collinear coefficient (massless, 5 legs, 3 scales)

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Automated toolchain: `QGRAF`, `LoopScalla`, `FeynCalc`, `FIRE`, `pySecDec` (`Alibrary`)

- number of master integrals: 5 (1-loop), 88 (2-loop), 24732 (3-loop)

⇒ 2-loop check works, 3-loop in progress

`FeynCalc 10` [Shtabovenko, Mertig, Orellana 23]

- greatly improved capabilities for multiloop calculations
- topology identification, optimized tensor reduction, detection of equivalent / scaleless integrals, derivation of Symanzik polynomials, support for differential equations, . . .

Conclusions

Project B2a: Automated calculations in SCET

- NNLO soft function for N-jettiness in hadronic collisions
- N3LL resummation for one-jettiness in Z+jet production
- `FeynCalc` goes multiloop
- Automated calculation of NNLO beam functions in momentum space

Project B1e: Power corrections in collider processes

- Non-perturbative corrections to event-shape variables
- Renormalon studies of top-quark observables
- Next-to-leading power soft-gluon corrections at one loop
- Interplay of soft-gluon and soft-quark corrections at next-to-leading power