From Factorisation to Cross Section Predictions

Projects B1a, B1b and B1e

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The 2024 Annual Meeting of the CRC TRR 257 “Particle Physics Phenomenology after the Higgs Discovery", 12th March 2024

Projects

- B1a: Bell, Czakon, Melnikov

  N3LO QCD predictions for production cross sections of colourless systems in association with a resolved jet

- B1b: Czakon, Heinrich, Worek

  Precision top-quark physics at the LHC

- B1e: Bell, Czakon, Melnikov

  Power corrections in collider processes

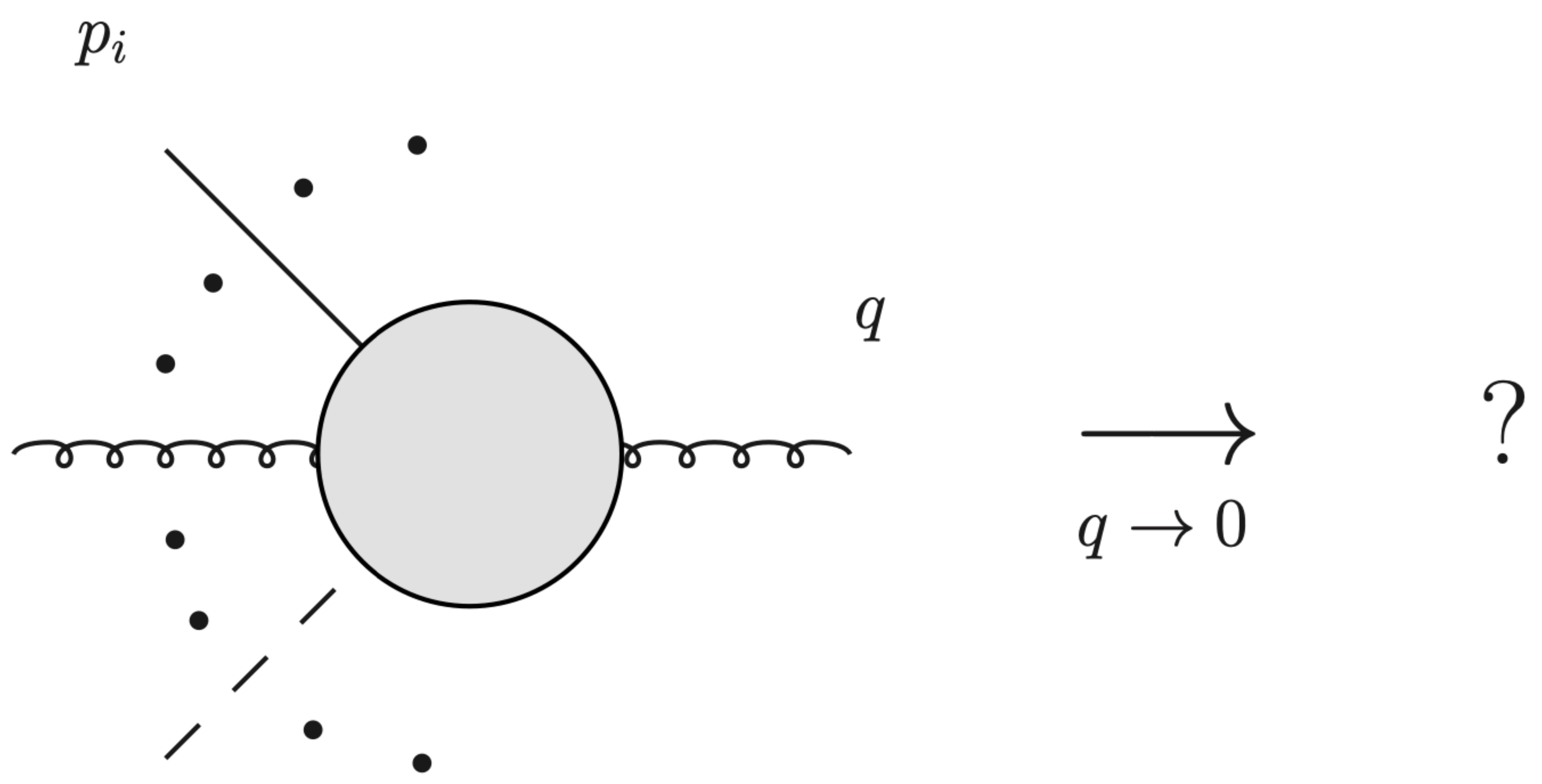
- In this talk:

  results from Aachen since last Annual Meeting including some results “in-between projects”

Power-Suppressed Effects

Subleading Soft-Gluon Effects at 1-Loop in QCD

- Classic problem:



- At leading power: eikonal approximation
  you need to calculate a soft current,
  but at least the structure is understood !

- Structure at next-to-leading power understood at tree-level QED by Low (1958), Burnett and Kroll (1968)

- Necessity to include virtual collinear enhancements at higher orders noticed by del Duca (1990)

- Extension to tree-level QCD described in 1404.5551, 1406.6987, 1406.6574

- Why bother (if you don't like pure theory) ?
  - needed to obtain cross sections approximations at subleading power in different kinematic variables
  - can be used to improve numerical stability in cross section calculations

Subleading Soft-Gluon Effects at 1-Loop in QCD

- Several attempts to understand one-loop QCD amplitudes (more results for photon emission):
  - based on SCET: 1412.3108, 1912.01585, 2112.00018
  - based on Feynman-diagram analysis: 1503.05156, 1610.06842

- Complete characterisation in Czakon, Eschment, Schellenberger, JHEP 12 (2023) 126

$$\left|M_g^{(1)}(\{p_i+\delta_i\},q)\right\rangle = \mathbf{S}^{(0)}(\{p_i\},\{\delta_i\},q)\left|M^{(1)}(\{p_i\})\right\rangle$$
$$+\mathbf{S}^{(1)}(\{p_i\},\{\delta_i\},q)\left|M^{(0)}(\{p_i\})\right\rangle + \int_0^1 \mathrm{d}x \sum_i \mathbf{J}_i^{(1)}(x,p_i,q)\left|H_{g,i}^{(0)}(x,\{p_i\},q)\right\rangle$$
$$+\sum_{i\neq j}\sum_{\substack{\tilde{a}_i\neq a_i\\ \tilde{a}_j\neq a_j}} \tilde{\mathbf{S}}^{(1)}_{a_ia_j\leftarrow\tilde{a}_i\tilde{a}_j,\,ij}(p_i,p_j,q)\left|M^{(0)}(\{p_i\})\Big|_{\substack{a_i\to\tilde{a}_i\\ a_j\to\tilde{a}_j}}\right\rangle + \int_0^1 \mathrm{d}x \sum_{\substack{i\\ a_i=g}} \tilde{\mathbf{J}}_i^{(1)}(x,p_i,q)\left|H_{\bar{q},i}^{(0)}(x,\{p_i\},q)\right\rangle + \mathcal{O}(\lambda)$$

$$\mathbf{P}_g(\sigma,c)\,\mathbf{S}^{(0)}(\{p_i\},\{\delta_i\},q) = -\sum_i \mathbf{T}_i^c \otimes \mathbf{S}_i^{(0)}(p_i,\delta_i,q,\sigma)\left|M^{(0)}(\{p_i\})\right\rangle ,$$

$$\mathbf{S}_i^{(0)} = \frac{p_i\cdot\epsilon^*}{p_i\cdot q} + \frac{1}{p_i\cdot q}\left[\left(\epsilon^* - \frac{p_i\cdot\epsilon^*}{p_i\cdot q}q\right)\cdot\delta_i + p_i\cdot\epsilon^*\sum_j \delta_j\cdot\partial_j + \frac{1}{2}F_{\mu\nu}\left(J_i^{\mu\nu} - \mathbf{K}_i^{\mu\nu}\right)\right]$$

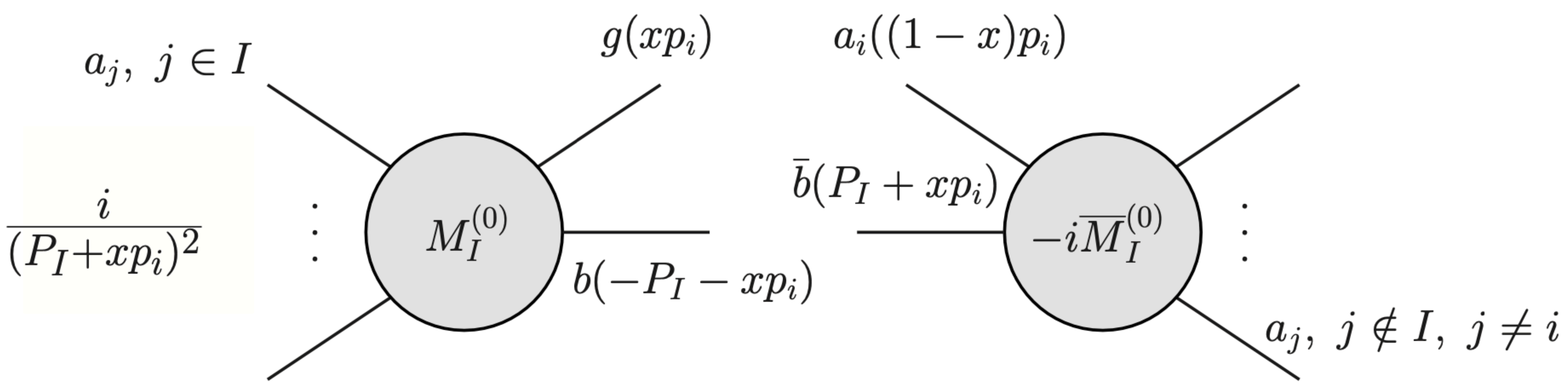
Subleading Soft-Gluon Effects at 1-Loop in QCD

Czakon, Eschment, Schellenberger, JHEP 12 (2023) 126

- Unexpected simplicity of contributions due to virtual collinear singularities

$$\mathbf{P}_g(\sigma,c)\,\mathbf{J}_i^{(1)}(x,p_i,q) \;=\frac{\Gamma(1+\epsilon)}{1-\epsilon}\left(-\frac{\mu^2}{s_{iq}}\right)^{\epsilon}\big(x(1-x)\big)^{-\epsilon}\epsilon^*(q,p_i,\sigma)\cdot\epsilon(p_i,-\sigma)\sum_{c'}\mathbf{P}_g(-\sigma,c')$$
$$\times\left[\left(\mathbf{T}_i^c\mathbf{T}_i^{c'}+\frac{1}{x}if^{cdc'}\mathbf{T}_i^d\right)\otimes\big(-2+x\big(1+\boldsymbol{\Sigma}_{g,i}\big)\big)\right]$$

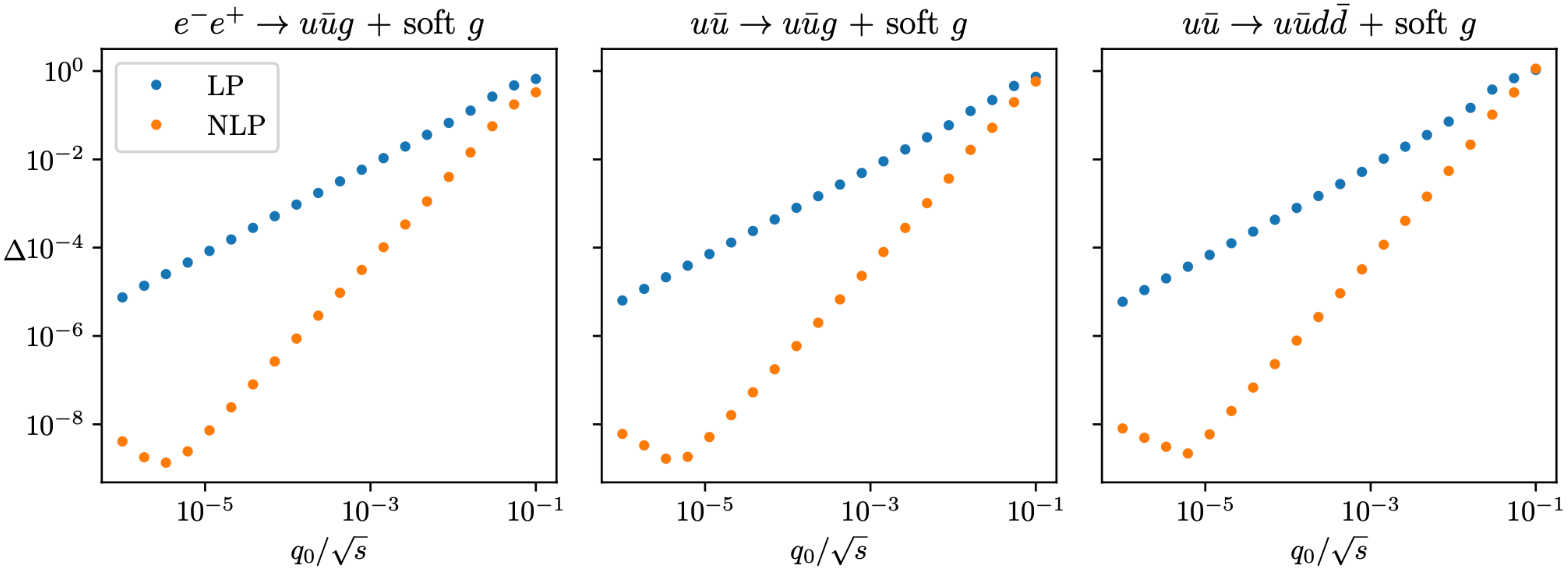
- Exact evaluation of convolutions in the general case through products of tree-level amplitudes



- Tree-level subleading collinear asymptotics
- General proof of correctness via expansion-by-regions

Subleading Soft-Gluon Effects at 1-Loop in QCD

- Numerical checks demonstrating potential gains in numerical stability in applications



Czakon, Eschment, Schellenberger, JHEP 12 (2023) 126

Jet Cross Sections at NNLO

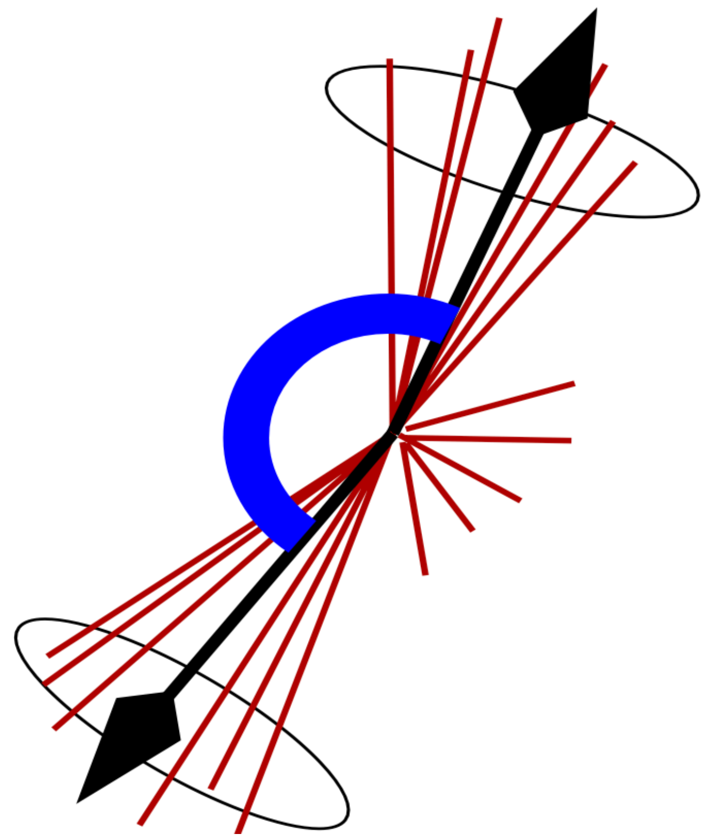
Transverse Energy-Energy Correlations at ATLAS

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet, JHEP 03 (2023) 129

ATLAS, JHEP 07 (2023) 85

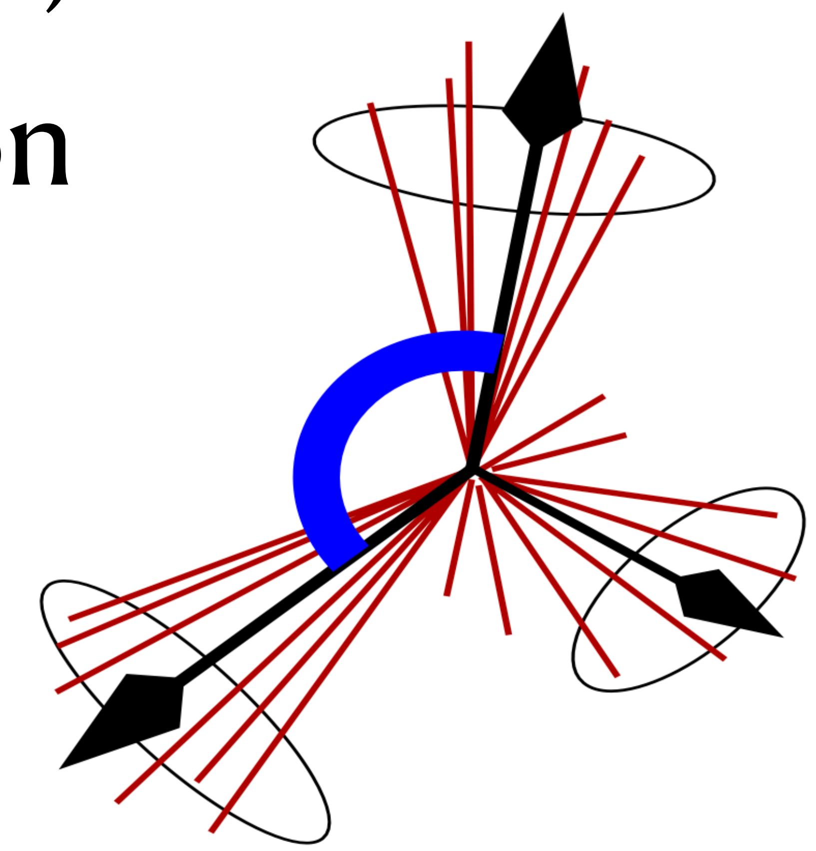
- TEEC : transverse-energy-weighted distribution of the azimuthal differences between jet pairs in the final state:

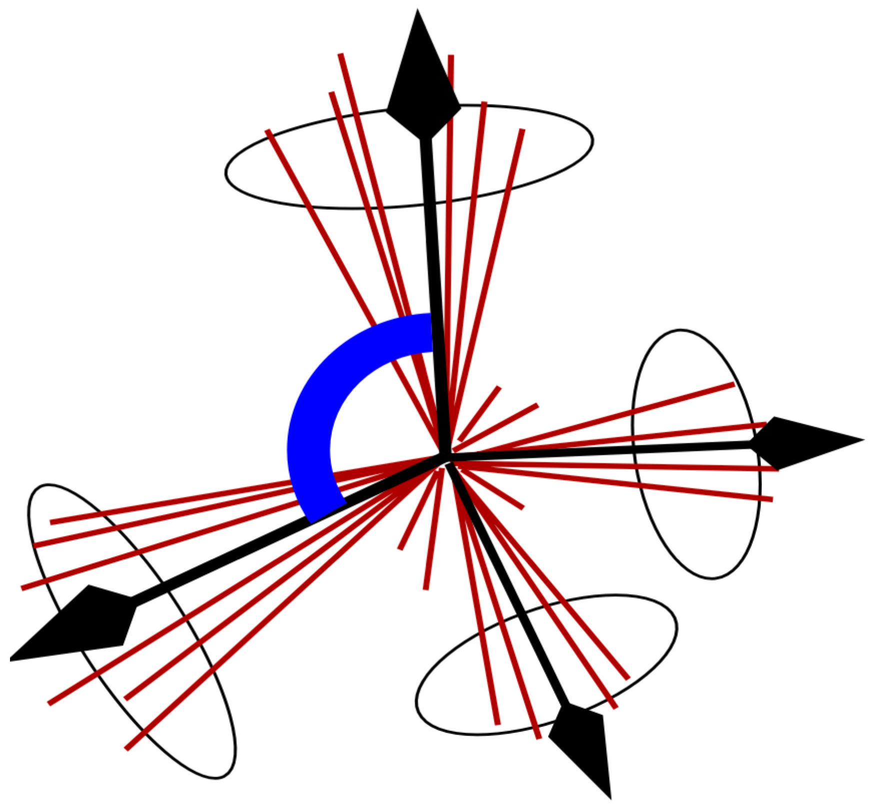
$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} \equiv \frac{1}{\sigma}\sum_{ij}\int \frac{d\sigma}{dx_{Ti}dx_{Tj}d\cos\phi} x_{Ti}x_{Tj}dx_{Ti}dx_{Tj} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E^A_{Ti}E^A_{Tj}}{\left(\sum_k E^A_{Tk}\right)^2}\delta(\cos\phi - \cos\varphi_{ij})$$



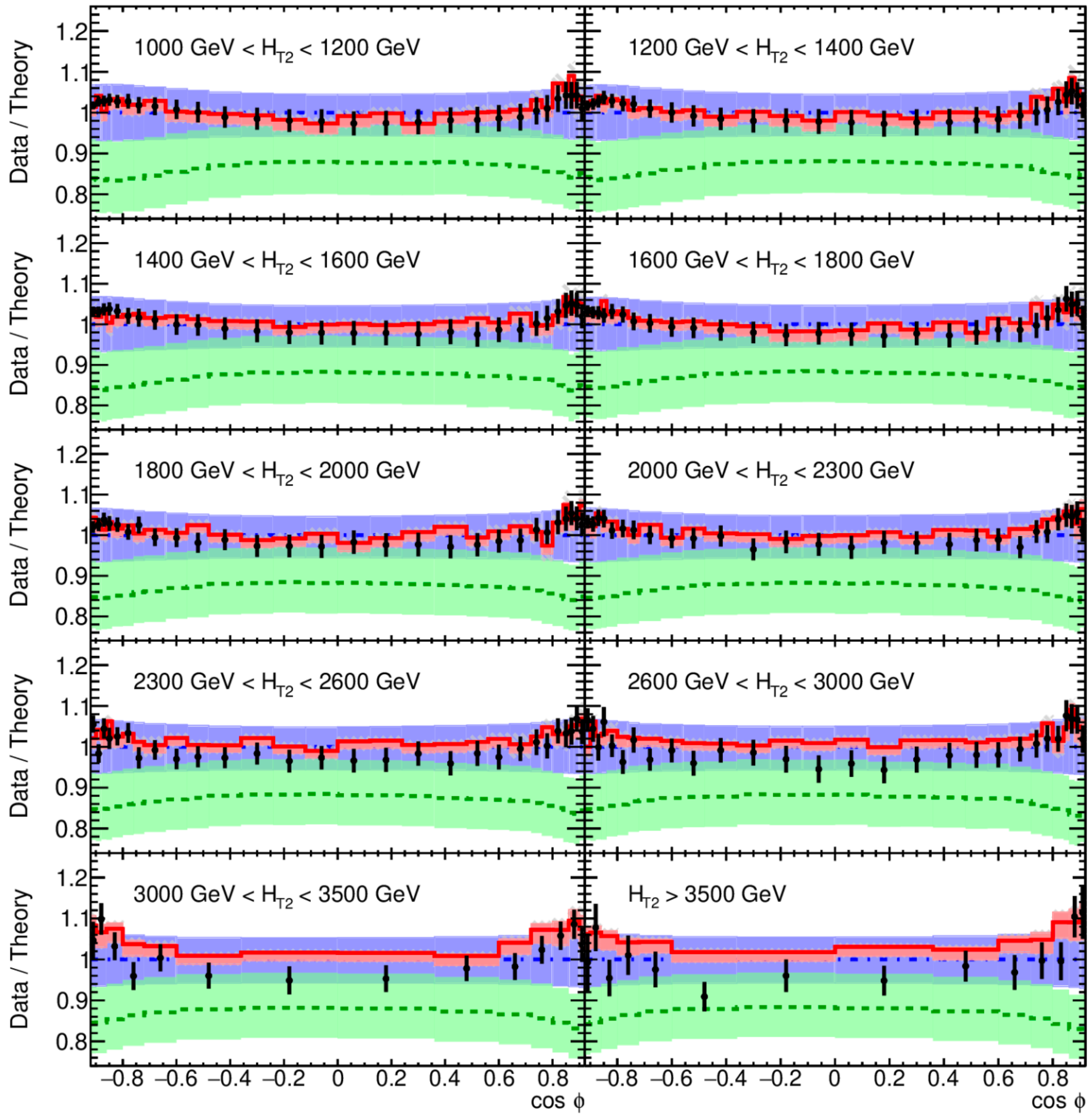
- ATEEC : difference between the forward ($\cos\phi > 0$) and the backward ($\cos\phi < 0$) part of the TEEC function

$$\frac{1}{\sigma}\frac{d\Sigma^{\text{asym}}}{d\cos\phi} = \frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi}\bigg|_{\phi} - \frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi}\bigg|_{\pi-\phi}$$





Transverse Energy-Energy Correlations at ATLAS



ATLAS JHEP 07 (2023) 85

Particle-level TEEC

$\sqrt{s}$ = 13 TeV; 139 fb^{-1}

anti-k_t R = 0.4

p_T > 60 GeV

$|\eta|$ < 2.4

$\mu_{R,F} = \hat{H}_T$

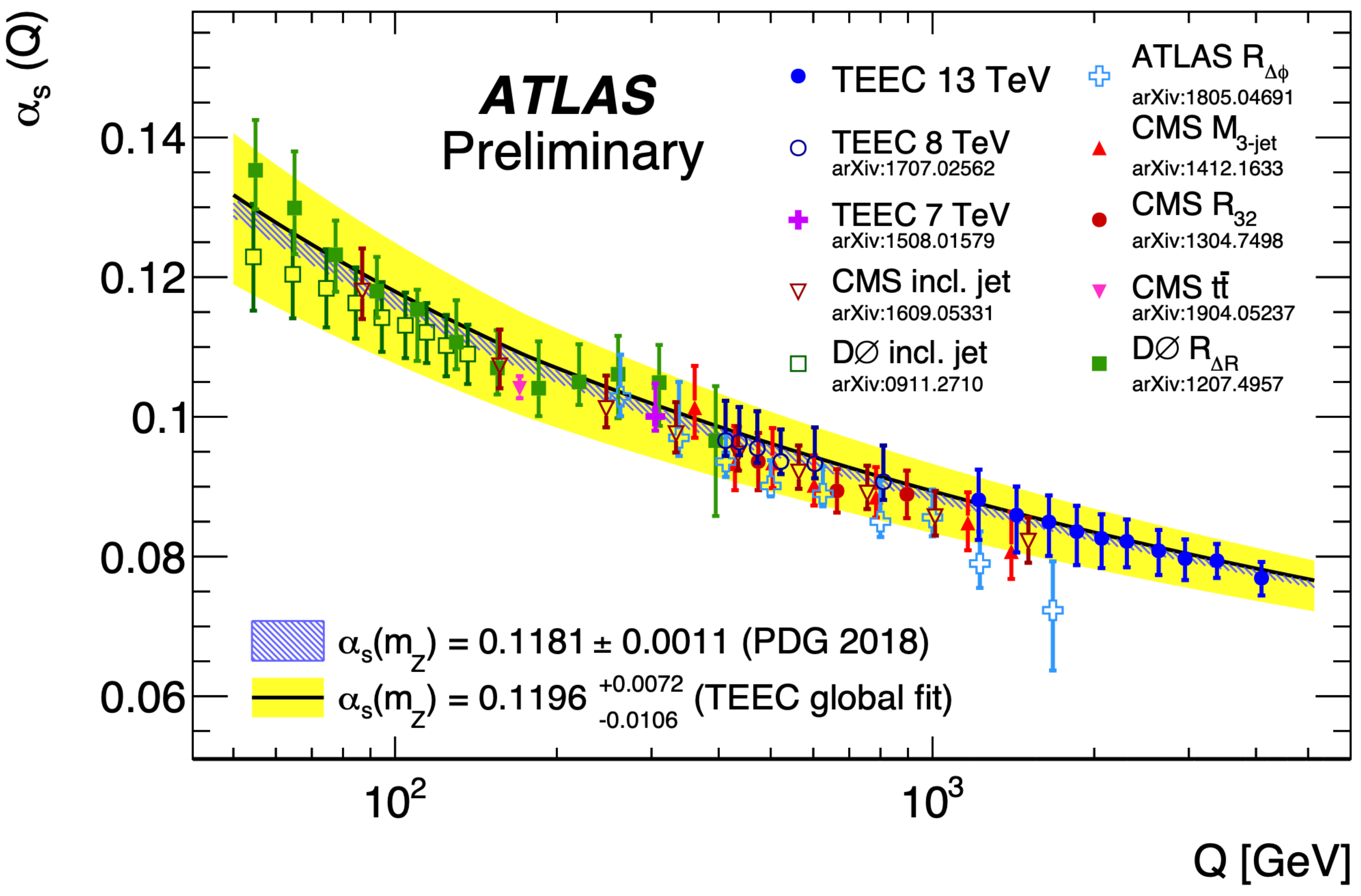
$\alpha_s(m_Z)$ = 0.1180

MMHT 2014 (NNLO)

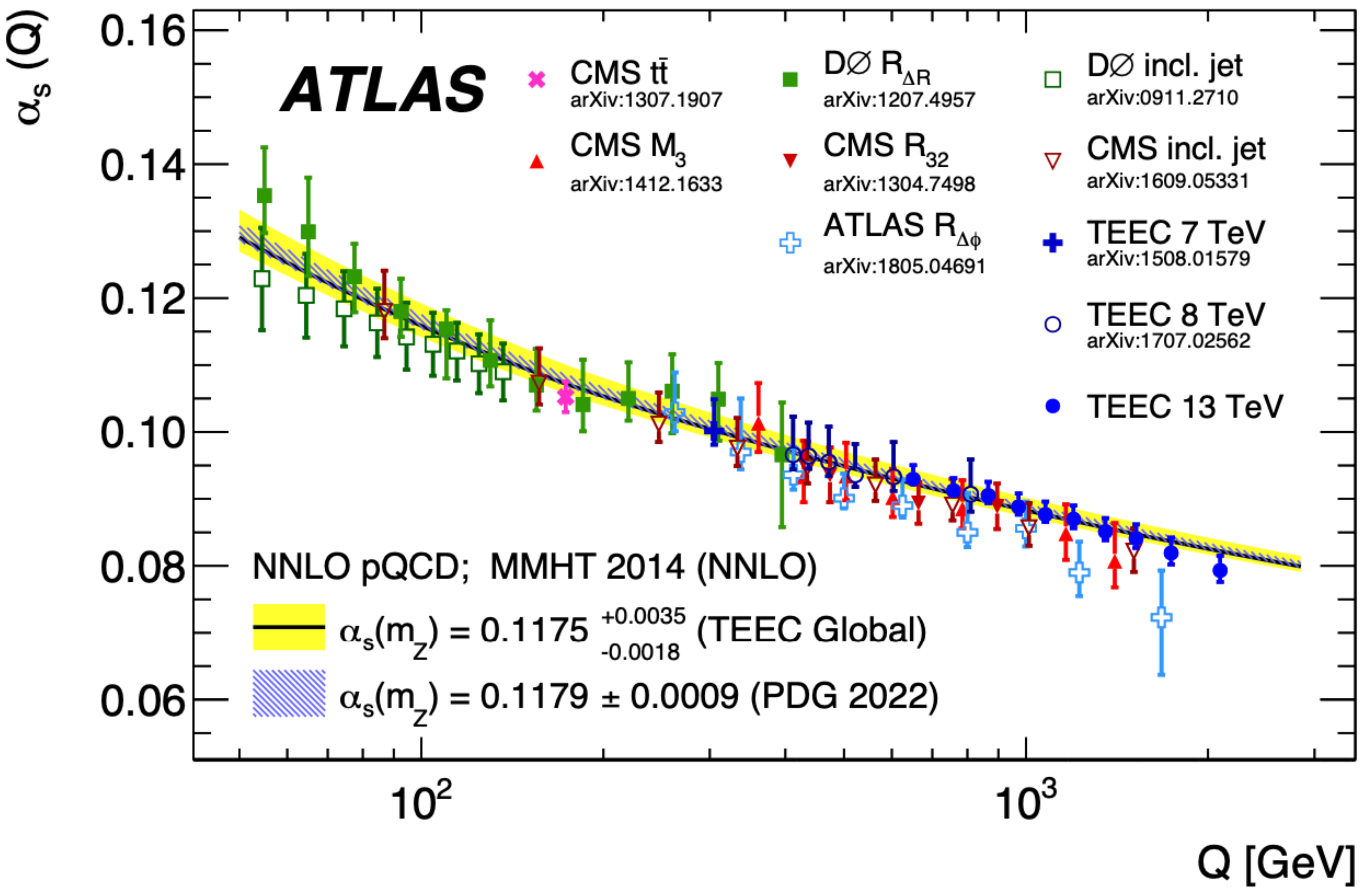
- Data
- LO
- NLO
- NNLO

Transverse Energy-Energy Correlations at ATLAS

ATLAS-CONF-2020-025
NLO QCD



ATLAS, JHEP 07 (2023) 85
NNLO QCD

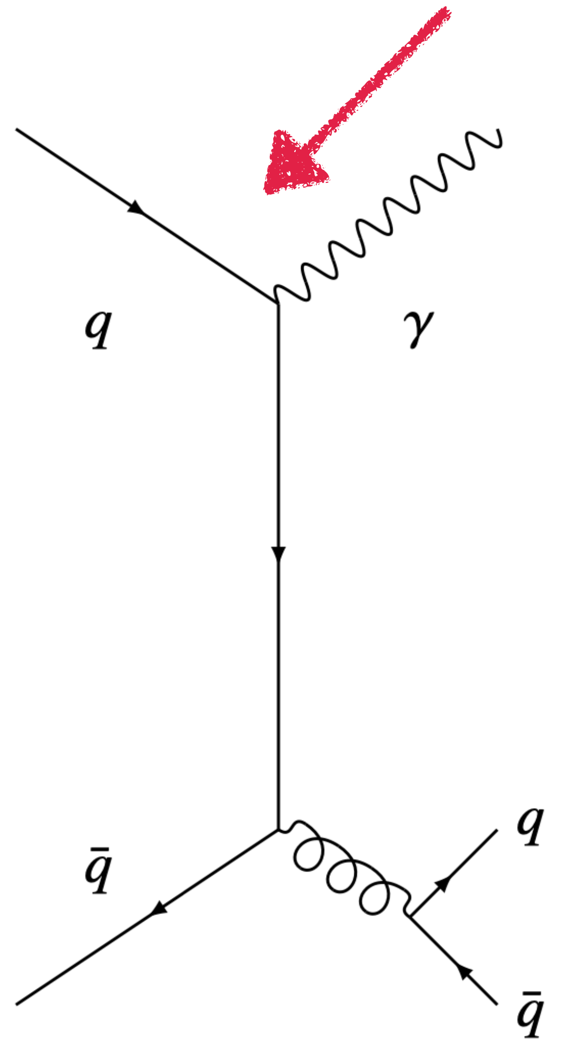


Isolated γ + di-jet at NNLO in QCD

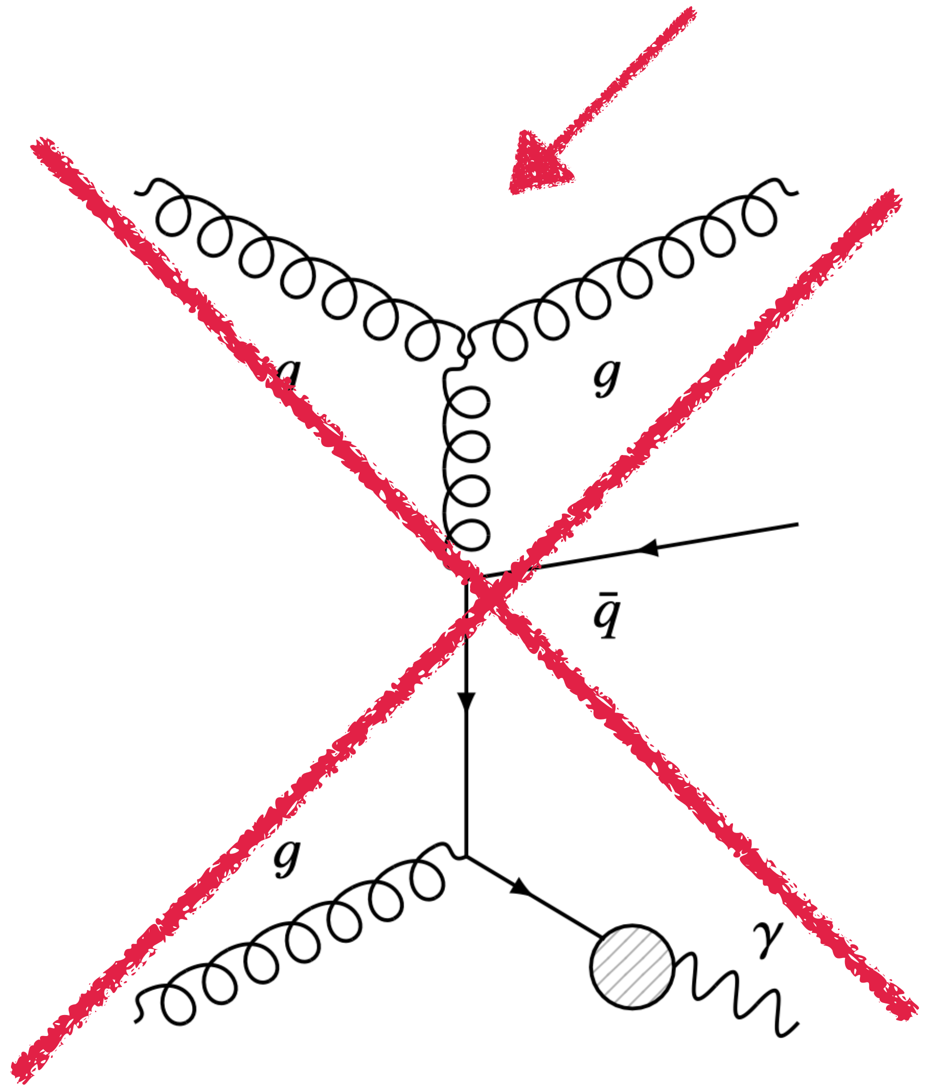
Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, JHEP 10 (2023) 071

- Highlight: first complete $2 \to 3$ process with full-color virtual amplitudes
- Analysis matching ATLAS measurements from JHEP 03 (2020) 179

Direct emission



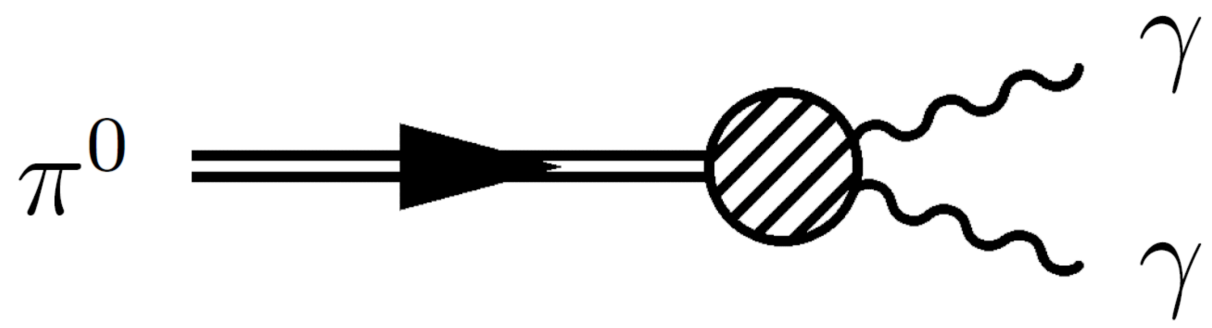
Fragmentation



Requirements on photon	$E_T^\gamma > 150\,\text{GeV}$, $\lvert\eta^\gamma\rvert < 2.37$ (excluding $1.37 < \lvert\eta^\gamma\rvert < 1.56$) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
Requirements on jets	at least two jets using anti-k_t algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100\,\text{GeV}$, $\lvert y^{\text{jet}}\rvert < 2.5$, $\Delta R^{\gamma\text{-jet}} > 0.8$		
Phase space	**total**	**fragmentation enriched** $E_T^\gamma < p_T^{\text{jet2}}$	**direct enriched** $E_T^\gamma > p_T^{\text{jet1}}$
Number of events	755 270	111 666	386 846

- Unwanted dominant source of photons: hadron decays

  $\Rightarrow$ remove by judicious cuts



Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, JHEP 10 (2023) 071

- Scale choice:

$$\mu_R = \mu_F = H_T = E_\perp(\gamma) + p_T(j_1) + p_T(j_2) \quad \text{and}$$
$$\mu_R = \mu_F = E_\perp(\gamma)\ ,$$

- Very interesting comparison: ATLAS used SHERPA predictions for reference
  - NLO-matched QCD parton-shower merged with LO photon+four-jet samples
  - In principle this corresponds to the double-real radiation contributions in the NNLO QCD predictions
  - $E_\perp(\gamma)$ used for renormalisation and factorisation scale

- Why is this process interesting otherwise?
  - Non-back-to-back Born configurations

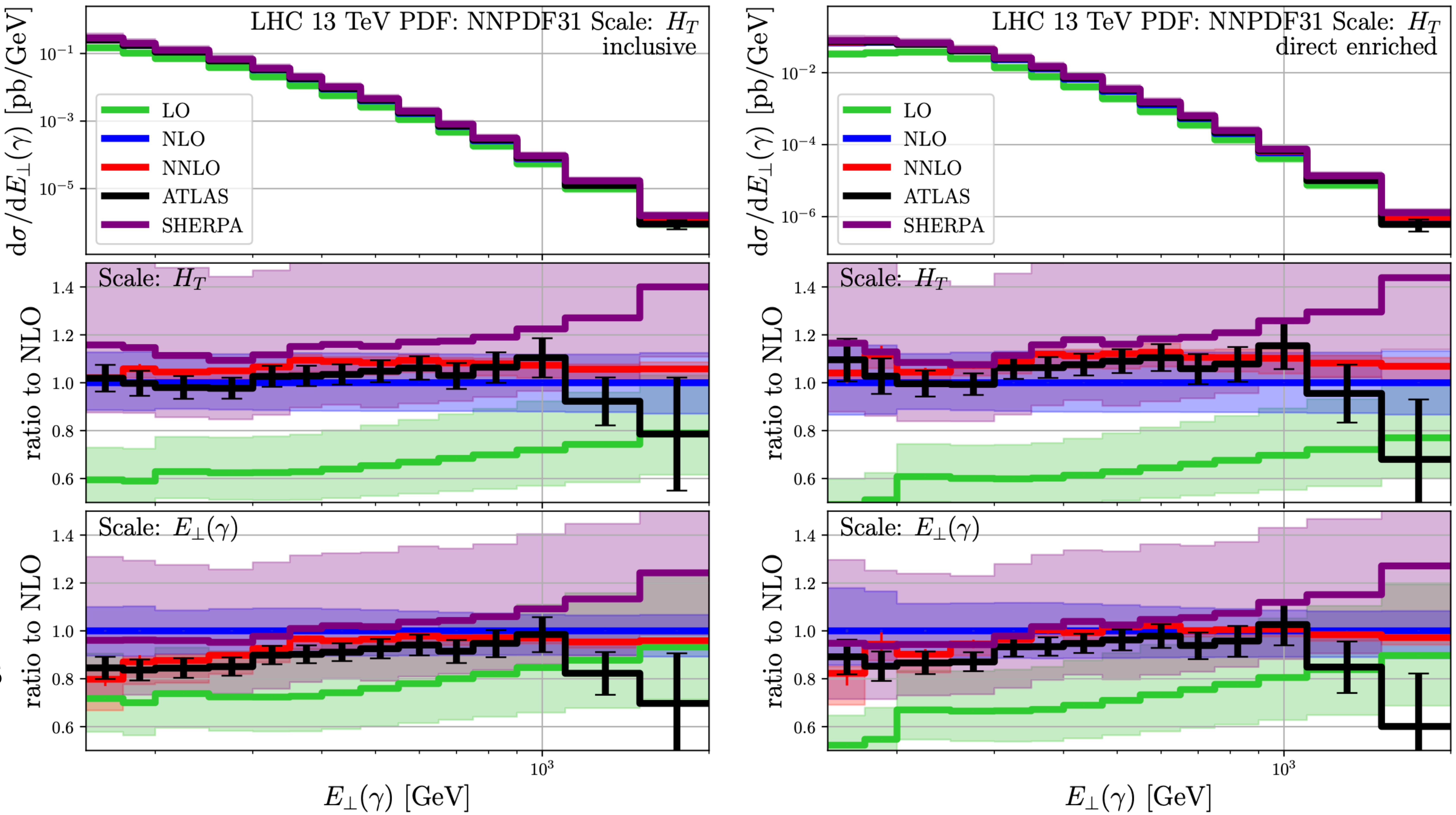
    → access to angular correlations between the photon and jets
  - Access to different kinematic regimes through distinguishable photon

    → enhance direct, high- or low-z fragmentation
  - Background process for BSM: $pp \to \gamma + Y(\to jj)$

Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, JHEP 10 (2023) 071

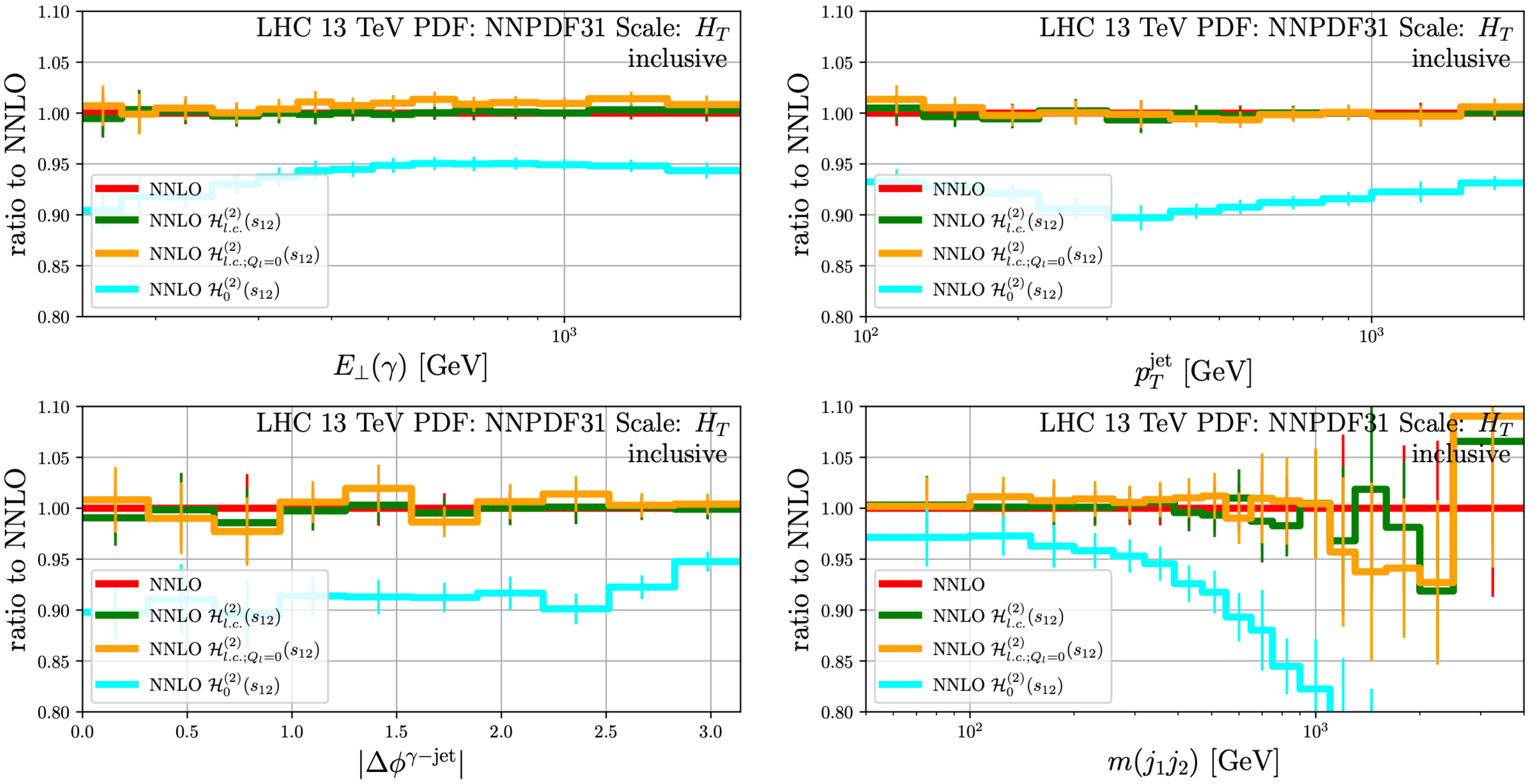
- Transverse photon energy
    1. NNLO corrections from 1% to 10%
    2. Improved description of data up to 1 TeV
    3. Larger scale uncertainties with the $E_{\perp}(\gamma)$ scale in the inclusive phase space
    4. Large experimental uncertainties beyond 1 TeV
    5. Default SHERPA predictions with merging are a poor description with large uncertainties



Isolated γ + di-jet at NNLO in QCD

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia, JHEP 10 (2023) 071

- Negligible size of the subleading color corrections

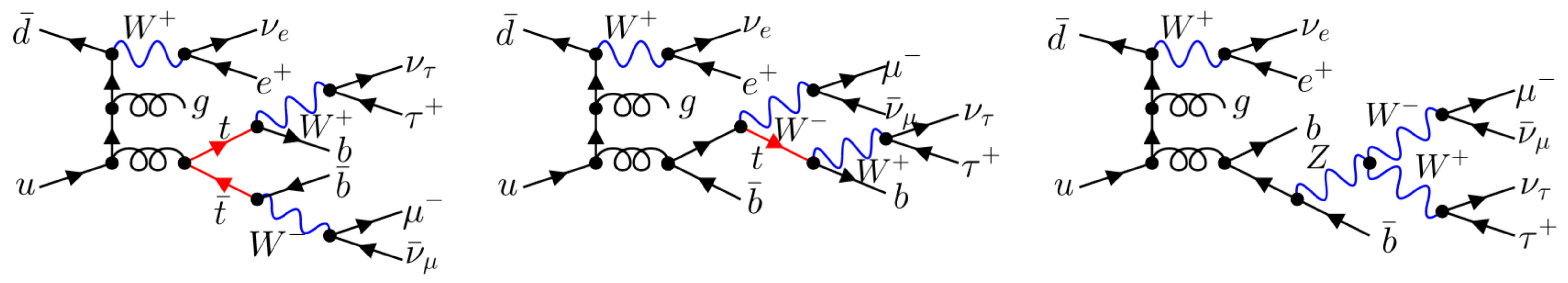


$t\bar{t} + X$ Cross Sections at NLO

Full Off-Shell Predictions

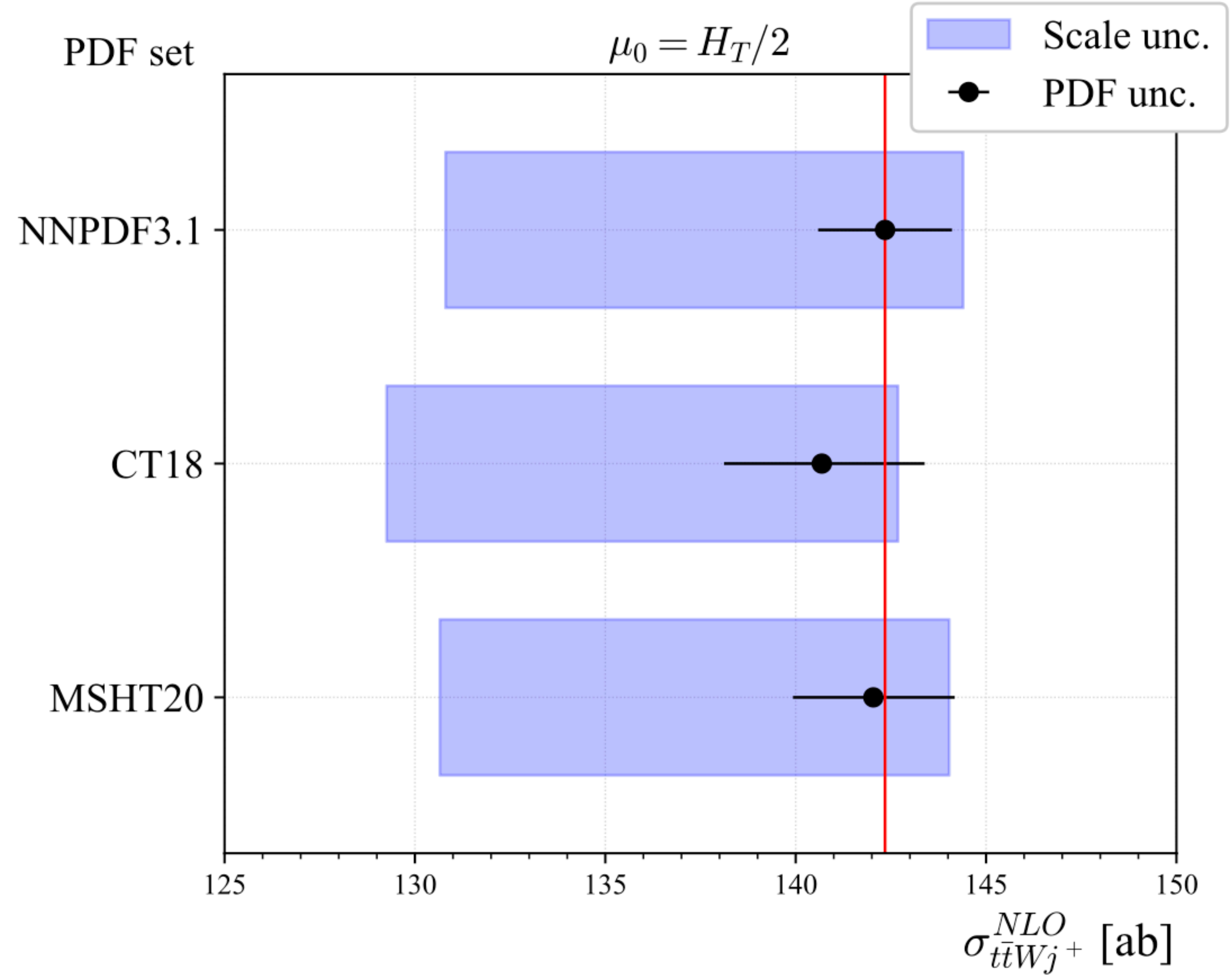
NLO $ttWj$

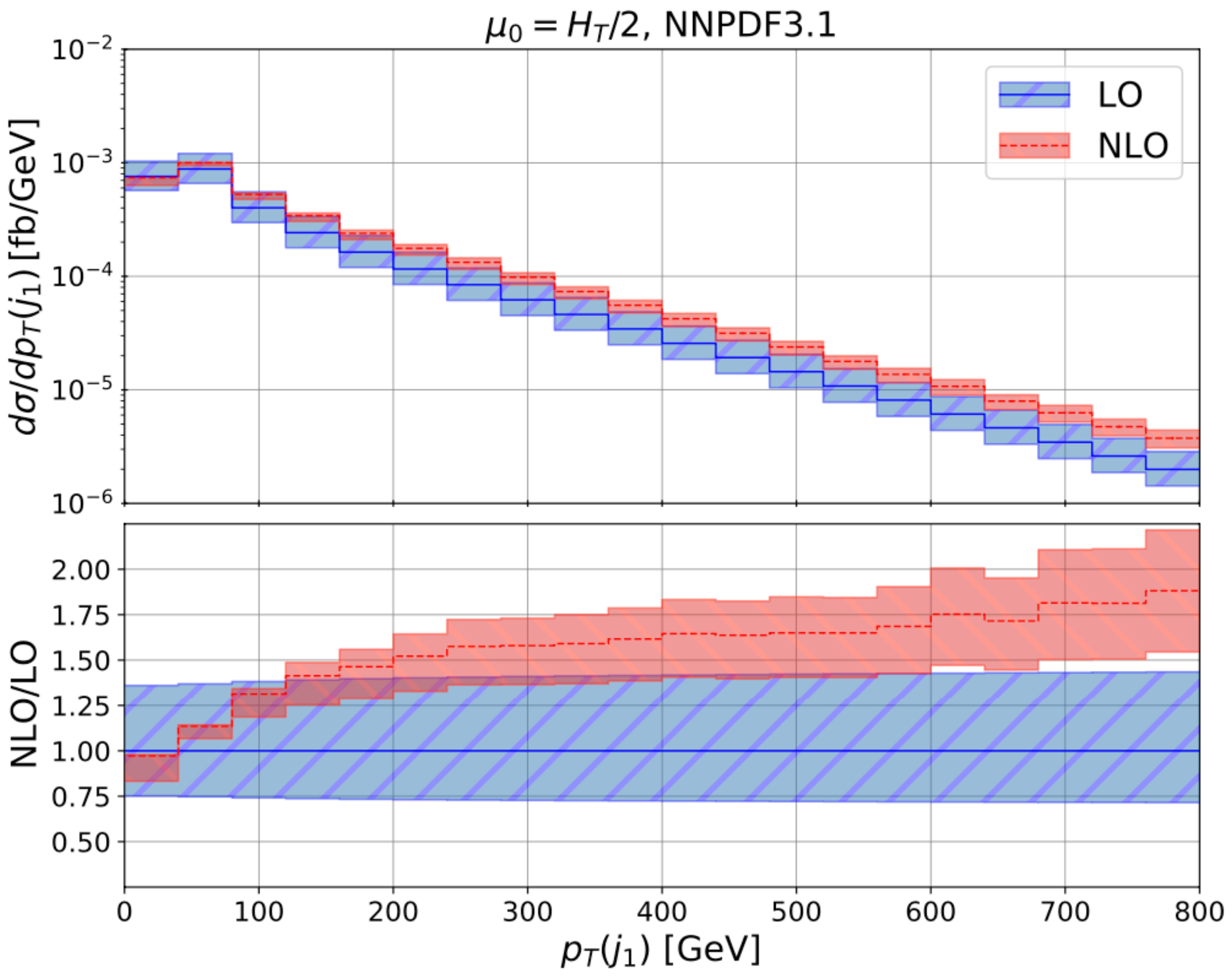
- Modelling of unstable particles ⇨ $ttWj$ production @ $\mathcal{O}(\alpha_s^4\alpha^6)$
  - ✳ *Full off-shell = DR + SR + NR + interferences + BW propagators*
  - ✳ NWA = DR restricts unstable t & W to on-shell states

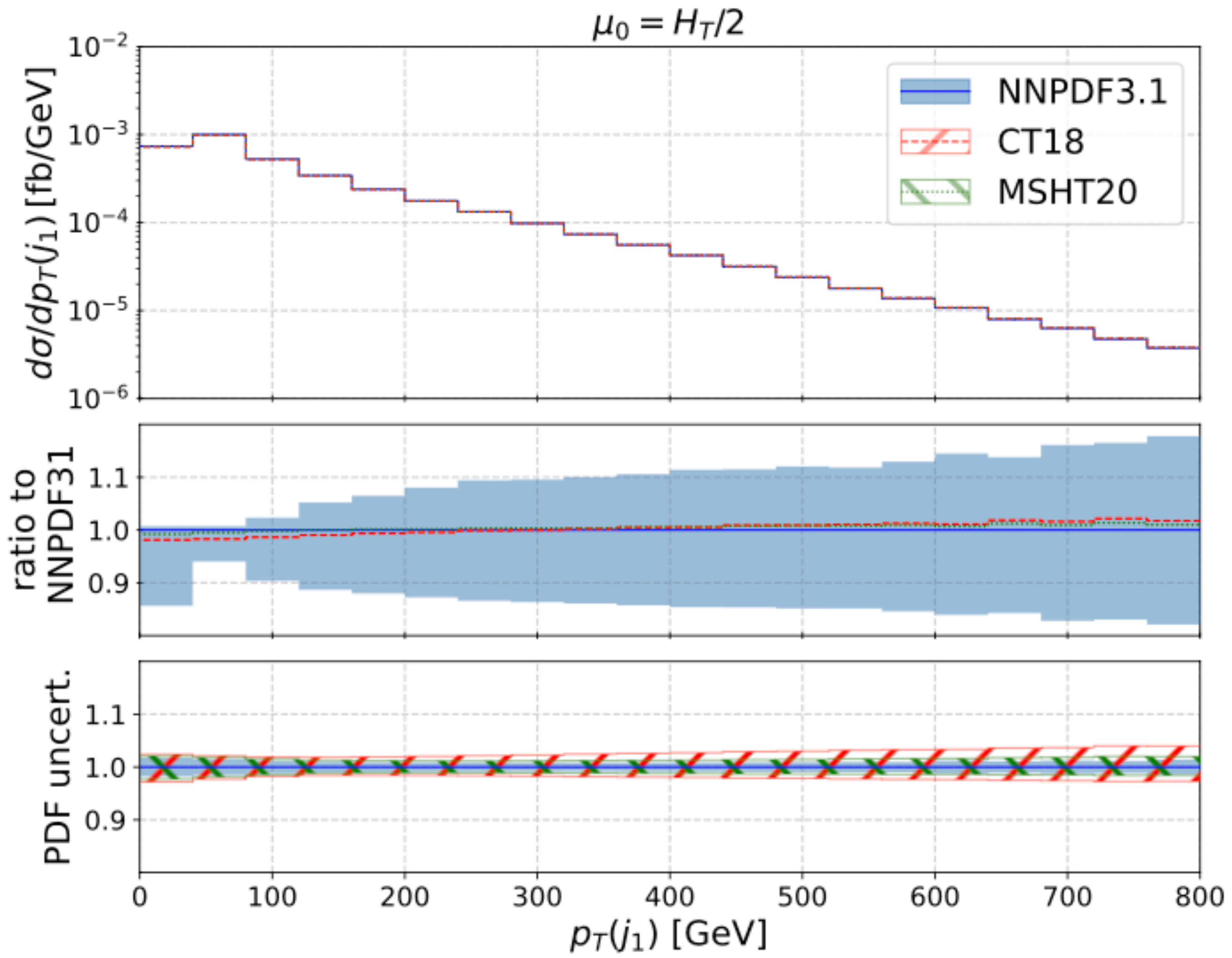


DR SR NR

$pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b}j + X$







Bi, Kraus, Reinartz & Worek
JHEP09 (2023) 026

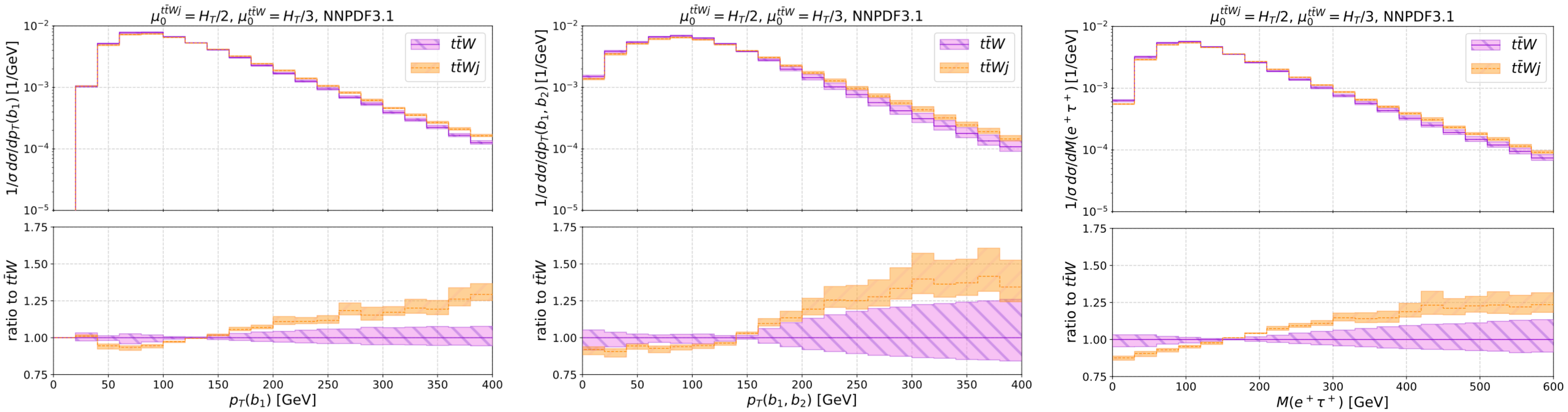
Additional Jet Activity in ttW Production

NLO ttW & $ttWj$

- Comparison between $pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b}j + X$ & $pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b} + X$

  ✴ For $p_T(j_1) > 25$ GeV & inclusive cuts more than half of $pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b} + X$ events are accompanied by additional hard jet

  ✴ At differential cross section level significant differences between two processes & large shape distortions due to presence of extra jet



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Additional Jet Activity in ttW Production

NLO ttW & $ttWj$

- NLO cross section for both processes as function of number of resolved jets Nj ➪ Resolved jets that pass all cuts

✳ Inclusive NLO results for $pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b}j + X$ & $pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b} + X$ have expected reduced theoretical uncertainties

✳ Observed large scale dependence for exclusive NLO samples

✳ Gives rise to concern as jet vetoes are widely used in experimental analyses @ LHC

$pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b}j + X$

(μ_R,μ_F)	$\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j=1)$ [ab]	$\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j=2)$ [ab]	$\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j\geq 1)$ [ab]
(μ_0,μ_0)	$78.6^{+13\%}_{-48\%}$	$63.7^{+56\%}_{-34\%}$	$142.3^{+1.4\%}_{-8.1\%}$
(μ_R,μ_F)	$\delta\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j=1)$ [ab]	$\delta\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j=2)$ [ab]	$\delta\sigma^{t\bar{t}W^+j}_{H_T/2}(N_j\geq 1)$ [ab]
$(2\mu_0,2\mu_0)$	$+9.9\,(+13\%)$	$-21.4\,(-34\%)$	$-11.5\,(-8.1\%)$
$(\mu_0/2,\mu_0/2)$	$-37.4\,(-48\%)$	$+35.7\,(+56\%)$	$-1.6\,(-1.1\%)$
$(2\mu_0,\mu_0)$	$+9.0\,(+11\%)$	$-18.0\,(-28\%)$	$-9.0\,(-6.3\%)$
$(\mu_0/2,\mu_0)$	$-28.7\,(-37\%)$	$+27.8\,(+44\%)$	$-0.9\,(-0.6\%)$
$(\mu_0,2\mu_0)$	$+3.5\,(+4.5\%)$	$-4.8\,(-7.5\%)$	$-1.2\,(-0.8\%)$
$(\mu_0,\mu_0/2)$	$-3.5\,(-4.5\%)$	$+5.5\,(+8.6\%)$	$+2.0\,(+1.4\%)$

$pp \to e^+\nu_e\,\mu^-\bar{\nu}_\mu\,\tau^+\nu_\tau\,b\bar{b} + X$

(μ_R,μ_F)	$\sigma^{t\bar{t}W^+}_{H_T/2}(N_j=0)$ [ab]	$\sigma^{t\bar{t}W^+}_{H_T/2}(N_j=1)$ [ab]	$\sigma^{t\bar{t}W^+}_{H_T/2}(N_j\geq 0)$ [ab]
(μ_0,μ_0)	$104.6^{+20\%}_{-44\%}$	$141.9^{+41\%}_{-27\%}$	$246.4^{+5.0\%}_{-7.0\%}$
(μ_R,μ_F)	$\delta\sigma^{t\bar{t}W^+}_{H_T/2}(N_j=0)$ [ab]	$\delta\sigma^{t\bar{t}W^+}_{H_T/2}(N_j=1)$ [ab]	$\delta\sigma^{t\bar{t}W^+}_{H_T/2}(N_j\geq 0)$ [ab]
$(2\mu_0,2\mu_0)$	$+21.1\,(+20\%)$	$-38.4\,(-27\%)$	$-17.2\,(-7.0\%)$
$(\mu_0/2,\mu_0/2)$	$-45.7\,(-44\%)$	$+58.1\,(+41\%)$	$+12.4\,(+5.0\%)$
$(2\mu_0,\mu_0)$	$+19.1\,(+18\%)$	$-32.3\,(-23\%)$	$-13.2\,(-5.3\%)$
$(\mu_0/2,\mu_0)$	$-40.0\,(-38\%)$	$+46.6\,(+33\%)$	$+6.6\,(+2.7\%)$
$(\mu_0,2\mu_0)$	$+4.1\,(+3.9\%)$	$-7.8\,(-5.5\%)$	$-3.8\,(-1.5\%)$
$(\mu_0,\mu_0/2)$	$-3.1\,(-2.9\%)$	$+8.7\,(+6.1\%)$	$+5.6\,(+2.3\%)$

Bi, Kraus, Reinartz & Worek
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Photons in Production & Decays

NLO $t\bar{t}\gamma\gamma$

$$pp \to \ell^+ \nu_\ell \, \ell^- \bar{\nu}_\ell \, b\bar{b} \, \gamma\gamma$$

$$d\sigma_{\rm Full} = \overbrace{d\sigma_{t\bar{t}\gamma\gamma} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t}}^{\sigma_{\rm Prod.}} + \overbrace{d\sigma_{t\bar{t}\gamma} \times \left(\frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}^{\sigma_{\rm Mixed}}$$
$$+ \underbrace{d\sigma_{t\bar{t}} \times \left(\frac{d\Gamma_{t\gamma\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma\gamma}}{\Gamma_t} + \frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}_{\sigma_{\rm Decay}} .$$

- Integrated fiducial cross-section level @ NLO in QCD

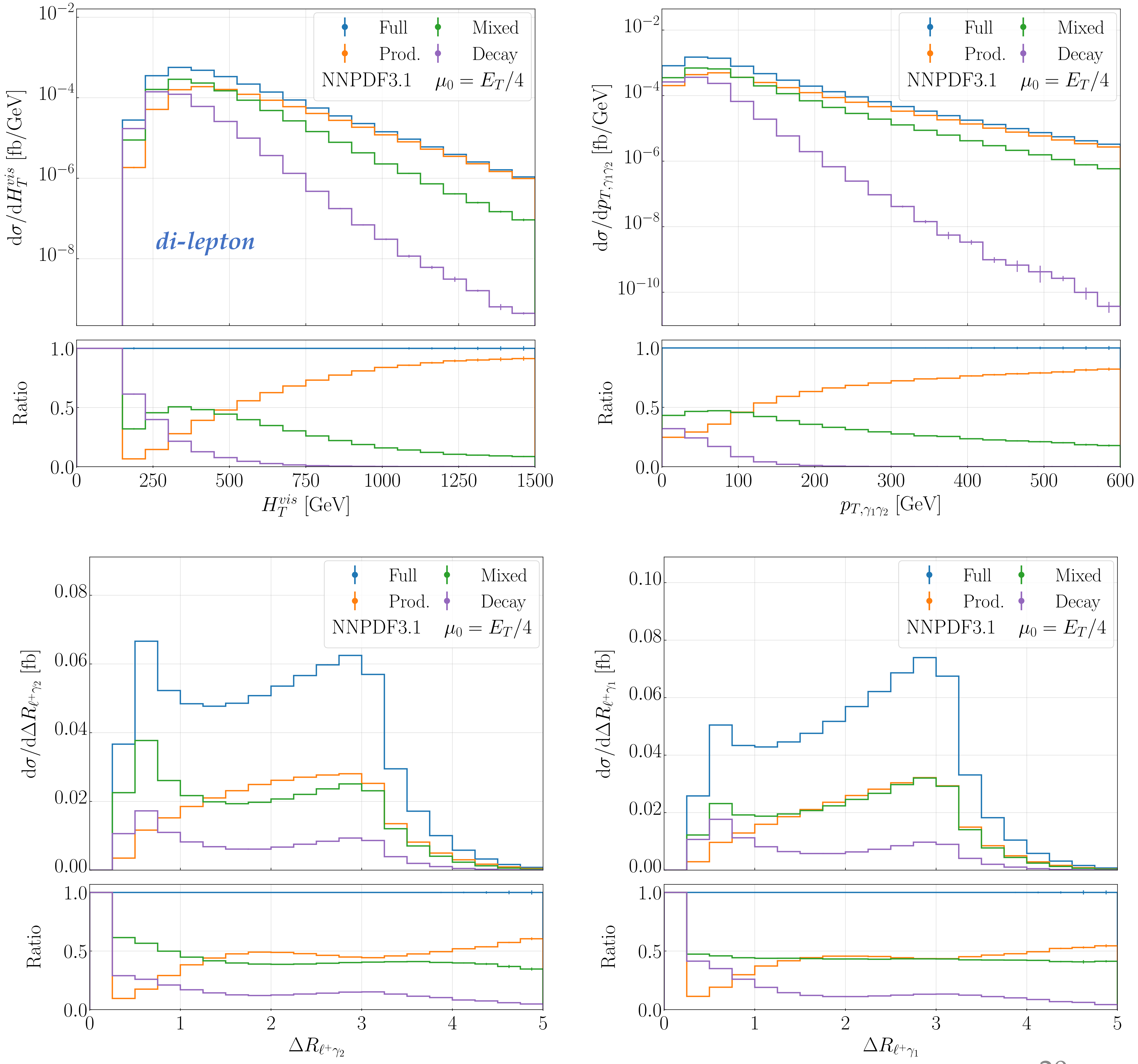
  $p_{T,b} > 25$ GeV, $p_{T,\gamma} > 25$ GeV:

  - *Mixed contribution* at the level of *44%*
  - *Prod. contribution* at the level of *40%*
  - *Decay contribution* is about half the size *16%*

- Differential fiducial cross-section level

  - Various phase-space regions with various effects

Stremmer, Worek
JHEP08 (2023) 179



Photons in Production & Decays

NLO $t\bar{t}\gamma\gamma$

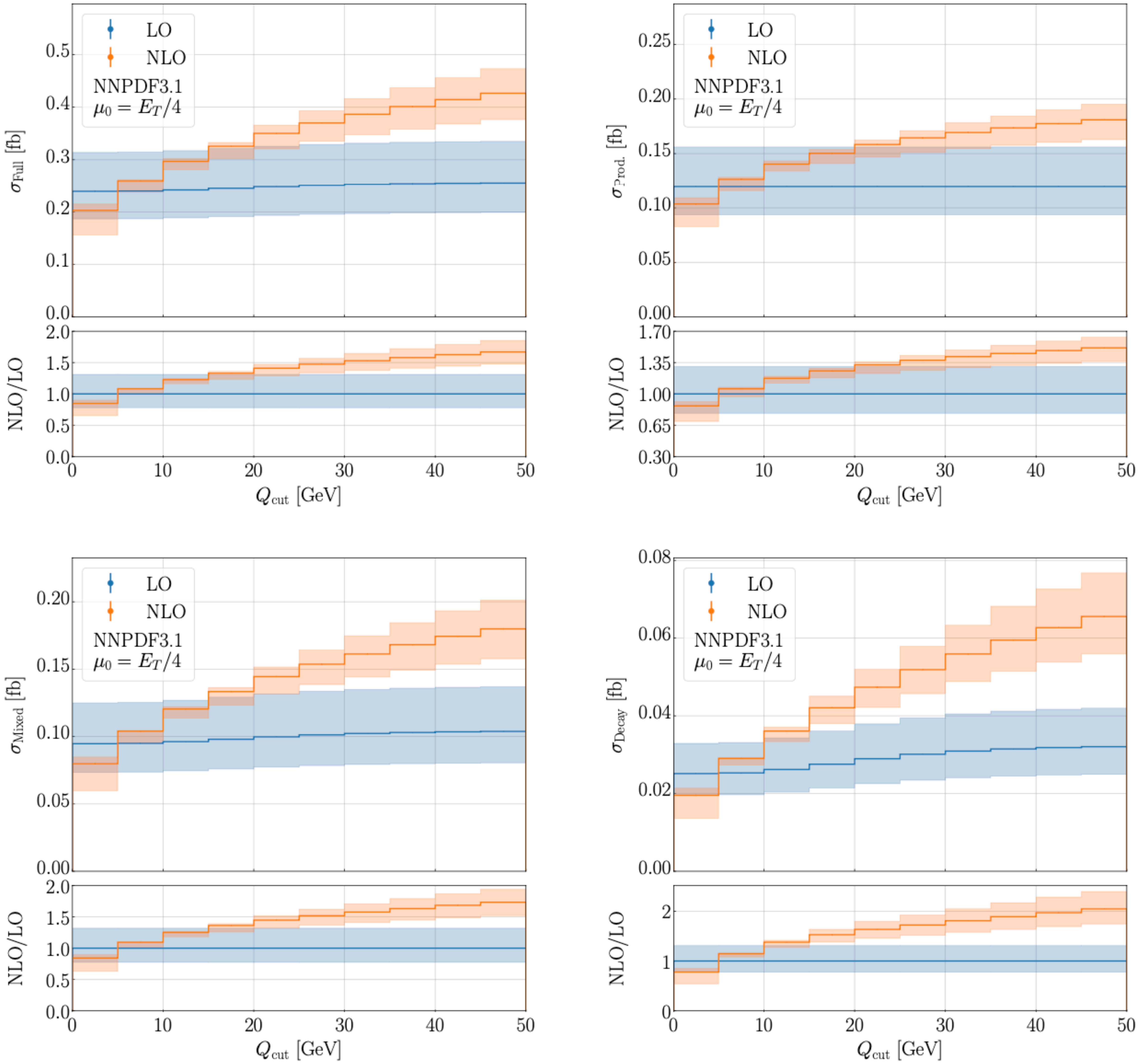
$$pp \to \ell^- \bar{\nu}_\ell \, jj \, b\bar{b} \, \gamma\gamma$$

$$d\sigma_{\text{Full}} = \overbrace{d\sigma_{t\bar{t}\gamma\gamma} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t}}^{\sigma_{\text{Prod.}}} + \overbrace{d\sigma_{t\bar{t}\gamma} \times \left(\frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}^{\sigma_{\text{Mixed}}}$$
$$\underbrace{+ \, d\sigma_{t\bar{t}} \times \left(\frac{d\Gamma_{t\gamma\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma\gamma}}{\Gamma_t} + \frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}_{\sigma_{\text{Decay}}} .$$

- For ***lepton + jet*** channel additional cut needed

  $$|m_W - M_{jj}| < Q_{\text{cut}} = 15 \text{ GeV}$$

- Suppress kinematical configurations from real radiation
  - ✳ Jets originating from W recombined into single jet
  - ✳ Extra jet from real radiation gives rise to second jet



Stremmer, Worek
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Photons in Production & Decays

NLO $t\bar{t}\gamma\gamma$

$$pp \to \ell^- \bar{\nu}_\ell \, jj \, b\bar{b} \, \gamma\gamma$$

$$d\sigma_{\text{Full}} = \overbrace{d\sigma_{t\bar{t}\gamma\gamma} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t}}^{\sigma_{\text{Prod.}}} + \overbrace{d\sigma_{t\bar{t}\gamma} \times \left(\frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}^{\sigma_{\text{Mixed}}}$$
$$+ \underbrace{d\sigma_{t\bar{t}} \times \left(\frac{d\Gamma_{t\gamma\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} + \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma\gamma}}{\Gamma_t} + \frac{d\Gamma_{t\gamma}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}\gamma}}{\Gamma_t} \right)}_{\sigma_{\text{Decay}}} .$$

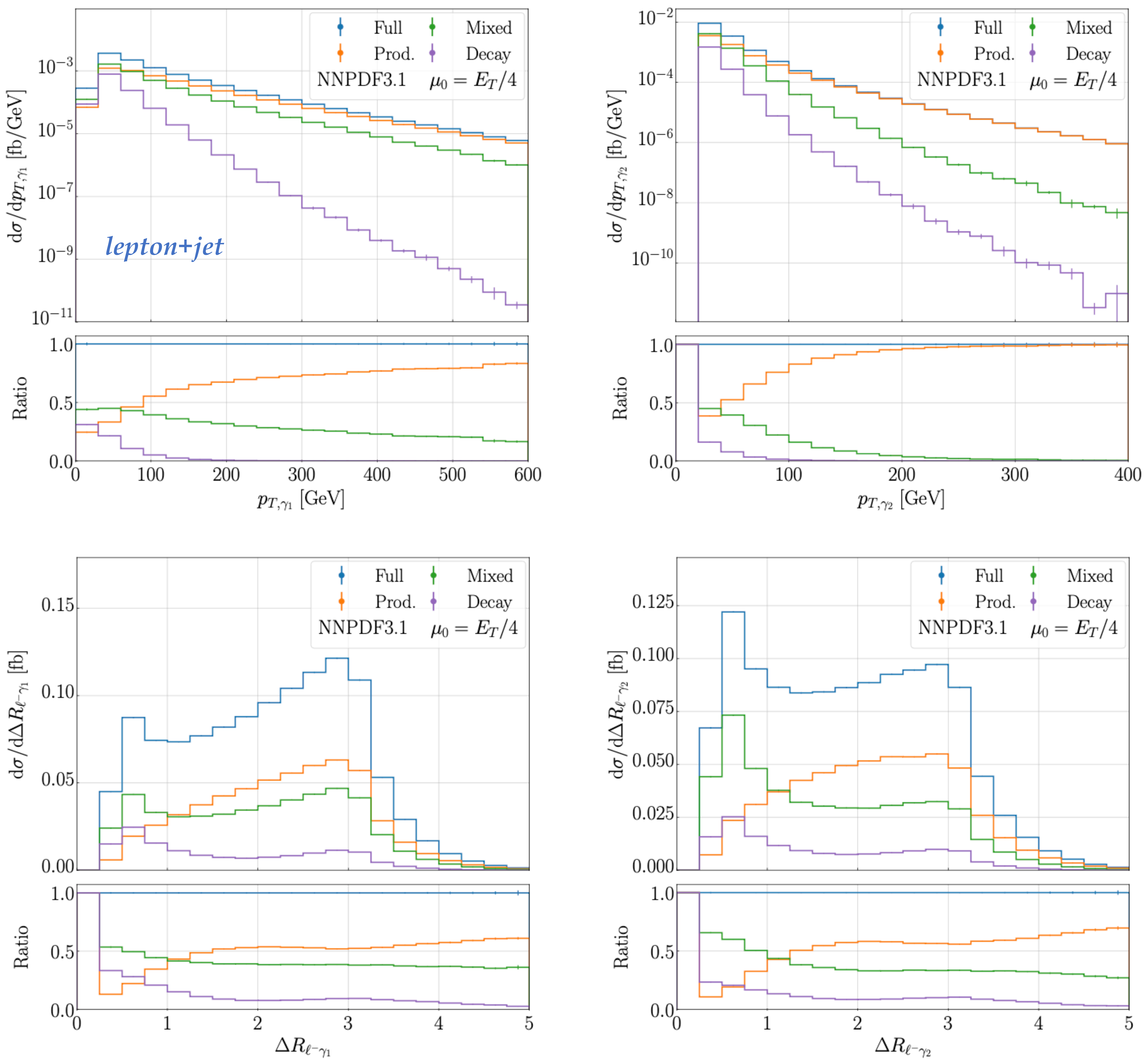
- Integrated fiducial cross-section level @ NLO in QCD with & without $|m_W - M_{jj}| < Q_{\text{cut}} = 15\,\text{GeV}$ $p_{T,b} > 25\,\text{GeV},\ p_{T,j} > 25\,\text{GeV},\ p_{T,\gamma} > 25\,\text{GeV}$:

✳ *Prod. contribution* at the level of *48%* ➪ *40%*

✳ *Mixed contribution* at the level of *40%* ➪ *43%*

✳ *Decay contribution* is about half the size *12%* ➪ *17%*

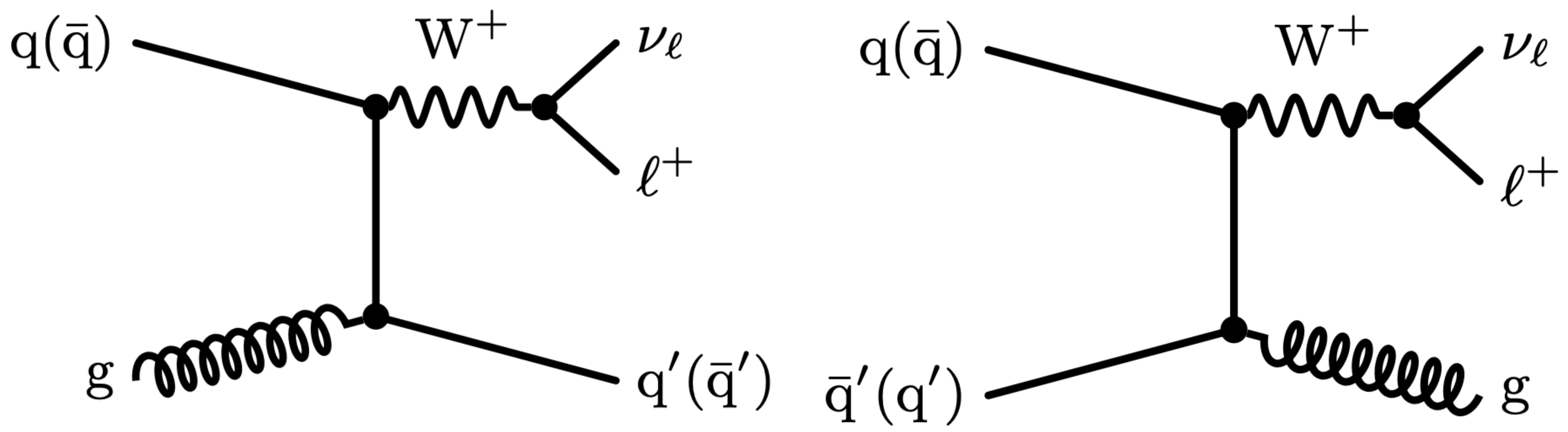
Stremmer, Worek
JHEP08 (2023) 179



The N3LO Project

Ambitious Plans

- Phenomenology of di-boson final states: $\gamma\gamma$, W^+W^-, ZZ, $W^\pm\gamma$, $Z\gamma$



- Predicitions for: $X + \text{jet}, X \in \{\gamma, W^\pm, Z, H\}$

- Methodology following the project: N-jettiness slicing

  Beam functions for N-jettiness at N3LO in perturbative QCD

  Baranowski, Behring, Melnikov, Tancredi, Wever, JHEP 02 (2023) 073

  One-loop corrections to the double-real emission contribution to the zero-jettiness soft function at N3LO in QCD

  Baranowski, Delto, Melnikov, Pikelner, Wang, 2401.05245

  N-jettiness soft function at next-to-next-to-leading order in perturbative QCD

  Agarwal, Melnikov, Pedron, 2403.03078

- How about a proper subtraction scheme? Need limits of amplitudes

  Revisiting the double-soft asymptotics of one-loop amplitudes in massless QCD

  Czakon, Eschment, Schellenberger, JHEP 04 (2023) 065

Conclusions

Achievements since last Annual Meeting

- Many results for phenomenologically relevant processes with gauge bosons, top quarks and jets in the final state at NLO and NNLO

- Theoretical progress on factorisation in QCD

- Substantial progress towards N3LO-precise predictions that will soon be reflected in publications