INTRODUCTION TO SMEFT

P3H SUMMER SCHOOL

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LECTURE 1

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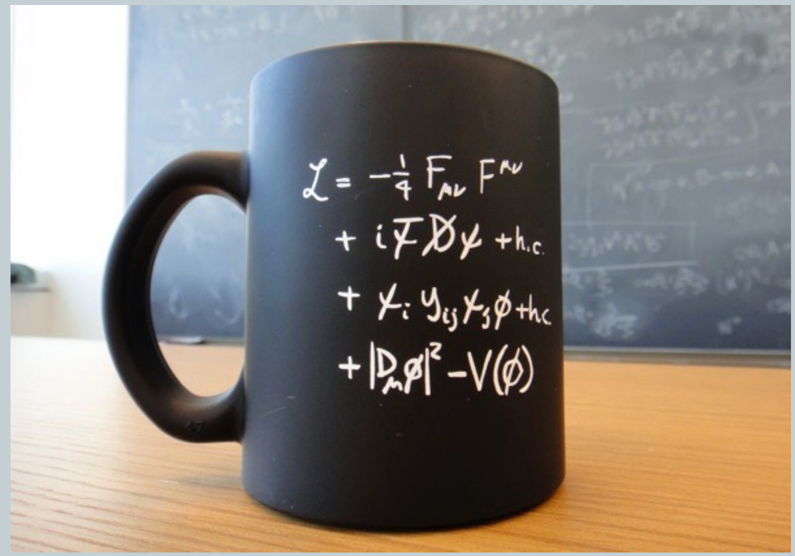
Please find all my typos!

THE SM IS SIMPLE AND PREDICTIVE

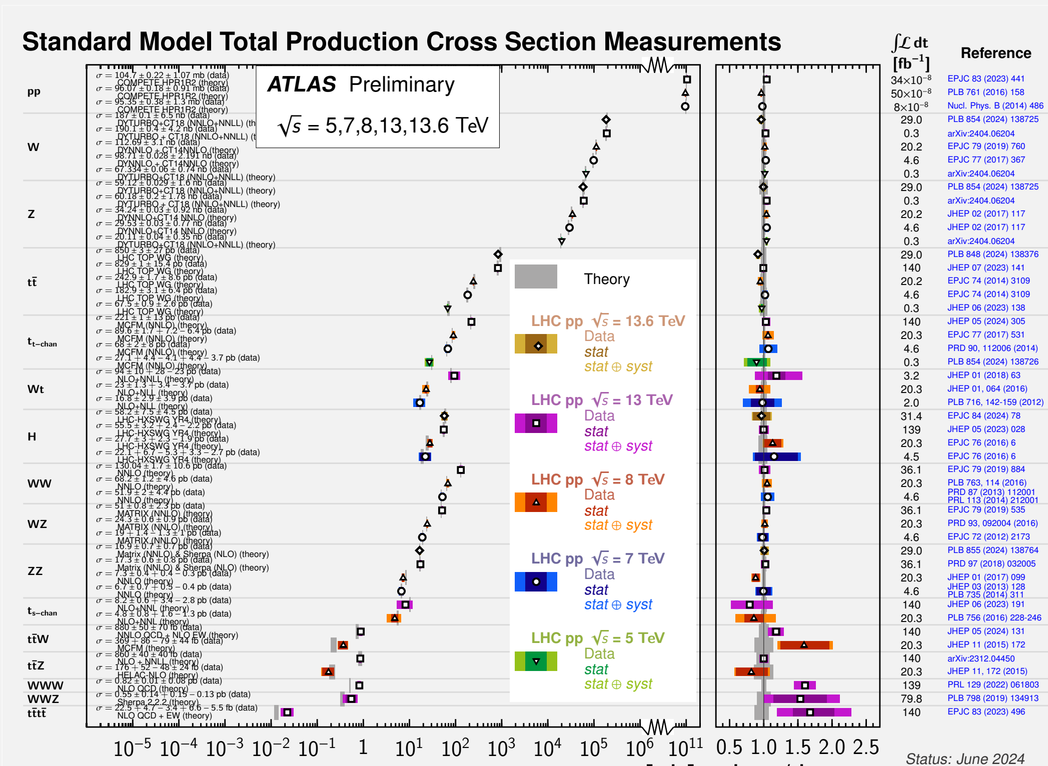
- SU(3) x SU(2) x U(1)
- Electroweak sector described in terms of masses and 3 inputs
  - Typically G_F, α, M_Z
- Particle couplings fixed

Only unknown parameter is Higgs mass

Testable model !



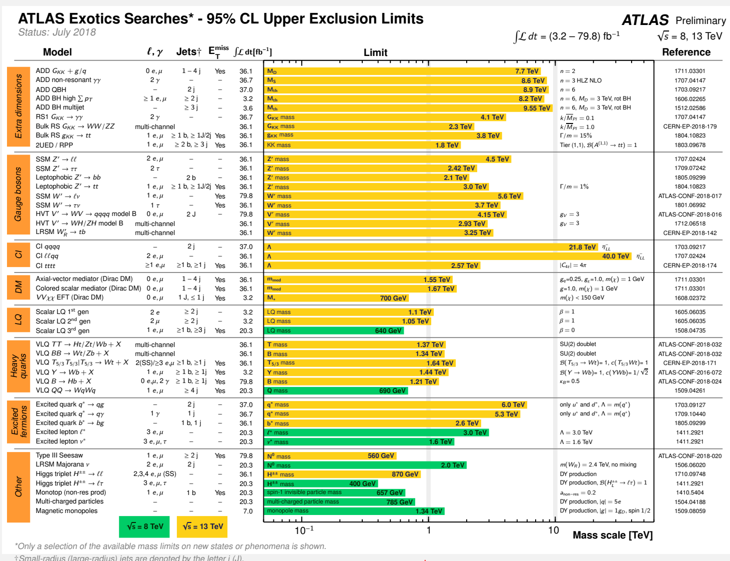
LHC MEASUREMENTS LOOK "SM-LIKE"

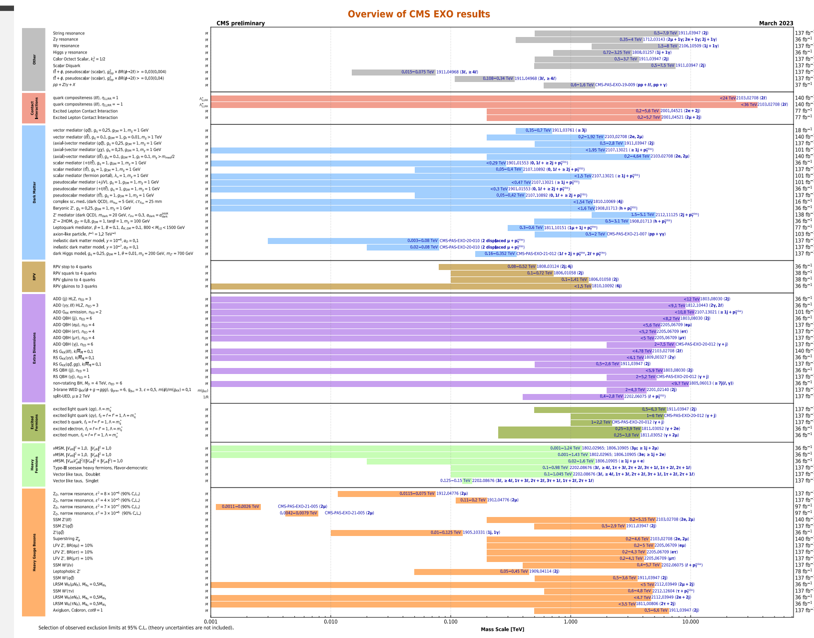


S. Dawson

Theory/experiment agreement over many orders of magnitude and for many different processes

NO NEW PARTICLES DISCOVERED *(YET?)*





Many limits exceed 1 TeV

IS THERE MORE?

- How do we know if the SM with the Higgs is just the low energy manifestation of some more complete model that exists at high scales?

USEFUL REFERENCES

- ***As Scales Become Separated: Lectures on Effective Field Theory***, Tim Cohen, 1903.03622
- ***Introduction to Effective Field Theories***, Aneesh Manohar, 1804.05863
- ***The Standard Model as an Effective Field Theory***, Ilaria Brivio and Michael Trott, 1706.08945
- **Feynman Rules for the Standard Model Effective Field Theory in R_ξ Gauges,** Dedes, Materkowska, Paraskevas, Rosiek, and Suxho, 1704.03888

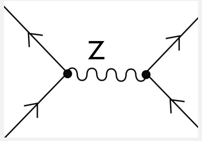
WHAT I DON'T DO

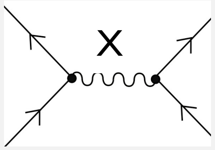
- I assume a basic knowledge of QFT and renormalization
- There are many automated tools for EFTS that I won't discuss
- There is tremendous progress in one-loop matching and dimension-8 effects

WHY EFTS?

- Electroweak (and TeV) scale physics seems to be well described by SM, suggesting that new physics essentially decouples
  - We don't know the source of dark matter, mases….
- EFTs can simplify things
- EFTS can help to understand large logarithms

HIGH SCALE DECOUPLING

- SM scattering: 

$A_{SM} \sim \frac{g^2}{M_Z^2}$
- Suppose there is a new particle X, with mass $M_X >> M_W$
- Contribution from X: 

$A_X \sim \frac{g_X^2}{M_X^2}$
- Scattering rate: $\sigma \sim \sigma_{SM} + \frac{g^2 g_X^2}{M_X^2} \to \sigma_{SM}$

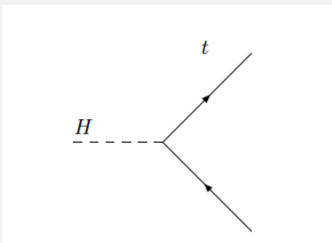
Effects of X vanish as $1/M_X^2$ for ***weak coupling***

Applequist-Carrazone decoupling theorem

What are exceptions to decoupling theorem?

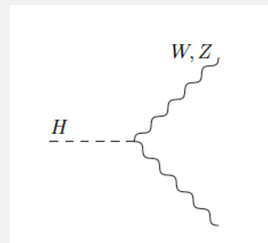
THE HIGGS IS DIFFERENT

- Particles whose couplings are proportional to mass don't decouple



$$-i\frac{m_f}{v}$$

See non-decoupling effect in $gg \rightarrow H$

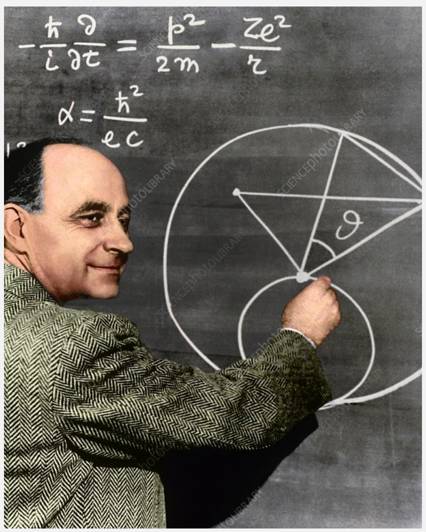


$$-2i\frac{M_V^2}{v}g^{\mu\nu} \qquad \epsilon_L(p_V) \sim \frac{p_V}{M_V}$$

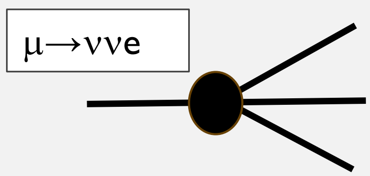
Longitudinal polarizations also change counting (growth with energy)

Suggests that Higgs and longitudinal gauge bosons interactions are good places to look for new physics

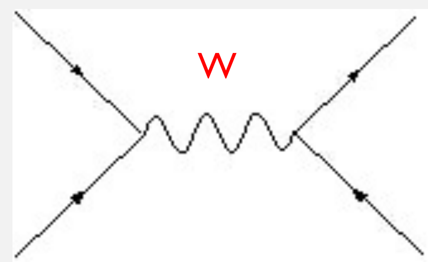
INDIRECTLY DISCOVER NEW PHYSICS



- Fermi theory (μ→ννe) becomes non-perturbative at E ~ 600 GeV
- W boson saves the day



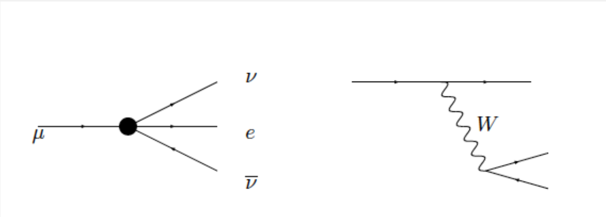
$G_F E^2 \rightarrow G_F M_W^2$



- Indirectly discover new physics
- Goal is to apply this lesson to TeV scale physics

OUR FIRST EFT

- Full theory is SM: Renormalizable, consistent dimension-4 theory



$$A_{\text{low energy}} = -\frac{G_F}{\sqrt{2}}(\overline{\psi}\gamma^\mu(1-\gamma_5)\psi)(\overline{\psi}\gamma_\mu(1-\gamma_5)\psi)$$

$$A_{\text{high energy}} = \frac{g^2}{2}(\overline{\psi}\gamma^\mu\frac{(1-\gamma_5)}{2}\psi)(\overline{\psi}\gamma_\mu\frac{(1-\gamma_5)}{2}\psi)\left(\frac{1}{q^2-M_W^2}\right)$$

Predict coefficients of low energy effective theory (G_F) in terms of UV physics (g, M_W)

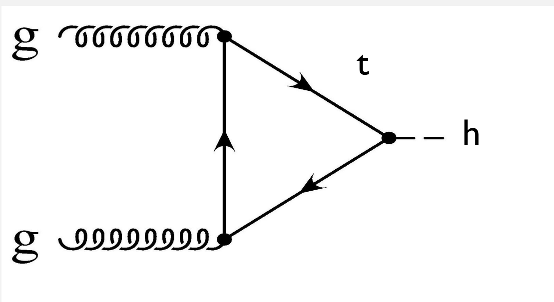
$$\frac{1}{q^2-M_W^2} = -\frac{1}{M_W^2}\left[1+\frac{q^2}{M_W^2}+\ldots\right]$$

$$q^2 << M_W^2 \rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

At matching scale (M_W) theories must give same result

OUR SECOND EFT

- High energy SM with top quark = full UV complete theory
- Low energy theory valid at scales << top quark (effective theory):



Full theory:

$$\sigma_{gg\to h} = \frac{\alpha_s^2}{1024\pi v^2} \mid \Sigma_q F_q\left(\frac{m_h^2}{4m_t^2}\right) \mid^2 \delta\left(1-\frac{m_h^2}{s}\right)$$

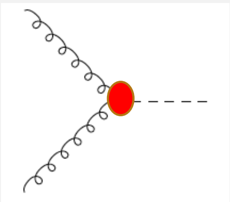
$$m_t^2 >> s \quad \to \frac{\alpha_s^2}{576\pi v^2}\delta\left(1-\frac{m_h^2}{s}\right)$$

- For heavy chiral fermion $F_{1/2} \to - 4/3$ independent of mass

Why does this show there cannot be a 4th generation of heavy chiral quarks?

OUR SECOND EFT

- This result yields the effective Lagrangian



$$L_{EFT} = \frac{\alpha_s}{12\pi v^2} \mid \phi \mid^2 G^A_{\mu\nu} G^{A,\mu\nu}$$

$$\phi^0 \rightarrow \frac{h+v}{\sqrt{2}}$$

- The effective Lagrangian reproduces the full theory $gg \rightarrow h$ large m_t result, but also contains an hhGG interaction whose strength is correlated with that of $gg \rightarrow h$
- This effective Lagrangian is invariant under SU(3) x SU(2) x U(1) and contains only SM fields

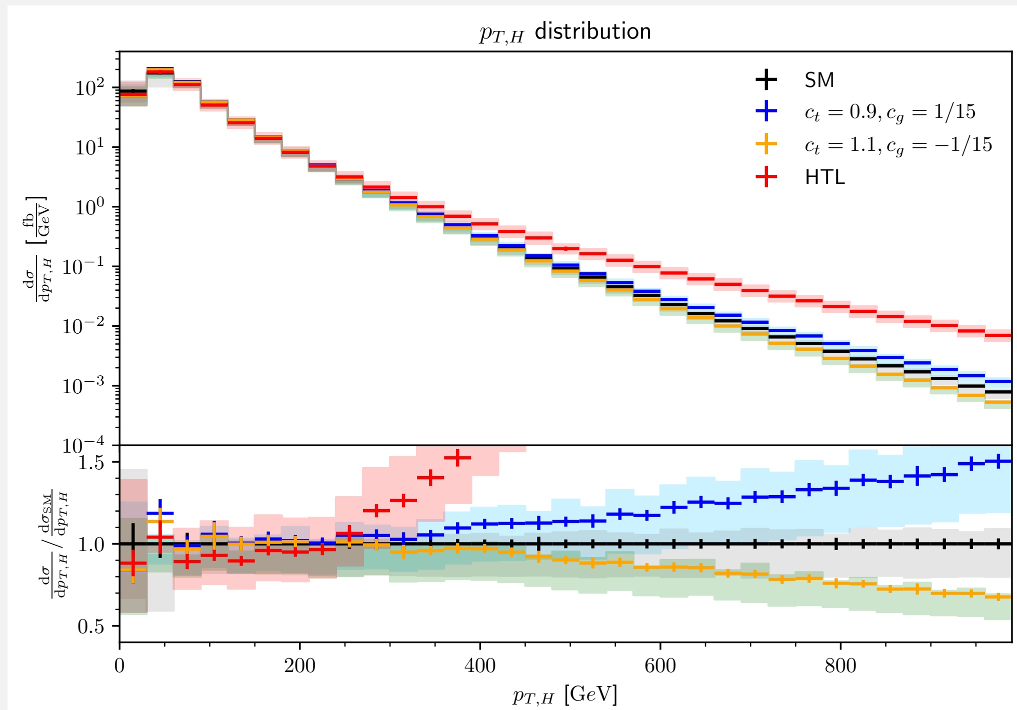
Historically, this operator used to calculate higher order corrections to $gg \rightarrow h$

MOMENTUM DEPENDENCE

Higgs + jet production at NLO

- Preview of things to come:
- Momentum dependent operators change shapes of distributions
- Effects largest at high p_T
- This operator can be generated with heavy colored scalars for example

$$L_{EFT} = \frac{\alpha_s c_g}{8\pi^2 v^2} \mid \phi \mid^2 G^A_{\mu\nu} G^{A,\mu\nu}$$



2409.05728

ASSUME A HIERARCHY OF SCALES

$\Lambda >> M_W$ where complete theory exists

- Any new particles or symmetries are at this scale
- Expect effects of heavy particles at low scales to be suppressed *(decoupling!)*

This is sad scenario where there is no intermediate scale physics

Only SM particles in theory at low scales

M_W

- Learn about high scale physics by measuring interactions of effective low energy theory
- ***We don't need to know the complete theory***

SMEFT: SM EFFECTIVE FIELD THEORY

- Assumptions: New physics decouples Λ >> v, E
- At the weak scale: SM SU(3) x SU(2) x U(1) symmetry and SM particles only
- New physics described by

$$L_{SMEFT} = L_{SM} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \frac{L_7}{\Lambda^3} + \frac{L_8}{\Lambda^4}$$

$$L_n = \Sigma_i C_i^n O_i^n$$

Assume Higgs is in an SU(2) doublet

This is the big assumption

- New physics contributions contained in coefficients C
- Operators form a complete basis (not unique)
- L_5 and L_7 are lepton number violating

ADVANTAGES OF SMEFT APPROACH

- Quantum field theory where calculations done order by order in loop expansion and in $1/\Lambda$ expansion
  - Compute cross sections without knowing high scale (UV) physics
- Systematically improvable
- At this level, SMEFT calculations are model independent
- Measurements interpreted in terms of SMEFT coefficients
- Can compare very different classes of measurements

Sounds good, but how does this work in practice?

And even more important, how model independent is this?

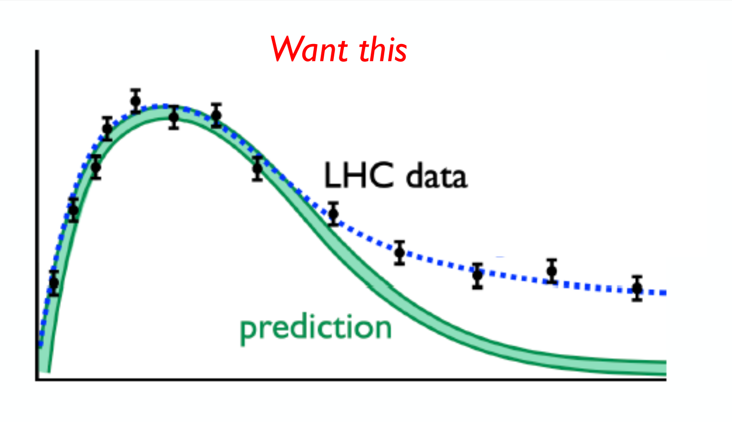
LEARNING FROM SMEFT

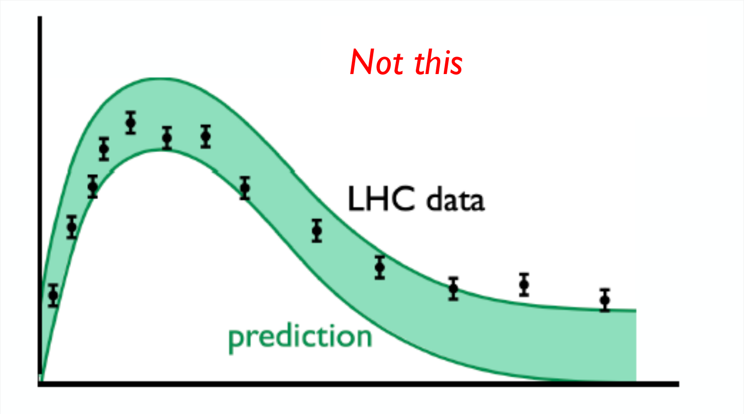
$$\text{Experiment} = \text{Theory}_{SM} + \sum \frac{x_i C_i^6}{\Lambda^2} + \ldots$$

Precise experimental measurements

Precise SM calculations

Precise SMEFT calculations





COUNTING DIMENSIONS

- Canonical dimensions in d=4

$$[\psi] \sim \frac{3}{2},\ [\phi] \sim 1,\ [W] \sim 1,\ [D_\mu] \sim 1$$

- SM is dimension 4: Allowed interactions

$$\phi^4,\ \phi\overline{\psi}\psi,\ D_\mu\phi D^\mu\phi,\ \overline{\psi}i\gamma^\mu D_\mu\psi,\ X^2_{\mu\nu}$$

  - Other forms vanish after integration over d^4x, or are related to these by integration by parts
- Only 1 dim-5 operator (for 1 generation) and it violates lepton number conservation
  - Generates Majorana neutrino mass

$$(\tilde{\phi}^\dagger L_L)^T C(\tilde{\phi}^\dagger L_L) \qquad \tilde{\phi} = i\sigma_2\phi \qquad L_L = \left(\begin{array}{c} \nu_L \\ e_L \end{array}\right) \qquad \phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right)$$

Usually start our EFT at dimension-6

COUNTING IN d=4-2e DIMENSIONS

- S matrix is dimensionless

$$[S] = 0 \quad [x] = -1 \quad [\partial_\mu] = [p_\mu] = [m] = 1$$

$$[S] = [\int d^d x L] = -d + [L] \longrightarrow [L] = d$$

- Find scaling of fields from kinetic energy

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \overline{\psi} i \partial \psi + ... \qquad [\phi] = 1 - \epsilon \quad [\psi] = \frac{3}{2} - \epsilon$$

- Couplings in Lagrangian should be dimensionless

$$L \sim \lambda \mu^{2\epsilon} \phi^4 + \frac{C}{\Lambda^2} \mu^{4\epsilon} \phi^6 + ...$$

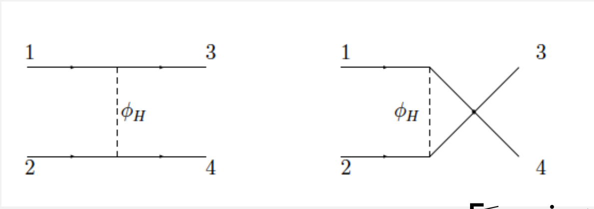
TOY MODEL

- Light fermion, ψ, light scalar, ϕ_L, and heavy scalar, ϕ_H, with $M_H >> m_L$

$$L_{UV} = i\overline{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial_\mu\phi_H)^2 - \frac{M_H^2}{2}\phi_H^2 + \frac{1}{2}(\partial_\mu\phi_L)^2 - \frac{m_L^2}{2}\phi_L^2 - \lambda_H\overline{\psi}\psi\phi_H - \lambda_L\overline{\psi}\psi\phi_L$$

- Integrate out ϕ_H using Feynman diagrams

$$\psi\psi \to \psi\psi$$



Fermi statistics

$$iA_{UV} = \overline{u}(p_3)u(p_1)\overline{u}(p_4)u(p_2)(-i\lambda_H)^2\frac{i}{(p_1-p_3)^2 - M_H^2} - (3 \leftrightarrow 4)$$

- Expand propagator

$$iA_{UV} = \overline{u}(p_3)u(p_1)\overline{u}(p_4)u(p_2)\left(i\frac{\lambda_H^2}{M_H^2}\right)\left[1 + \frac{(p_1-p_3)^2}{M_H^2} + \mathcal{O}\left(\frac{p^4}{M_H^4}\right) + ...\right] - (3 \leftrightarrow 4)$$

*plus interactions with purely light fields that are present in both UV theory and low energy IR theory

TOY MODEL #2

- To LO in external momentum, low energy (IR) theory is* $L^0_{EFT} = i\overline{\psi}\partial\!\!\!/\psi + \frac{C}{2}(\overline{\psi}\psi)(\overline{\psi}\psi)$

$\psi\psi \to \psi\psi \qquad iA_{IR} = \overline{u}(p_3)u(p_1)\overline{u}(p_4)u(p_2)iC - (3 \leftrightarrow 4)$

- Match coefficients in UV and IR theory:

$$A_{UV} = A_{IR} \Rightarrow C = \frac{\lambda_H^2}{M_H^2}$$

- Next order in the momentum expansion:

$$L^1_{EFT} = i\overline{\psi}\partial\!\!\!/\psi + \frac{\lambda_H^2}{2M_H^2}(\overline{\psi}\psi)(\overline{\psi}\psi) + C_2(\partial_\mu\overline{\psi}\partial^\mu\psi)(\overline{\psi}\psi)$$

← Postulate this form

dim-6 dim-8

* Plus ϕ_L terms….

TOY MODEL, #3

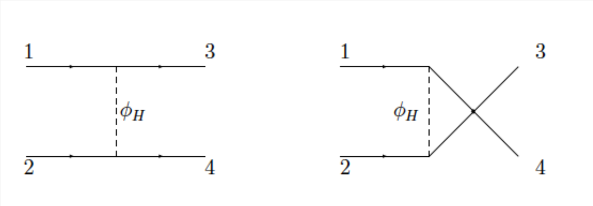
- Effective Lagrangian must be valid on and off shell
- Use any convenient choice of momentum: take $p_i^2=0$, so amplitude depends on $p_i \cdot p_j$

$$iA^8_{IR} = iC_2(p_1 \cdot p_3 + p_2 \cdot p_4)\overline{u}(p_3)u(p_1)\overline{u}(p_4)u(p_2) - (3 \leftrightarrow 4)$$

- Term from propagator in UV theory is $-2i\frac{\lambda_H^2}{M_H^4}p_1 \cdot p_3$
- Matching UV and IR

$C_2=-\lambda_H^2/M_H^4$

Systematic expansion in powers of $1/M_H^2$



* Conservation of momenta: $p_1 \cdot p_3 = p_2 \cdot p_4$

TOY MODEL, #4

- Now we have effective Lagrangian in terms of light fields,

$$L^1_{EFT} = i\overline{\psi}\partial\!\!\!/\psi + \frac{\lambda_H^2}{2M_H^2}(\overline{\psi}\psi)(\overline{\psi}\psi) - \frac{\lambda_H^2}{M_H^4}(\partial_\mu\overline{\psi}\partial^\mu\psi)(\overline{\psi}\psi)$$

← Dimension-8 operator

- Is this a unique prescription? Could have written:

$$(\partial^2\overline{\psi}\psi)(\overline{\psi}\psi)\,,(\overline{\psi}\partial^2\psi)(\overline{\psi}\psi)\,,(\partial_\mu\overline{\psi}\psi)(\overline{\psi}\partial^\mu\psi)\,,(\partial_\mu\overline{\psi}\partial^\mu\psi)(\overline{\psi}\psi)$$

- Integration by parts gives one relationship among coefficients
- $(\partial^2\psi)$ does not contribute on-shell, so two new operators are sufficient
  - Only one contributed at tree level

Different basis possible: same physics

No unique prescription

* Also ϕ_L

TOY MODEL, #5

- We matched coefficients at high scale, M_H (H stands for "heavy" here)
- Calculate amplitude at one loop in dim-6 effective theory (no ϕ_H)

$$L_{EFT} = i\overline{\psi}_0 \not\partial \psi_0 + \frac{C_0}{2}(\overline{\psi}_0\psi_0)(\overline{\psi}_0\psi_0) - \lambda_L \overline{\psi}_0 \psi_0 \phi_L$$

- Calculate renormalization coefficients as usual

$$\psi_0 = \sqrt{Z_\psi}\psi \quad C_0 = C\mu^{2\epsilon} Z_c$$

See 1006.2142 details of calculation of Z's

ψ_0 is unrenormalized field, etc

TOY MODEL, #6

- Bare quantities cannot depend on scale $$0 = \mu \frac{d}{d\mu} C_0$$

- EFTs assume large separation of scale: $m_L << M_H$

$$C(m_L) = C(M_H) - \frac{3\lambda_L^2}{8\pi^2} C \log\left(\frac{M_H}{m_L}\right)$$

- Always assume large logarithms

This example used a diagrammatic approach…. But there is a simpler way

MORE TOY MODELS

- Suppose there was a very massive heavy gauge singlet scalar, ϕ_H, and ϕ is SM-like scalar doublet

$$L_{UV} = \frac{1}{2}(\partial_\mu \phi_H)^2 - \frac{M_H^2}{2}\phi_H^2 - A \mid \phi \mid^2 \phi_H - \frac{\kappa}{2} \mid \phi \mid^2 \phi_H^2 - \frac{\mu}{6}\phi_H^3 - \frac{\eta_H}{4}\phi_H^4 + \text{SM HIGGS TERMS}$$

- Re-write

$$L_{UV} = -A \mid \phi \mid^2 \phi_H + \phi_H \left[-\frac{1}{2}\partial_\mu \partial^\mu - \frac{M_H^2}{2} - \frac{\kappa}{2} \mid \phi \mid^2 \right] \phi_H + \mathcal{O}(\phi_H^3)$$

- "Integrate out ϕ_H" using equations of motion, $\partial L / \partial \phi_H = 0$, assuming $M_H >> m_h$

* ϕ potential is usual SM one

arxiv:1412.1837

A LOT OF PHENO
WITH THIS MODEL

DECOUPLING TOY MODEL

- Linearized equations of motion give classical solution:

$$\phi_H^c \sim -\frac{A \mid \phi \mid^2}{\partial^2 + M_H^2 + \kappa \mid \phi \mid^2}$$ Just rearrangement

$$\sim -\frac{1}{M_H^2} A \mid \phi \mid^2 + \frac{1}{M_H^4}\left(\partial^2 + \kappa \mid \phi \mid^2\right) A \mid \phi \mid^2$$

$$M_H >> m_h$$

- Substitute into original Lagrangian to generate new operators

$$L_{EFT} = -A \mid \phi \mid^2 \phi_H^c + \frac{1}{2}\phi_H^c\left(-\partial^2 - M_H^2 - \kappa \mid \phi \mid^2\right)\phi_H^c - \frac{\mu}{6}(\phi_H^c)^3 - \frac{\eta_H}{4}(\phi_H^c)^4$$

$$\sim \frac{1}{2M_H^2} A^2 \mid \phi \mid^4 + \frac{A^2}{M_H^4} O_H + \left(-\frac{\kappa A^2}{2M_H^4} + \frac{\mu A^3}{6M_H^6}\right) O_\phi$$

$$O_H = \frac{1}{2}(\partial_\mu \mid \phi \mid^2)^2$$

$$O_\phi = (\phi^\dagger \phi)^3$$

O_ϕ shifts Higgs trilinear coupling

*m_h is SM Higgs doublet field

TOY MODEL CONTINUED

- Modification of SM quartic Higgs coupling: $\lambda_h \rightarrow \lambda_h - \frac{A^2}{2M_H^2}$
- In consistent EFT, many new effects….but suppressed by $1/M_H^2$

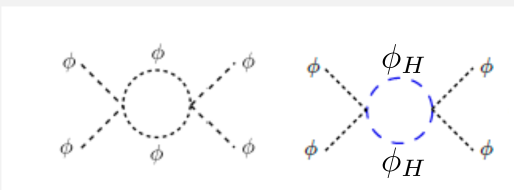
Idea is that UV model can be determined by measuring pattern of coefficients

TOY MODEL AT ONE-LOOP, #1

- Light and heavy scalar:

$$L_{UV} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \phi_H\left[-\frac{1}{2}\partial_\mu\partial^\mu - \frac{M_H^2}{2} - \frac{\kappa}{2} \mid \phi \mid^2\right]\phi_H$$

- Calculate $\phi\phi \rightarrow \phi\phi$ in $\overline{MS}$ (ie drop poles). (Also t- and u- channel, plus tree level)



$$A_{UV} = -6\lambda + \frac{27}{8\pi^2}\lambda^2\left[\log\left(\frac{\bar{\mu}^2}{m^2}\right) + \frac{2}{3}\right] + \frac{3}{8\pi^2}\kappa^2 \log\left(\frac{\bar{\mu}^2}{M^2}\right)$$

- No choice of scale eliminates logs

$$\overline{\mu} \equiv \mu^2 \frac{e^{\gamma_E}}{(4\pi)}$$

How does decoupling work?

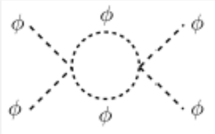
*Integral computed at threshold

TOY MODEL AT ONE- LOOP, #2

- Now compute $\phi\phi \to \phi\phi$ in EFT

$$L_{EFT} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \frac{C_{\phi 4}}{4}\phi^4$$

$$A_{EFT} = 6C_{\phi 4} + \frac{27}{8\pi^2}C_{\phi 4}^2\left[\log\left(\frac{\bar{\mu}^2}{m^2}\right) + \frac{2}{3}\right]$$



- Note Lagrangian coefficients can be different in EFT (it's a different theory)
- At matching scale, Λ: $A_{EFT} = A_{UV}$
- At tree level: $C_{\phi 4} = -\lambda$
- At one-loop:

$$\delta C_{\phi 4}(\Lambda) = \frac{\kappa^2}{16\pi^2}\log\left(\frac{\Lambda^2}{M^2}\right)$$

Matching has no logarithmic dependence on low scale, m

Take Λ~M to fix log small

MORE ON SCALES

- Since matching is done at Λ, low energy amplitude is:

$$A_{EFT} = 6C_{\phi 4}(\Lambda) + \frac{27}{8\pi^2} C_{\phi 4}(\Lambda)^2 \left[\log\left(\frac{\Lambda^2}{m^2}\right) + \frac{2}{3}\right]$$

- RGE running of $C_{\phi 4}$ from Λ to μ_L

$$C_{\phi 4}(\mu_L) = C_{\phi 4}(\Lambda) + \frac{9}{16\pi^2} C_{\phi 4}(\Lambda)^2 \log\left(\frac{\mu_L^2}{\Lambda^2}\right)$$

- No large logs in EFT amplitude

$$A_{EFT} = 6C_{\phi 4}(\mu_L) + \frac{27}{8\pi^2} C_{\phi 4}(\mu_L)^2 \left[\log\left(\frac{\mu_L^2}{m^2}\right) + \frac{2}{3}\right]$$

* Can resum logarithms

SCALES AND THE EFT

UV Model $\rightarrow$ C(Λ) **Matching**

$$\frac{dC_i(\mu)}{d\log\mu} = \Sigma_j \frac{1}{16\pi^2}\gamma_{ij}C_j$$

Running

Note that γ_{ij} is not diagonal in general

EFT$\rightarrow$ C(M_W) **Precision**

Assume large separation of scales

RUNNING AND NEW PHYSICS

- Anomalous dimensions of dim-6 operators to NLO known $$\frac{dC_i(\mu)}{d\log\mu} = \Sigma_j \frac{1}{16\pi^2}\gamma_{ij}C_j$$
- Solve:

$$C_i(\mu) = C_i(M_Z) + \frac{1}{16\pi^2}\gamma_{ij}C_j \log\left(\frac{\mu}{M_Z}\right)$$

- Operator mixing can generate new effects
- Example: $C_{\phi D}$ is T parameter (isospin violation). Toy model generated $C\phi$ but not $C_{\phi D}$
- Running of C_ϕ:

$$C_\phi(\mu) = C_\phi(M_Z) + \frac{1}{16\pi^2}\left[(..)C_\phi + (...)C_{\phi D} + ...\right]\log\left(\frac{\mu}{M_Z}\right)$$

FINDING A BASIS

- Suppose we have a light scalar ϕ
- Write all possible terms to dimension-6

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \frac{C_\phi}{\Lambda^2}\phi^6 + \frac{C_d}{\Lambda^2}\phi^3 \partial^2 \phi$$

- A basis requires that we eliminate all operators that can be removed by field redefinitions or equations of motion

$$\frac{\partial L}{\partial \phi} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu \phi)} \right) = 0 \longrightarrow \partial^2 \phi = -m^2 \phi - \lambda \phi^3$$

$$L \longrightarrow \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \left(\frac{\lambda}{4} + m^2 \frac{C_d}{\Lambda^2} \right)\phi^4 + \frac{C_\phi - C_d \lambda}{\Lambda^2}\phi^6$$

- Only 1 non-redundant dimension-6 operator
- Same physics

FINDING A BASIS

- Could alternatively make a field redefinition and recover the same L
  - Physical predictions are independent of field redefinitions

$$\phi \rightarrow \phi + \frac{C_d}{\Lambda^2}\phi^3$$

- Or finally could do it diagrammatically
- At linear level field redefinition and equations of motion are equivalent
- Operators that can be eliminated with equations of motion are called *redundant operators* and are not needed to compute physical observables