

A gravity driven inverse cascade controls the size distribution of raindrops

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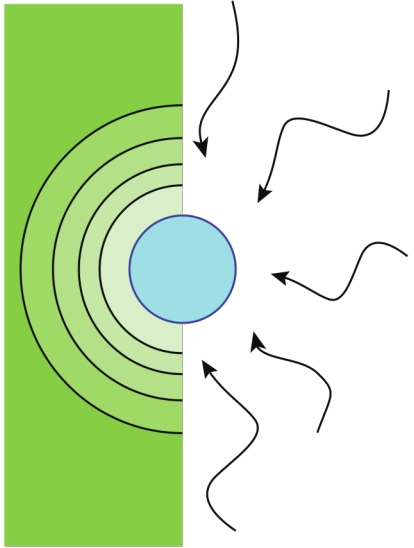
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The warm rain process

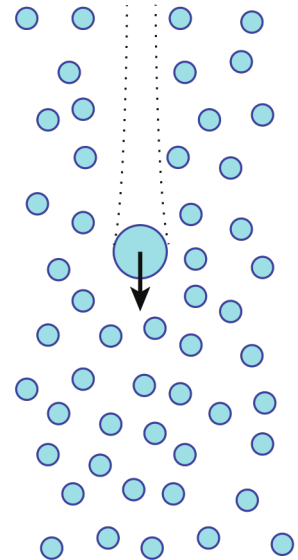
Condensation growth



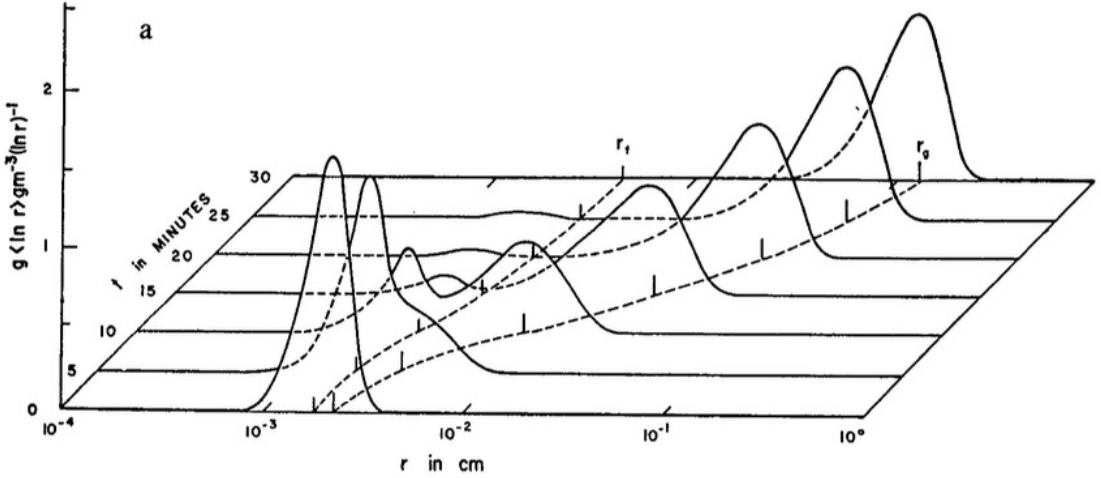
Collision-coalescence:

$$U^t \propto \sqrt{R}$$

Berry & Reinhardt, 1974



What controls the rain initiation time and the DSD?

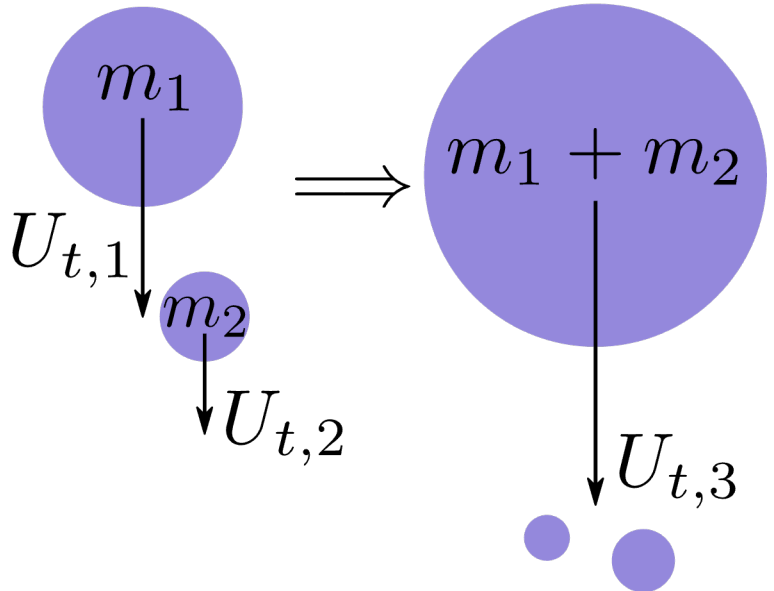


Statistical description of growth by collision

$$U^t \propto \sqrt{R}$$

$$\frac{\partial n}{\partial t} = \frac{1}{2} \int_{m_1+m_2=m} dm_1 dm_2 K_{12} n(m_1) n(m_2)$$

Nonlinear growth process:



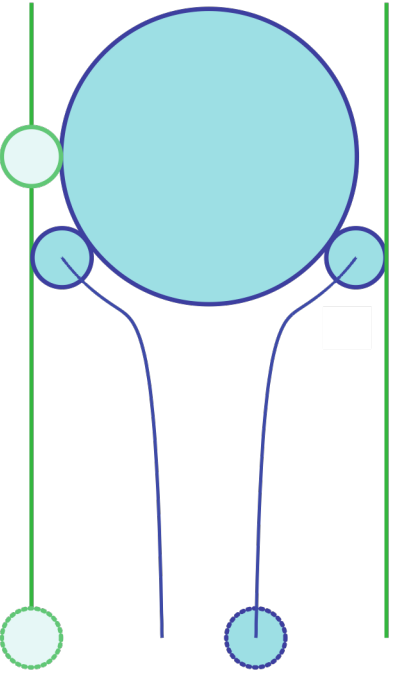
$$- \int_0^{\infty} dm_1 K_{12} n(m) n(m_1)$$

Collision kernel:

$$K_{12} = \underbrace{\pi(R_1 + R_2)^2}_{\text{cross-section}} \underbrace{(U_1^t - U_2^t)}_{\text{relative velocity}}$$

A collision kernel from μm - to mm -size droplets

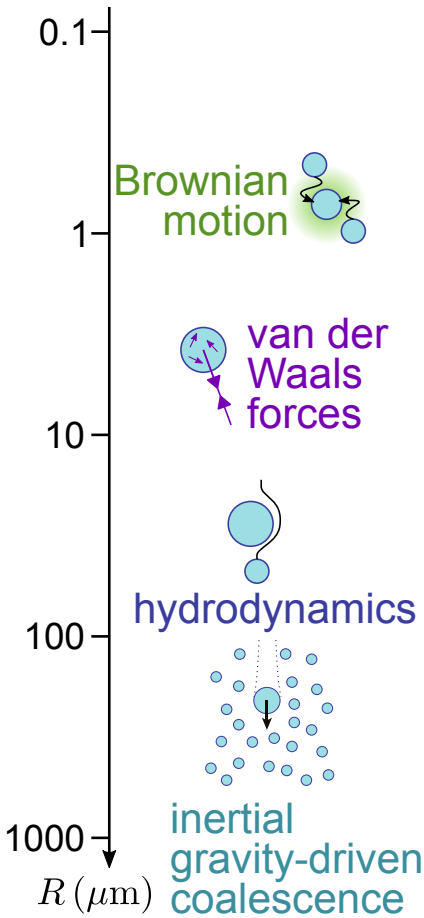
$$St = \frac{\tau_{inertia}}{\tau_{forces}}$$



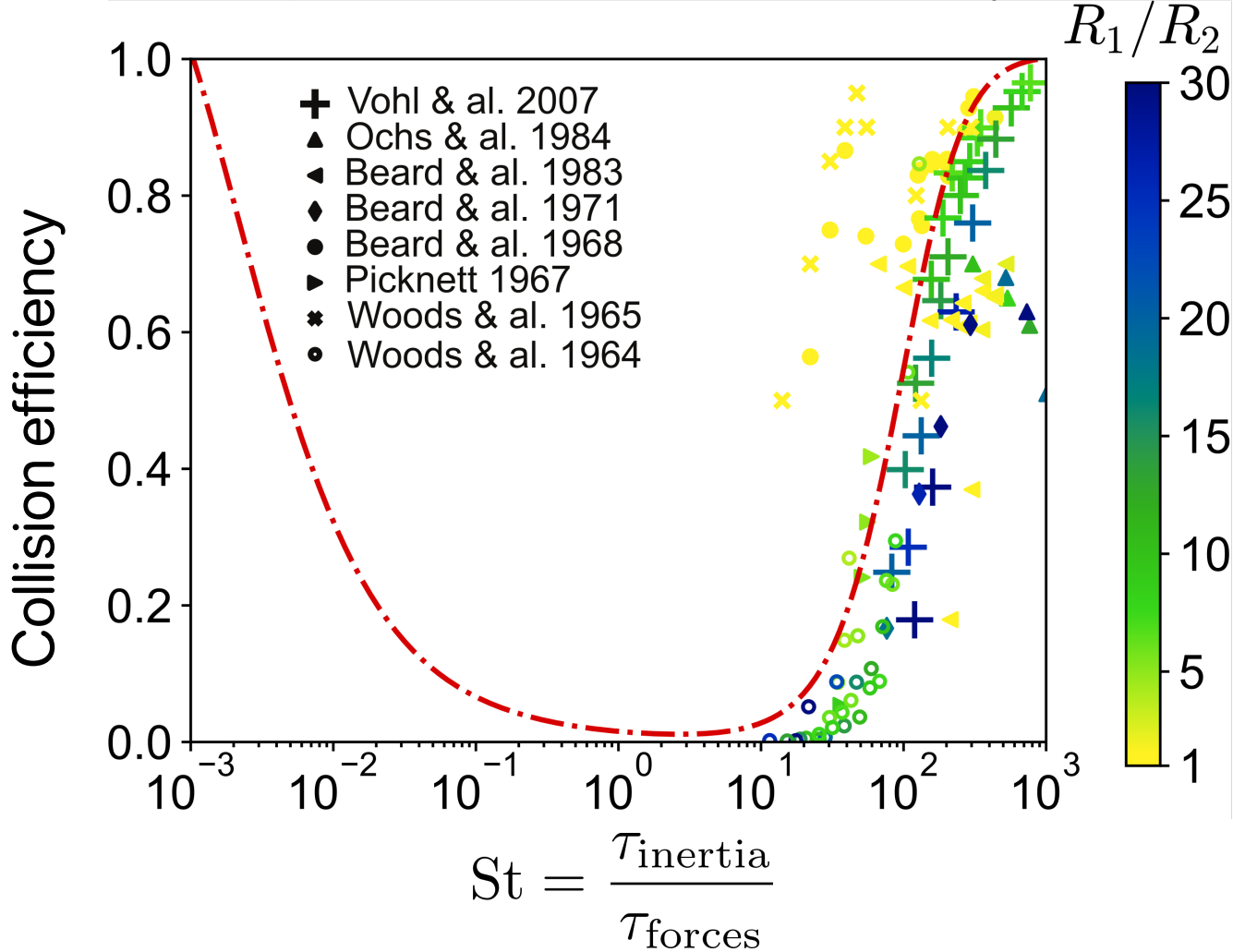
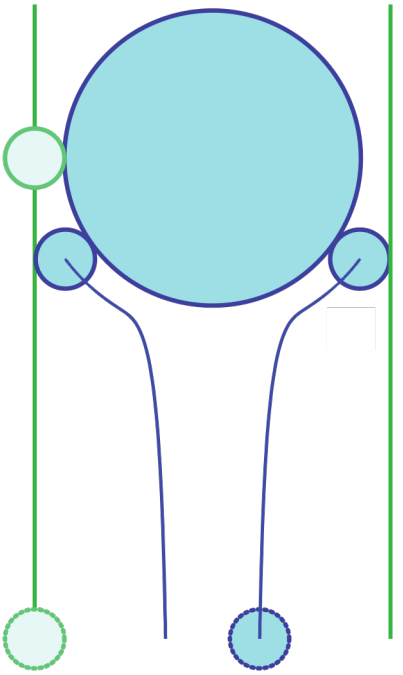
- Monte-Carlo simulation of binary droplet collision
- Taking into account finite Reynolds effects, non-continuum effects in the lubrication film, van der Waals interactions and Brownian motion



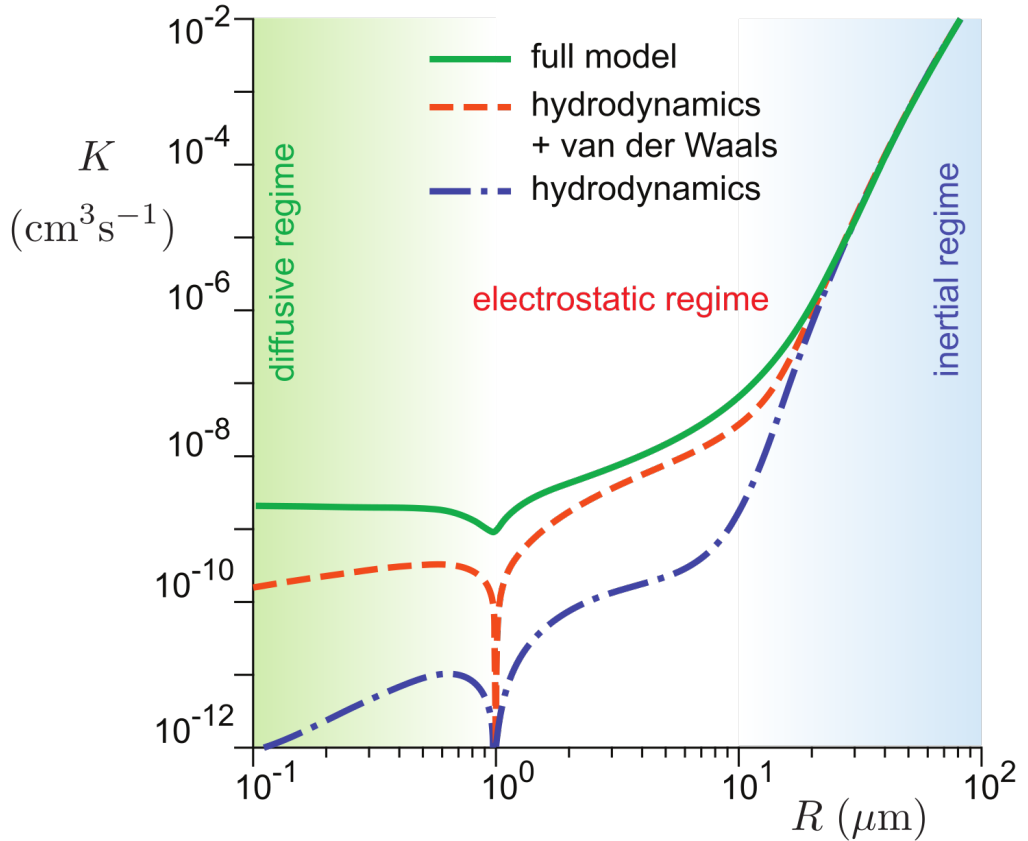
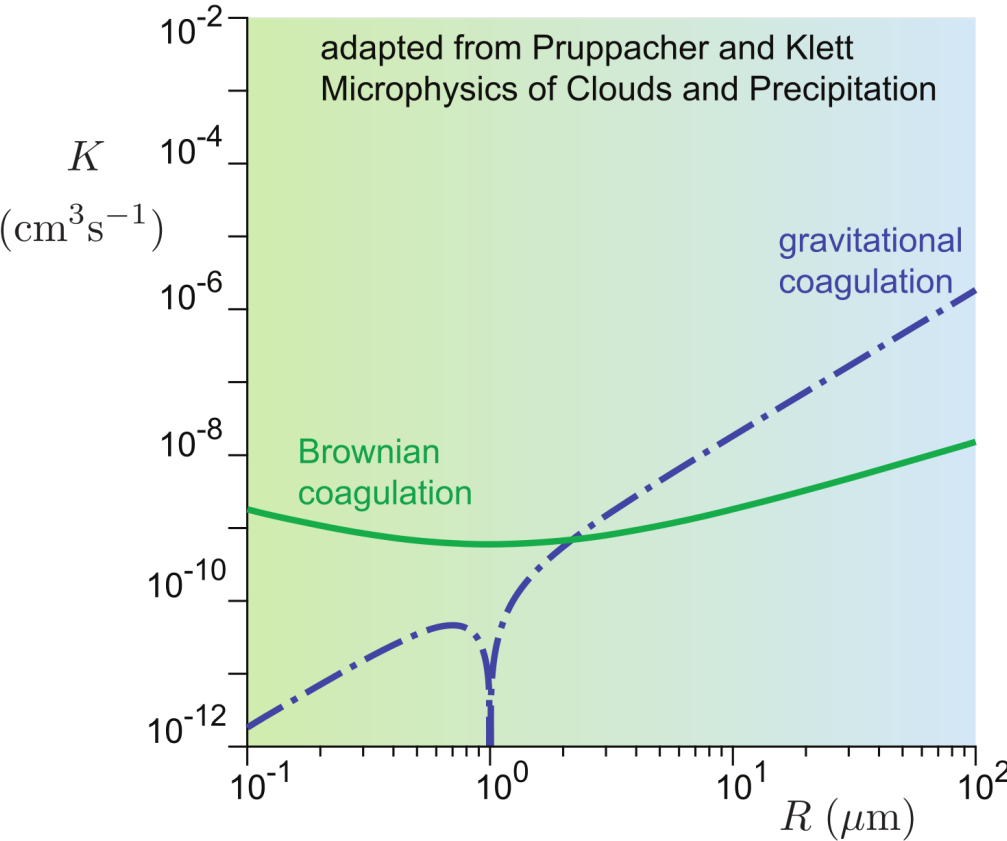
F. Poydenot, B. Andreotti, JFM 2024



A collision kernel from μm - to mm -size droplets

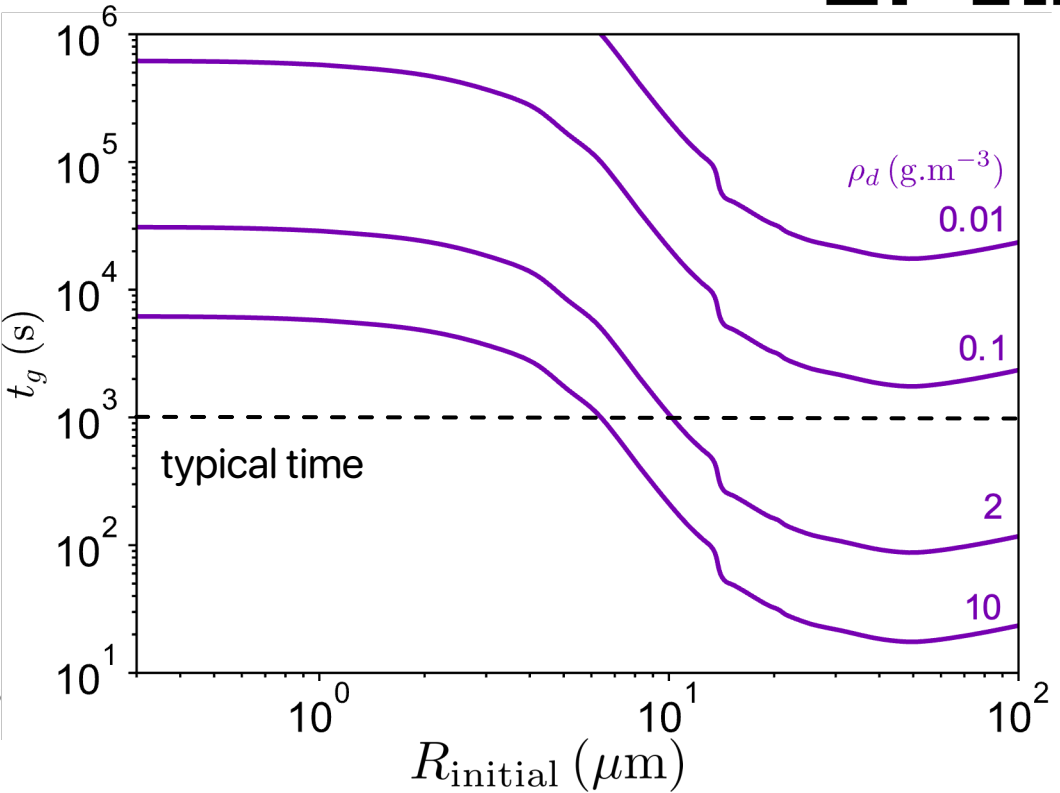
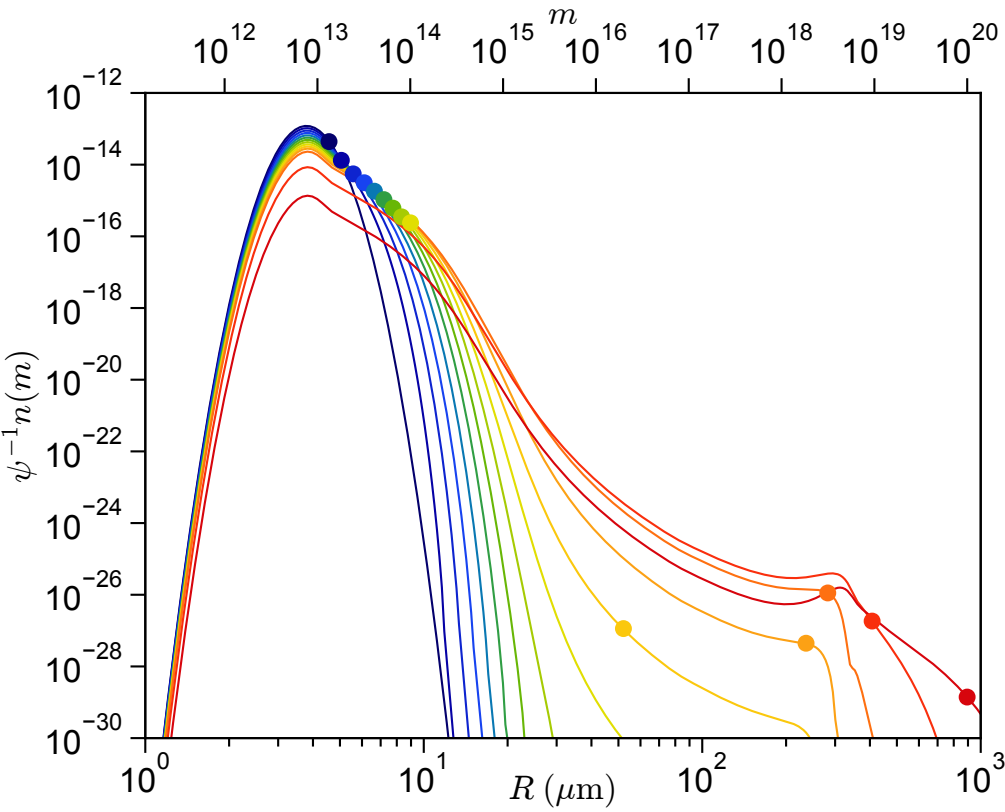


A new picture of the collision kernel



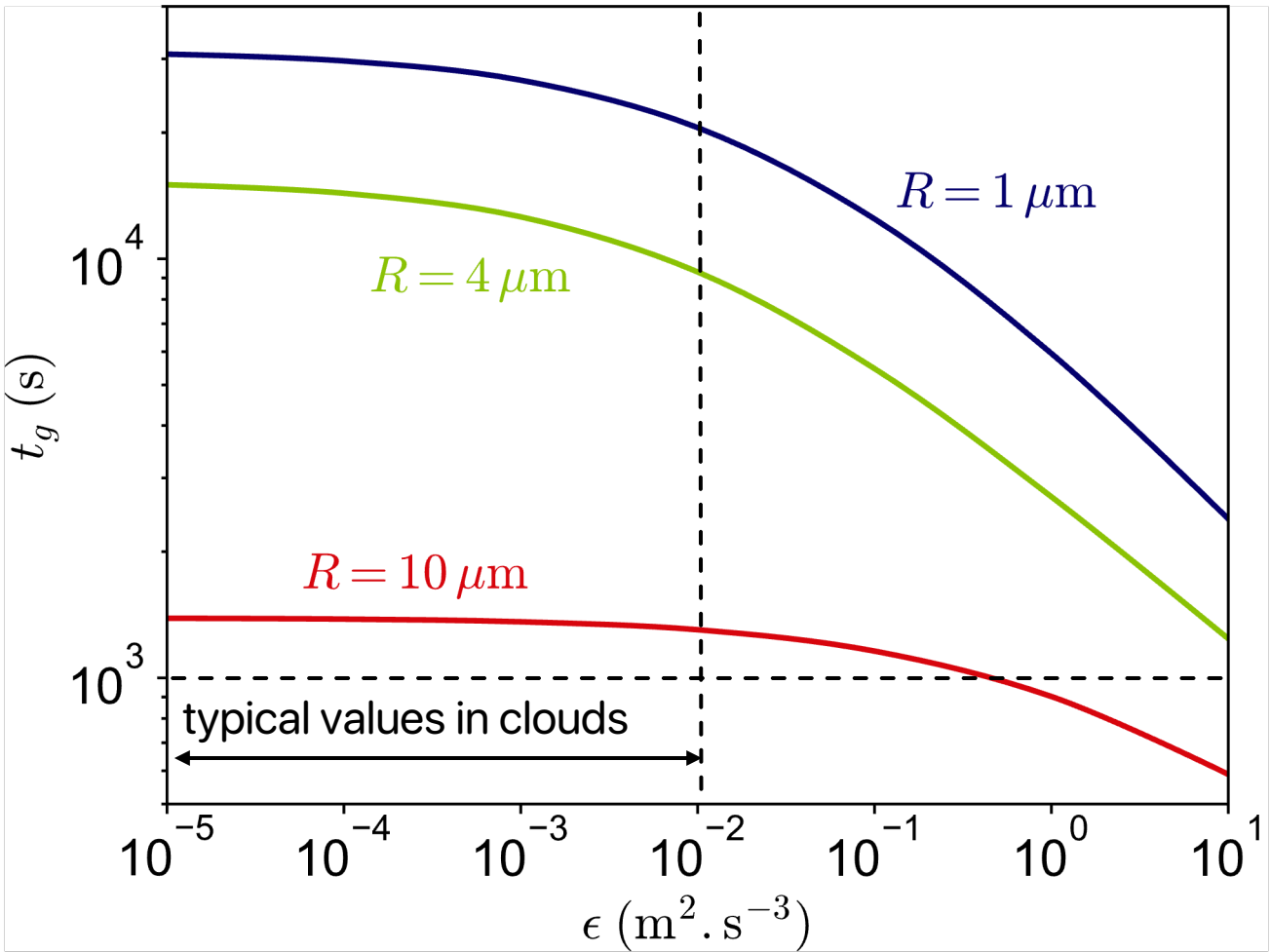
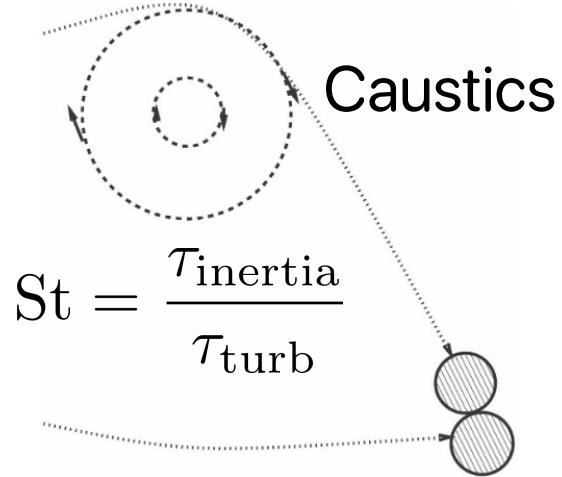
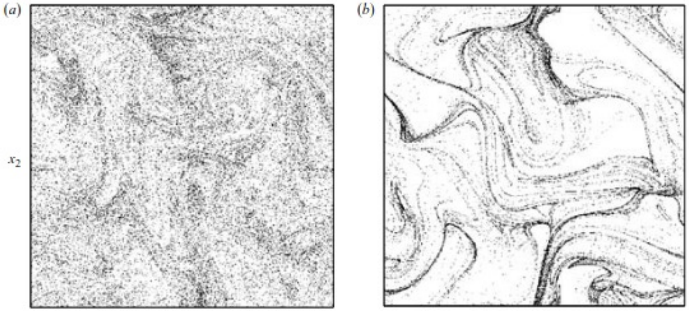
Rain initiation time

Idealized homogeneous box model
F. Poydenot, B. Andreotti, PRF 2024



Collisions due to turbulence

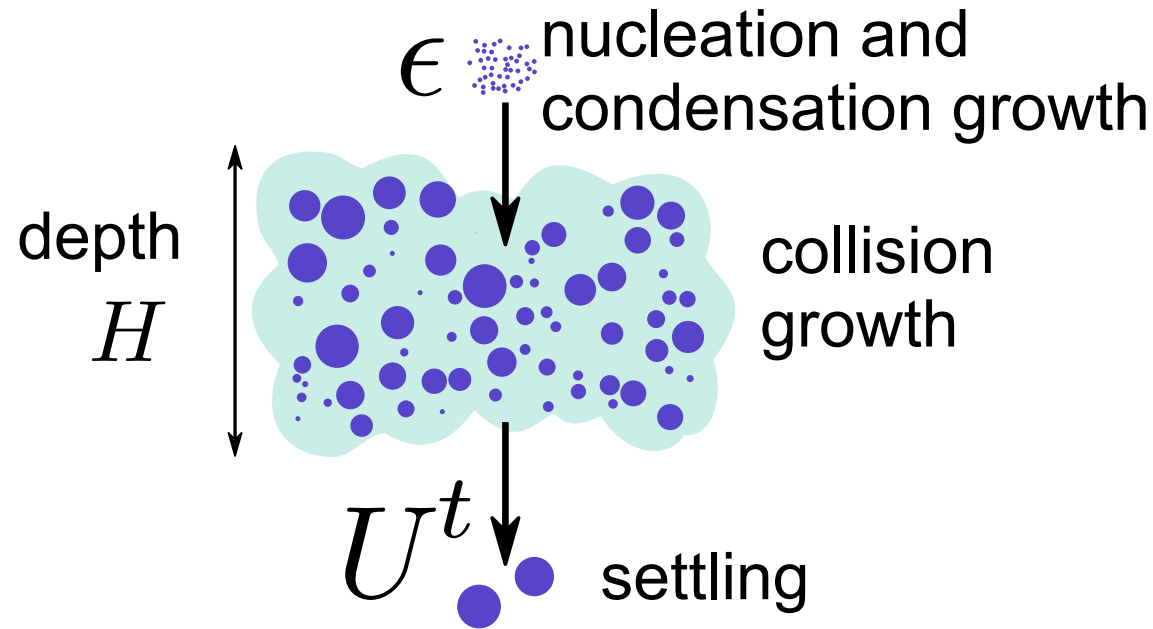
Clustering



Bec 2004
 Falkovich & Pumir 2007
 Pumir & Wilkinson 2016

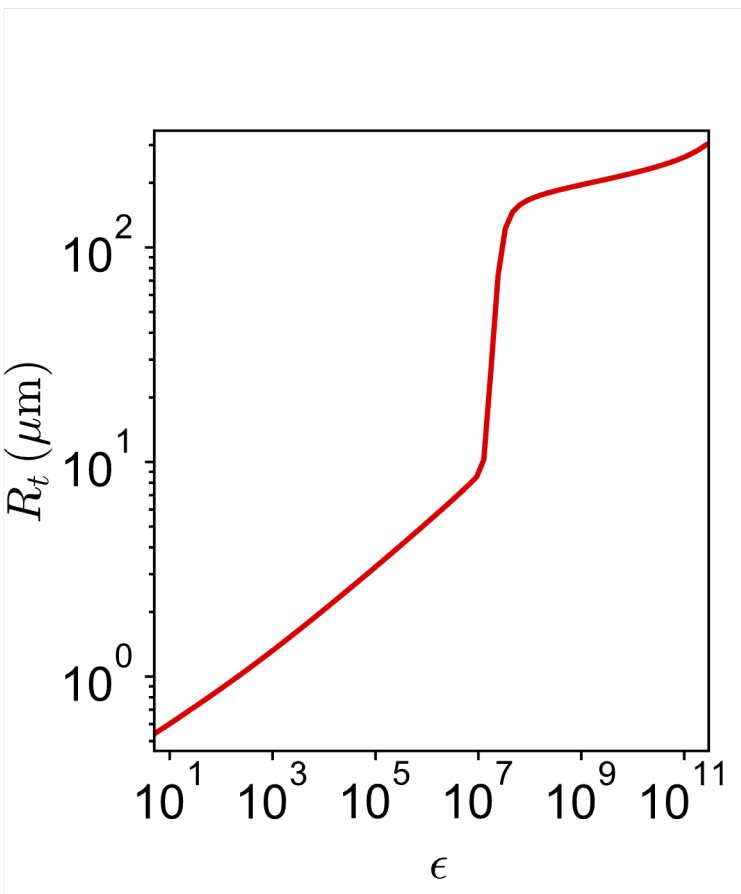
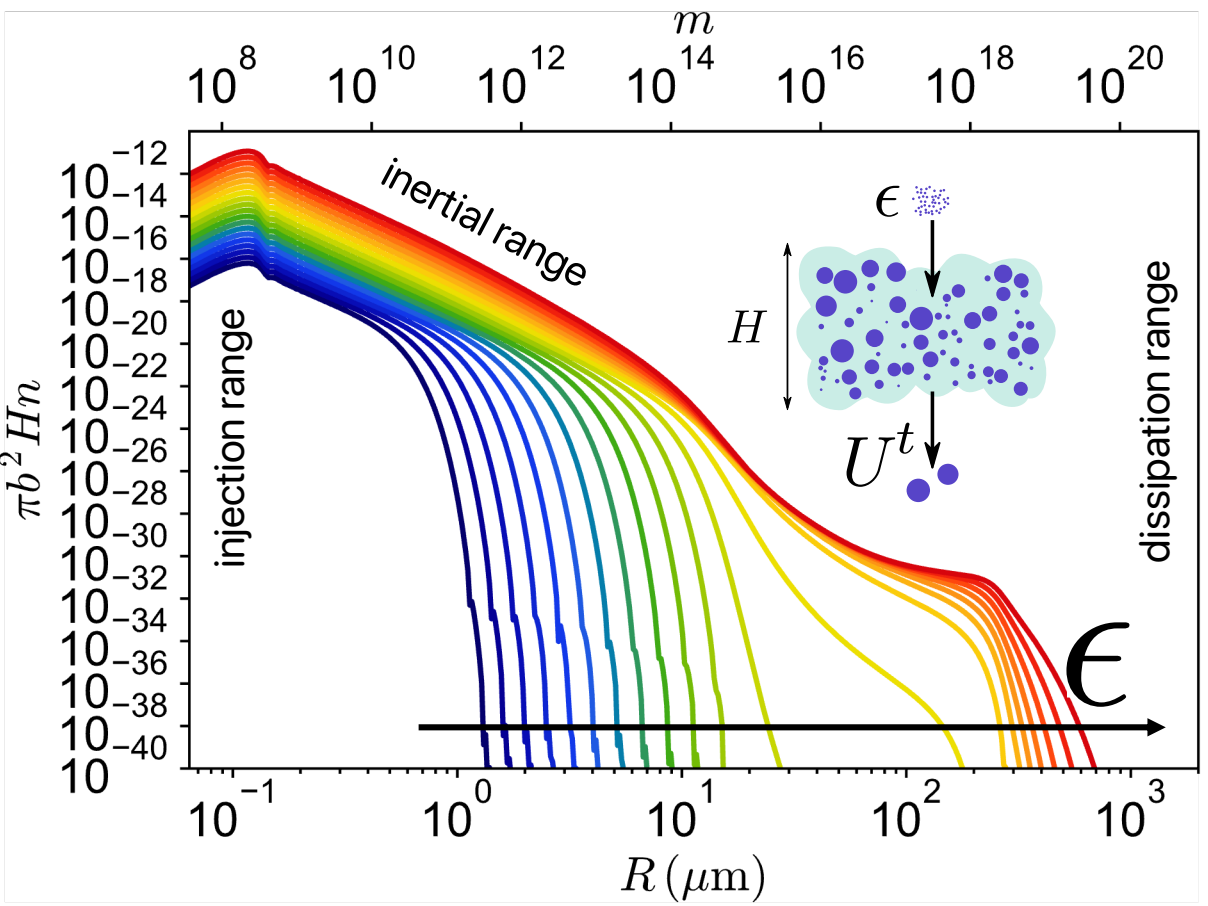
Well-mixed steady-state layer

F. Poydenot, B. Andreotti, in prep.



$$\frac{\partial n}{\partial t} = \epsilon + \text{collision terms} - \frac{U^t(m)}{H} n(m)$$

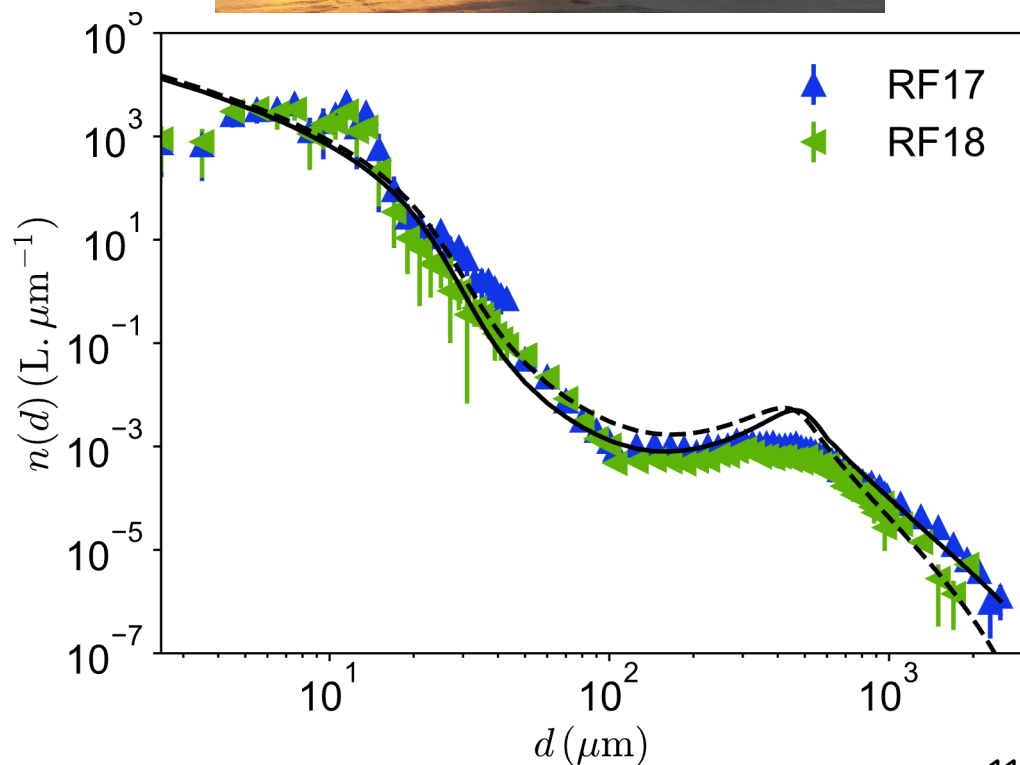
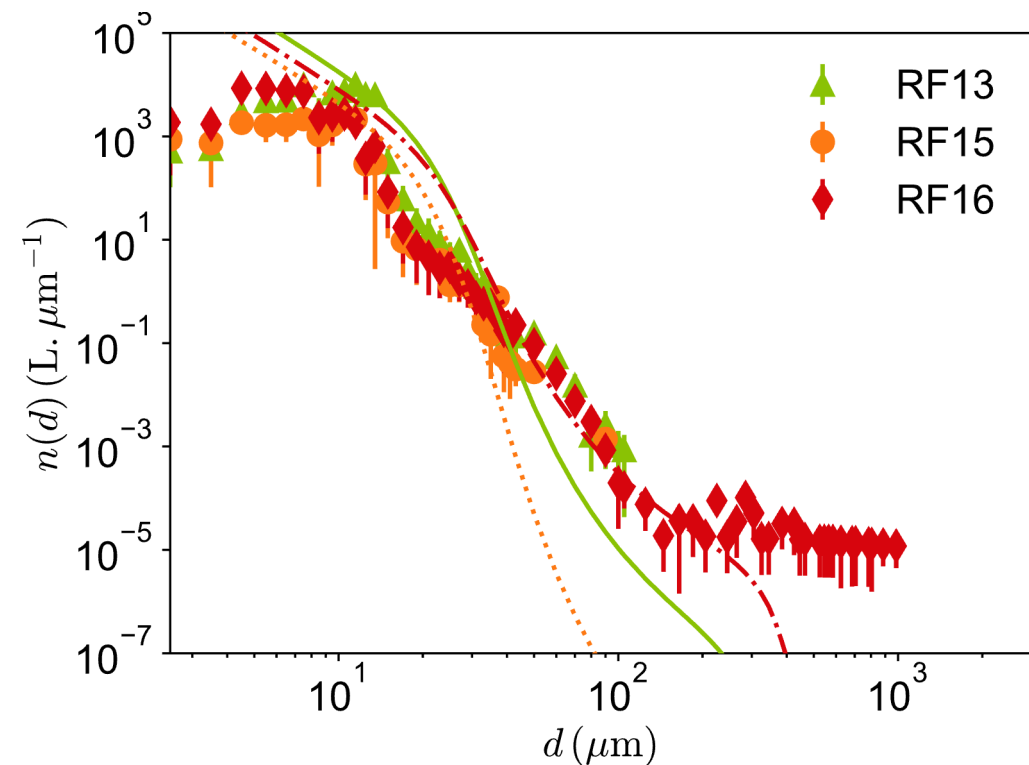
Inverse size cascade



DSDs from the EUREC⁴A campaign at Barbados

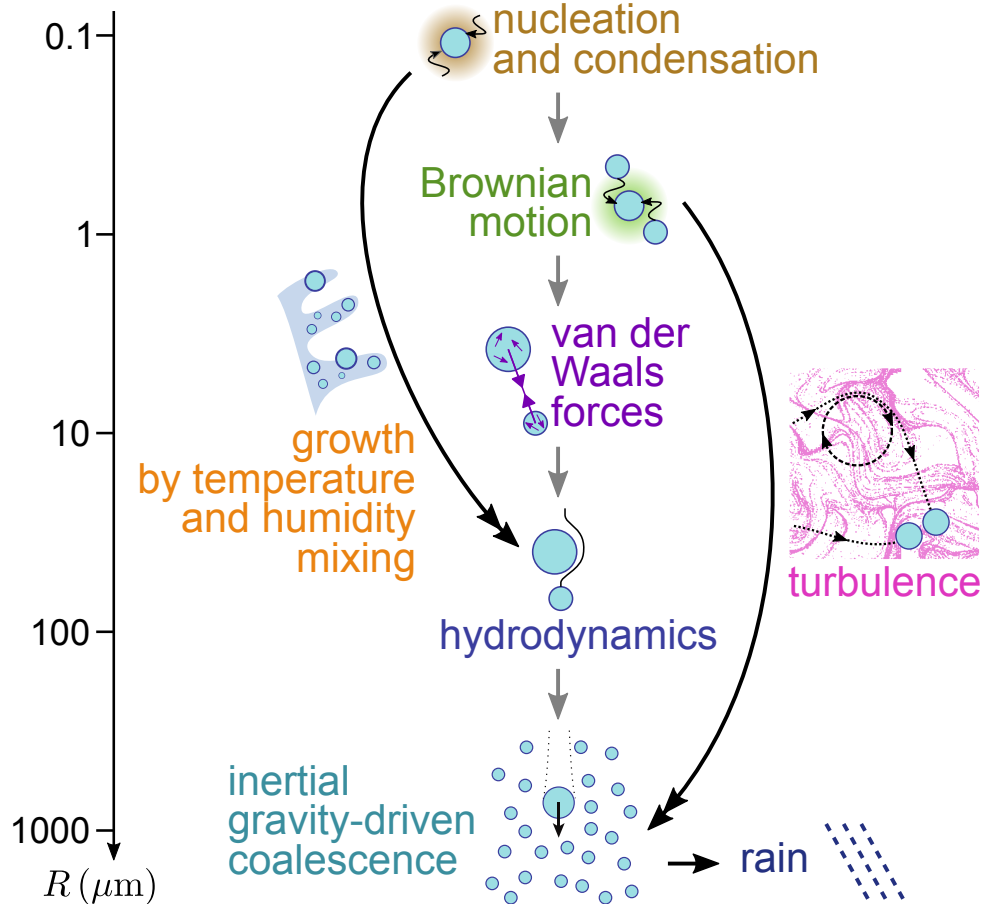
Bony et al. 2022, Caudoux et al. 2021, Coutris 2021

- In-situ airborne DSD measurements at cloud base
- Cloud depth derived from radar measurements



Take home messages

- Van der Waals forces bridge the gap between Brownian motion and inertial collisions
- We can think of the rain DSD as the result of a collision cascade, at the crossover between growth and settling



Inverse size cascade for a self-similar kernel

$$K(\alpha m_1, \alpha m_2) = \alpha^{5/6} K(m_1, m_2)$$

