Classicality of Strong Quantum Coupling: Implication for Hierarchy Problem

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Why New Physics?

Motivations:

- Elegance, simplicity, predictivity;
- Trying to explain existing phenomena (Dark matter, Dark energy, unification with quantum gravity...)
- Naturalness

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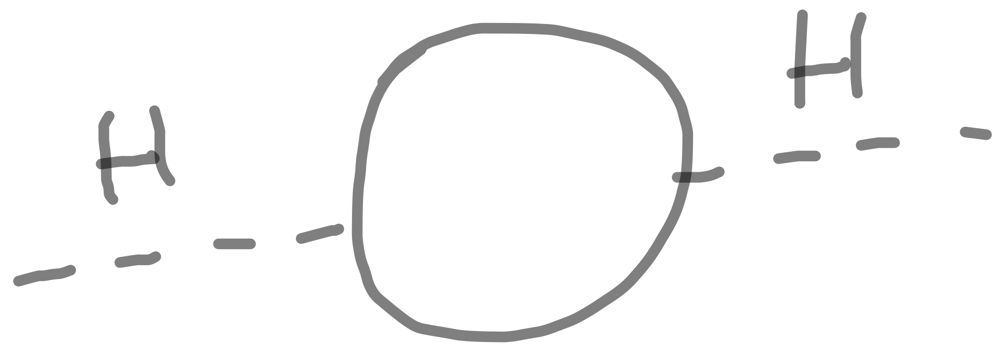
Naturalness problems

① UV-sensitivity

e.g. Higgs mass



② Vacuum super-selection

e.g. vacuum θ-angle in QCD

$$\theta F \tilde{F}$$

The hierarchy problem has a meaning because of gravity:

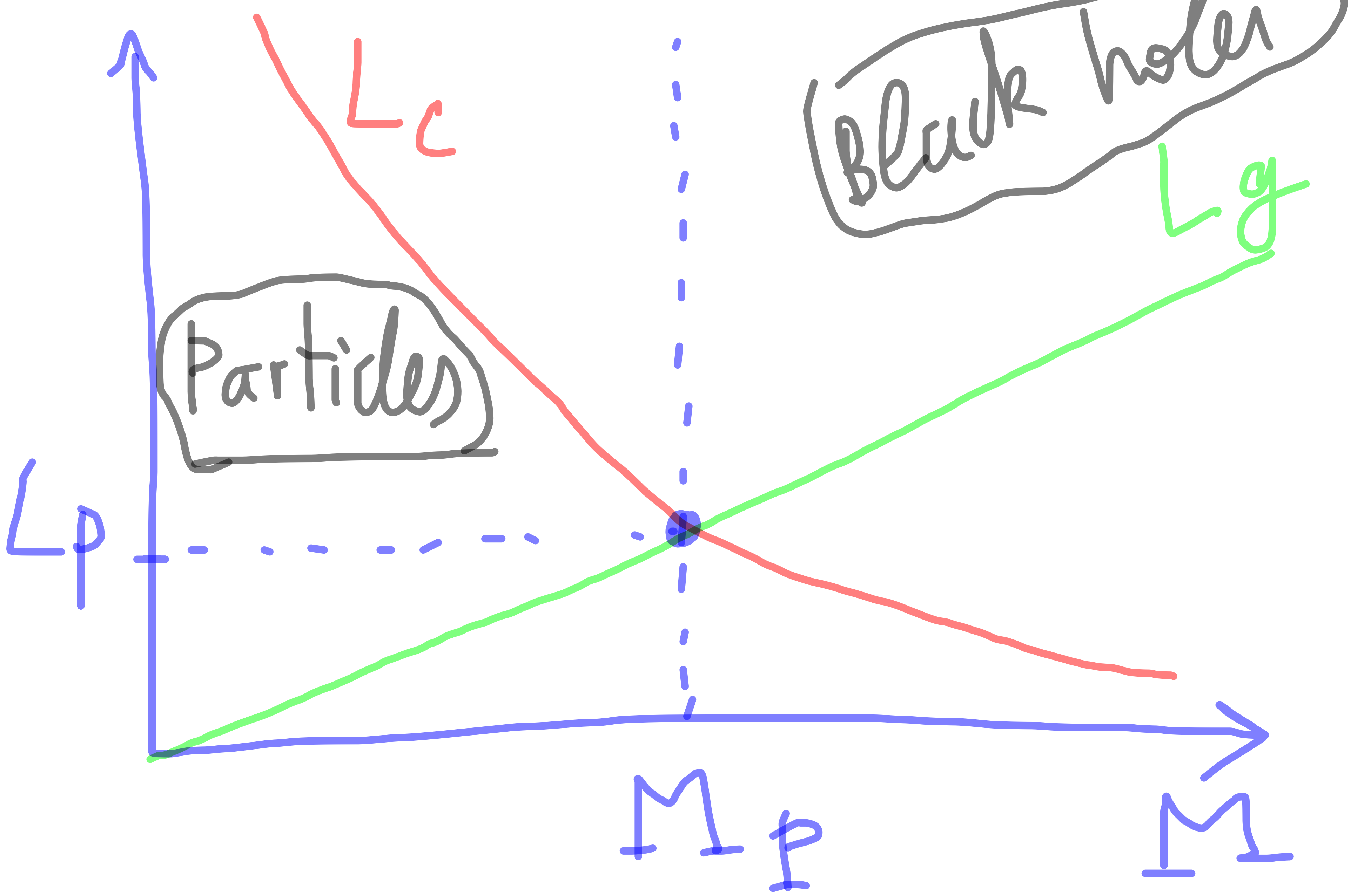
$$M_P \equiv \frac{\hbar}{L_P}, \quad L_P^2 \equiv \hbar G_N$$

$M_P \sim 10^{19}\,\mathrm{GeV}$, $L_P \sim 10^{-33}\,\mathrm{cm}$

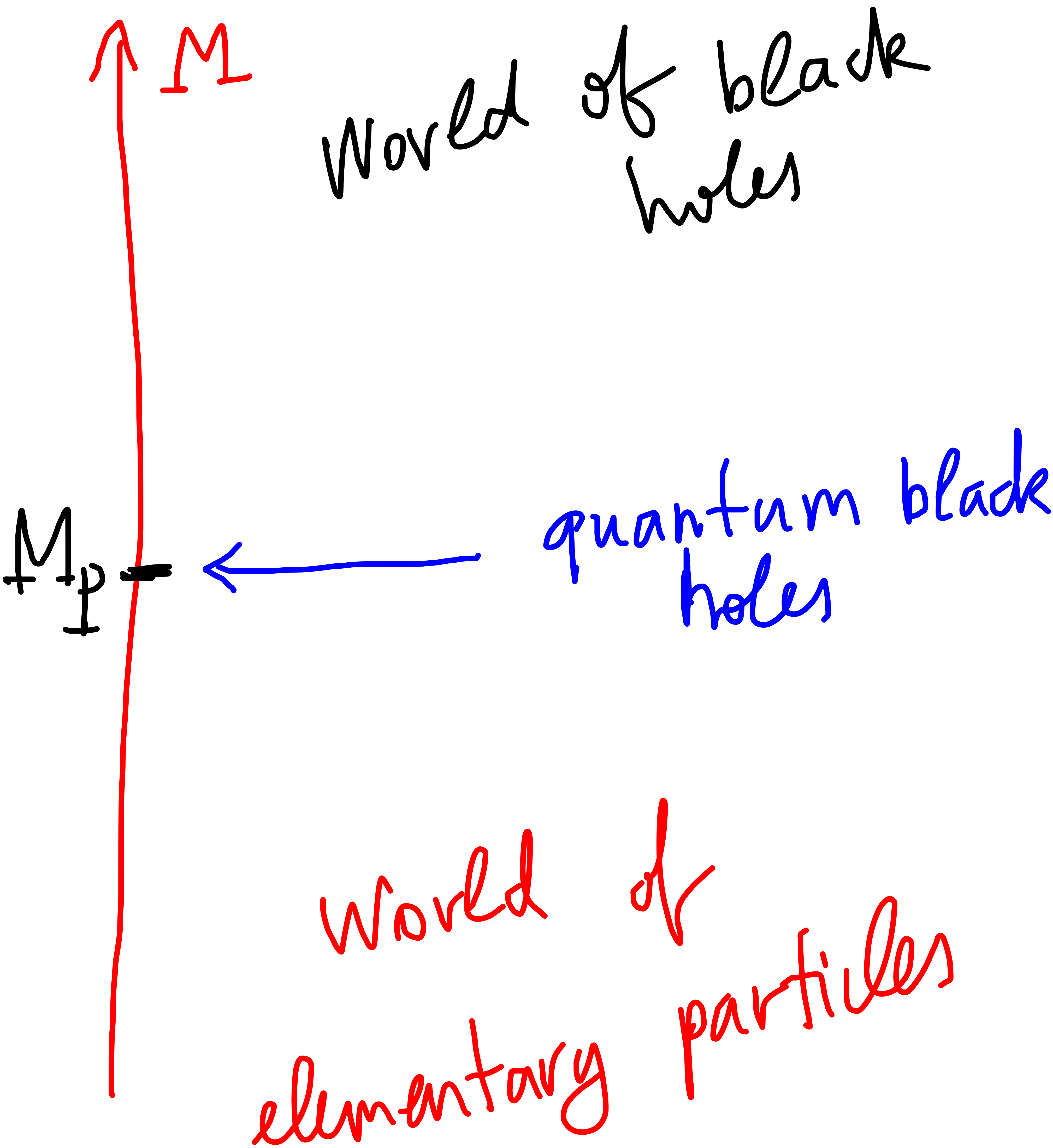
Particles heavier than M_P do not exist: they are black holes!

A particle of mass M has two length scales:

$$L_c \equiv \frac{\hbar}{M} \qquad L_g \equiv \frac{M}{M_p^2}\hbar$$



$L_c = L_g = L_p$ for $M = M_p$


M
World of black holes
M_P
quantum black holes
World of elementary particles

Hierarchy problem:

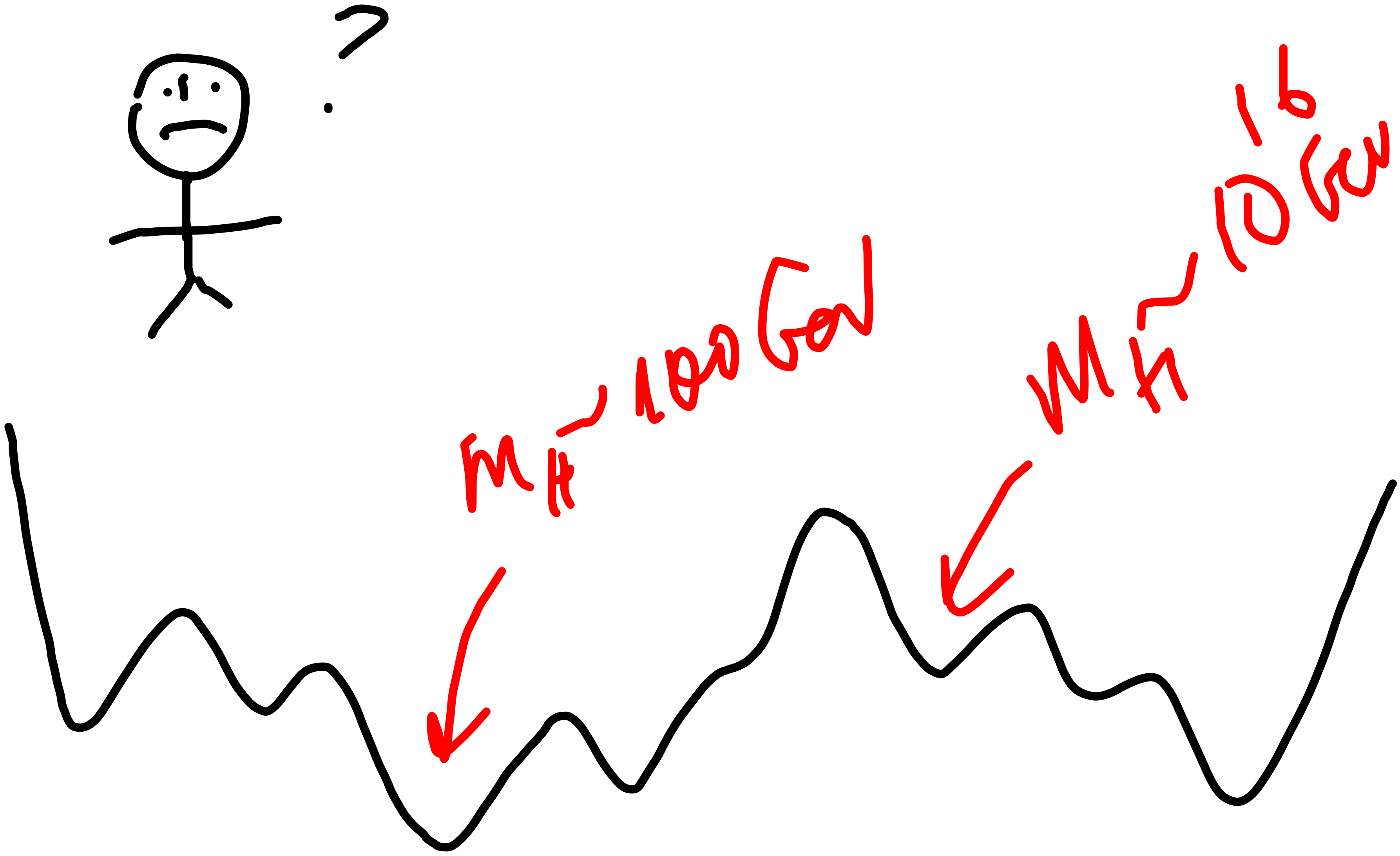
Why $m_H \ll M_P$?

or

Why Higgs is not a quantum black hole?

Can the hierarchy problem be promoted into a problem of vacuum selection?

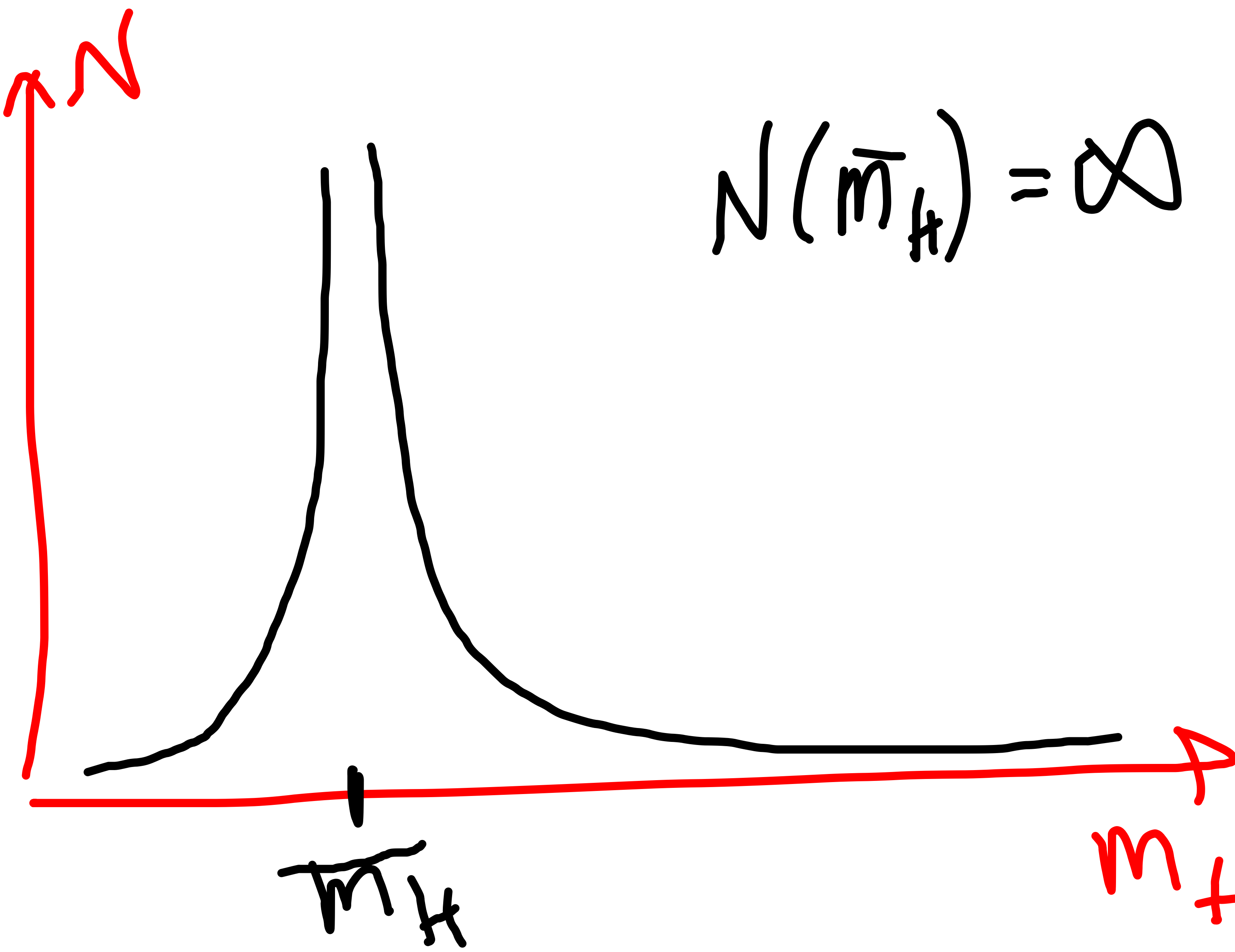
Instead of picking up one among many theories, we pick up one vacuum among many vacua of the same theory.



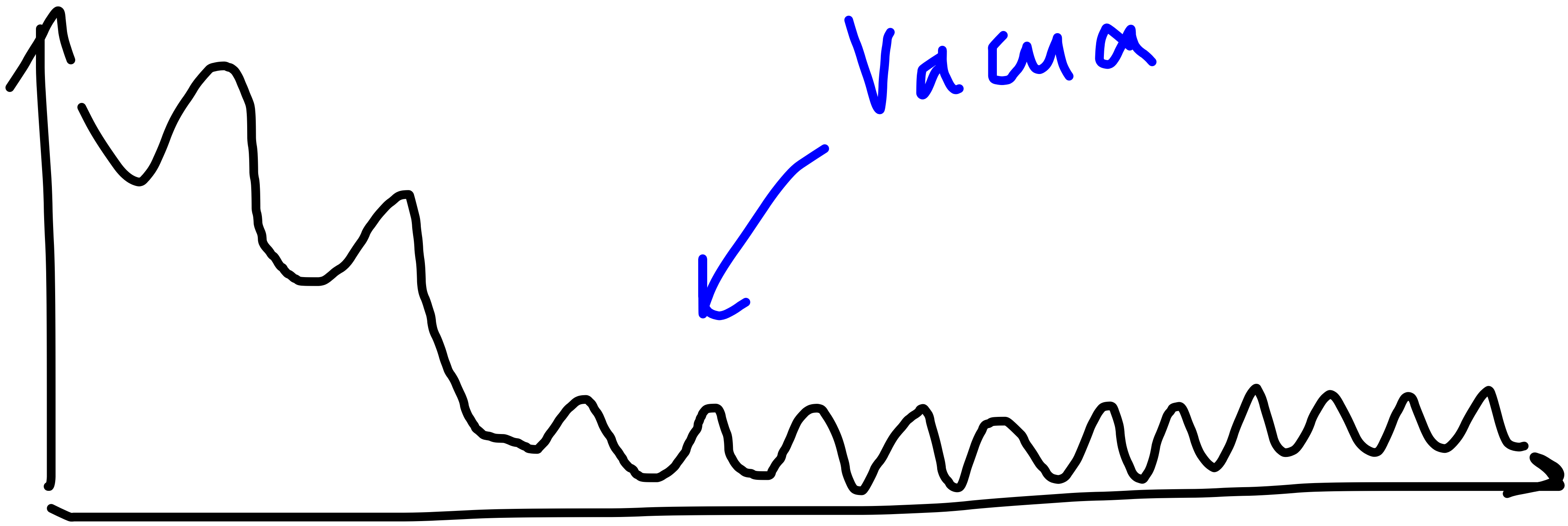
So what is the vacuum selection mechanism?

Vacuum attractor mechanism:

Higgs mass (and VEV) controls the number density of vacua G.D., A.Vilenkin '03



$N(\bar{m}_H) = \infty$!



Singularity at $m_H = \overline{m}_H$

Vacuum with $m_H \sim 100$ GeV has ∞ entropy?!

If the hierarchy problem is not a vacuum seletion problem, then there must be new physics around TeV scale.

This new physics can be weakly-coupled

or

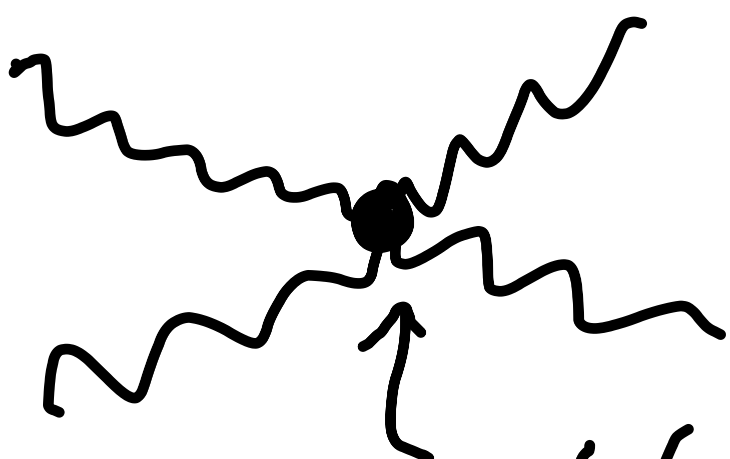
strongly-coupled.

Let me focus on strongly-coupled case.

If some couplings become strong around TeV, LHC cannot miss it.

What happens in such a case?

In quantum field theory the couplings depend on energy scale E.



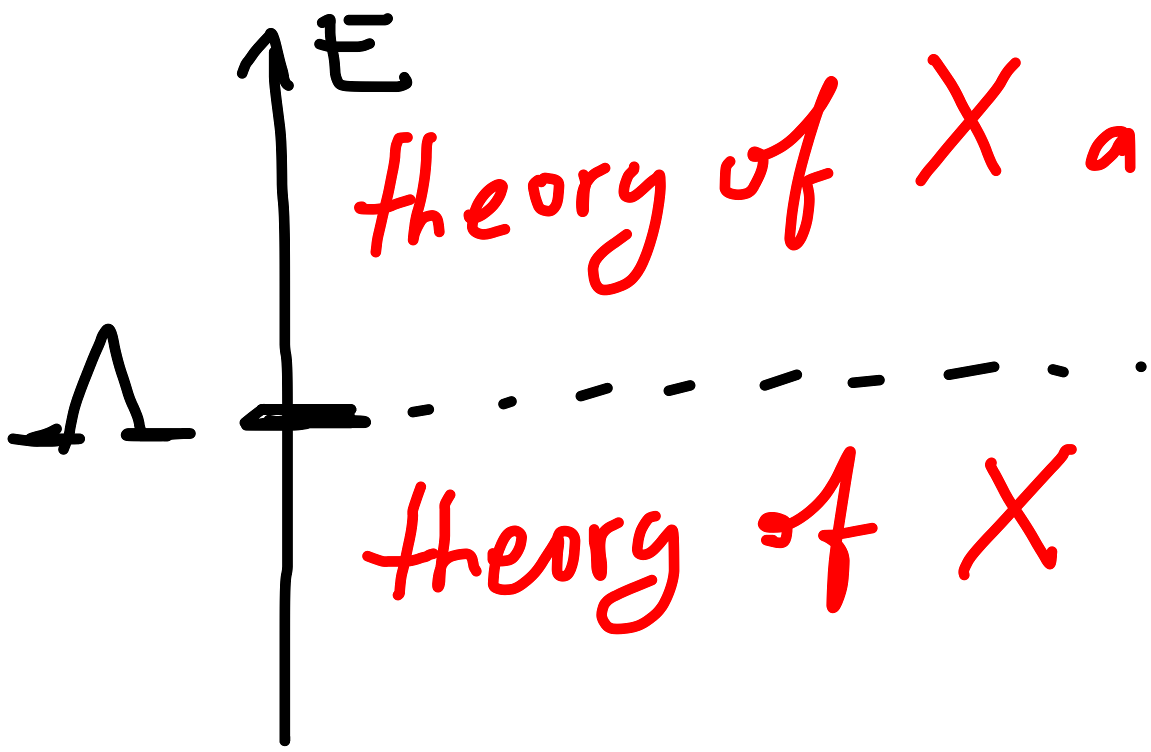
$\alpha(E)$

What happens when a theory hits the strong coupling at some scale $E = \Lambda$?

$\alpha(\Lambda) \sim 1$

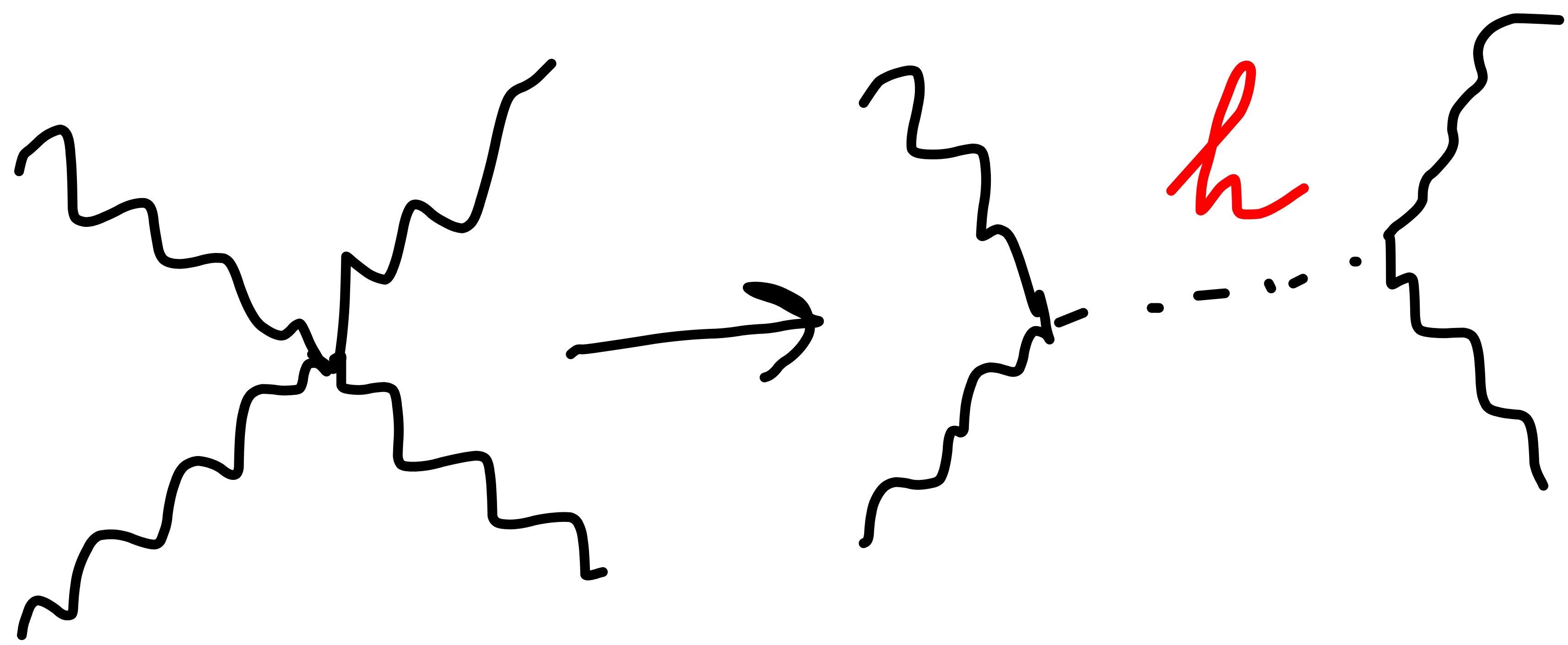
It becomes a theory of something else: new degrees of freedom enter the game.

⊛ The old degrees of freedom can coexist with new ones



Example: Higgs in the SM

Due to this, Higgs restores perturbative unitarity violated by longitudinal W-s



or

$\circledast$ theory may completely change

theory of Y-s

Λ

theory of X-s

Examples: QCD and Gravity

QCD

theory of quarks, gluons

Λ_{QCD}

theory of mesons, glueballs

GRAVITY

theory of black holes

M_P

theory of particles:
quarks, gravitons, ...

From both sides the degrees of freedom become strongly coupled at the scale Λ.

e.g. Classical black holes (of mass $M \gg M_p$) are very weakly-coupled, but for $M \sim M_p$ become strongly coupled.

Particles exhibit opposite behaviour.

So the scale Λ is a regulator and this solves the hierarchy problem!

What should we see if $\Lambda \sim \text{TeV}$?

Classicalization

G.D., Giudice, Gomez, Kehagias

Imagine the following game. You are given power to invent laws of nature.

Imagine a theory with 4-point interaction

with coupling $\alpha(E)$ that gets strong above scale Λ

$$\alpha(E \gg \Lambda) \gg 1$$

Consider a head-on collision of 2 ϕ-quanta of center of mass energy $\sqrt{s} \gg \Lambda$.

Then, $\alpha(\sqrt{s}) \gg 1$

Your task is to invent the rule of the game that avoids paradox (i.e. violation of unitarity).

The rules are:

1. You are not allowed to invent new elementary quanta -

and

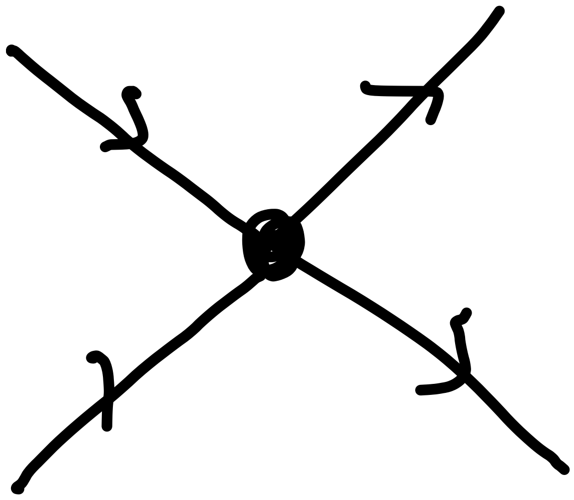
2. You must respect all the basic rules of quantum field theory (e.g. conservation of energy, no negative norm, etc...)

The right thing to do is to redistribute energy $\sqrt{s}$ among N quanta, in such a way that their coupling is weak

$$\alpha\left(\frac{\sqrt{s}}{N}\right) \ll 1$$

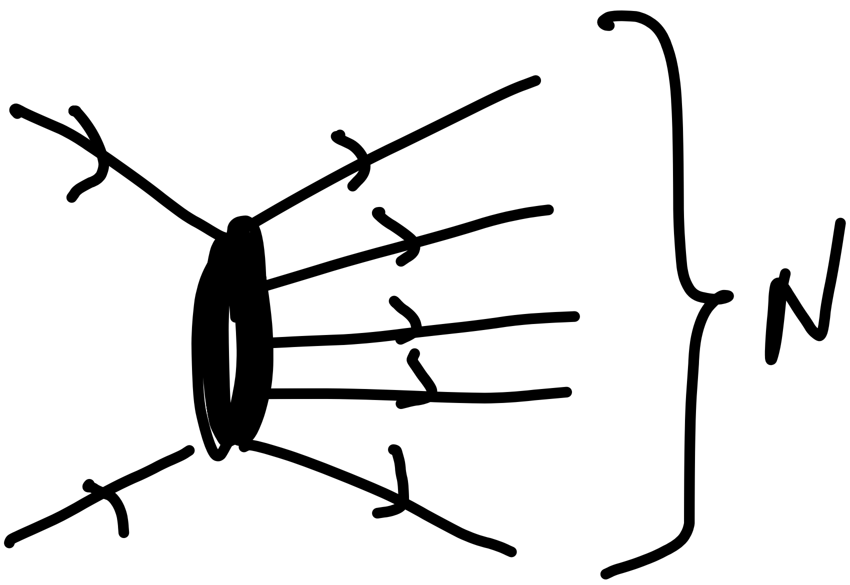
But, the states with $N \gg 1$ are approximatelly classical.

Hence, Classicalization!



$\alpha(\sqrt{s}) \gg 1$

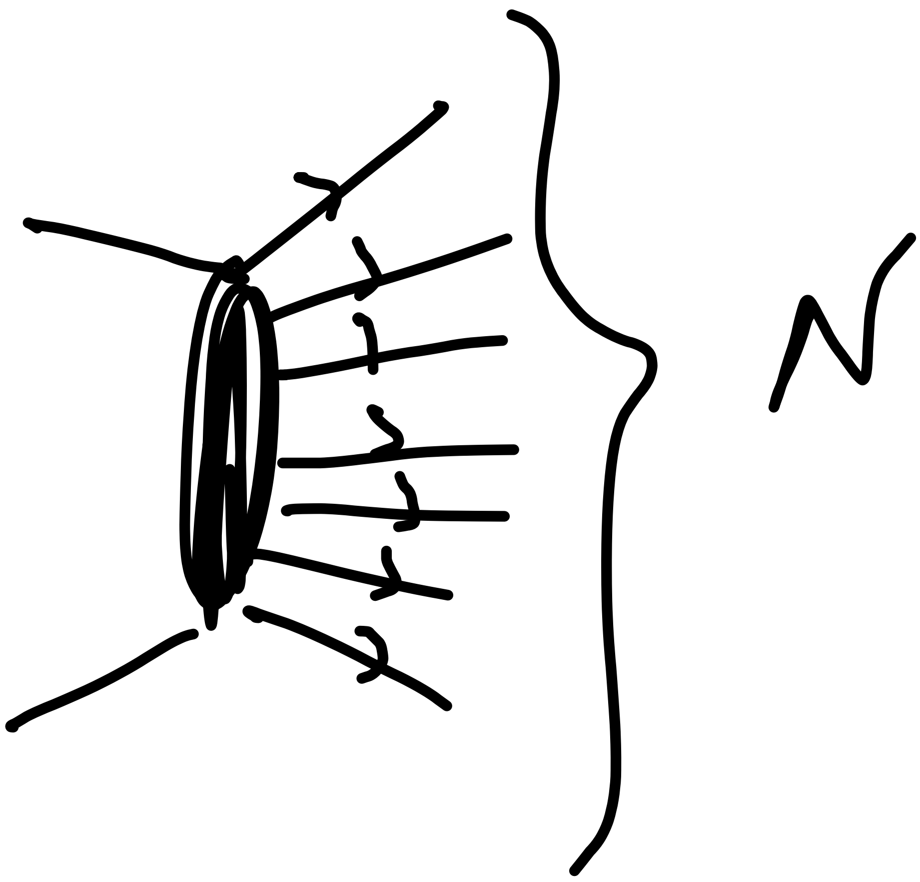
Instead:



$\alpha\left(\frac{\sqrt{s}}{N}\right) \ll 1$!

Thus, classicalization is the way the theory shields itself from entering the strong coupling domain, by means of redistributing total energy ($\sqrt{s} \gg \Lambda$) among many soft quanta for which the coupling is weak $\frac{\sqrt{s}}{N} \ll \Lambda$,

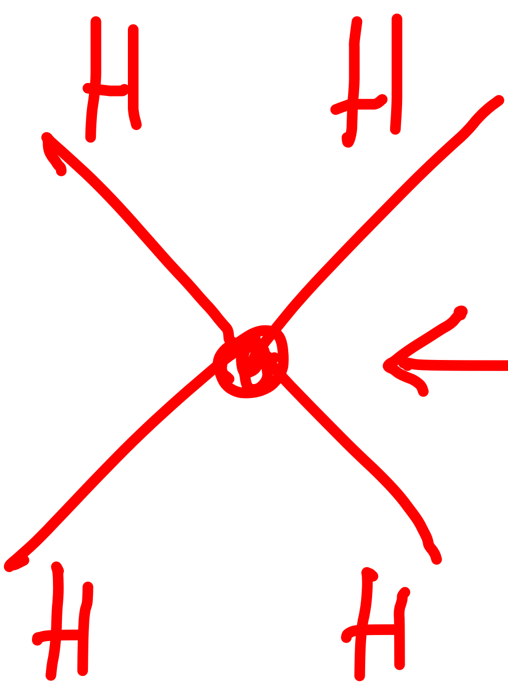
$$\alpha\left(\frac{\sqrt{s}}{N}\right) \ll 1$$



Almost classical state for $N \gg 1$

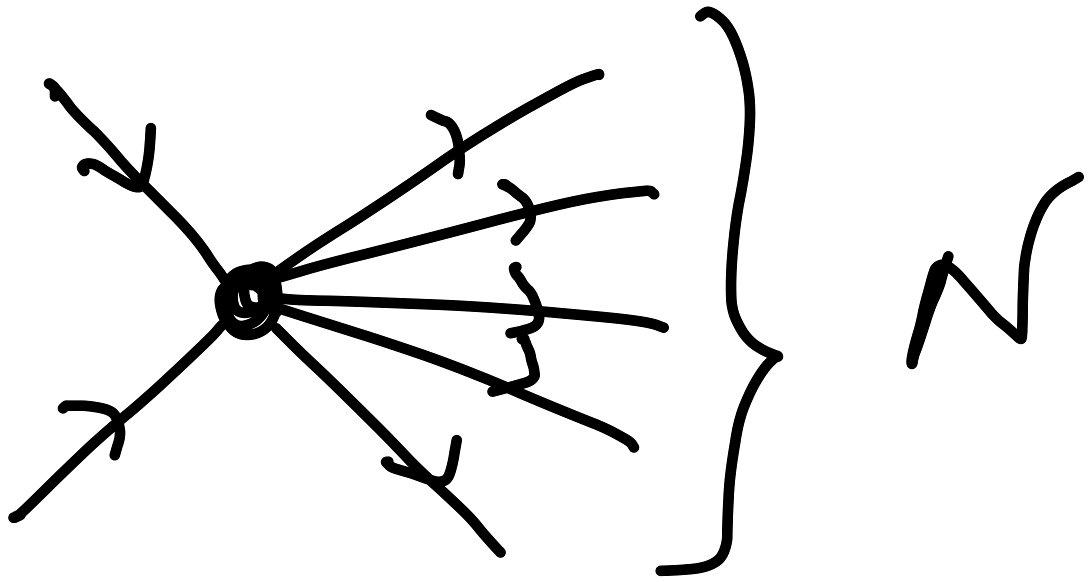
Simplest solution to Hierarchy Problem?

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda^4}\left(\partial_\mu H^\dagger \partial^\mu H\right)^2$$



$$\alpha = \left(\frac{\sqrt{s}}{\Lambda}\right)^4$$

<u>Naive !</u>



Classicalization radius

$$r_* = \frac{\hbar}{\Lambda}\left(\frac{\sqrt{s}}{\Lambda}\right)^{\frac{1}{3}}$$

$$N = \alpha^{-1}\left(\frac{\hbar}{r_*}\right)$$

$$\hat{H}_0 = \nu\, \hat{b}^\dagger \hat{b} + \hat{a}_0^\dagger \hat{a}_0$$

$$\hat{H}_{tr} = g_\nu\, \hat{b}^\dagger \hat{a}_0^\nu + h.c.$$

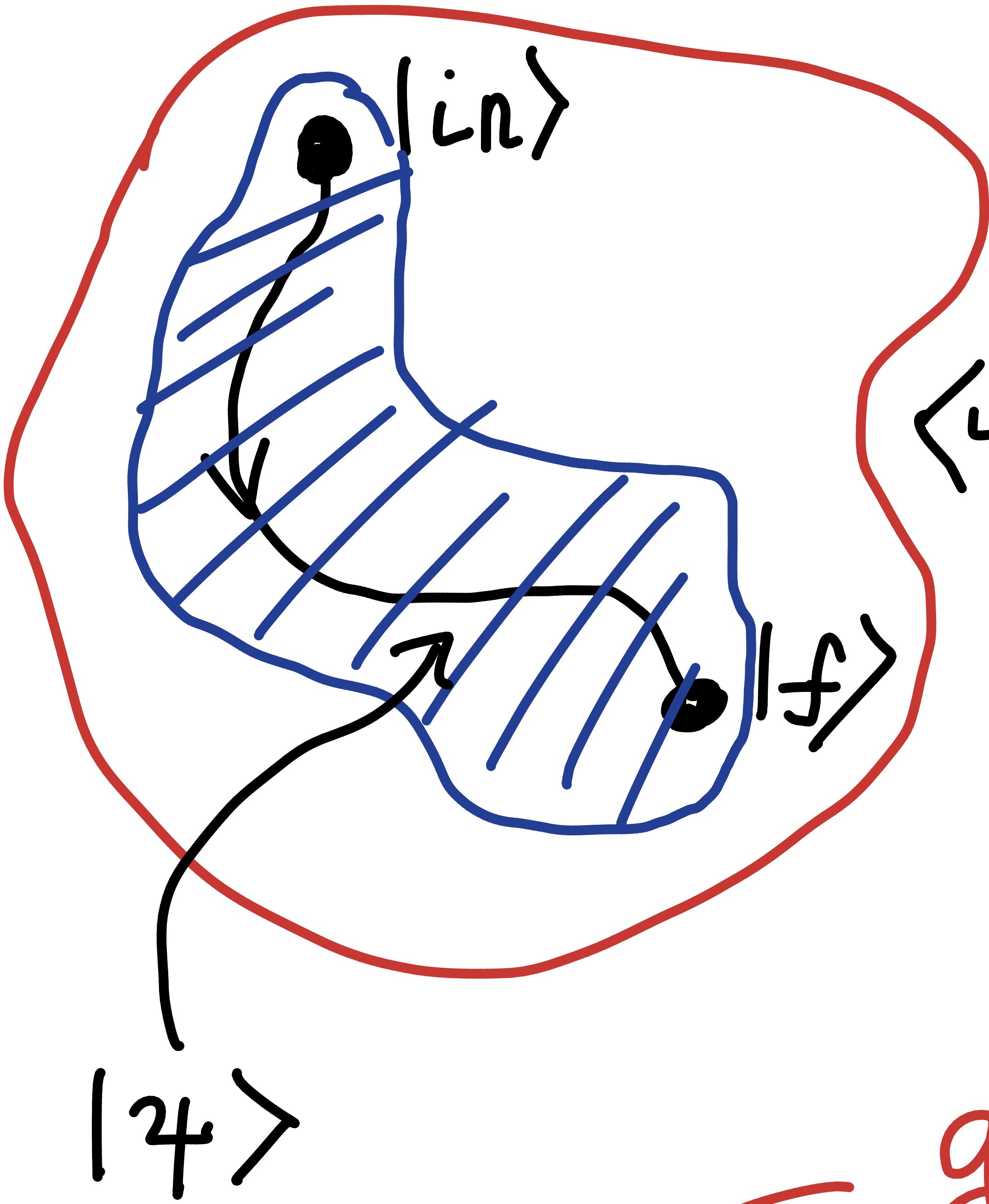
$$[\hat{a}_0, \hat{a}_0^\dagger] = [\hat{b}_0, \hat{b}_0^\dagger] = 1$$

$$|in\rangle_Q = |1\rangle_b \otimes |0\rangle_{a_0}$$

$$|f\rangle = |0\rangle_b \otimes |N\rangle_{a_0}$$

$$\hat{H}_{tr} |in\rangle = g_\nu \sqrt{N!}\, |f\rangle$$

$$|\langle in| \hat{H}_{tr} |f\rangle|^2 = g_\nu^2\, N!$$



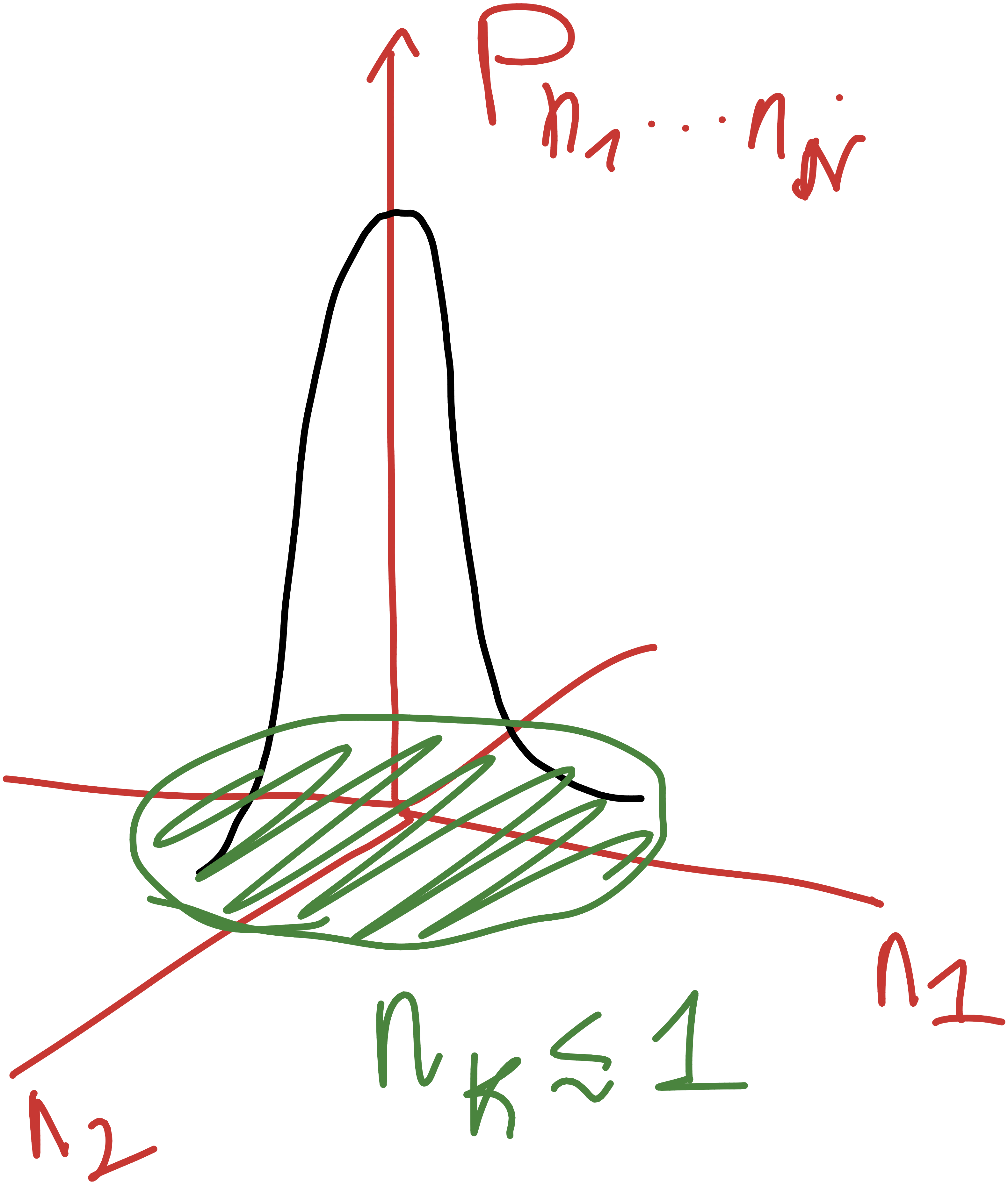
Condition:

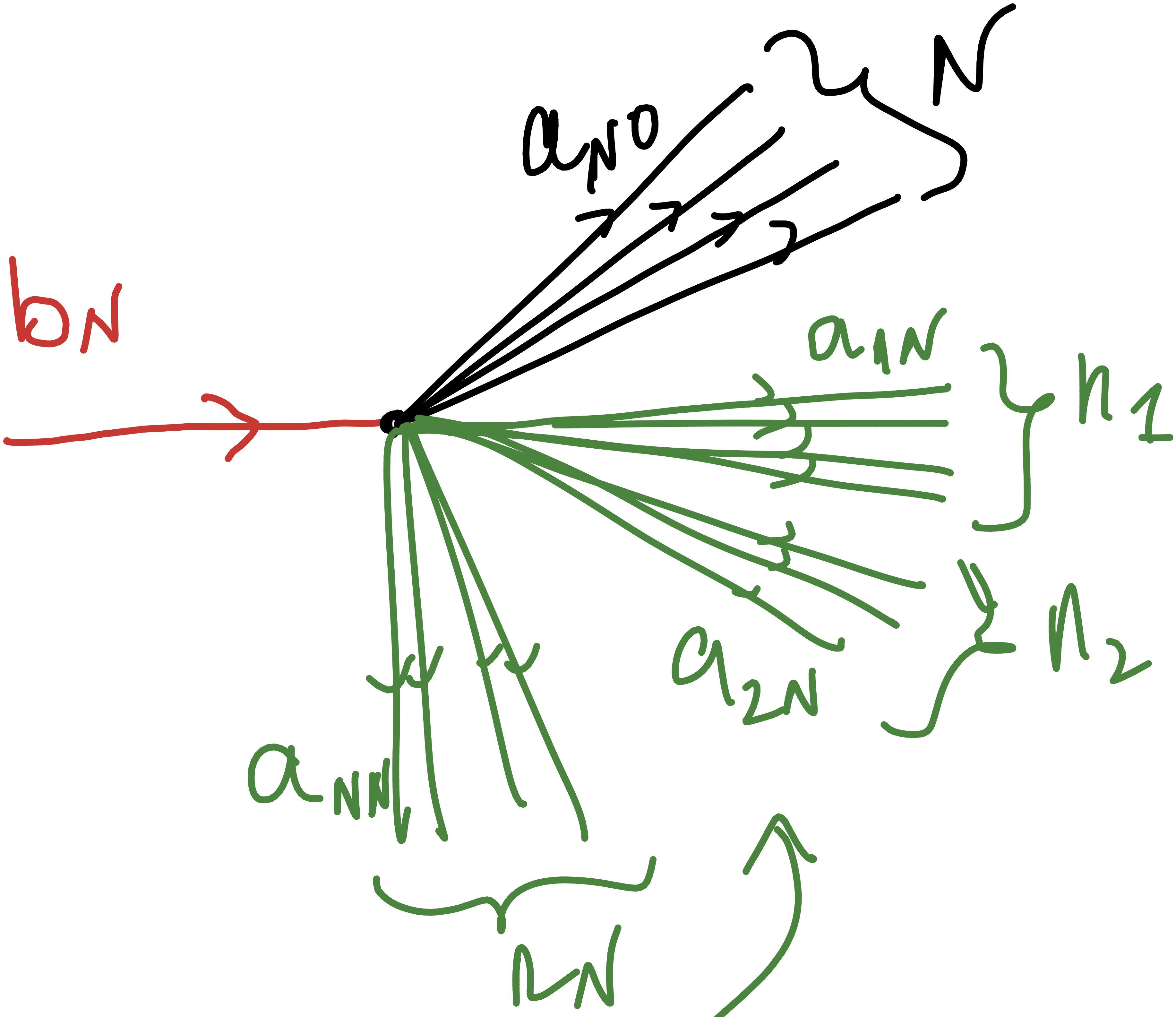
$$\langle \psi | \hat{H}_{tr} | \psi \rangle \lesssim \langle \psi | \hat{H}_0 | \psi \rangle$$

$$g^2 \lesssim N^{-N}$$

$$|\langle in | \hat{H}_{tr} | f \rangle|^2 \sim e^{-N} \;!$$

$$|1_b\rangle \longrightarrow |N\rangle_{n_1 \cdots n_N}$$



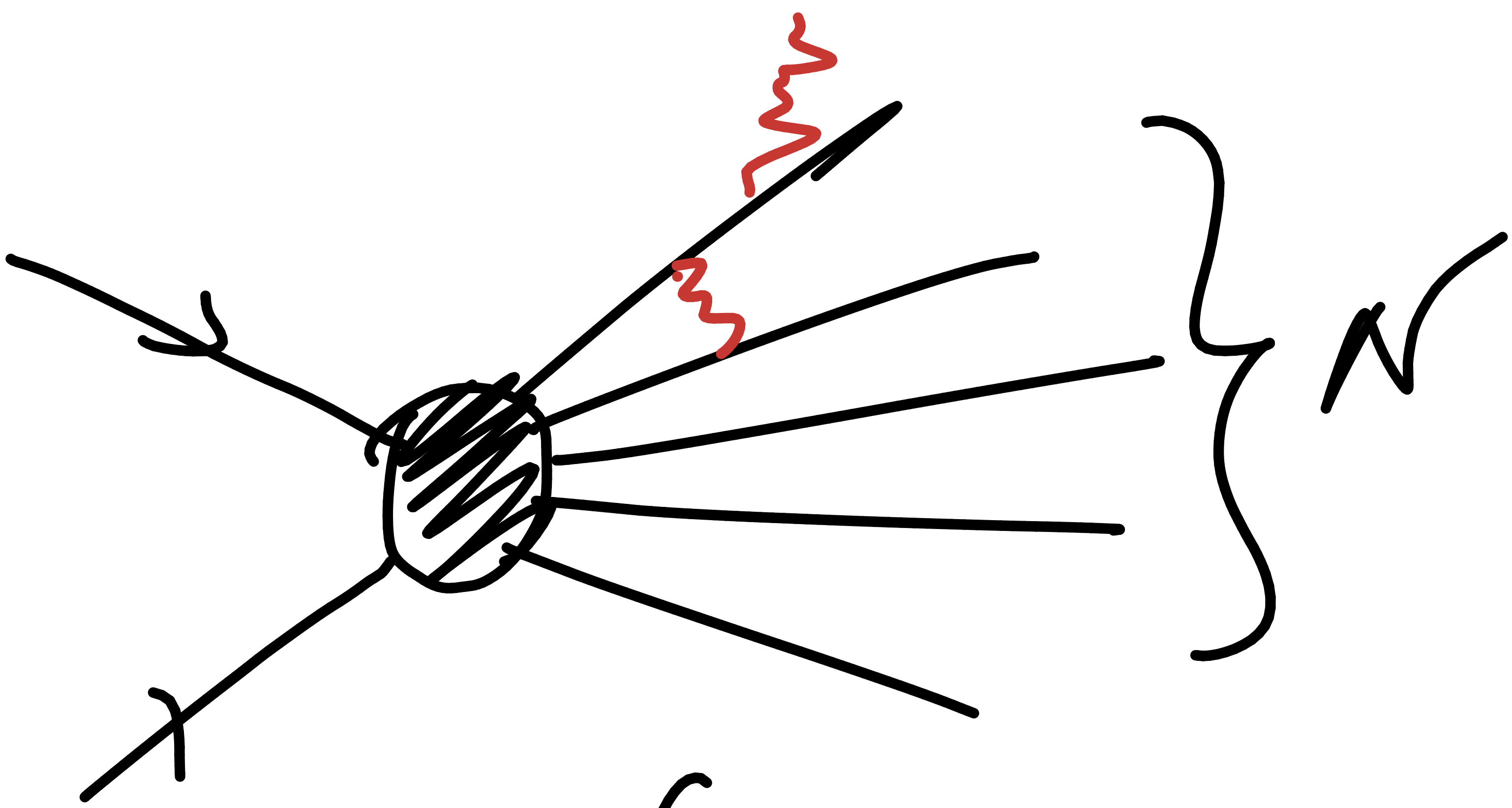


Dressing by super-soft modes.

$|1\rangle_{\text{Quant}} \longrightarrow |N\rangle_{\text{class}}$

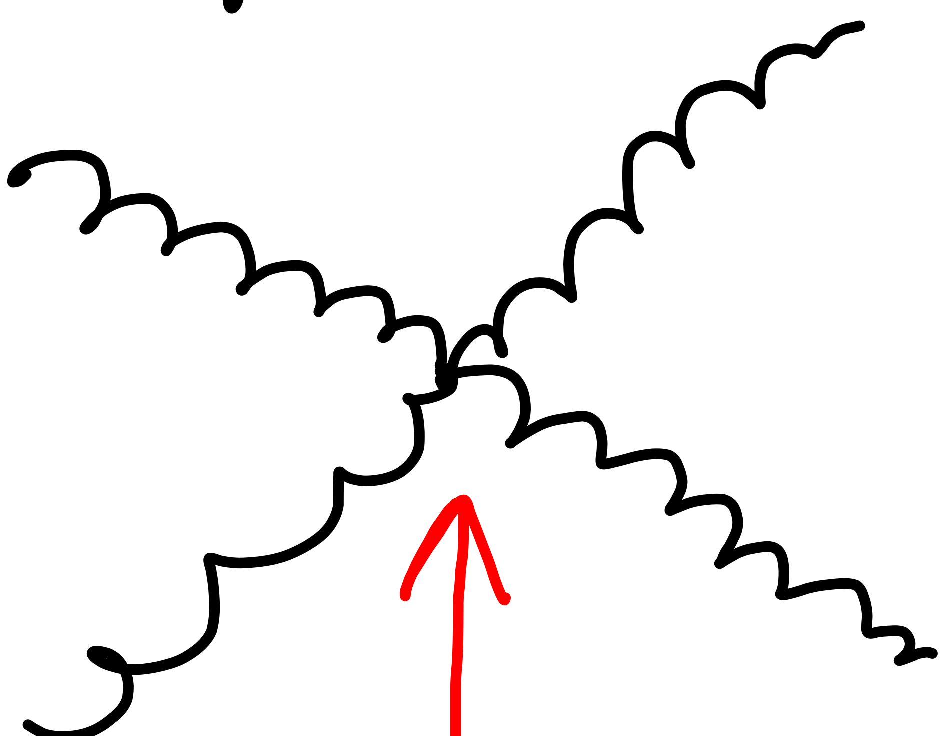
Very similar to $2 \to N$ graviton amplitudes

G.D., Gomez, Isermann, Lüst, Stieberger '15; Addazi, Bianchi, Veneziano '16



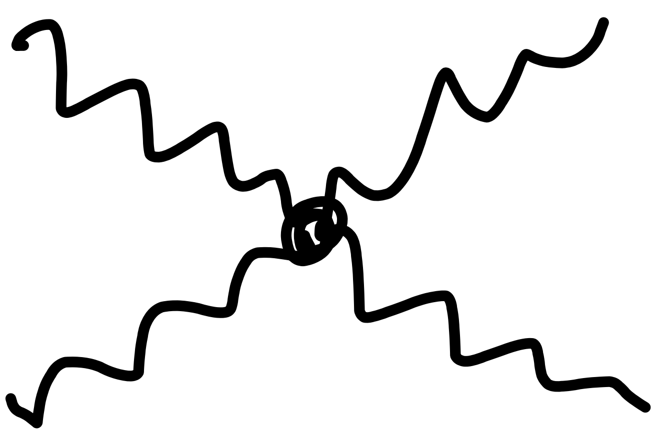
$P \sim e^{N} \times$ dressing.

Gravity is a quantum theory of a particle (graviton) of $m = 0$ and spin $= 2$



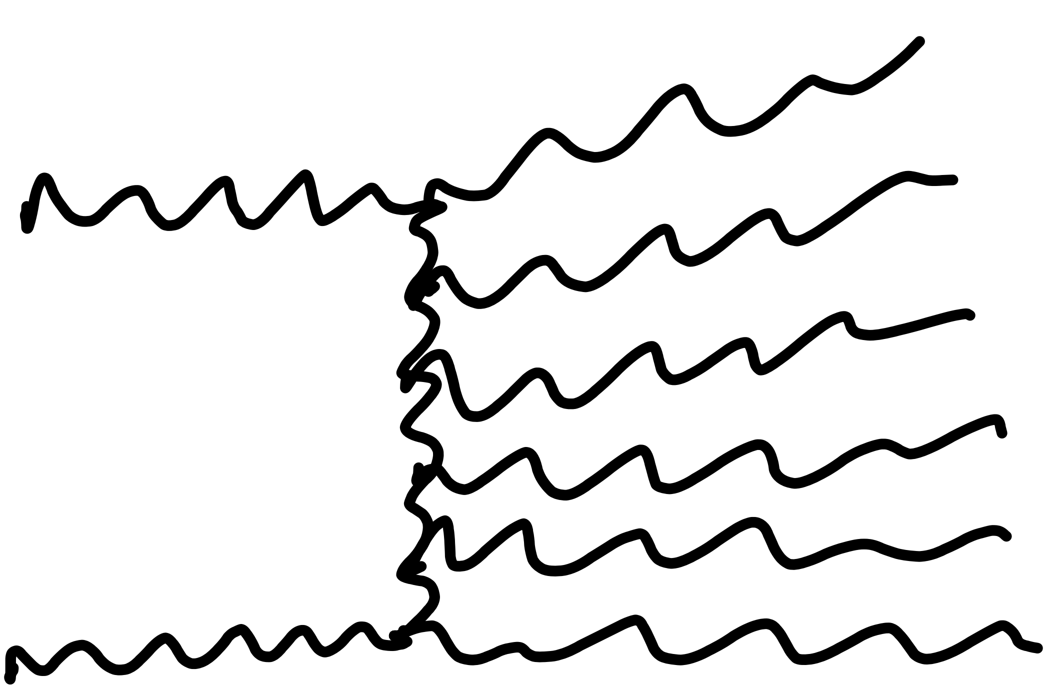
$$\alpha_{gr} \equiv \hbar G_N \bar{\lambda}^{-2}$$

Above the cutoff the naive coupling becomes strong



$$\alpha_{\text{naive}} \sim \frac{s}{M_p^2}$$

But, in reality the theory becomes a theory of many **soft quanta**



$N = \alpha_{\text{naive}}$

$$\alpha = \frac{1}{N} = \frac{M_p^2}{s} \; !$$

It is commonly accepted that black holes should be produced in trans-Planckian scattering

e.g.

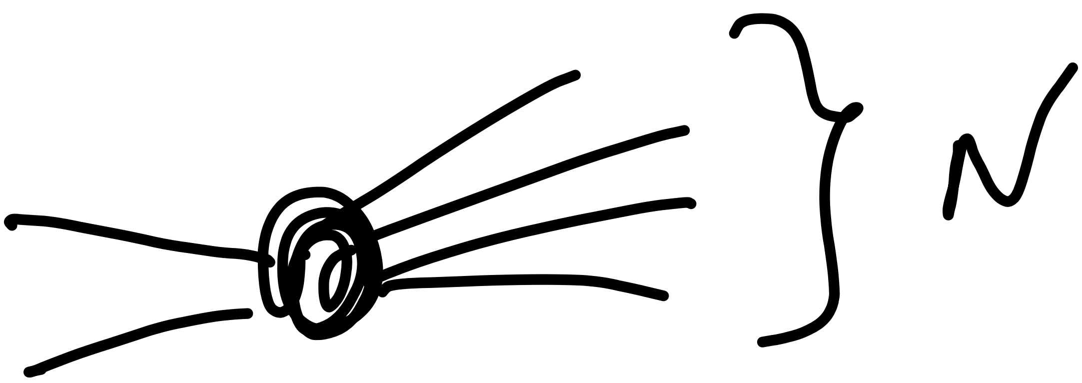
$$e^+ + e^- \longrightarrow BH$$

('t Hooft; Amati, Ciafaloni, Veneziano; Gross, Mende, ...)

Was even predicted at LHC (Antoniadis, Arkani-Hamed, Dimopoulos, GD)

We have such a microscopic theory which predicts that the relevant process is

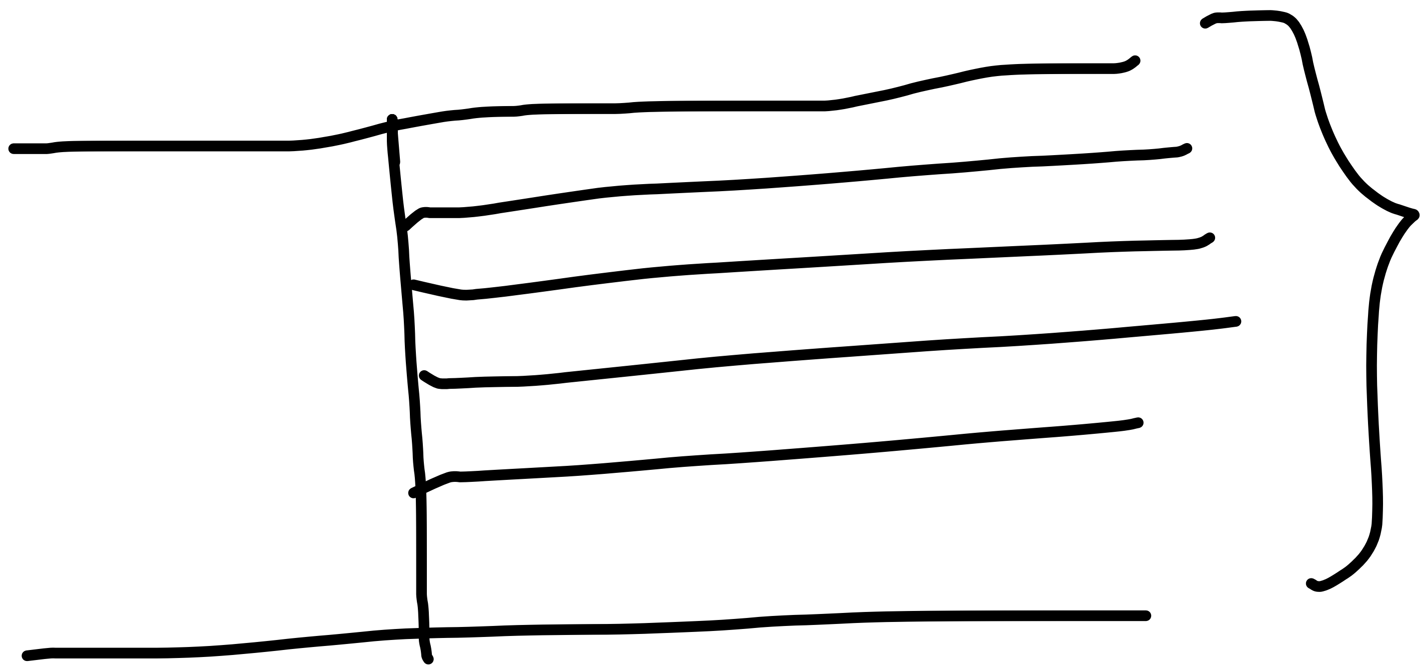
$$2 \to N \text{ gravitons}$$



with $N = \frac{s}{M_P^2} \gg 1$

2 → N graviton scattering

GD, Gomez, Isermann, Lüst, Stieberger, hep-th/1409.7405

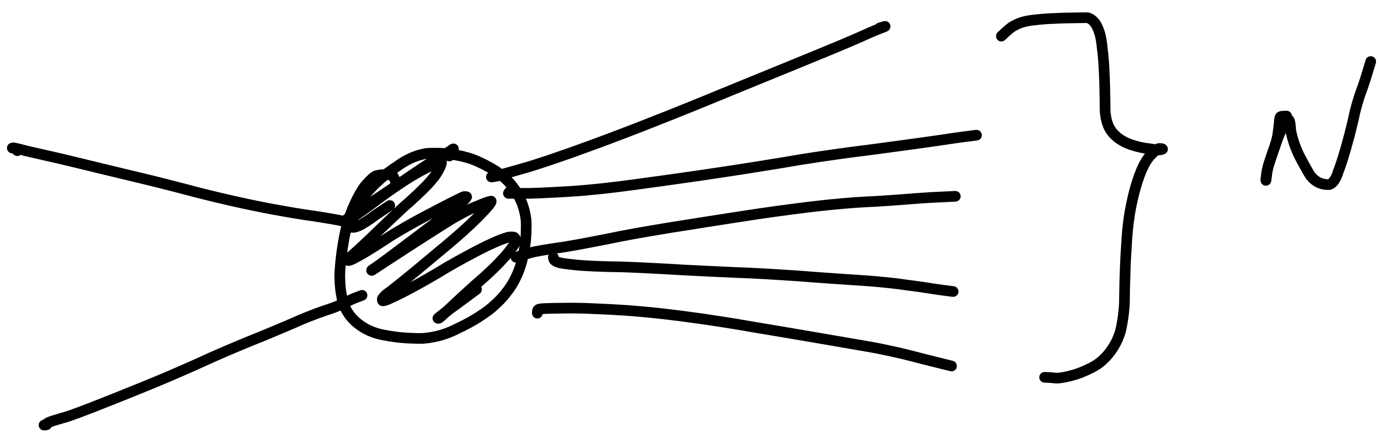


N

$+ \ldots$

In our kinematic regime loops are suppressed by $\sim \frac{1}{N}$

for $2 \to N$ amplitude
we get

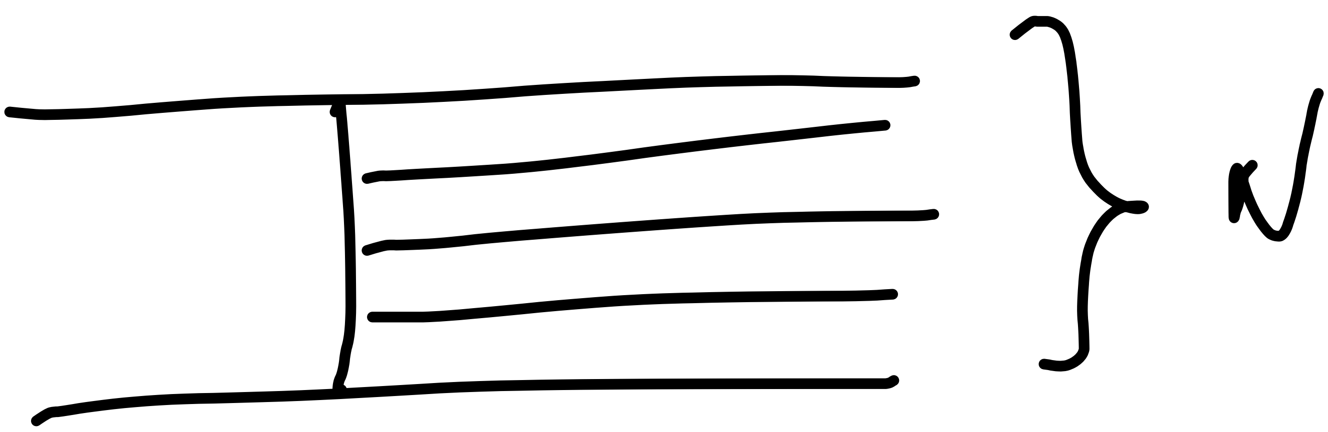


$$\sigma_{2\to N} = \frac{s}{M_p^4}\left(\frac{1}{N}\right)^N N! = \frac{s}{M_p^4} e^{-N}$$

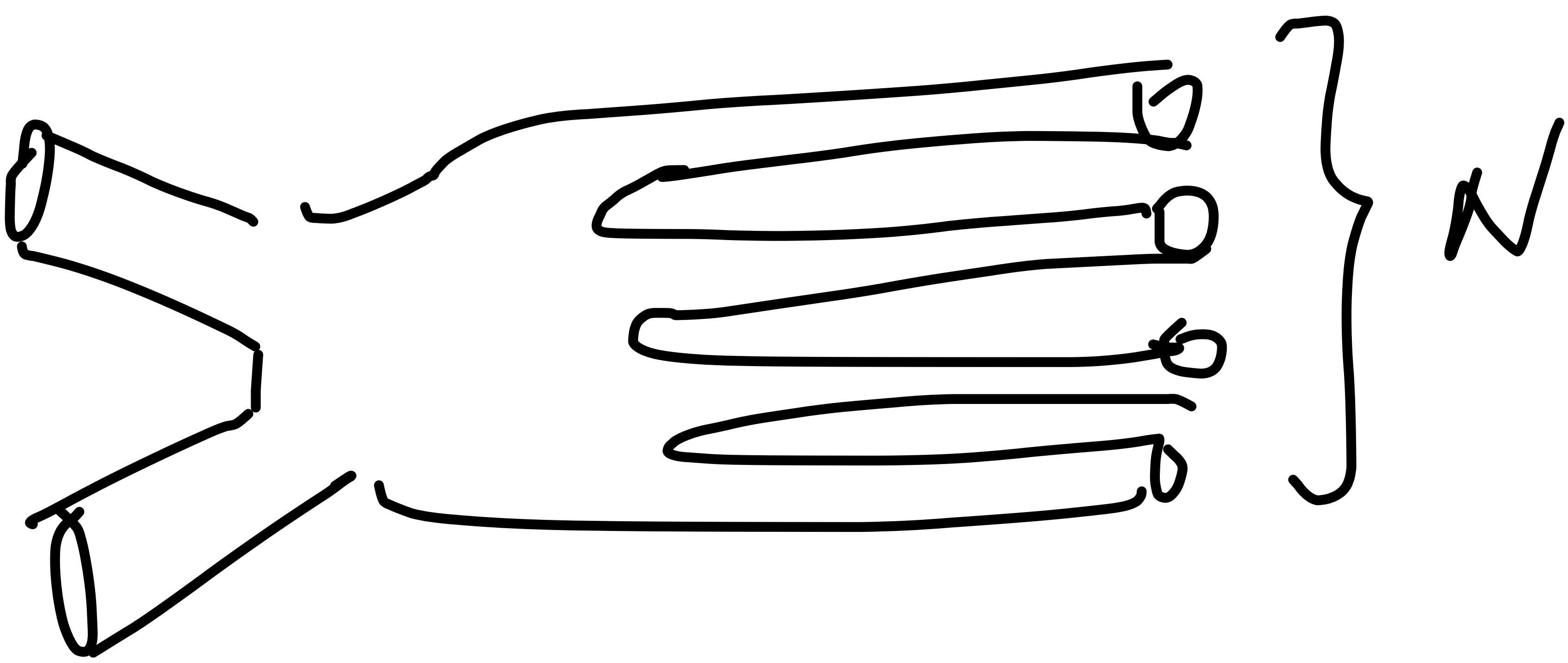
This exactly matches
the black hole entropy
factor!

Our result are UV-insensitive:

We get the same result in field theory



and string theory



So if the solution to the hierarchy problem is due to strong coupling (without Wilsonian UV--completion), LHC should observe the transition to multi-particle classicalization physics.

A tower of resonances becoming longer leaved with higher masses.

Lagrangian:

$$\mathcal{L} = \mathcal{L}_{SM} + H^2\left(m^2 - \frac{F^2}{M_p^2}\right)$$

$$-\lambda H^4 - C_{\alpha\beta\gamma} J_T^{\alpha\beta\gamma} Q(H)$$

where:

$C_{\alpha\beta\gamma} \leftarrow$ 3-form

$$F \equiv \partial_\alpha C_{\beta\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$$

$$Q \equiv M_p^2 \left\{ \left(\frac{H \bar{q}_L q_R}{M_p^4}\right)^n - \left(\frac{\bar{q}_L q_R \bar{q}_L q_R}{M_p^6}\right)^k \right\}$$