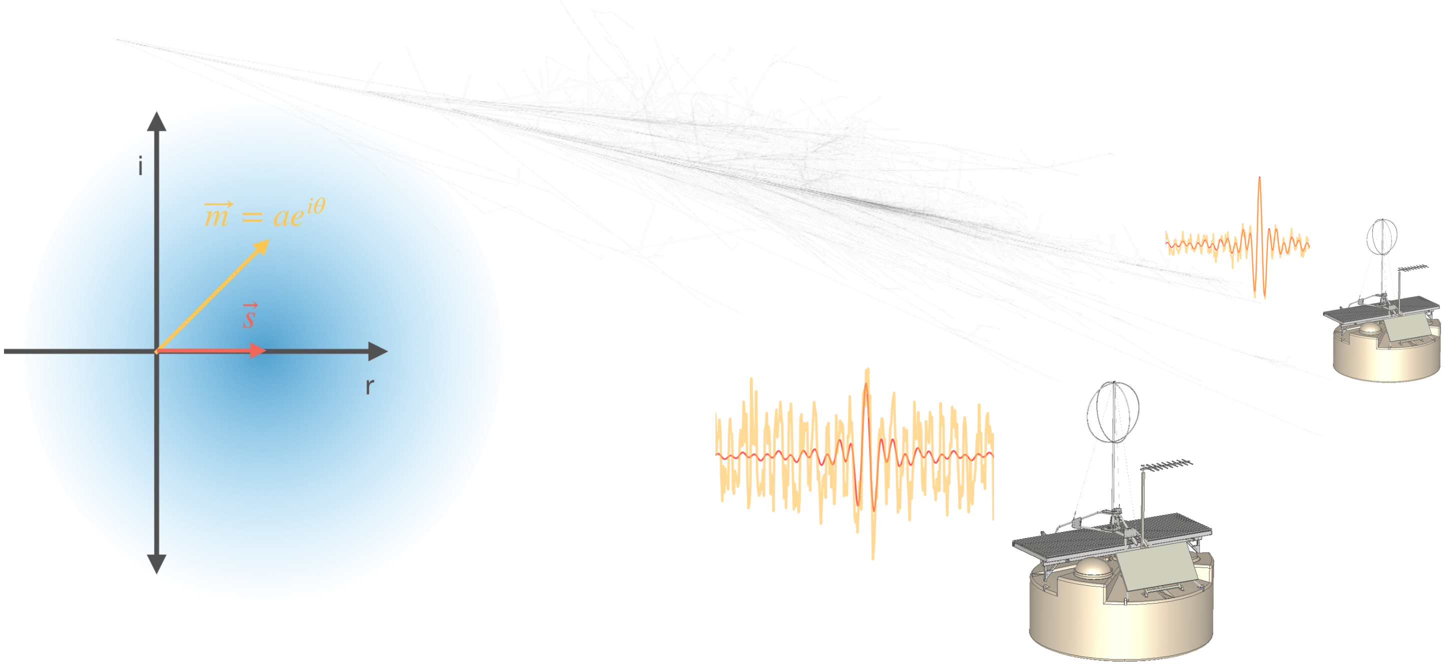
DPG 2024, Karlsruhe 05.03.24

Radio signal and uncertainty estimation

S. Martinelli*, Dr. T. Huege, Dr. D. Ravignani, Dr. H. Schoorlemmer for the Pierre-Auger collaboration



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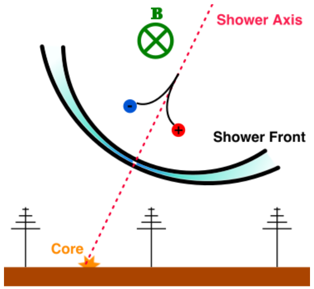
PIERRE AUGER OBSERVATORY

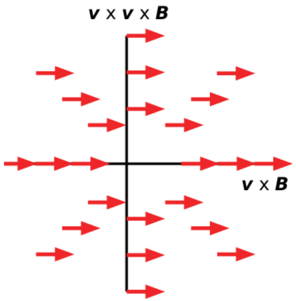
HIRSAP Helmholtz International Research School for Astroparticle Physics

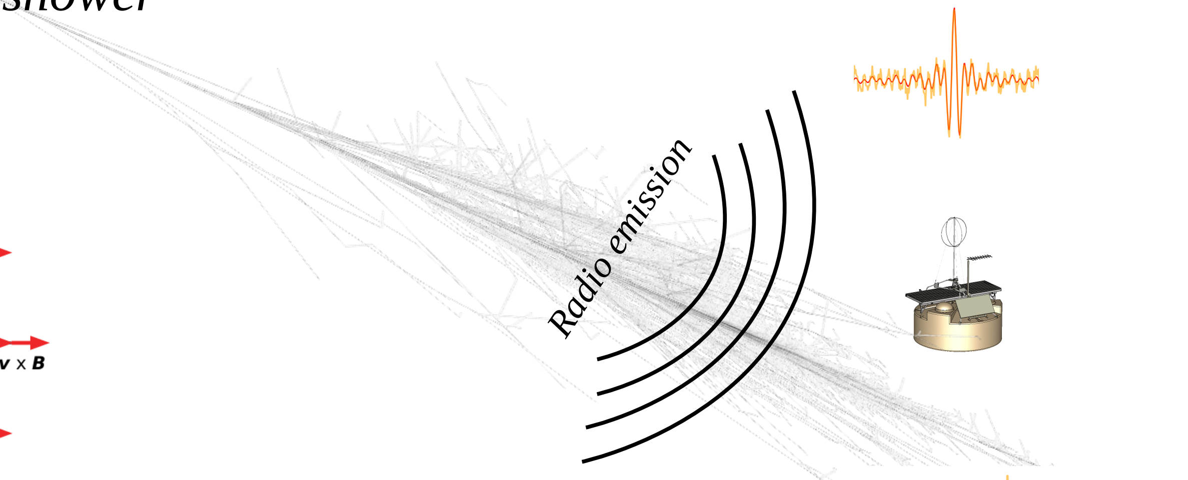
KIT Karlsruher Institut für Technologie

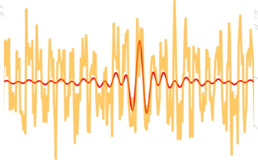
UNSAM

Cosmic ray (CR) initiating a shower



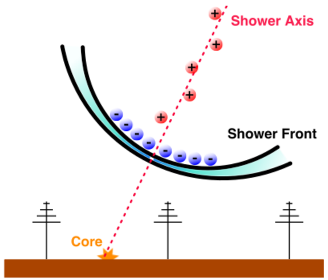


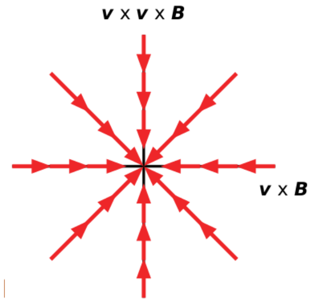


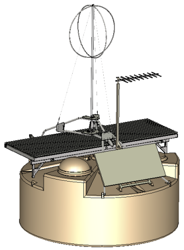


See previous talk by Simon Strähnz

This talk: how to determine the signal in the presence of noise





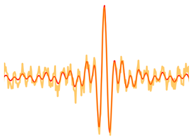


Figures adapted from K. D. de Vries, O. Scholten, K. Werner, Macroscopic geomagnetic radiation model; polarization effects and finite volume calculations, Nucl. Instr. Meth. A 662, Supplement 1 (2012) S175 – S178.

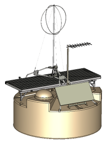
This talk: how to determine the signal in the presence of noise

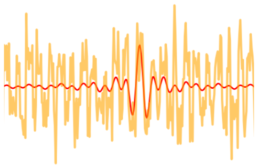
Radio measurements of CR pulses have both an amplitude and a phase

- The CR signal and the random noise can add up constructively or destructively
- Recovering of the underlying signal for small signal-to-noise ratio (SNR) is non-trivial

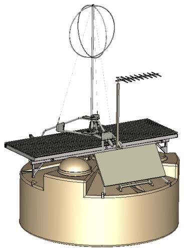


large SNR





small SNR



This talk: how to determine the signal in the presence of noise

Radio measurements of CR pulses have both an amplitude and a phase

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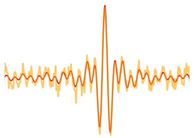
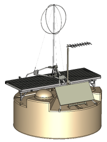
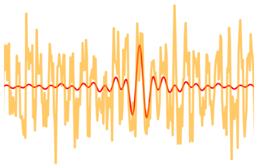
Energy fluence

- Energy deposit per unit area [eV/m^2]
- Typically reconstructed by noise subtraction (Auger, LOFAR...)

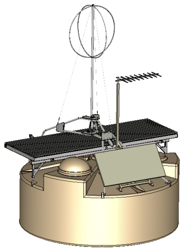


large SNR

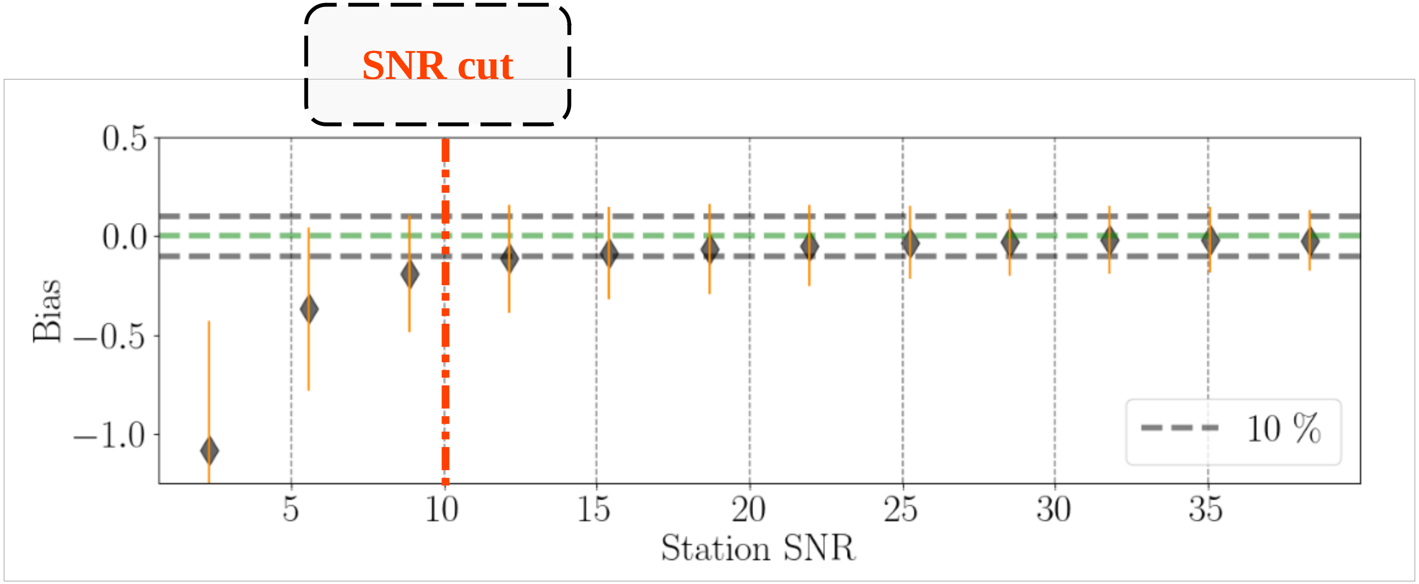




small SNR



Energy Fluence Reconstruction: Noise Subtraction Method



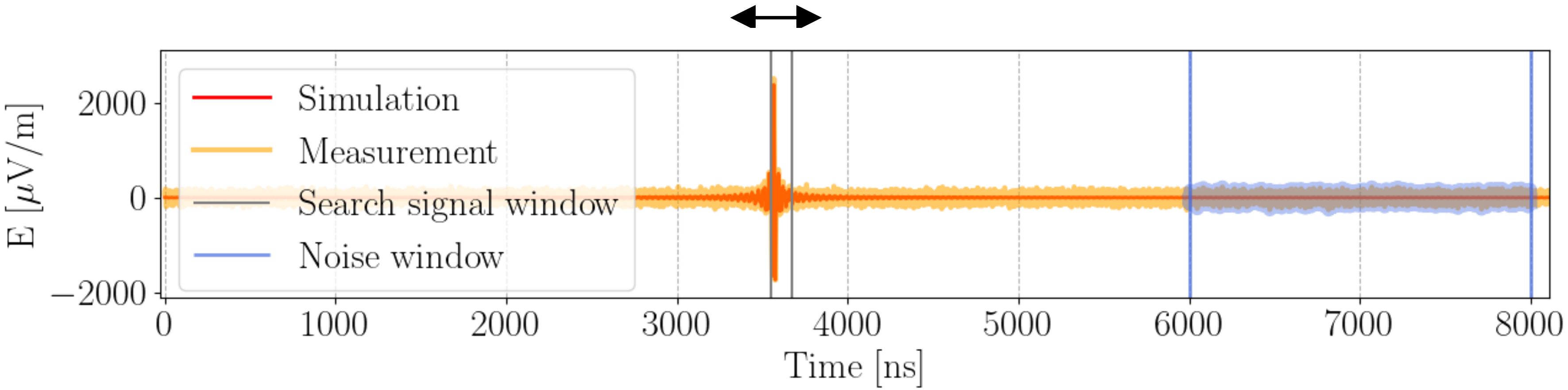
- Method unbiased for SNR>10 (cut)
- Uncertainties underestimated

How does the method work?

n-th pol.

Definition of the search-signal

Trigger constrains the signal search

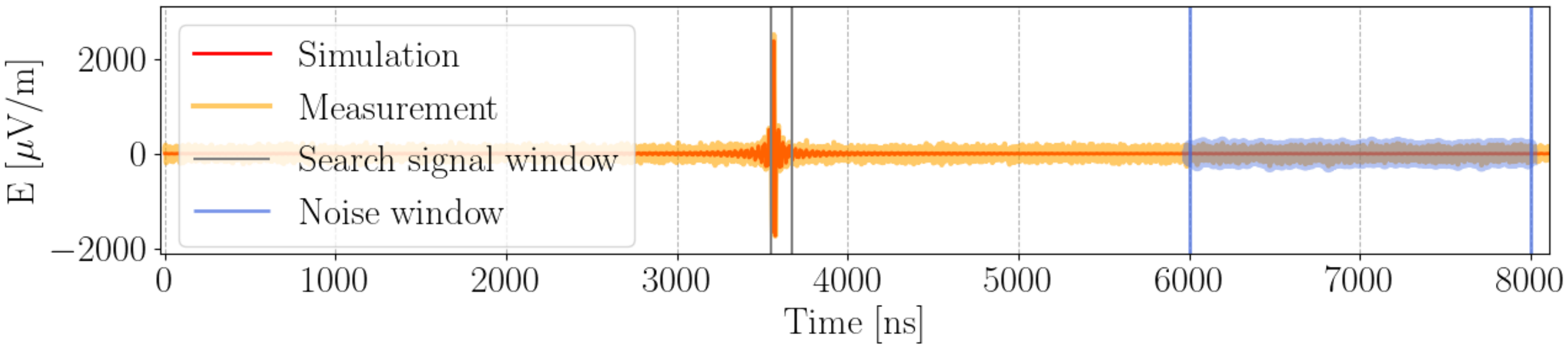


n-th pol.

Definition of the search-signal

Definition of the noise window

Here we do not expect a signal



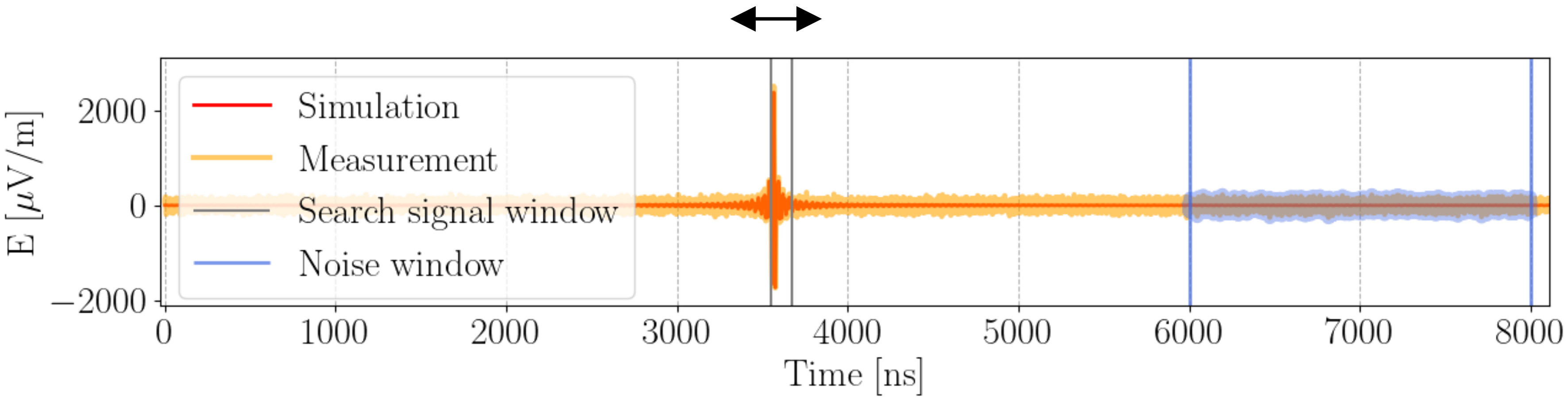
n-th pol.

Definition of the search-signal

Definition of the noise window

Find the peak in the search-signal window

Pulse Finding Algorithm (e.g. maimum of the Hilbert envelope over the polarisations)



Energy Fluence Reconstruction: Noise Subtraction Method

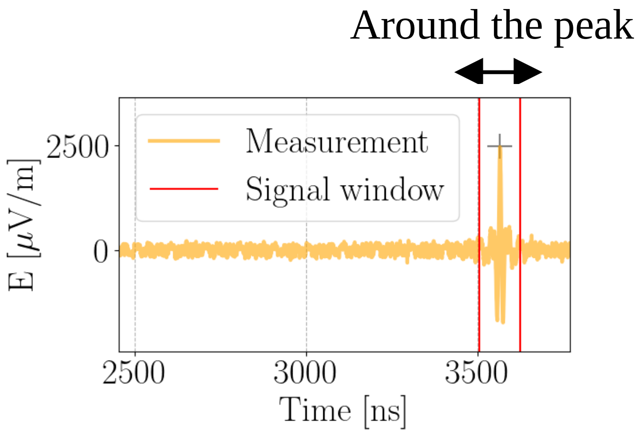
n-th pol.

Definition of the search-signal

Definition of the noise window

Find the peak in the search-signal window

Definition of the signal window



n-th pol.

Definition of the search-signal

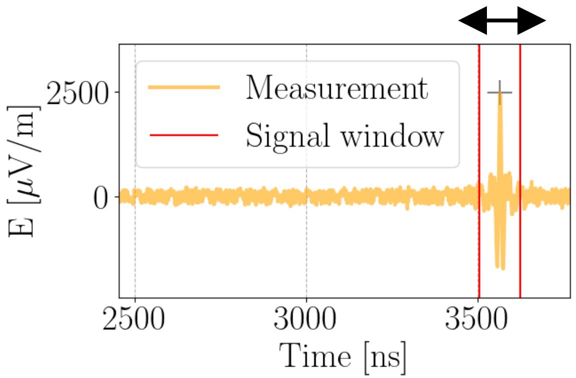
Definition of the noise window

Find the peak in the search-signal window

Definition of the signal window

Fluence in the signal window

f ~ sum over the amplitudes squared



$$\hat{f}^{\mathrm{pol}} = \epsilon_0\, c\, \Delta t \left(\sum_{t_j = t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} + \ldots \right)$$

n-th pol.

Definition of the search-signal

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Find the peak in the search-signal window

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Fluence in the signal window

$$\hat{f}^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \left(\sum_{t_j = t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} + \ldots \right)$$

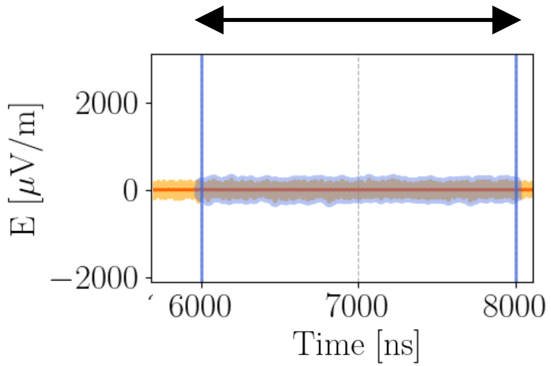
Subtraction of the noise fluence

$$\hat{f}^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \left(\sum_{t_j = t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} - \frac{t_4 - t_3}{t_2 - t_1} \sum_{t_j = t1}^{t_2} A(t_j)^2_{\mathrm{pol}} \right)$$

Norm.

(uncertainty model: backup)

f ~ sum over the amplitudes squared



The estimator can be negative!

n-th pol.

Definition of the search-signal

Definition of the noise window

Find the peak in the search-signal window

Definition of the signal window

Fluence in the signal window $$\hat{f}^{\mathrm{pol}} = \epsilon_0\, c\, \Delta t \left(\sum_{t_j=t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} + \ldots \right)$$

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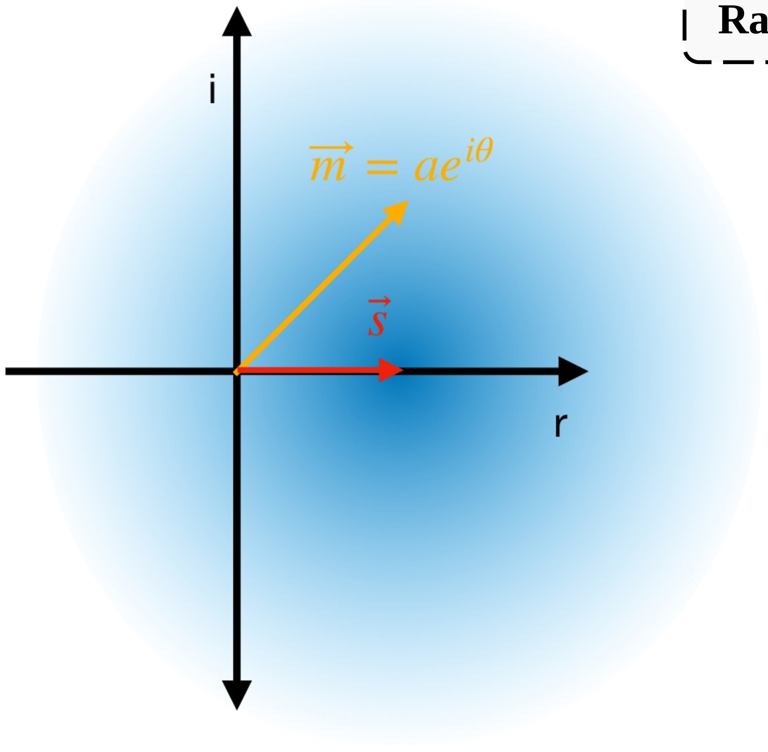
antenna station $$\hat{f}^{\mathrm{station}} = \sum_{\mathrm{pol}} \hat{f}^{\mathrm{pol}}$$

The estimator can be negative!

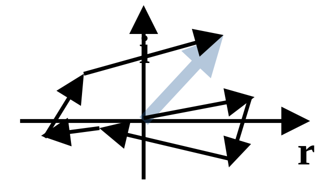
Can we do better than this?



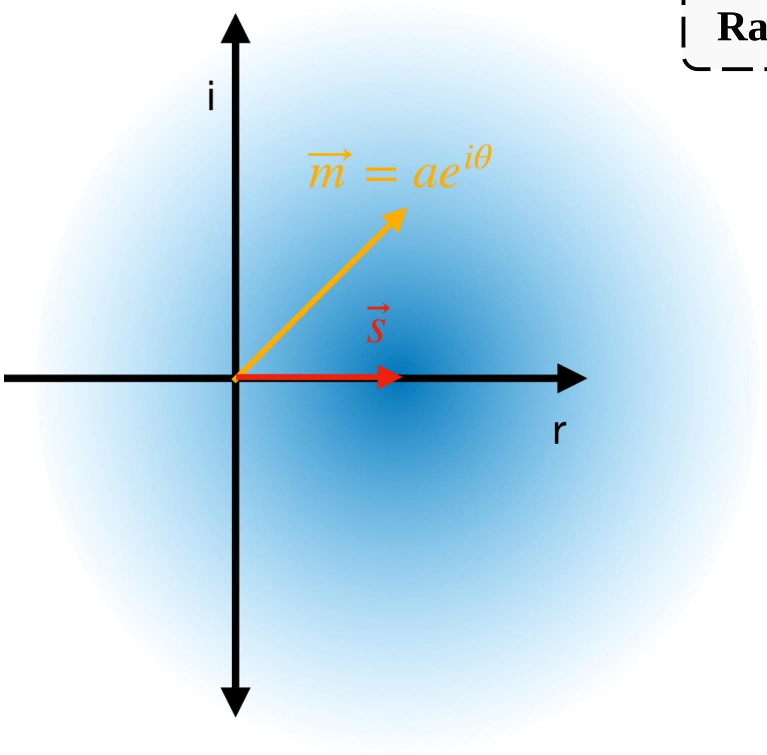
Radio measurements have both an amplitude and a phase

- Our **measurement** can be expressed as the sum of **constant known phasor s** and a **random phasor sum** (Rayleigh-distributed noise).

random phasor sum =



Reference: Chapter 2.9 from J. W. Goodman, Statistical Optics (2015)



Radio measurements have both an amplitude and a phase

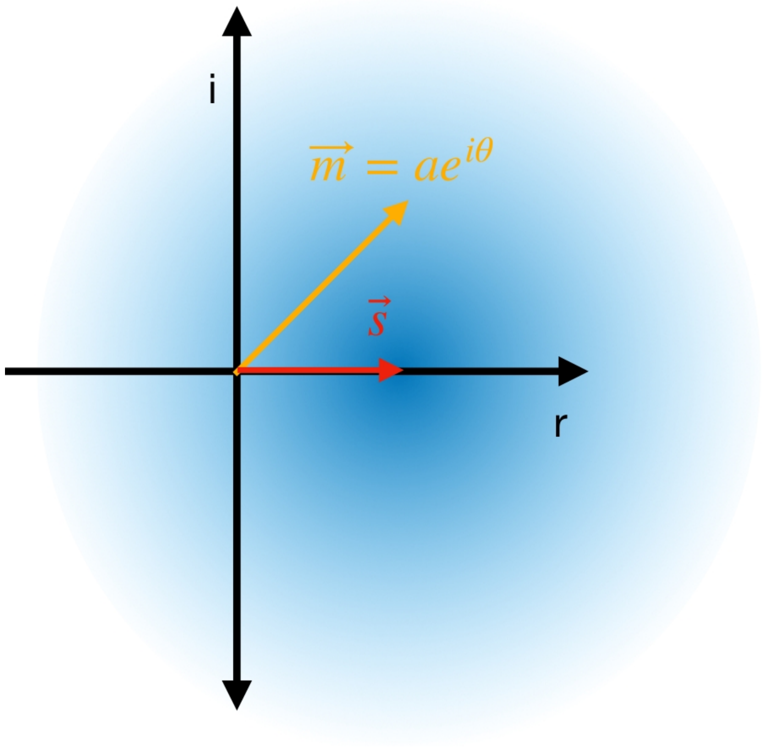
- Our **measurement** can be expressed as the sum of **constant known phasor s** and a **random phasor sum** (Rayleigh-distributed noise).

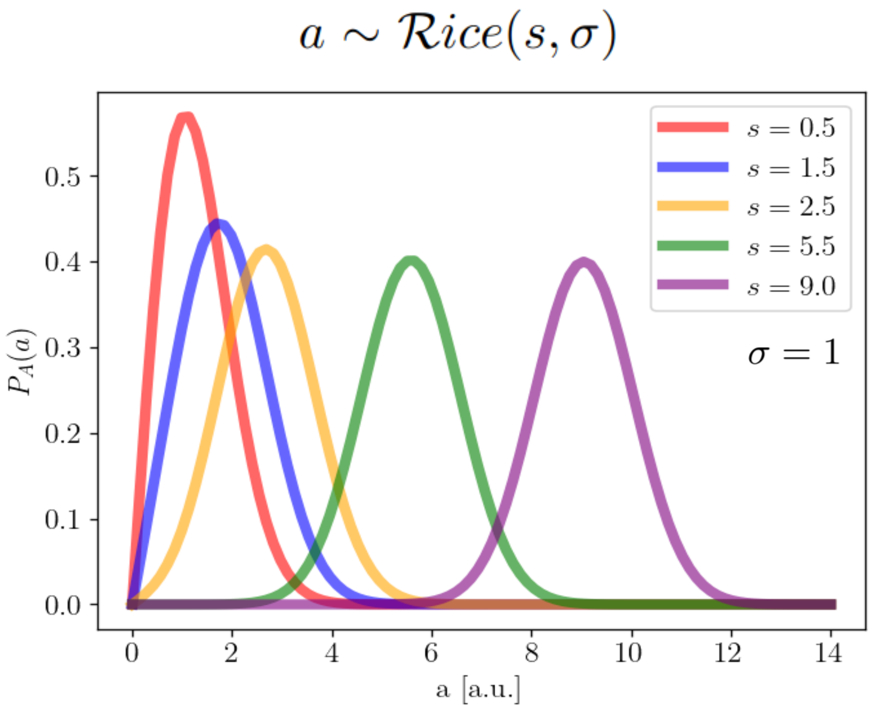
- The marginal p.d.f. of the measured **amplitude** is the Rice distribution.

$$p_A(a|s,\sigma) = \begin{cases} \frac{a}{\sigma^2} \cdot \exp\left(-\frac{a^2+s^2}{2\sigma^2}\right) \cdot I_0\left(\frac{as}{\sigma^2}\right) & \text{a} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Reference: Chapter 2.9 from J. W. Goodman, Statistical Optics (2015)

Radio Measurements: Rice Distribution





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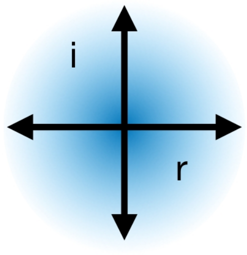
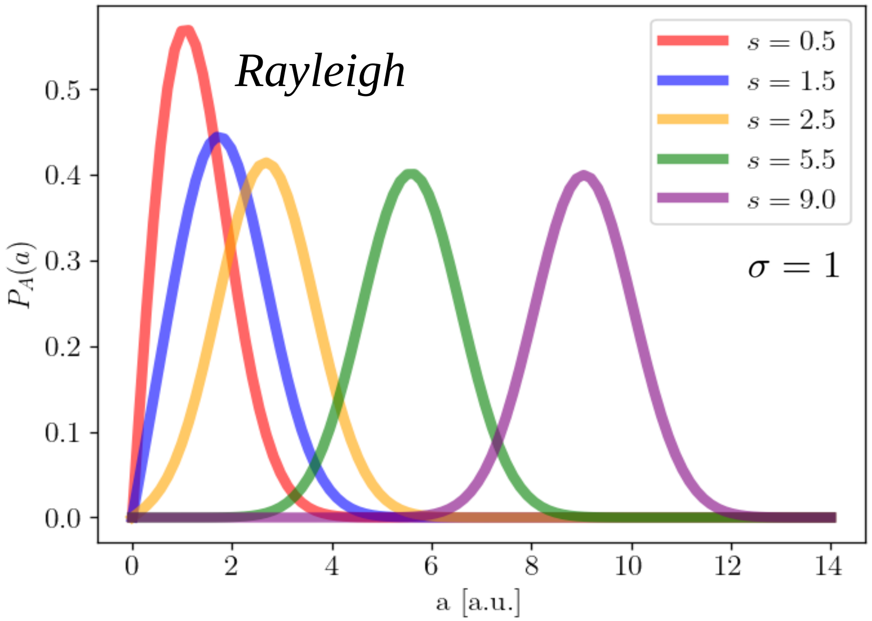


No signal

$$a \sim \mathcal{R}ice(s, \sigma)$$



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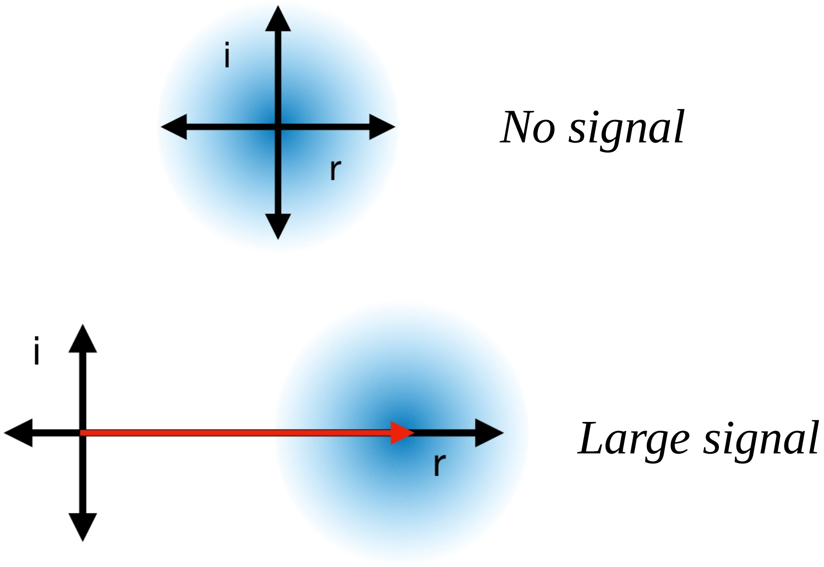

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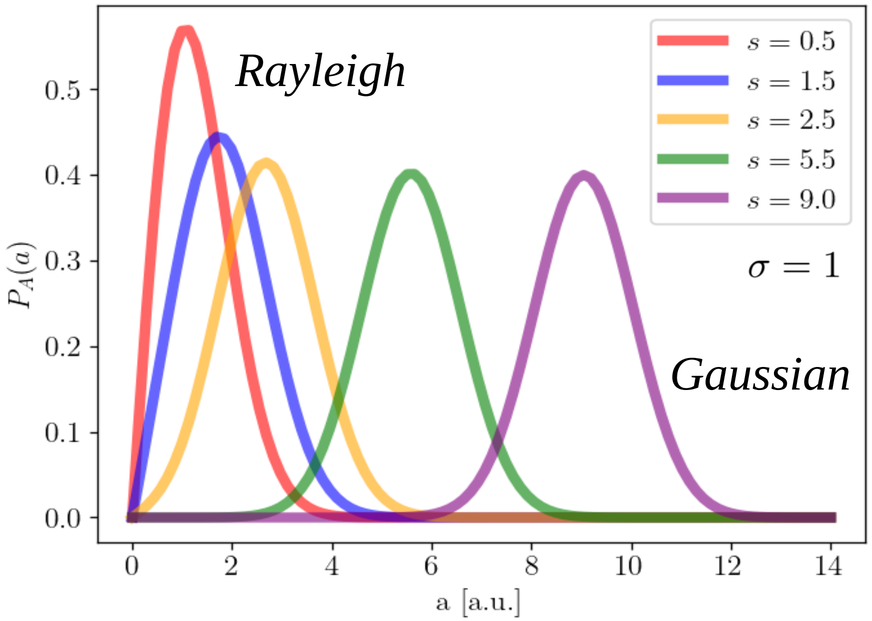

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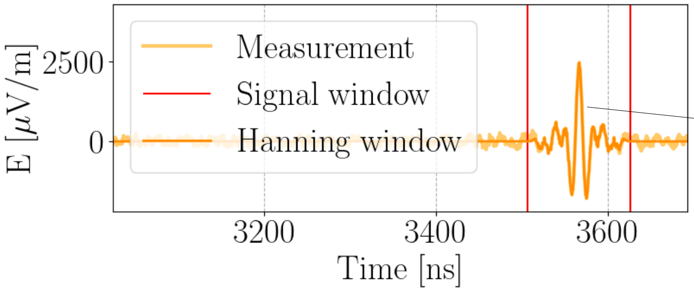
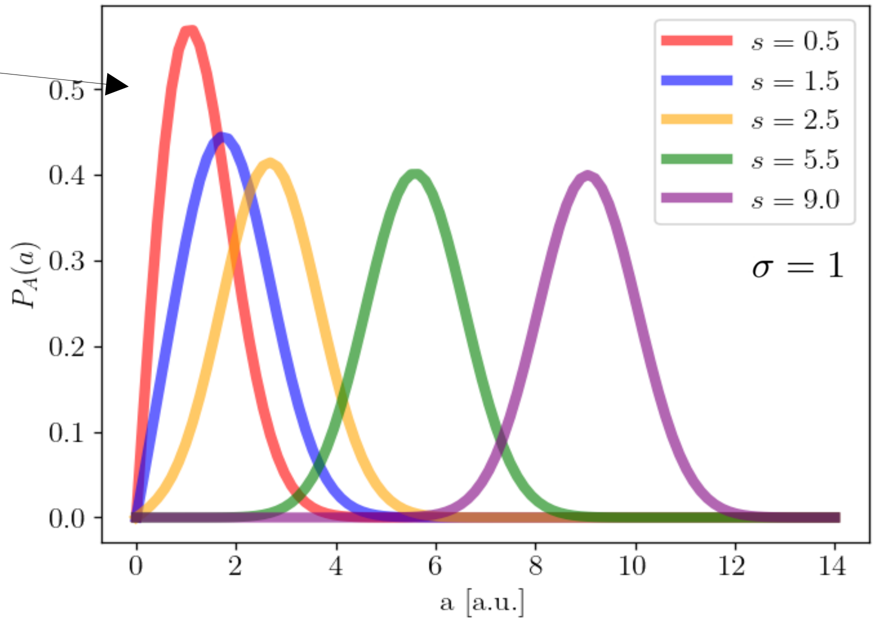
Radio Measurements: Rice Distribution

Time Domain



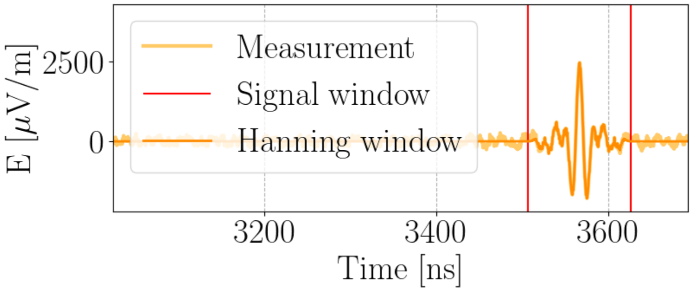
$$a \sim \mathcal{R}ice(s, \sigma)$$



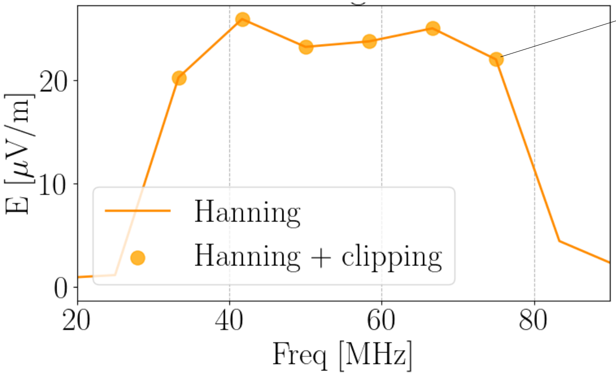
Formalism valid for both time and frequency domains

Radio Measurements: Rice Distribution

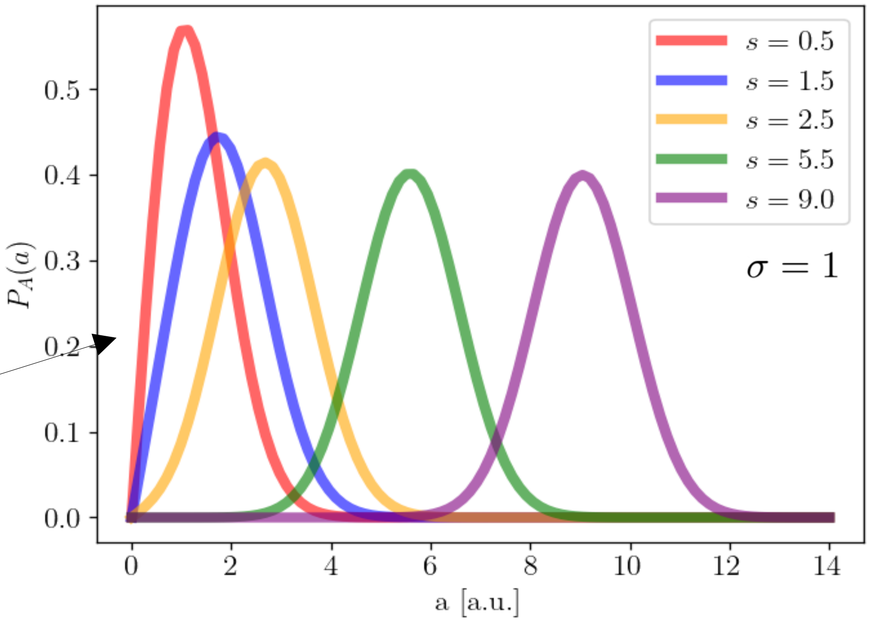
Time Domain



Fourier Transform
(after some signal processing)

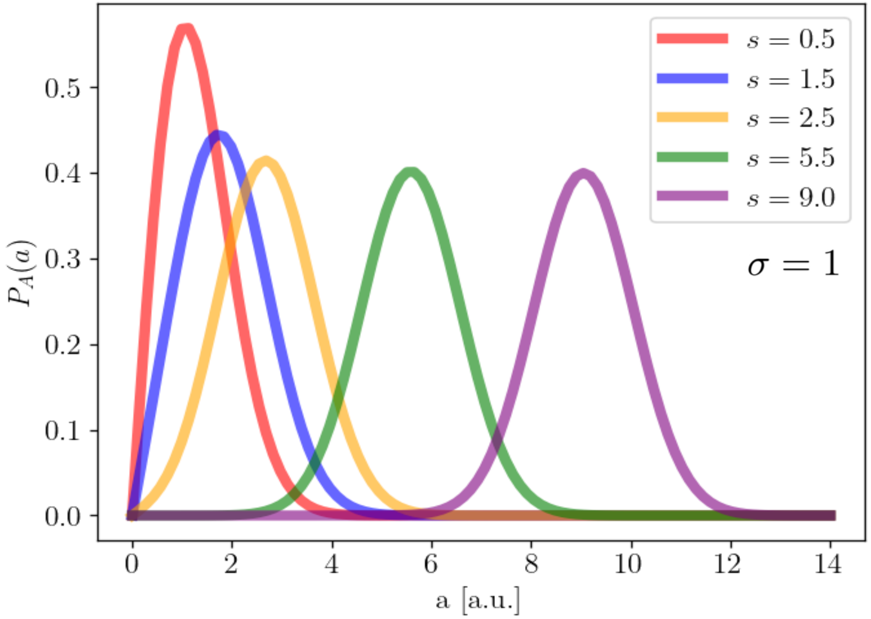


$$a \sim \mathcal{R}ice(s, \sigma)$$



Formalism valid for both time and frequency domains

$$a \sim \mathcal{R}ice(s, \sigma)$$



How can we use this theory to build a better estimator of the fluence?

n-th pol — Evaluation of the fluence in the frequency domain

$$f^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \sum_{j=0}^{N-1} A(t_j)^2_{\mathrm{pol}} = 2\epsilon_0 \, c \, \frac{\Delta t}{N} \sum_{j=0}^{M-1} |D(v_j)_{\mathrm{pol}}|^2$$

(Parseval's theorem)

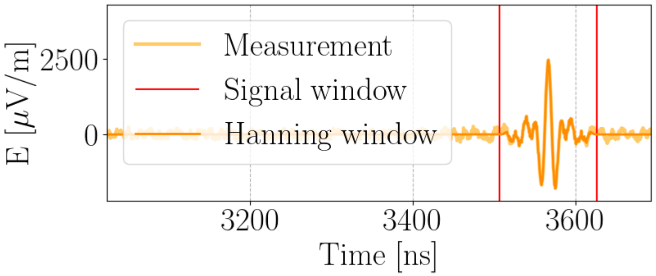
Fourier Transform

Building a New Fluence Estimator

n-th pol

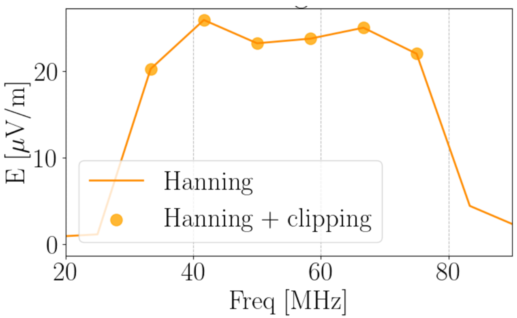
Evaluation of the fluence in the frequency domain

Definition of signal window



signal processing

Fourier Transform



Building a New Fluence Estimator

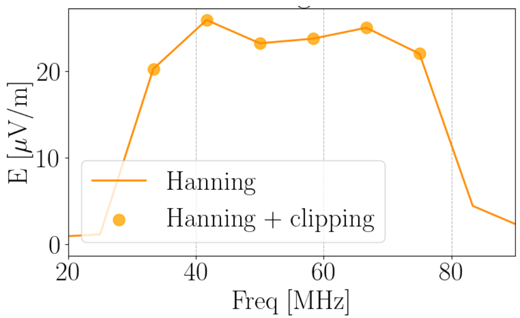
n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies

$$\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$$



n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

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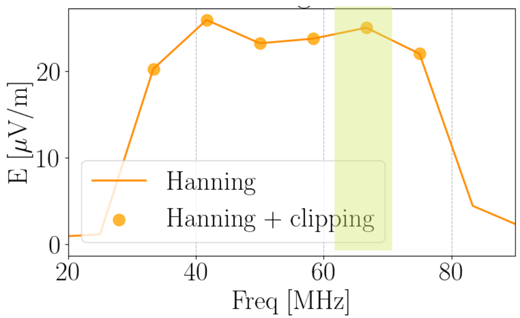
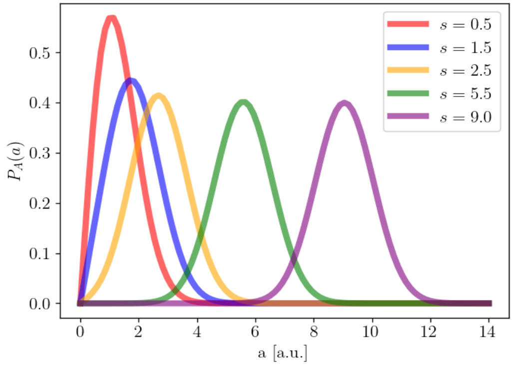


j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$



Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

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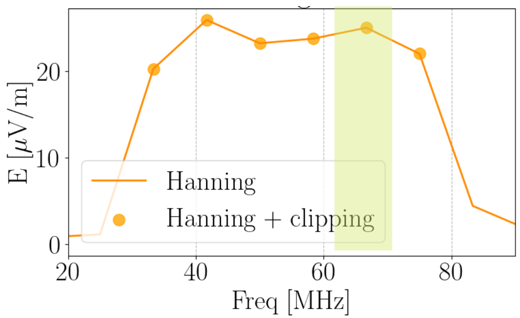


j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

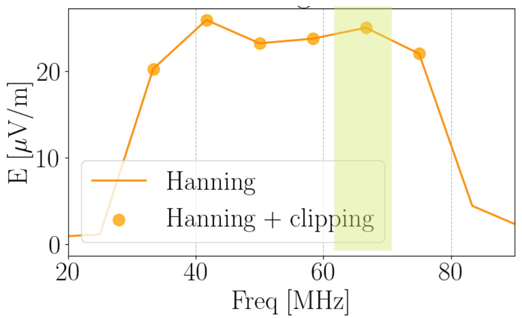
$\hat{f} \propto a^2$ $a^2 \sim ?$

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

Derivation:

$b = a/\sigma$ Change of variable by scaling the measured amplitude by the noise level of the bin

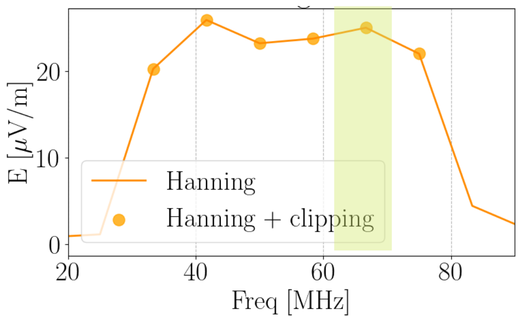
Adimensional

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$\hat{f} \propto a^2$ $a^2 \sim ?$

$b = a/\sigma$

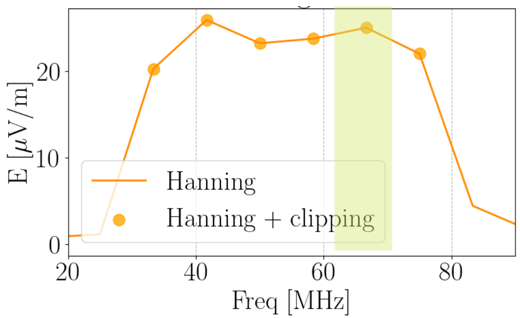
$snr = (s/\sigma)^2$ *snr* at bin level, to not be confused with the SNR of the full measurement!

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

$b = a/\sigma$

$snr = (s/\sigma)^2$

$$b \sim \mathcal{R}ice(\sqrt{snr}, 1) = \chi_{\text{nc}}(DF = 2, \lambda = \sqrt{snr})$$

Degrees of Freedom

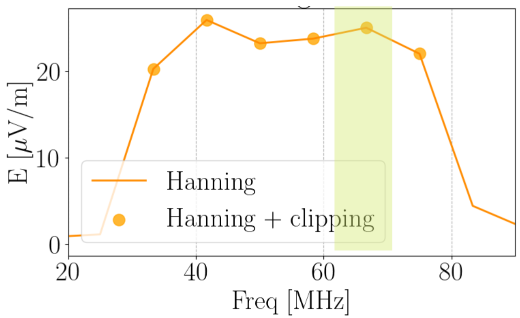
Non-centrality parameter

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

Derivation:

$b = a/\sigma$ → $b^2 \sim \chi^2_{\text{nc}}(DF = 2, \lambda = snr)$ Well-known distribution!

$snr = (s/\sigma)^2$

n-th pol

Evaluation of the fluence in the frequency domain

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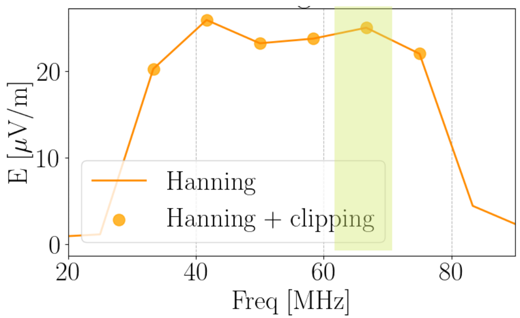


j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

Derivation:

$b = a/\sigma$

$snr = (s/\sigma)^2$

$\longrightarrow \quad b^2 \sim \chi^2_{\mathrm{nc}}(DF = 2, \lambda = snr)$

$E(b^2) = 2 + snr$ Expected value

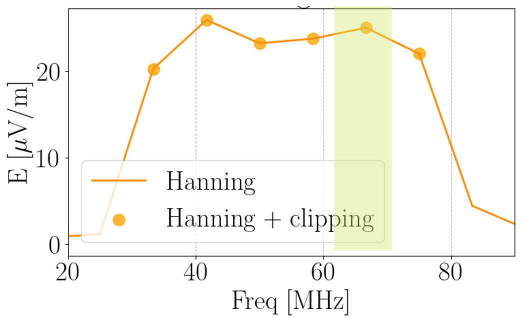
$Var(b^2) = 2\,(2 + 2\,snr)$ Variance

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2$ $\quad a^2 \sim ?$

Derivation:

$b = a/\sigma$

$snr = (s/\sigma)^2$

$\longrightarrow$ $b^2 \sim \chi^2_{\text{nc}}(DF = 2, \lambda = snr)$

$E(b^2) = 2 + snr$

$Var(b^2) = 2\,(2 + 2\,snr)$

$\xrightarrow{b^2 = a^2/\sigma^2}$

$E(a^2) = 2\sigma^2 + s^2$

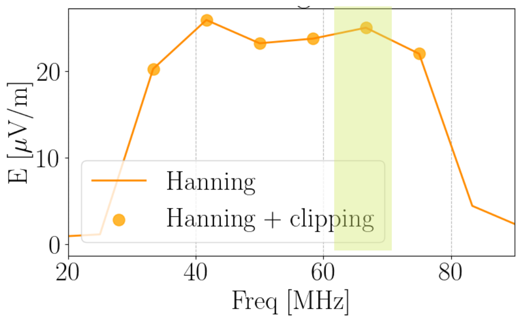
$Var(a^2) = 2\,\sigma^4(2 + 2\,snr)$

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

Derivation:

$$E(a^2) = 2\sigma^2 + s^2 \longrightarrow \hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

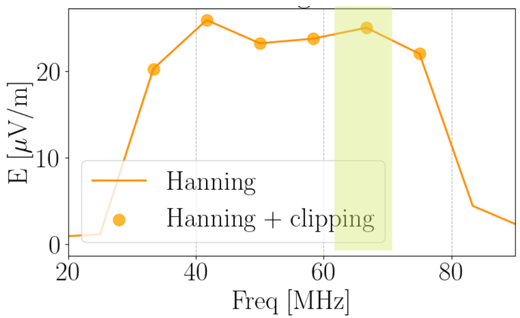
Always positive-valued!

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$



j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$\hat{f} \propto a^2 \qquad a^2 \sim ?$

Derivation:

$$E(a^2) = 2\sigma^2 + s^2 \longrightarrow \hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(a^2) = 2\,\sigma^4(2 + 2\,snr) \qquad Var(\hat{f}) = Var(a^2)$$

Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$$

$$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$$

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

j-th bin

$a \sim \mathcal{R}ice(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$$

$snr = (s/\sigma)^2$ ► $s^2 \approx \hat{f}$

$\sigma^2 \sim$?

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$

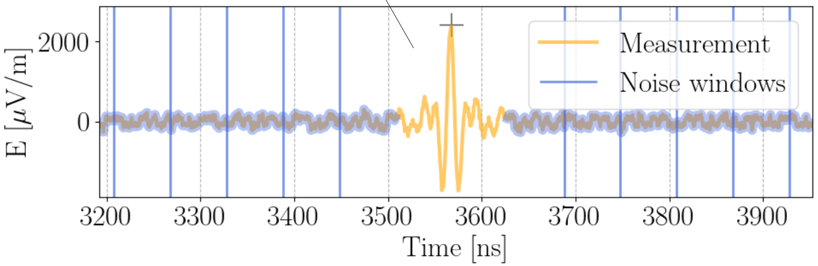
$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$

$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$

$\sigma^2 \sim\ ?$

N_{noise} windows along the whole trace

Signal window excluded



Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

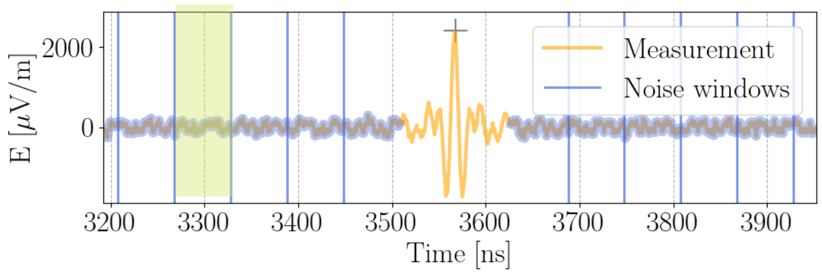
$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$

$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$

$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$

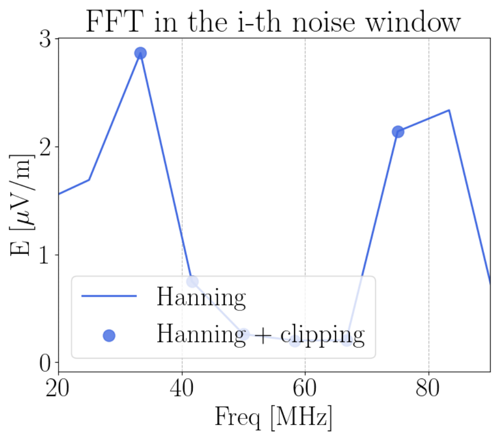
$\sigma^2 \sim\ ?$

Repeated for all the N_{noise} windows



signal processing

Fourier Transform



n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$

$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$

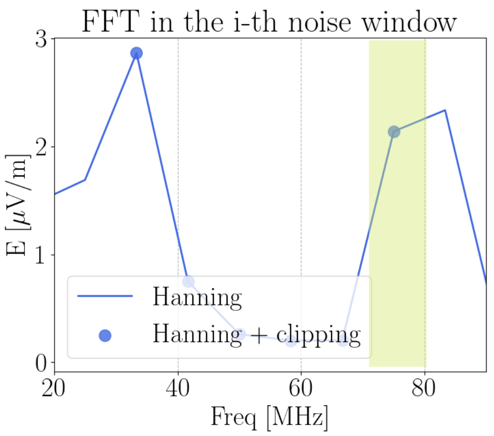
$\sigma^2 \sim$?

Derivation:

$$E(a^2) = 2\sigma^2 + s^2$$

$s = 0$

$$\sigma^2 = \frac{E(a^2)}{2}$$



Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$

$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$

$\sigma^2 \sim$?

Derivation:

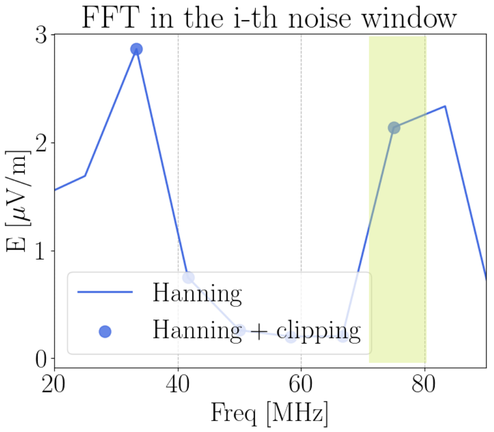
$$E(a^2) = 2\sigma^2 + s^2$$

$s = 0$

$$\sigma^2 = \frac{E(a^2)}{2}$$

Sample mean of the statistic over the noise windows:

$$\hat{\sigma}^2 = \frac{1}{2}\left(\frac{1}{N_{\text{noise}}} \sum_{i=0}^{N_{\text{noise}}-1} a_i^2\right)$$



Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$

$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$

$\sigma^2 \sim$?

Derivation:

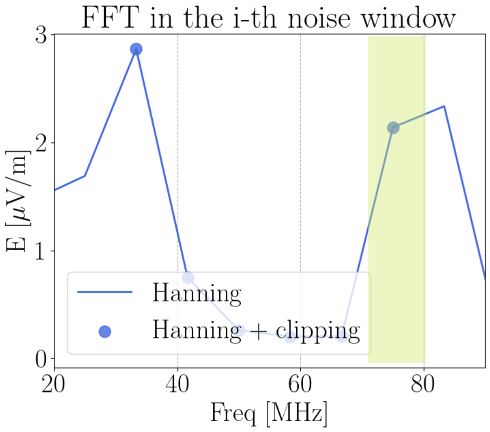
$$E(a^2) = 2\sigma^2 + s^2$$

$s = 0$

$$\sigma^2 = \frac{E(a^2)}{2}$$

Sample standard deviation over the noise windows:

$$\delta(\hat{\sigma}^2) = \frac{1}{2} s(a_i^2)/\sqrt{N_{\text{noise}}}$$ (small)



n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$$a \sim \mathcal{R}ice(s, \sigma)$$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\hat{f}) = 2\,\sigma^4 (2 + 2\,snr)$$

$$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$$

$$\hat{\sigma}^2 = \frac{1}{2}\left(\frac{1}{N_{\mathrm{noise}}} \sum_{i=0}^{N_{\mathrm{noise}}-1} a_i^2\right)$$

We have all the ingredients at bin level

Building a New Fluence Estimator

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$$a \sim \mathcal{Rice}(s, \sigma)$$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$$

$$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$$

$$\hat{\sigma}^2 = \frac{1}{2}\left(\frac{1}{N_{\text{noise}}} \sum_{i=0}^{N_{\text{noise}}-1} a_i^2\right)$$

We can now evaluate the fluence at trace level ...

n-th pol

Evaluation of the fluence in the frequency domain

Definition of signal window

Fluence estimator ~ sum over the frequencies $\hat{f}^{\text{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

Definition of noise windows

Noise estimator over the windows

j-th bin

$a \sim \mathcal{Rice}(s, \sigma)$

$$\hat{f} = \begin{cases} a^2 - 2\sigma^2 & a^2 \geq 2\sigma^2 \\ 0 & \text{otherwise} \end{cases}$$

$$Var(\hat{f}) = 2\,\sigma^4(2 + 2\,snr)$$

$$snr = (s/\sigma)^2 \rightarrow s^2 \approx \hat{f}$$

$$\hat{\sigma}^2 = \frac{1}{2}\left(\frac{1}{N_{\text{noise}}} \sum_{i=0}^{N_{\text{noise}}-1} a_i^2\right)$$

antenna station

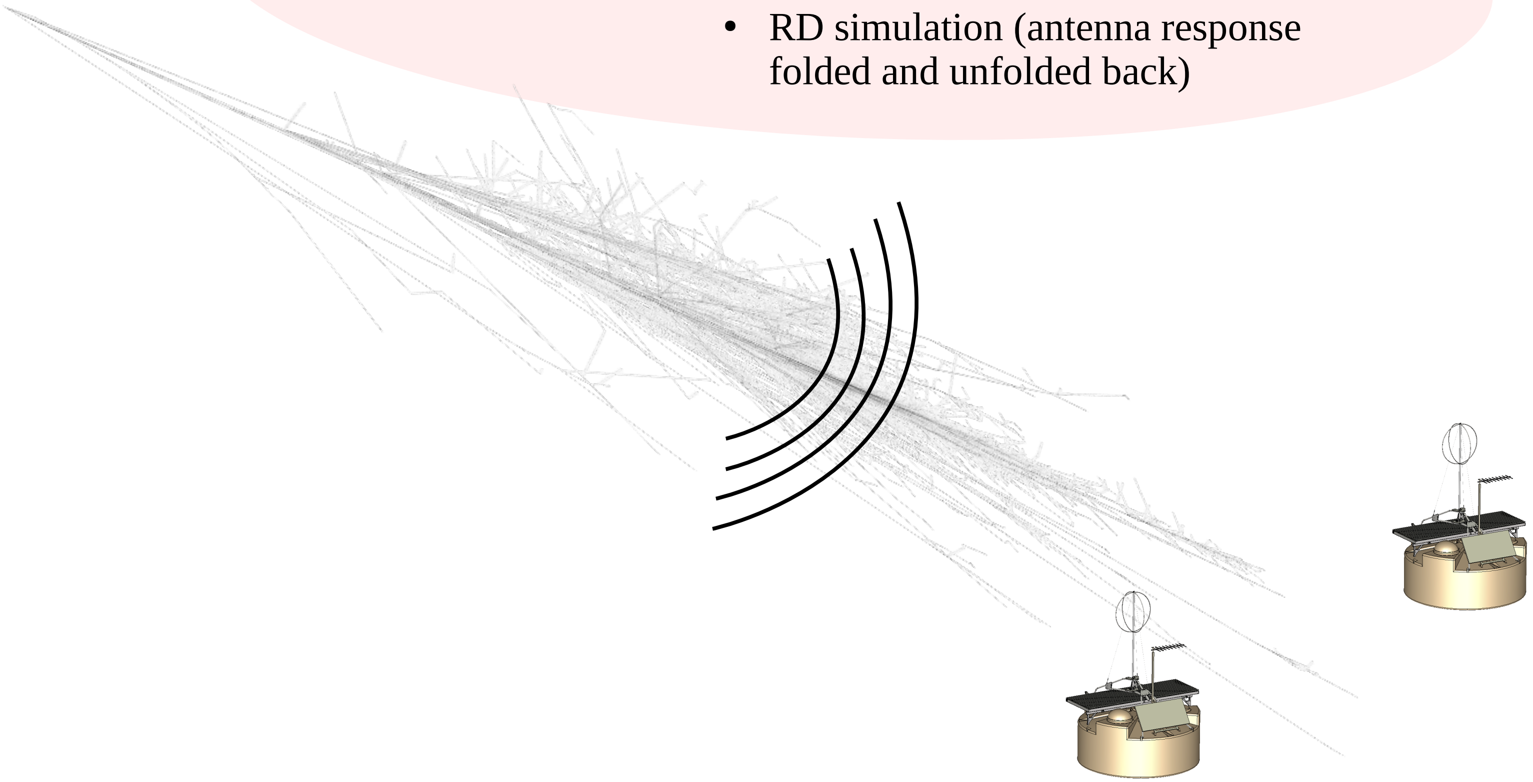
$$\hat{f}^{\text{station}} = \sum_{\text{pol}} \hat{f}^{\text{pol}}$$

… and at the antenna level

Now we can compare the new method and the noise-subtraction method

RD Simulations

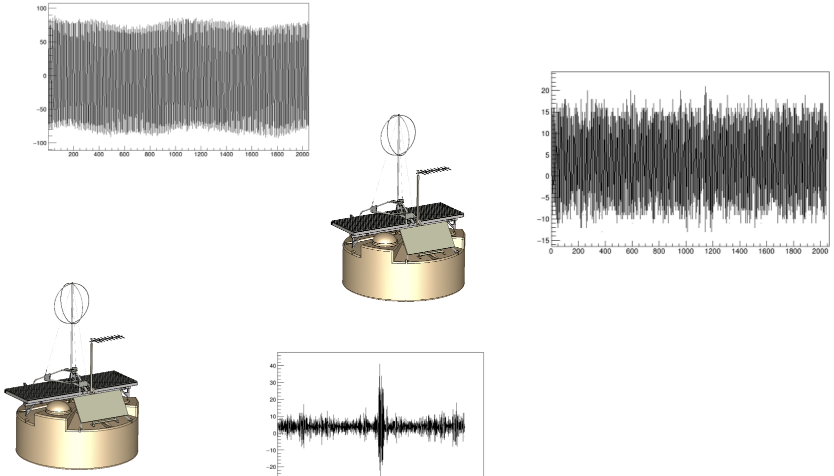
- 8000 proton/iron/nitrogen/helium CORSIKA/CoREAS simulations
- RD simulation (antenna response folded and unfolded back)



RD Simulations

- RD traces recorded over one year from the stations of the Engineering Array
    - Cleaned from the showers signals and corrupted traces

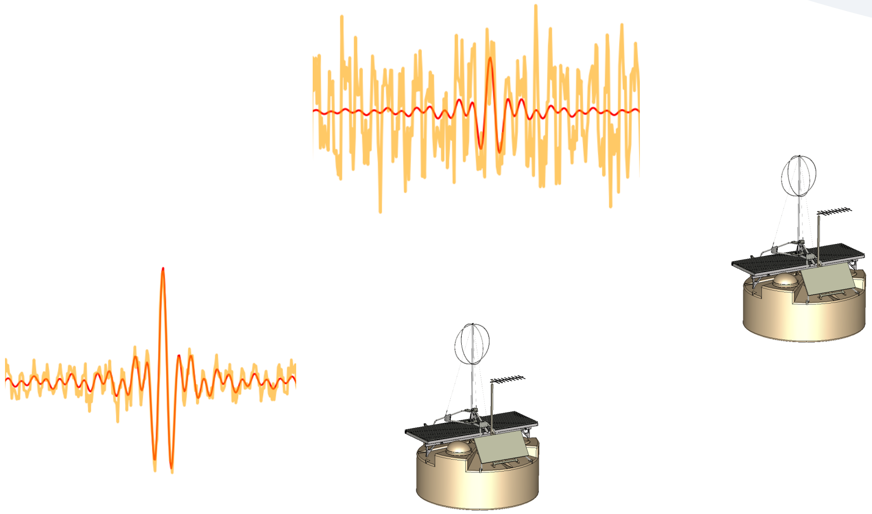
RD Noise Library



RD Simulations

Simulated RD Measurements

RD Noise Library



RD Simulations

Simulated RD Measurements

RECONSTRUCTION

→ New method $\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$

→ Noise subtraction method

$$\hat{f}^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \left(\sum_{t_j=t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} - \frac{t_4 - t_3}{t_2 - t_1} \sum_{t_j=t1}^{t_2} A(t_j)^2_{\mathrm{pol}} \right)$$

RD Simulations

Simulated RD Measurements

New method

$$f^{\mathrm{pol}} = K \sum_{j=0}^{M-1} s^2(\nu_j)$$

New method

$$\hat{f}^{\mathrm{pol}} = K \sum_{j=0}^{M-1} \hat{f}_j$$

REFERENCE VALUES vs RECONSTRUCTION

Noise subtraction method

$$f^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \left(\sum_{t_j=t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} \right)$$

Noise subtraction method

$$\hat{f}^{\mathrm{pol}} = \epsilon_0 \, c \, \Delta t \left(\sum_{t_j=t_3}^{t_4} A(t_j)^2_{\mathrm{pol}} - \frac{t_4 - t_3}{t_2 - t_1} \sum_{t_j=t1}^{t_2} A(t_j)^2_{\mathrm{pol}} \right)$$

Different reference values for different methods

RD Simulations

Simulated RD Measurements

New method

New method

REFERENCE VALUES vs RECONSTRUCTION

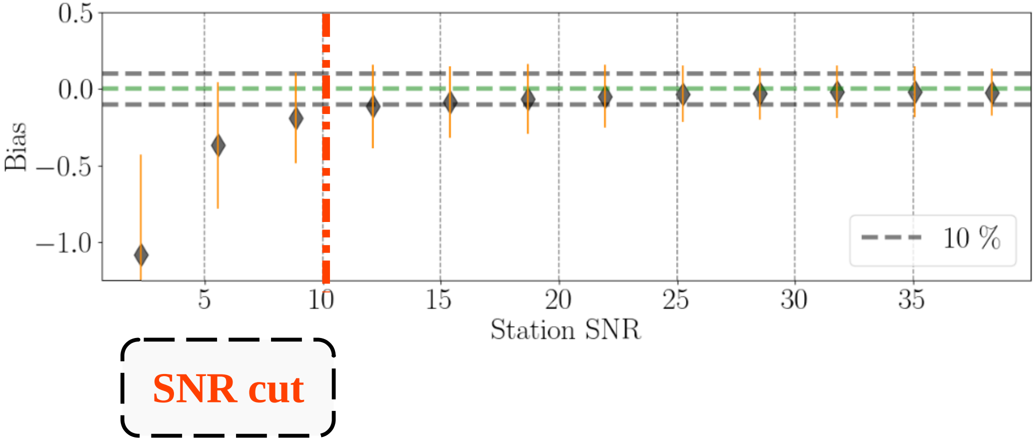
Noise subtraction method

Noise subtraction method

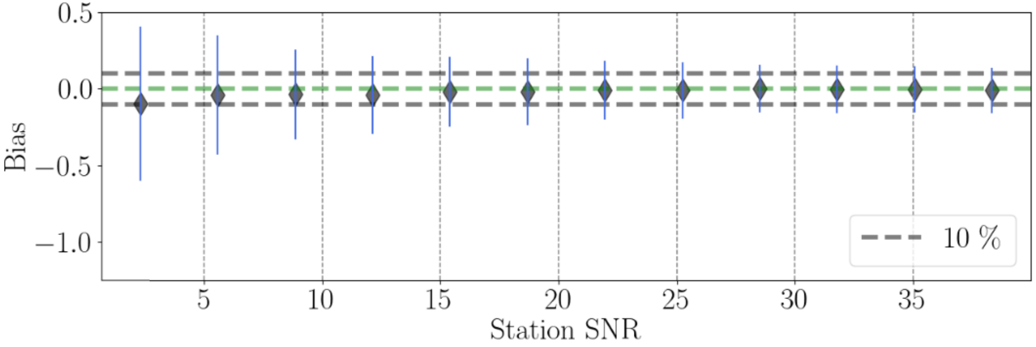
Quality cuts:

- Stations affected by thinning artifacts from simulation
- Stations where the pulse-finding algorithm “fails” (>2ns)

Noise subtraction method

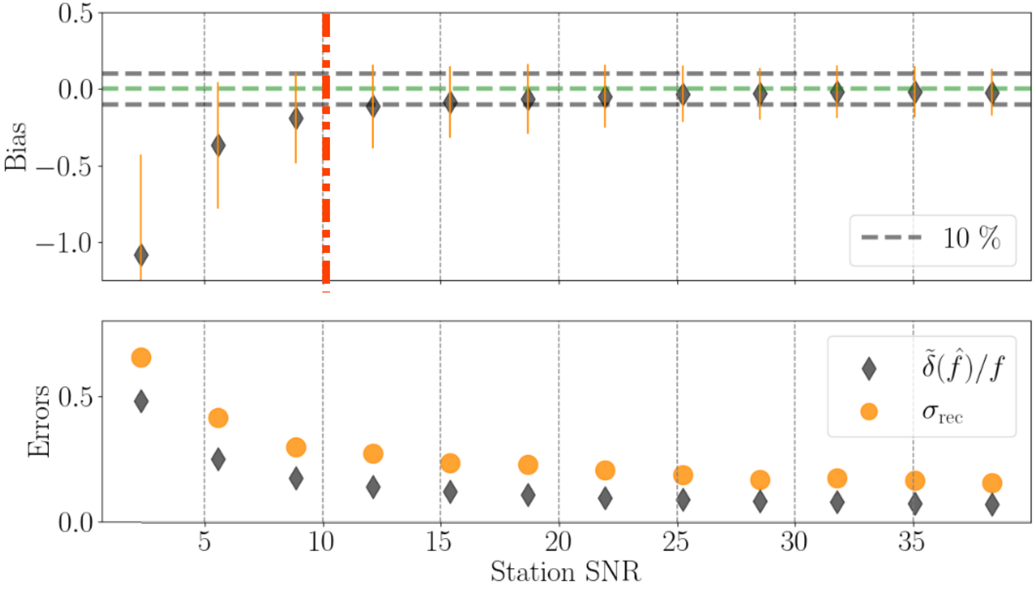


New method



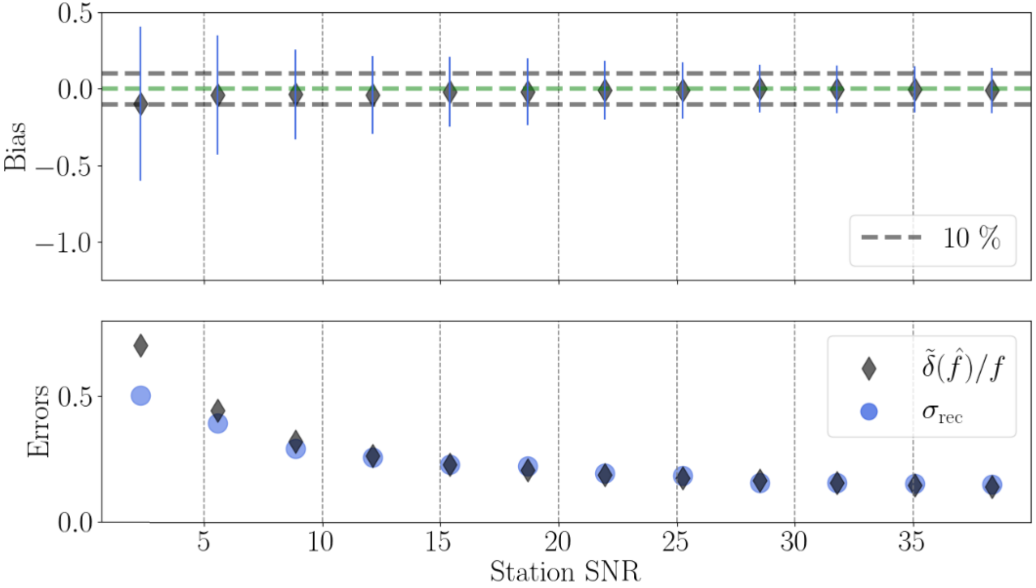
On average the new method is unbiased even at small SNR

Noise subtraction method

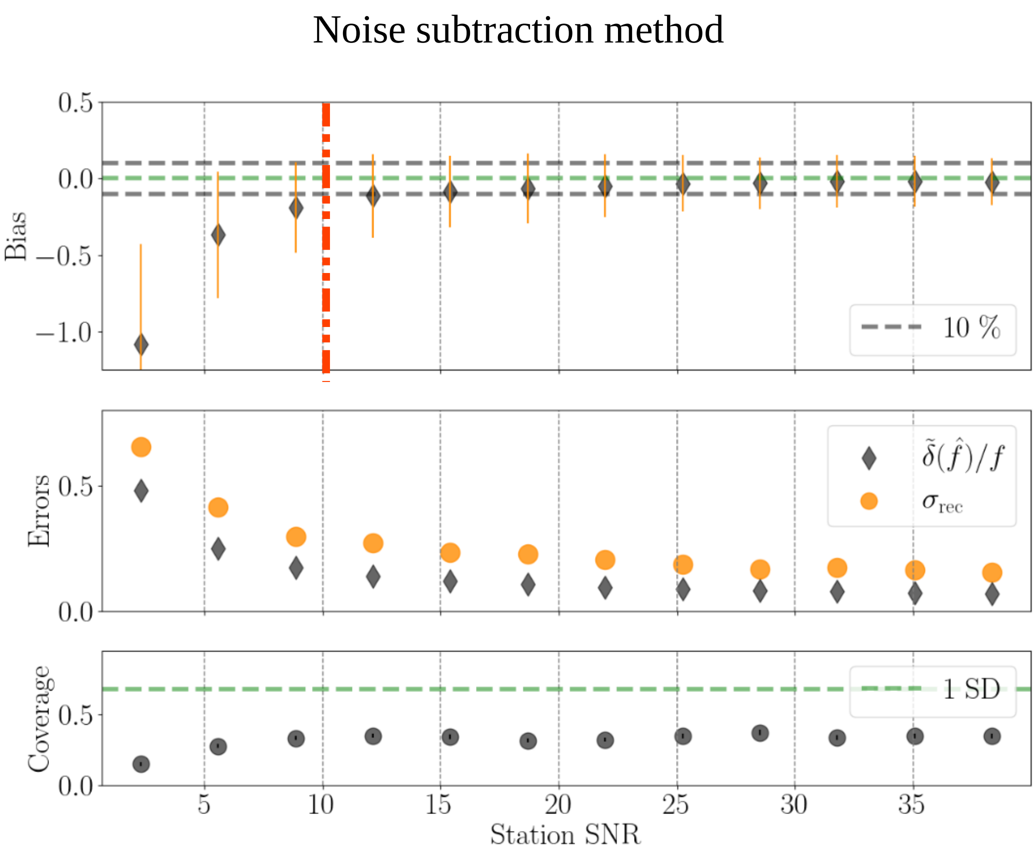


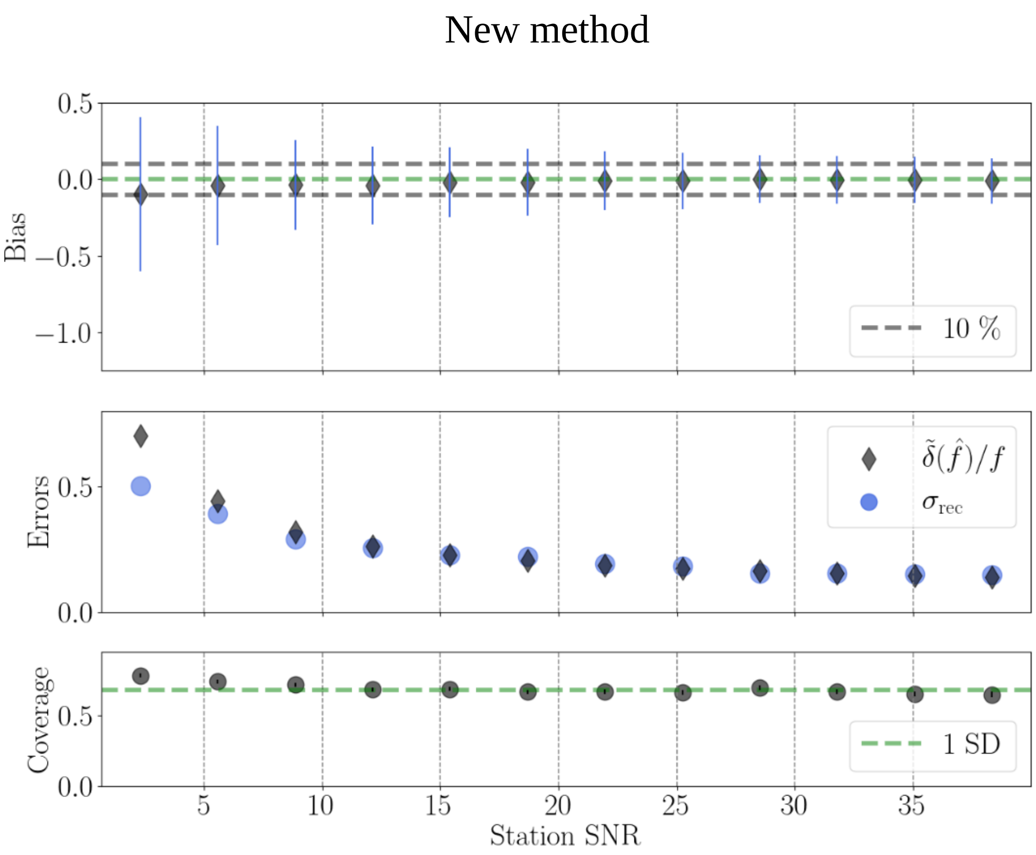
The relative errors are smaller than the reconstruction resolution of the same bin

New method



The relative errors of the new method reflect better the reconstruction resolution



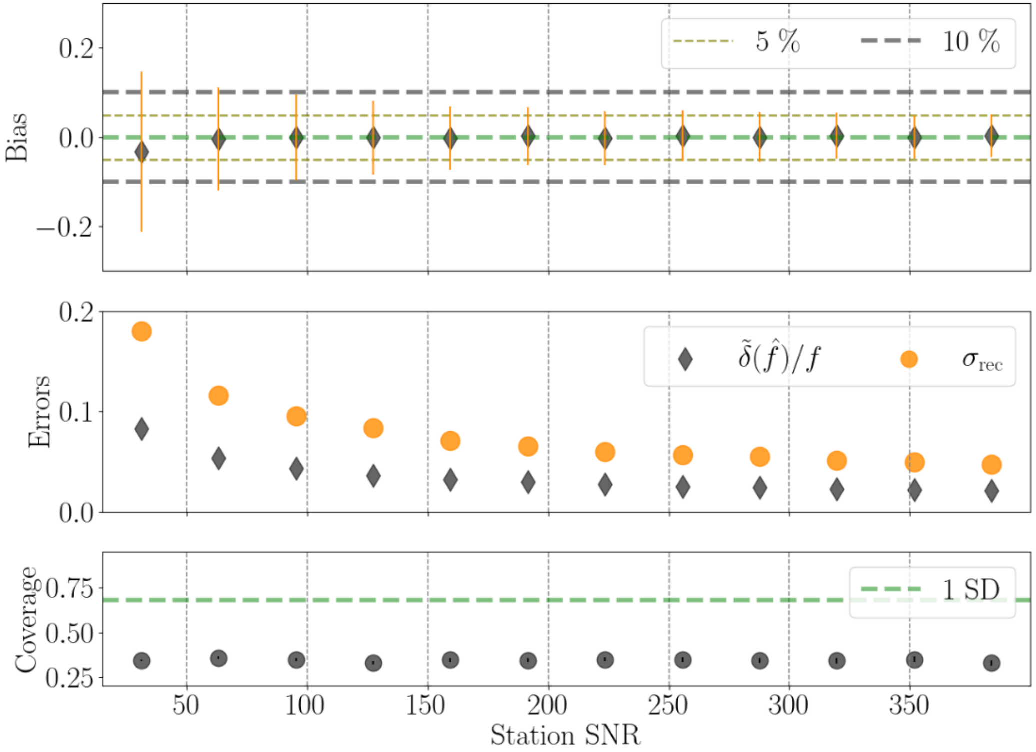


The noise subtraction method underestimates the uncertainties

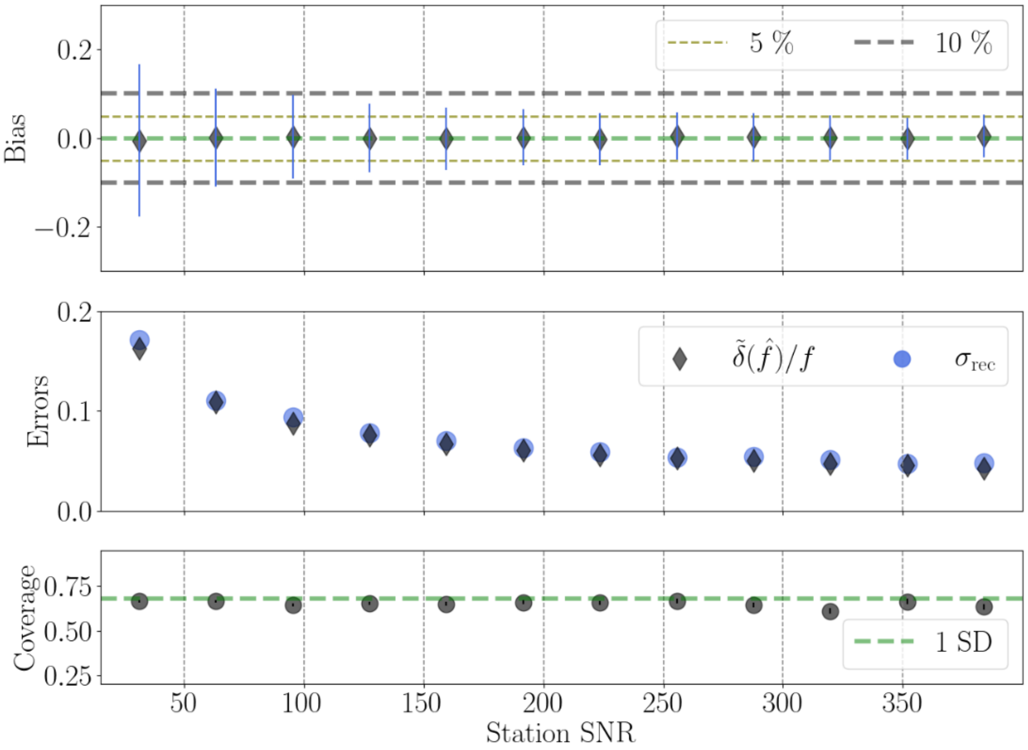
Coverage of the errors fluctuates around 68%

Comparison of the Methods

Noise subtraction method



New method



The noise subtraction method strongly underestimates the uncertainties at any SNR.

Conclusions

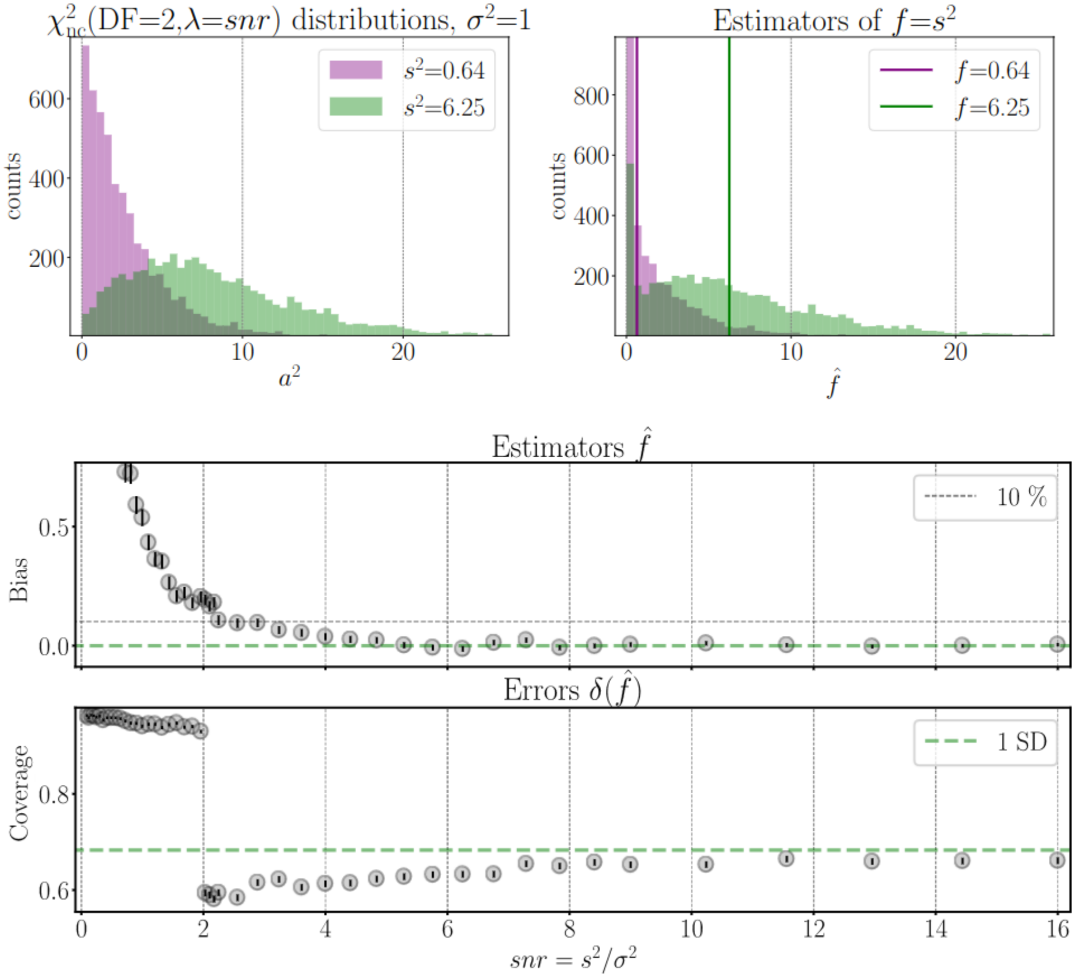
- The new fluence estimation shows a smaller bias than the conventional method for small SNR values (on average less than 10%)
- At higher SNRs, the bias is comparable (on average less than 5%)
- The new method correctly estimate the uncertainties at any SNR (coverage about 68%)

… & Outlooks

- Paper soon
- SNR cut can be lowered/removed
- Improvements on the elm. Energy reconstruction
- Possible employment of frequency spectrum modeling

Backup

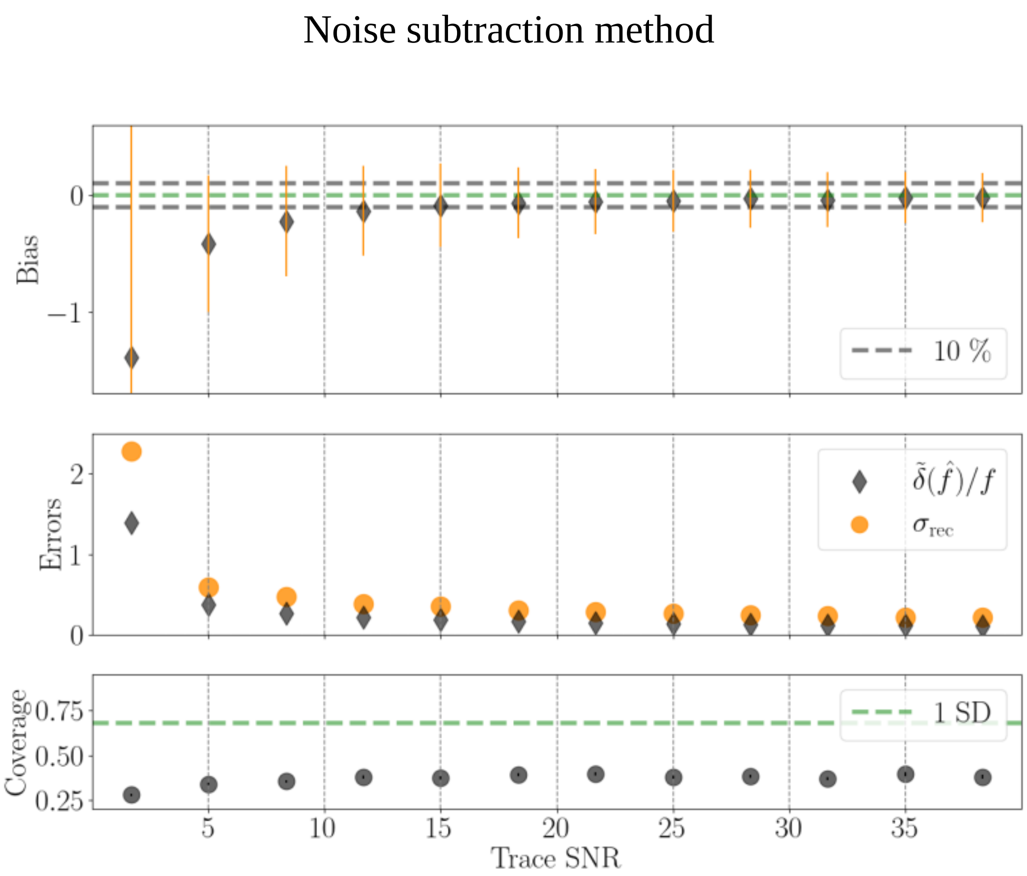
Monte Carlo toy: intrinsic bias

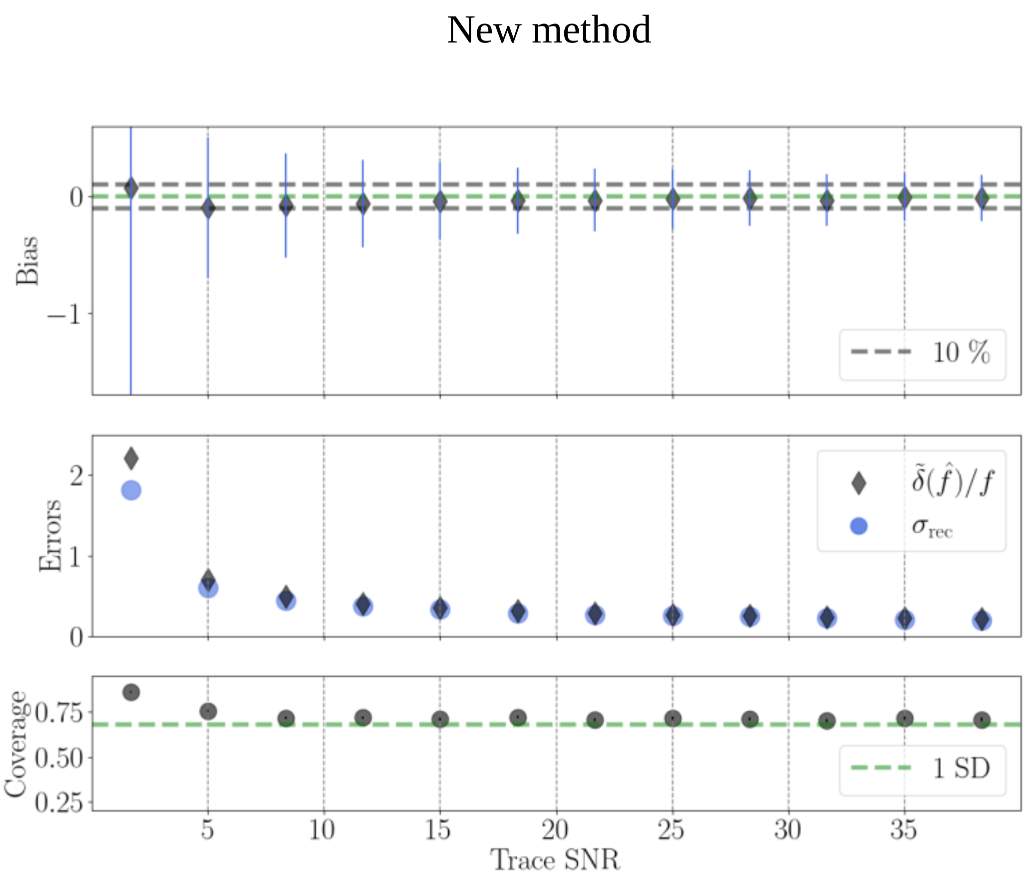


N=5000

$$\left(snr_{\mathrm{MC}}, (a^k_{\mathrm{MC}})^2\right) \rightarrow \left(\hat{f}^k, \delta(\hat{f}^k)\right)$$

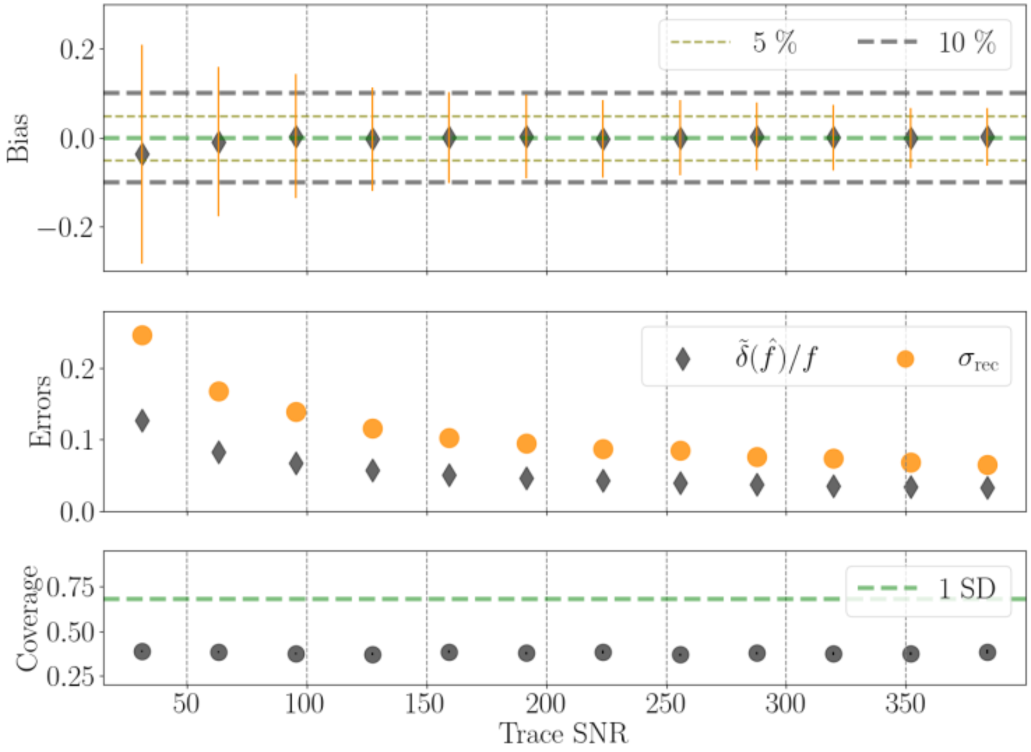
Repeating by fixing snr to several values


Noise subtraction method
Bias
0
−1
10 %
Errors
2
1
0
$\tilde{\delta}(\hat{f})/f$
$\sigma_{\rm rec}$
Coverage
0.75
0.50
0.25
1 SD
5
10
15
20
25
30
35
Trace SNR

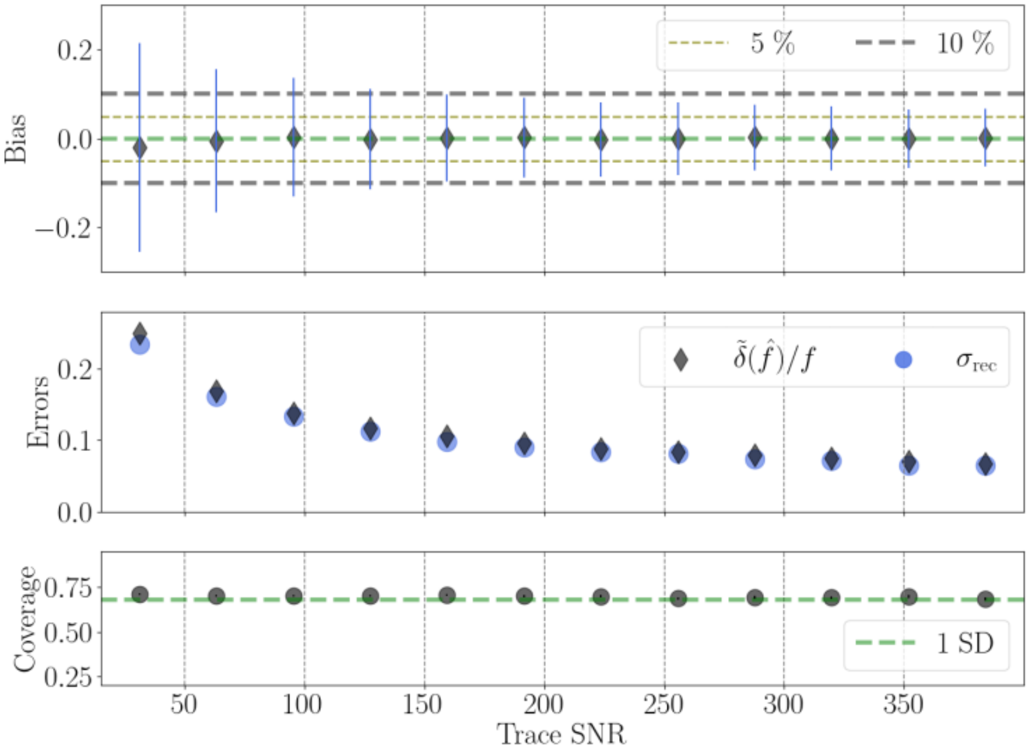

New method
Bias
0
−1
10 %
Errors
2
1
0
$\tilde{\delta}(\hat{f})/f$
$\sigma_{\rm rec}$
Coverage
0.75
0.50
0.25
1 SD
5
10
15
20
25
30
35
Trace SNR

Results (trace level)

Noise subtraction method

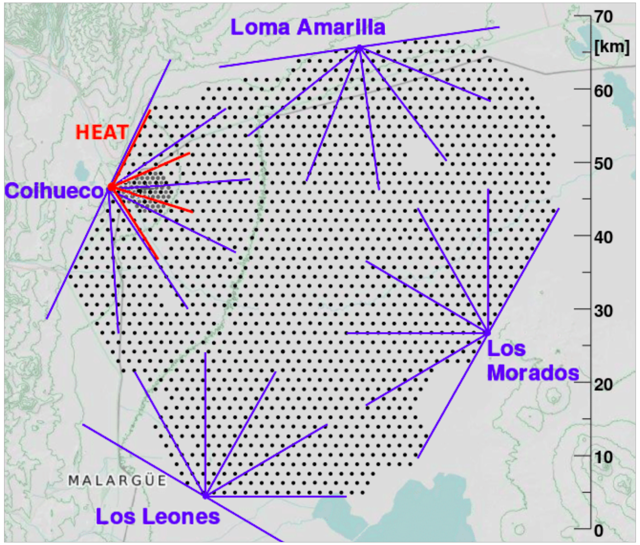


New method

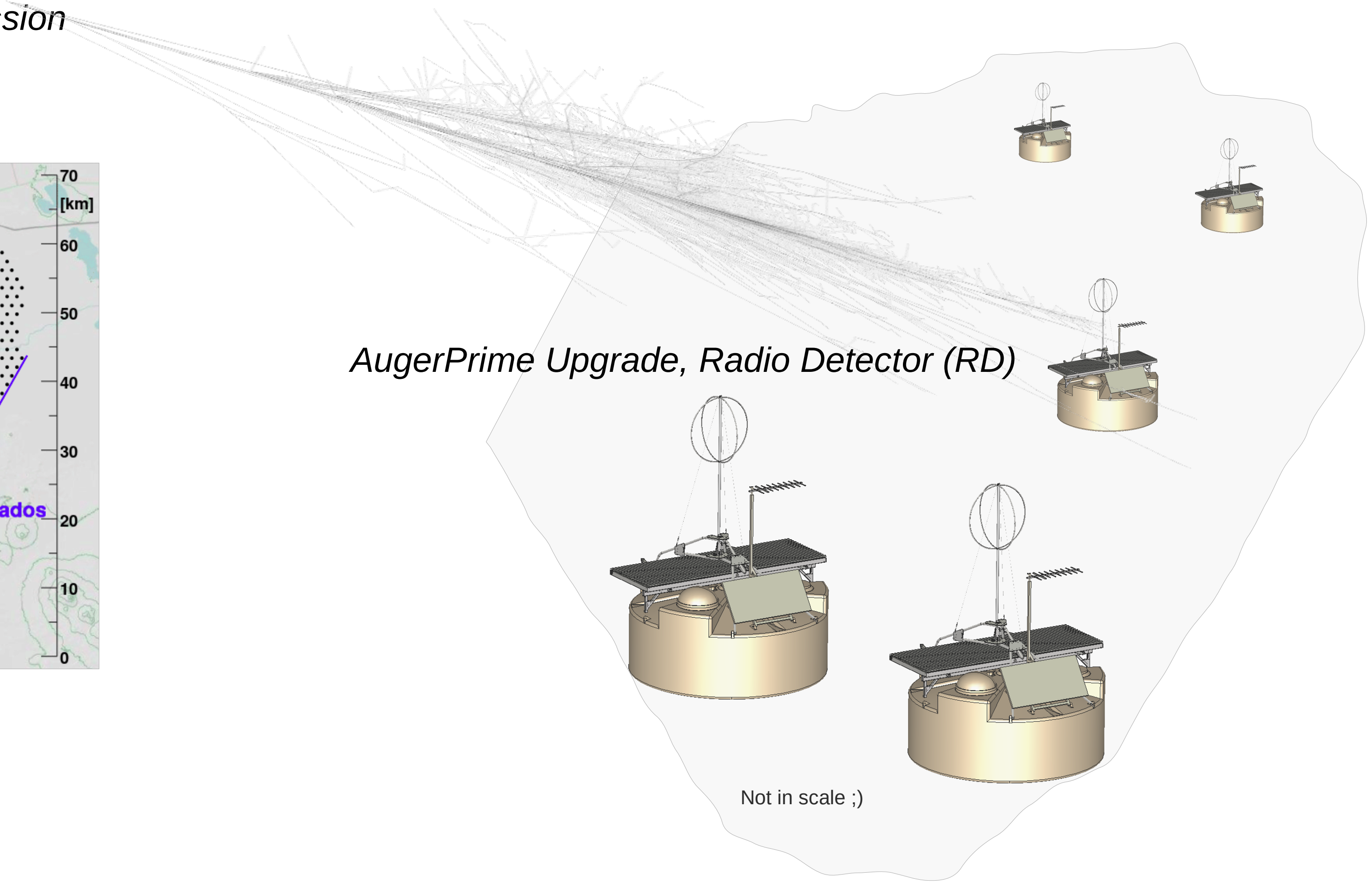


CR initiating EAS

elm. component => radio emission



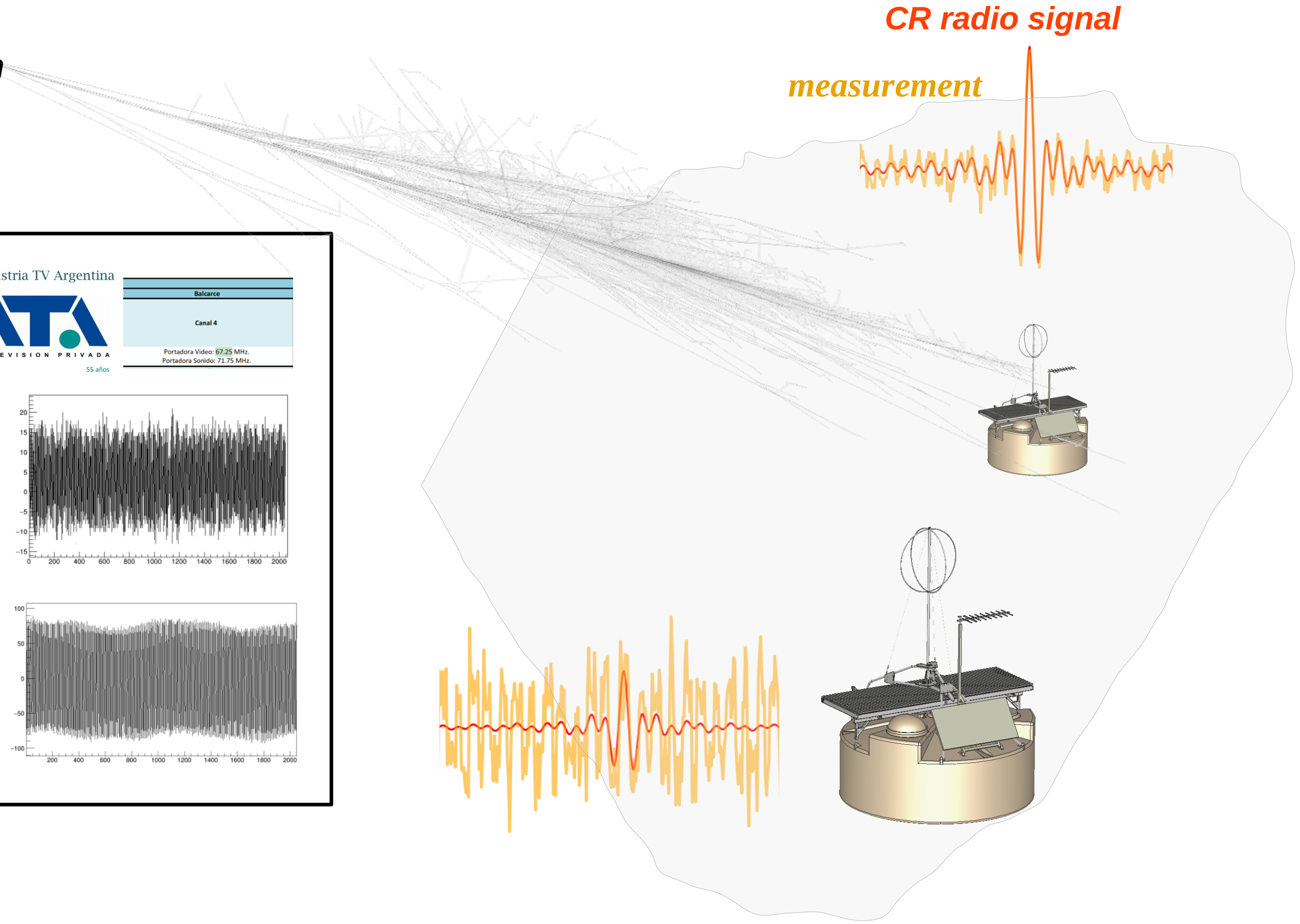
Pierre Auger Observatory

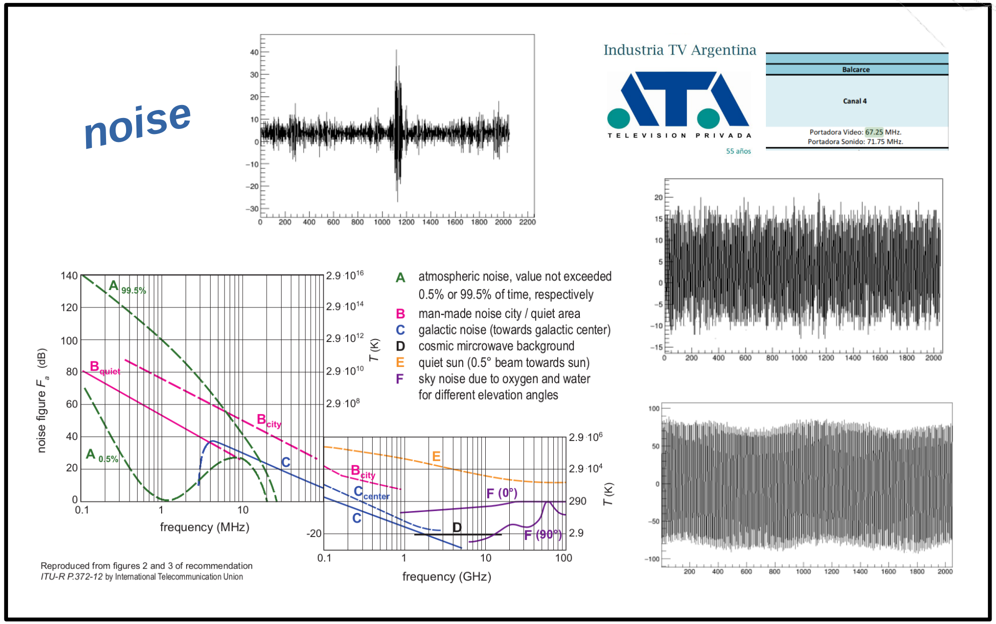


AugerPrime Upgrade, Radio Detector (RD)

CR initiating shower

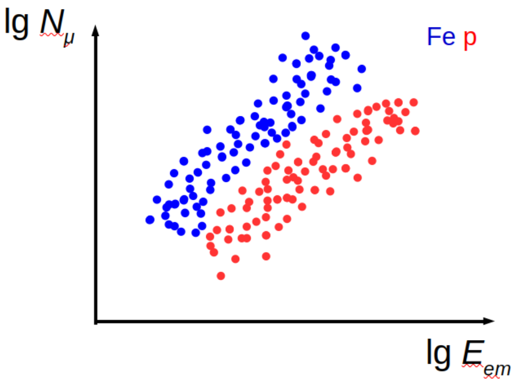
elm. component => radio emission

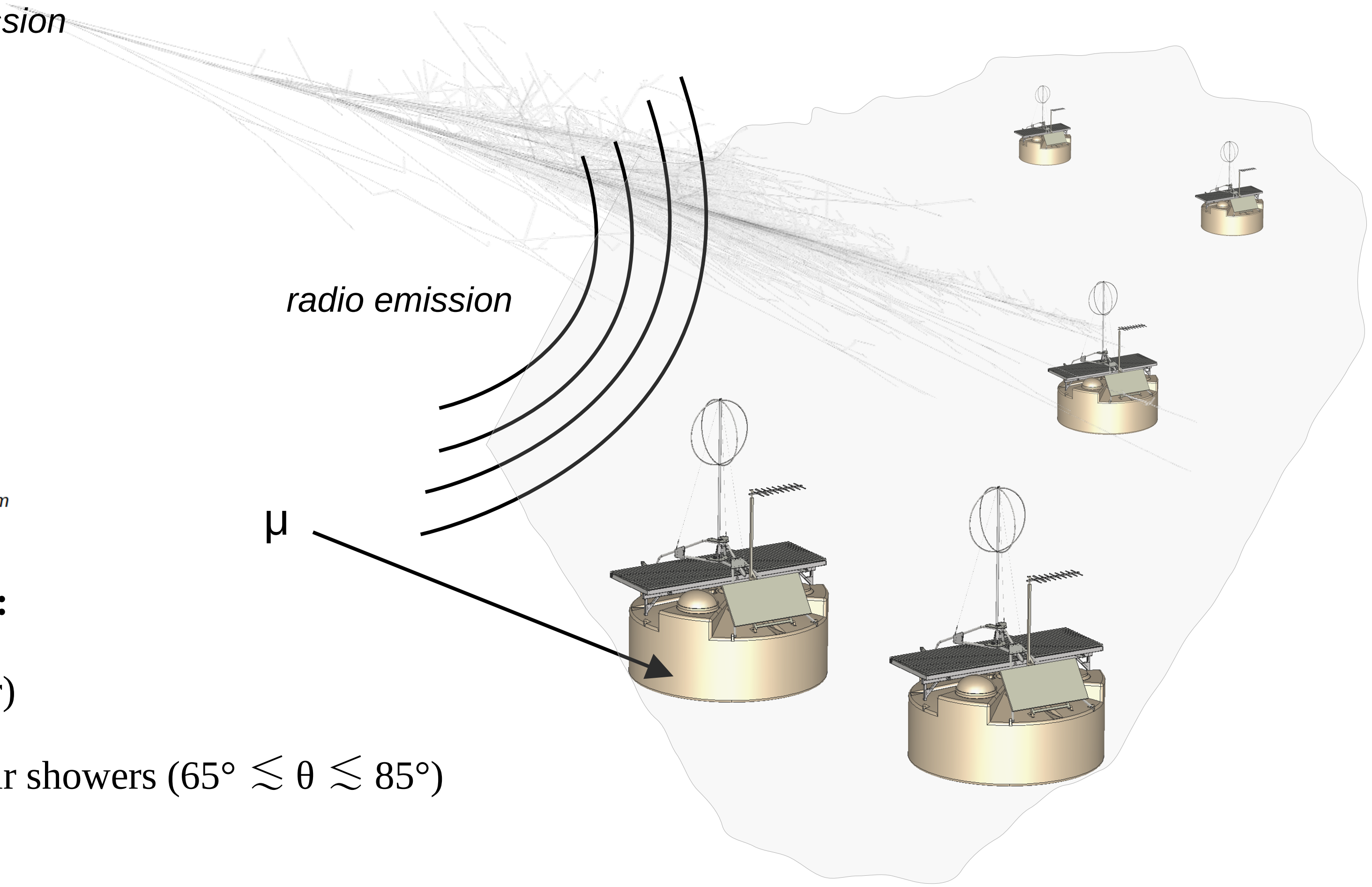




CR initiating shower

elm. component => radio emission



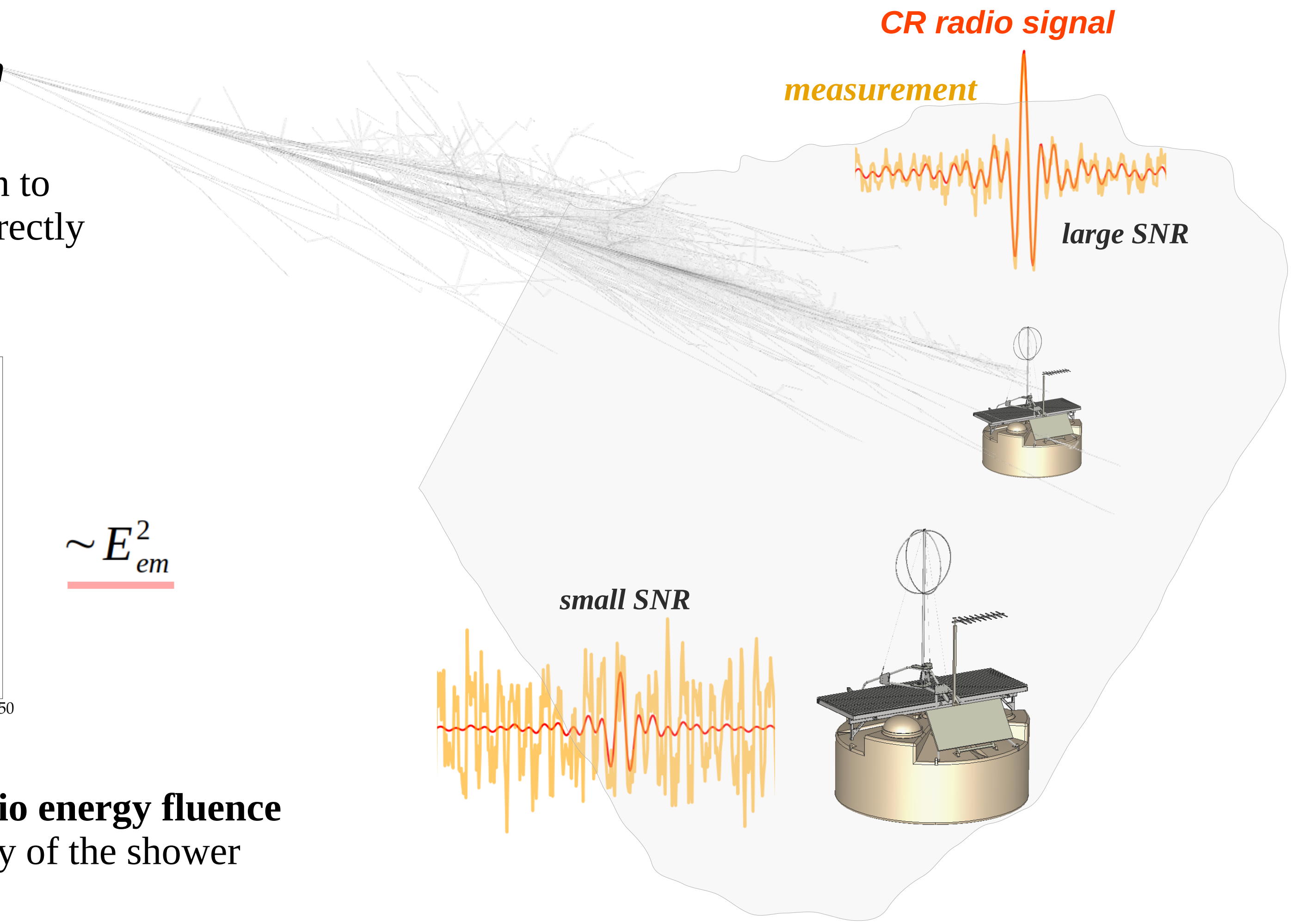


Mass composition studies:
RD + WCD
(Water Cherenkov Detector)

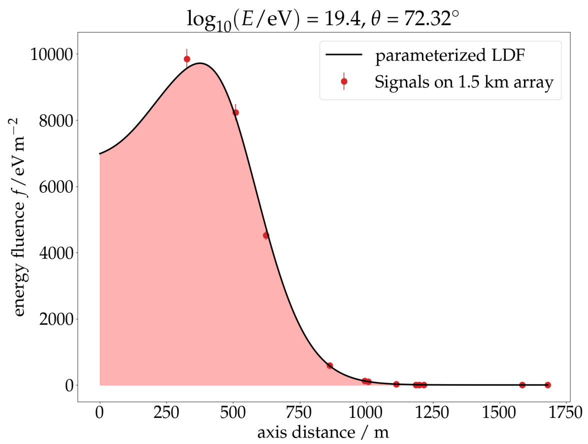
Works well with inclined air showers ($65° \lesssim \theta \lesssim 85°$)

CR initiating shower

elm. component => radio emission



Goals: improve the signal estimation to recover the excluded radio data; correctly evaluate the uncertainties



$\sim E_{em}^2$

Application: LDF fitting of the **radio energy fluence** [eV m^{-2}] to estimate the elm. Energy of the shower

Reference: C. Glaser's PhD thesis

5.5.2 Uncertainty of the energy fluence

We estimate the uncertainty of the energy fluence by assuming that the measured electric-field amplitude $A(t)$ is the sum of the cosmic-ray radio pulse $S(t)$ and noise $e(t)$. Furthermore, we assume that the noise $e(t)$ is Gaussian distributed with mean $\mu = 0$ and standard deviation $\sigma = \sigma_e$. The energy fluence of A is then given by the equation

$$f(A) = \epsilon_0 c \Delta t \sum_{t_1}^{t_2} A(t_i)^2 = \epsilon_0 c \Delta t \sum_{t_1}^{t_2} [S(t_i) + e(t_i)]^2 = \epsilon_0 c \Delta t \sum_{t_1}^{t_2} [S(t_i)^2 + 2S(t_i)e(t_i) + e(t_i)^2] \tag{5.16}$$

and the expectation value of $f(A)$ is

$$\begin{aligned} \langle f(A) \rangle =& \epsilon_0 c \Delta t \sum_{t_1}^{t_2} A(t_i)^2 = \epsilon_0 c \Delta t \sum_{t_1}^{t_2} [\langle S(t_i)^2 \rangle + 2 \langle S(t_i) e(t_i) \rangle + \langle e(t_i)^2 \rangle] \\ =& \epsilon_0 c \Delta t \sum_{t_1}^{t_2} \Bigg[\langle S(t_i) \rangle^2 + \underbrace{Var(S(t_i))}_{=0} + 2 \langle S(t_i) \rangle \underbrace{\langle e(t_i) \rangle}_{=0} \\ & + 2 \underbrace{Cov(S(t_i), e(t_i))}_{=0} + \underbrace{\langle e(t_i) \rangle^2}_{=0} + \underbrace{Var(e(t_i))}_{\sigma_e^2} \Bigg] \\ =& \epsilon_0 c \Delta t \sum_{t_1}^{t_2} [\langle S(t_i) \rangle^2 + \sigma_e^2] \, . \end{aligned} \tag{5.17}$$

Hence, the best estimate of the energy fluence of the radio signal S is indeed

$$f(S) = \epsilon_0 c \Delta t \sum_{t_1}^{t_2} [A(t_i)^2 - \sigma_e^2] \tag{5.18}$$

as defined in Eq. (5.8) where σ_e^2 is also calculated from the electric-field trace in a part where no signal is present. Following a similar calculation we can estimate the uncertainty of $f(S)$ by computing $\sigma_f^2 = Var(f) = \langle f^2 \rangle - \langle f \rangle^2$. After several lines of calculation it follows that

$$\sigma_f^2 = 4 f \, \epsilon_0 c \, \Delta t \, \sigma_e^2 + 2 \, (\epsilon_0 c)^2 \, (t_2 - t_1) \, \Delta t \, \sigma_e^4 \, . \tag{5.19}$$