



Composition studies of TeV cosmic rays with HAWC

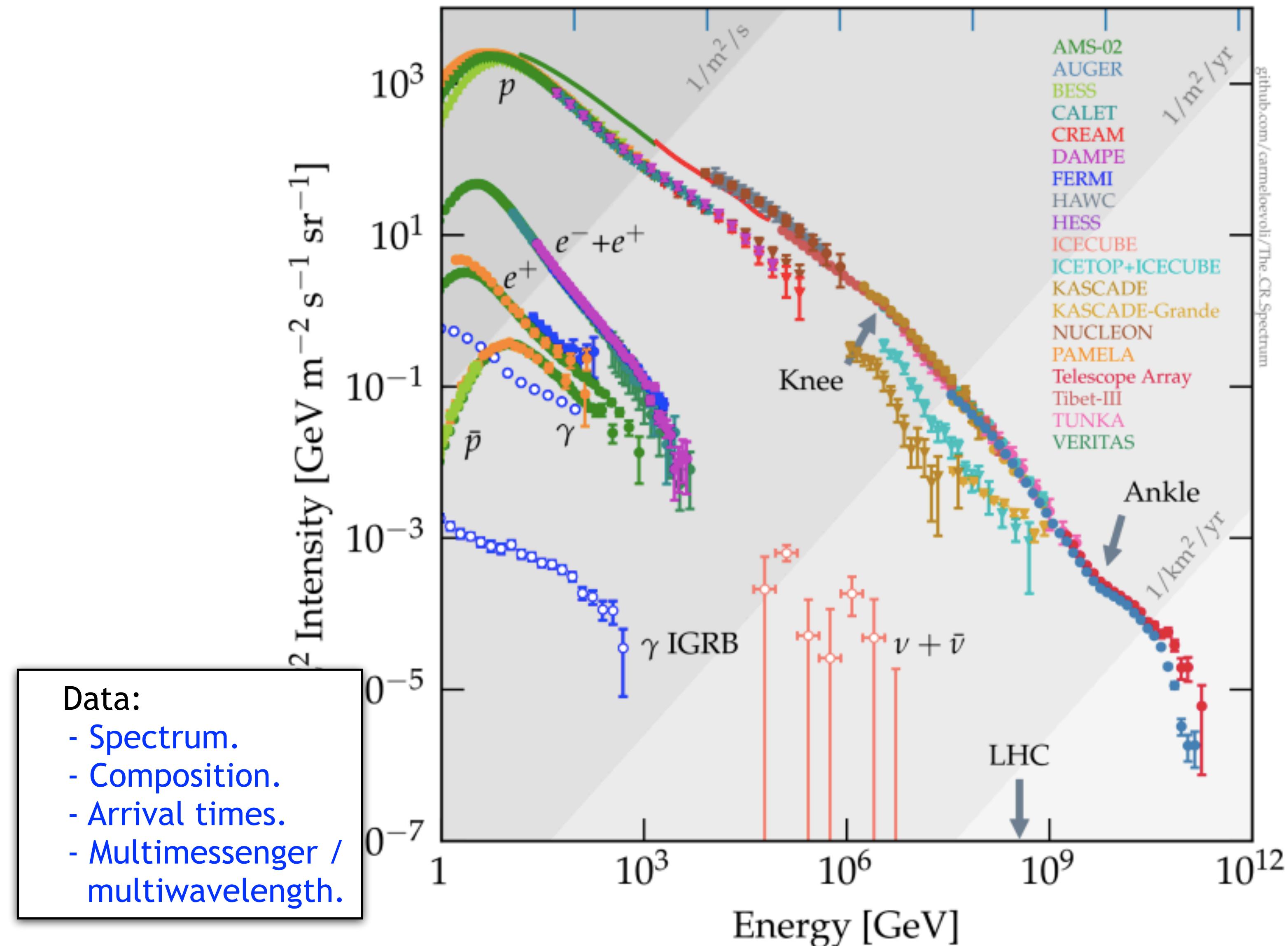
J.C. Arteaga-Velázquez for the HAWC Collaboration
Universidad Michoacana, Morelia, Mexico



1) Motivation

- Energy: 100 MeV - 100 EeV.
- Spectrum follows roughly a power law
 $F(E) = E^{-\gamma}$.
- Composed mainly by atomic nuclei:
 - Atomic nuclei (99 %) :
H (85%), He (3%), $Z \geq 3$ (3%)
 - Electrons (1 %)
 - Traces of antiparticles (e^+ y anti p)
- Origin in cataclysmic galactic ($E < 10^{17}$ eV) and extragalactic ($E > 10^{17} - 10^{18}$ eV) events.
- Unknown questions:
 - Sources.
 - Propagation.
 - Acceleration mechanism.

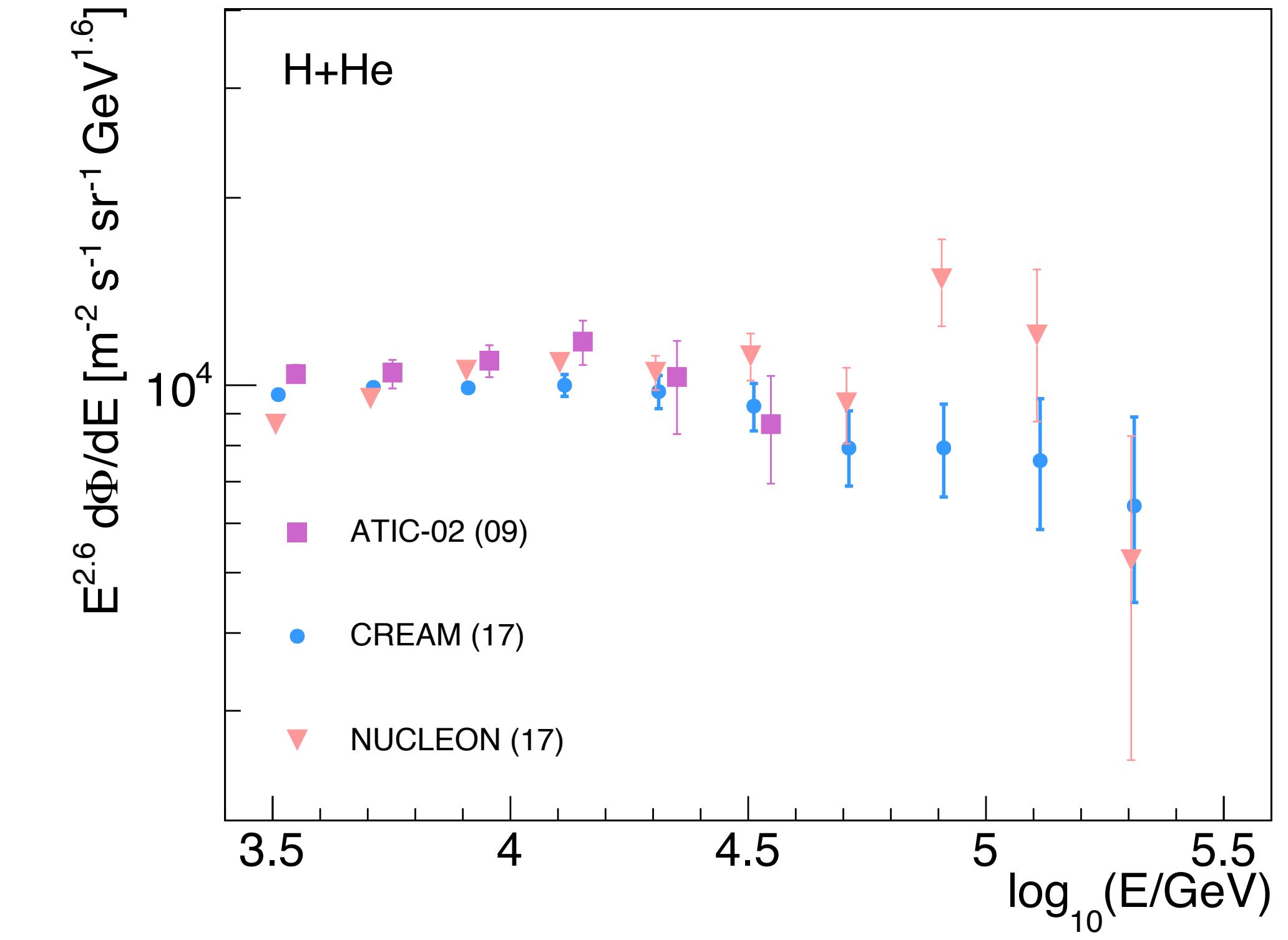
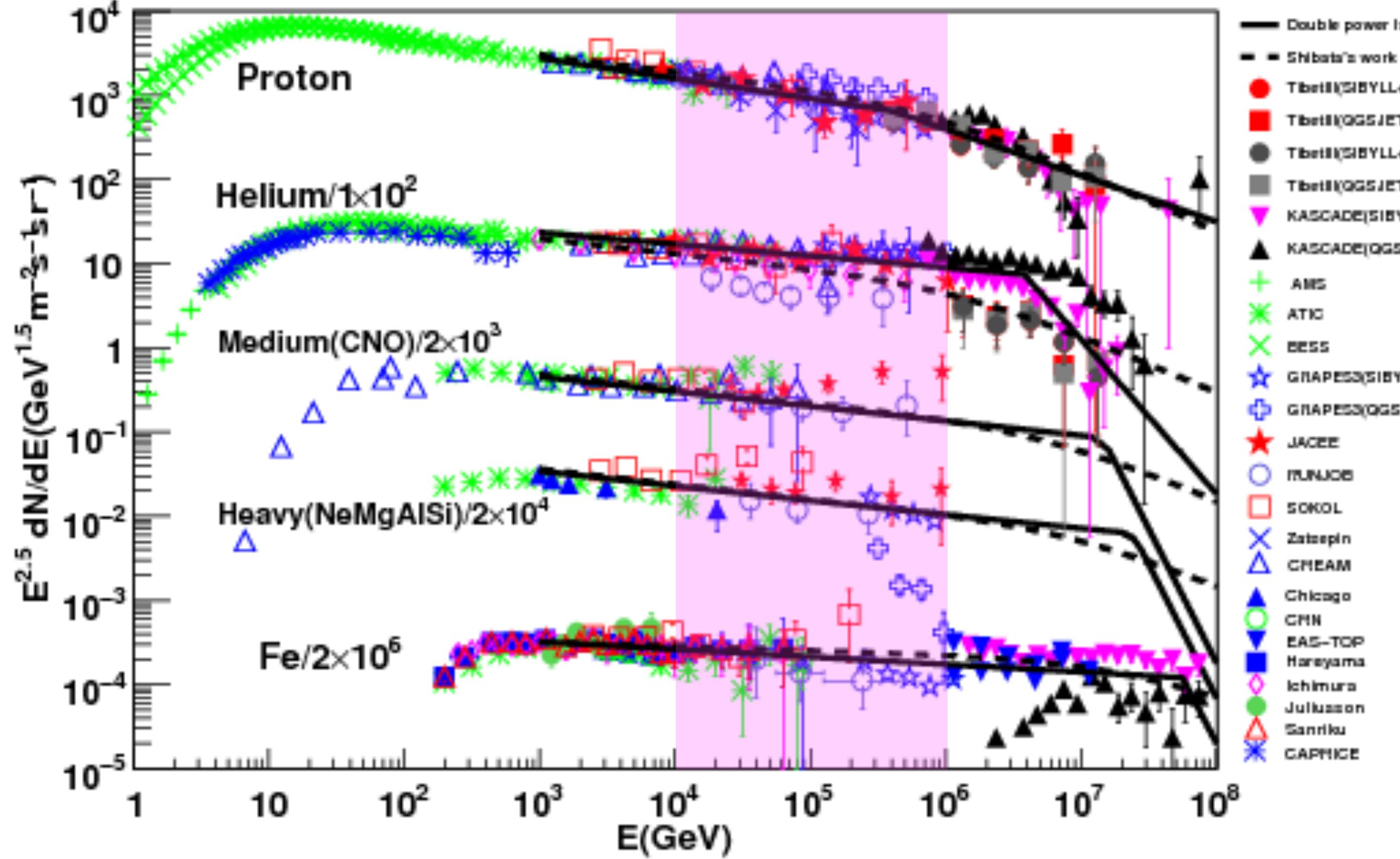
Cosmic rays



1) Motivation

Research of the cosmic ray composition of cosmic rays for **E = 10 TeV - 1 PeV**

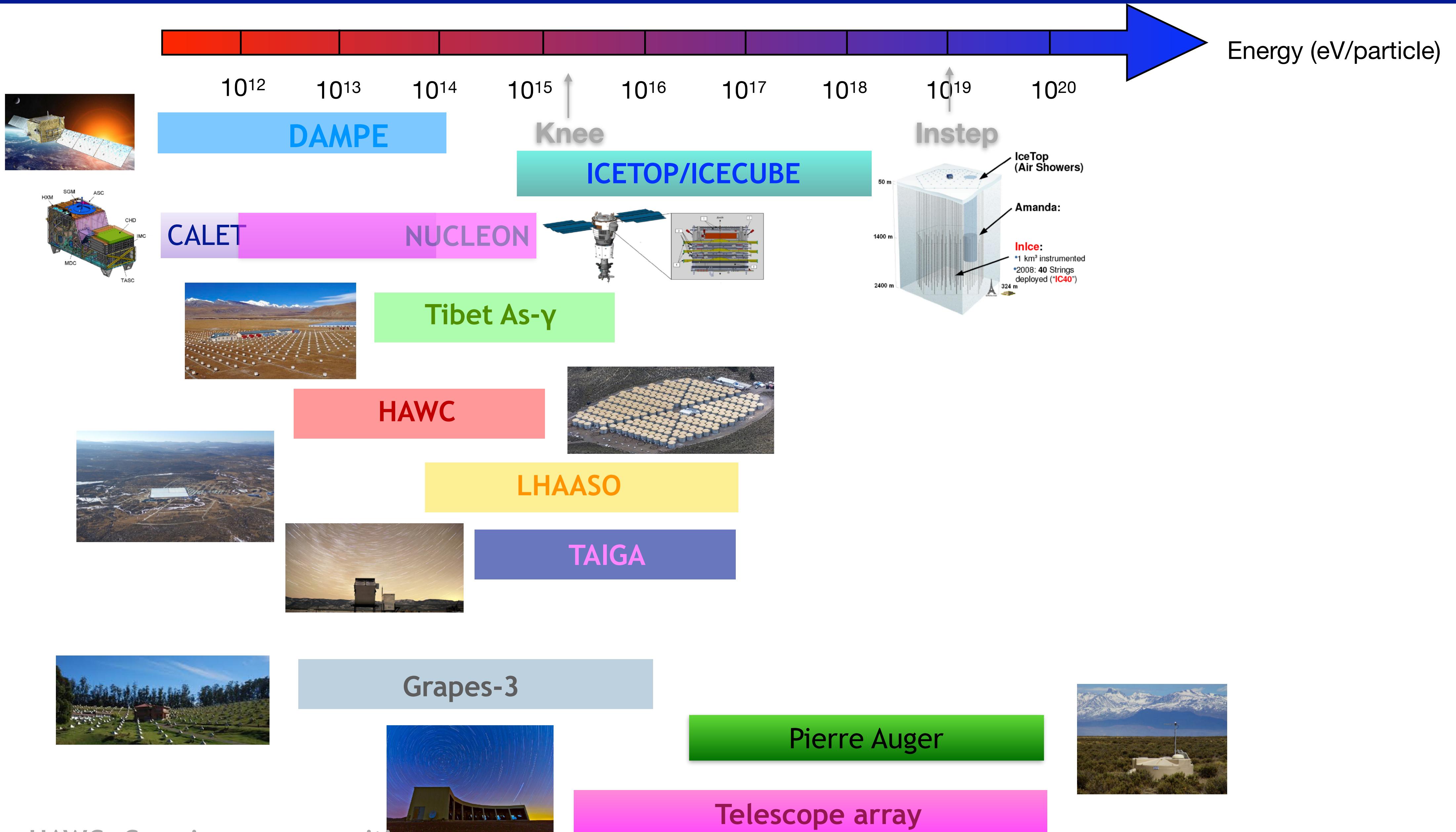
H. Hu, arXiv:0911.3034 [astro-ph.HE]



- Region at the limit between direct/indirect detection
- Barely studied
- Detailed exploration has just started

Early measurements with ATIC-02, CREAM-II/III and NUCLEON hinted the existence of a break in spectrum of H+He nuclei @ O(10 TeV).

1) Motivation



2) The HAWC observatory



Ubicación:

- Sierra Negra Volcano, Puebla, Mexico
- $18^{\circ} 59' N$ and $97^{\circ} 18' W$
- 4100 m s.n.m. (640 g/cm^2)
- Closer to Xmax of hadronic EAS at TeVs ($560-425 \text{ g/cm}^2$ for Fe/H @ 1 PeV)

2) The HAWC observatory

UNITED STATES



Los Alamos National Laboratory
Pennsylvania State University
Michigan Technological University
University of Maryland
University of Wisconsin-Madison
Michigan State University
University of Utah
University of New Mexico
Stanford University

GERMANY



Max-Planck Institute for Nuclear Physics
Friedrich Alexander Universität

ITALY



INFN Padova

POLAND



Institute of Nuclear Physics
Polish Academy of Sciences

MEXICO



Universidad Nacional Autónoma de México
Universidad Autónoma de Chiapas
Universidad Michoacana de San Nicolás de Hidalgo
Instituto Nacional de Astrofísica, Óptica y Electrónica
Universidad de Guadalajara
Tecnológico de Monterrey
Universidad Politécnica de Pachuca
Instituto Politécnico Nacional
Universidad Autónoma del Estado de Hidalgo
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SOUTH KOREA



University of Seoul
Sungkyunkwan University

CHINA



Shanghai Jiao Tong University

THAILAND



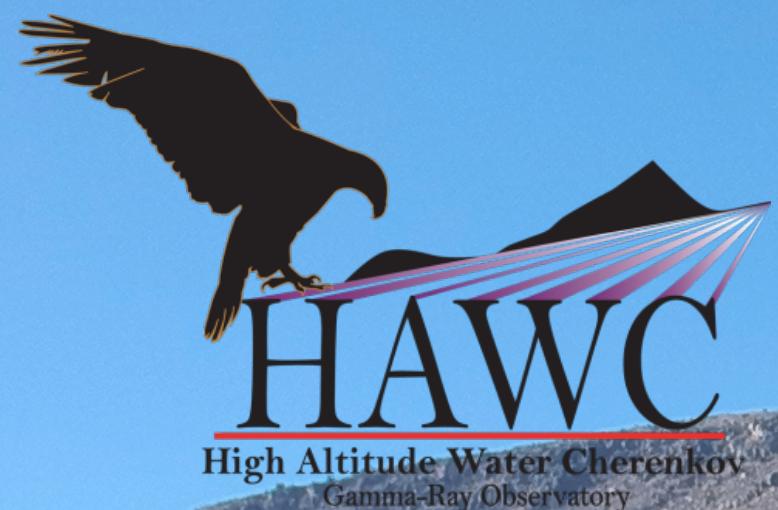
Chulalongkorn University

COSTA RICA



Universidad de Costa Rica

2) The HAWC observatory



Sierra Negra Volcano

Large Millimeter telescope

γ - and cosmic-ray detector:

- ▶ Air-shower observatory
- ▶ Ground-based Cherenkov array
- ▶ $E = 100 \text{ GeV} - 100 \text{ TeV}$ (up to 1 PeV for Cosmic rays)
- ▶ Trigger rate: 25 kHz
- ▶ 99.9% of EAS are hadronic
- ▶ Instantaneous field of view of 2 sr
- ▶ Monitors 2/3 of sky in one day
- ▶ Duty cycle: 95%

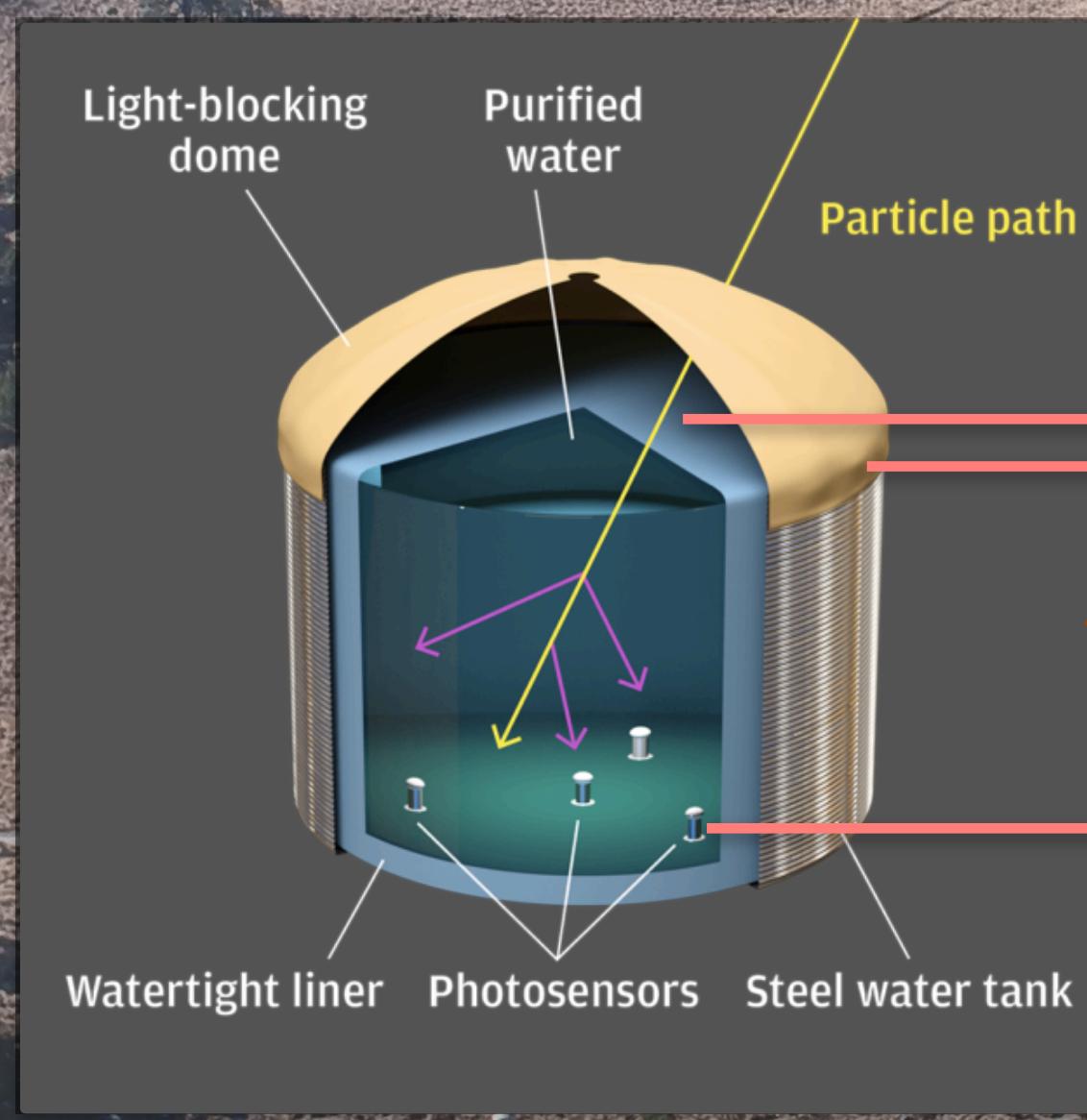
HAWC

<https://www.hawc-observatory.org>

2) The HAWC observatory

HAWC central detector

- 22 000 m² compact array
- 300 detectores Cherenkov tanks (7.3 m Ø x 4.5 m high)
- Optically isolated
- 200,000 ℥ of water
- 4 PMTs: 3 x 8-in, 1 x 10-in



Outrigger detector:

- Area: 4 times central detector
- 345 Cherenkov detectors (1.55 m Ø x 1.65 m height)
- 3,110 ℥ of water
- One 8-in PMT



<https://www.hawc-observatory.org>

2) The HAWC observatory



11/2010



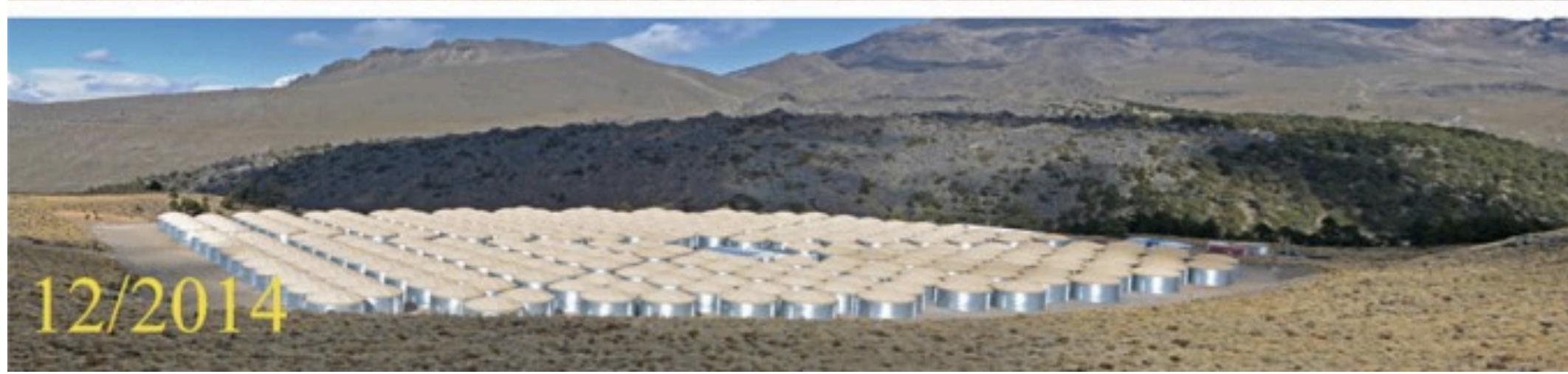
02/2012



08/2012



05/2013

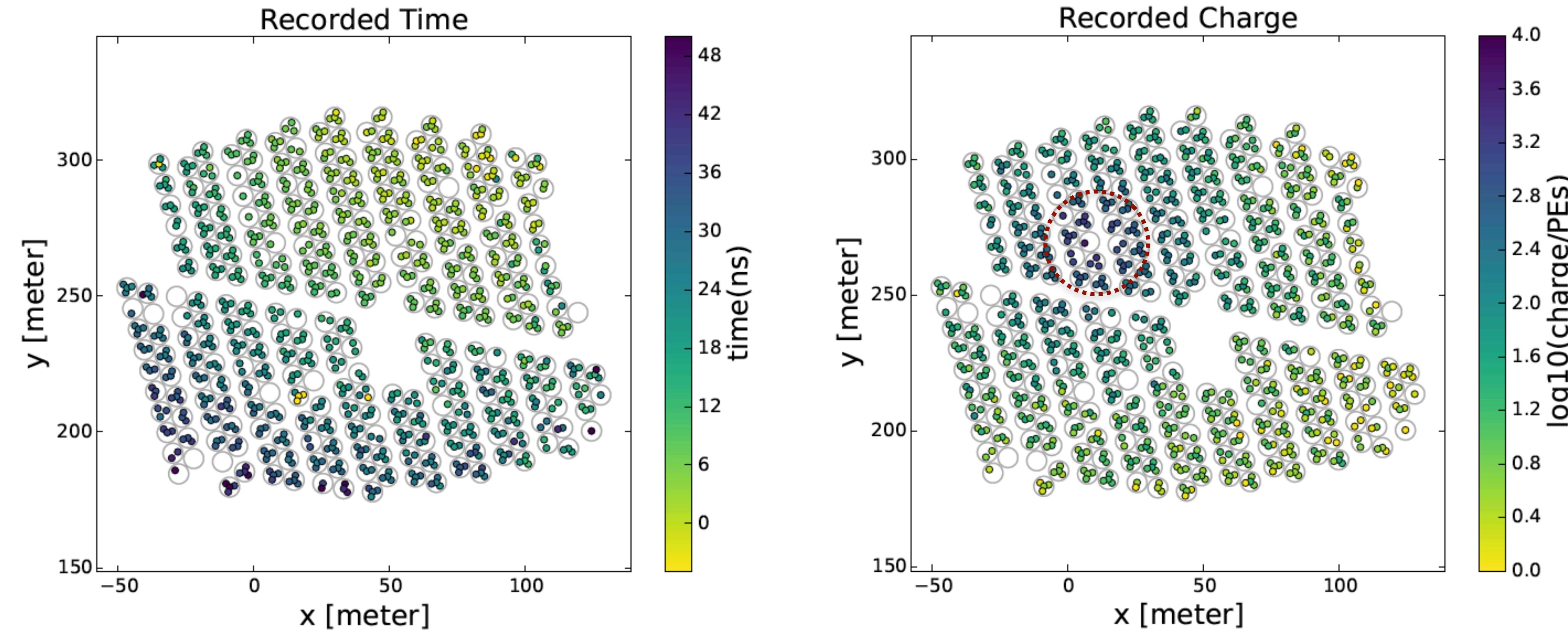


12/2014

- ▶ Full detector inaugurated in March 20, 2015.
- ▶ Outrigger since 2018.



2) The HAWC observatory



- From hit times at PMTs, deposited charged, number of PMT's with signal:
 - ▶ Core location, (X_c, Y_c)
 - ▶ Arrival direction, θ
 - ▶ Fraction of hit PMT's, f_{hit}
 - ▶ Lateral charge profile, $Q_{\text{eff}}(r)$
 - ▶ ...

[HAWC Coll., ApJ 843 (2017) 39]

2) The HAWC observatory

Lateral age parameter (s)

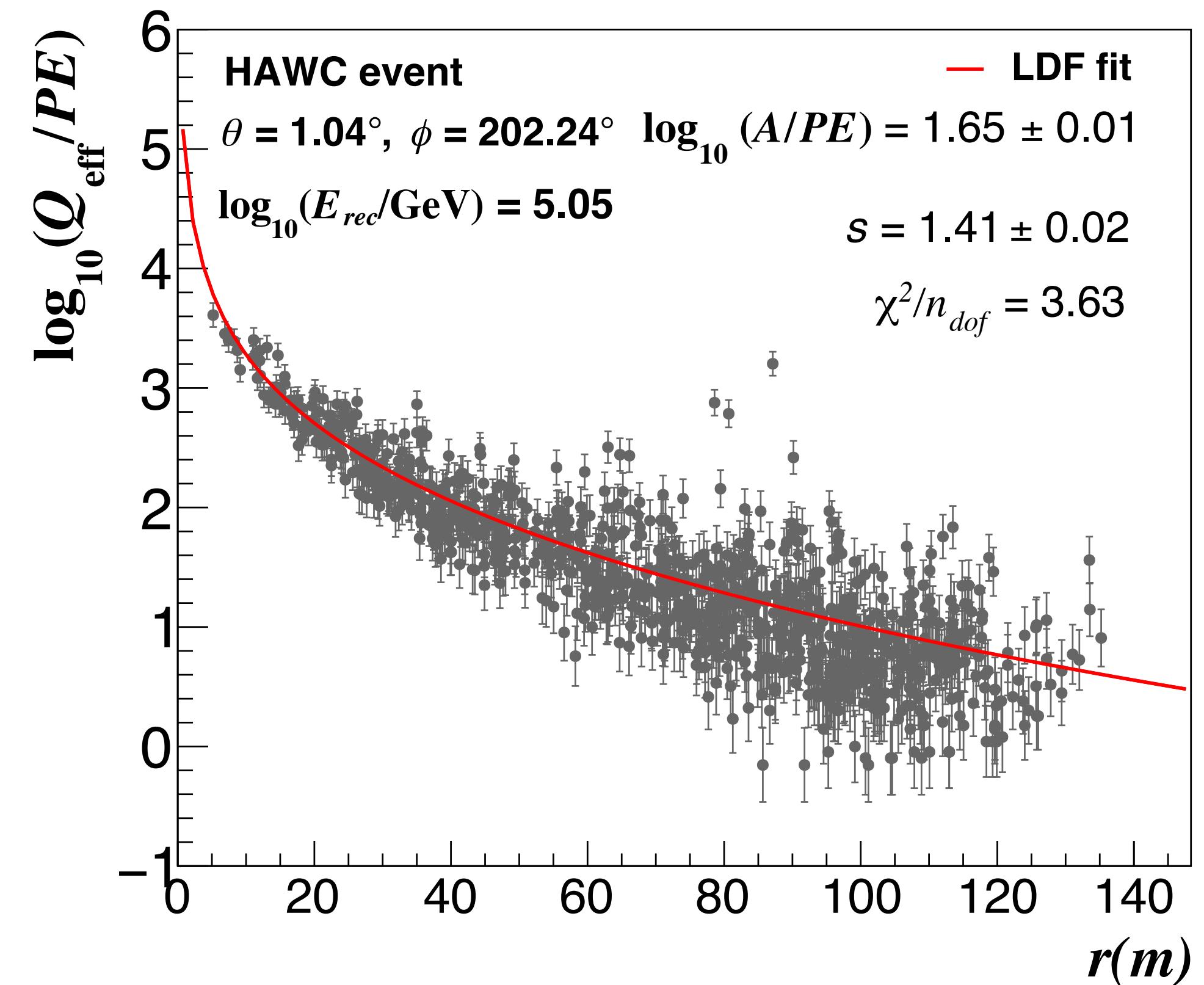
- Obtained event-by-event
- Fit of $Q_{eff}(r)$ with a NKG-like function:

$$f_{ch}(r) = A \cdot (r/r_0)^{s-3} \cdot (1 + r/r_0)^{s-4.5}$$

with $r_0 = 124.21$ m.

A , s are free parameters

[HAWC Collab., APJ 881 (2017); J.A. Morales Soto et al., PoS(ICRC2019) 359 (2019)]

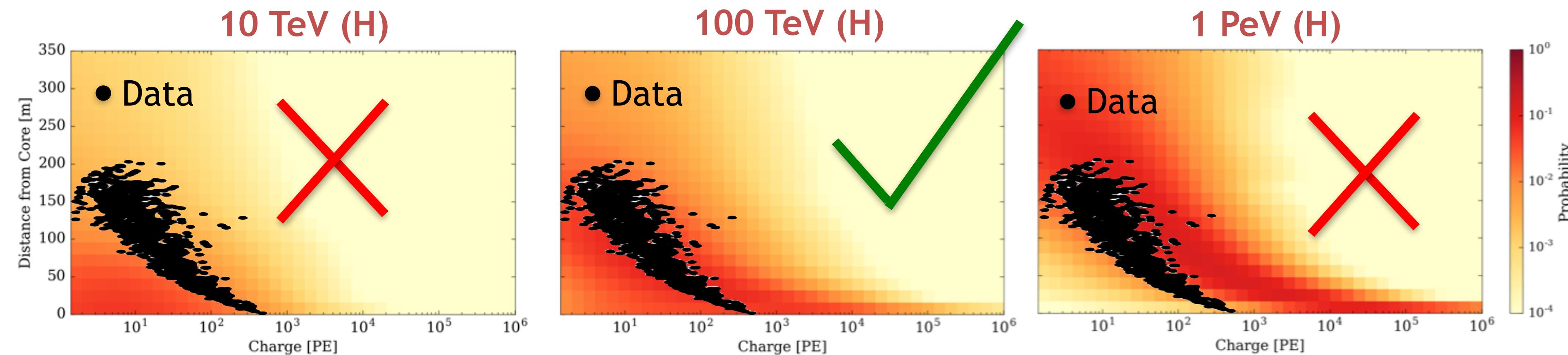


[HAWC Collab., PRD 105 (2022)]

2) The HAWC observatory

EAS primary energy estimation [HAWC Collab., PRD 96 (2017) 122001]

- Based on the maximum likelihood method
- Comparison of the **lateral distribution of the event** with MC templates for **protons**.

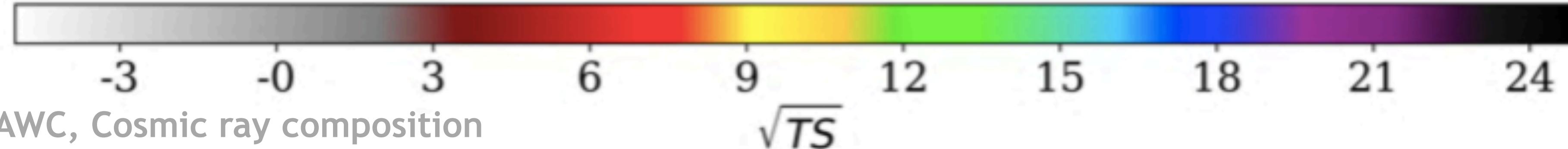
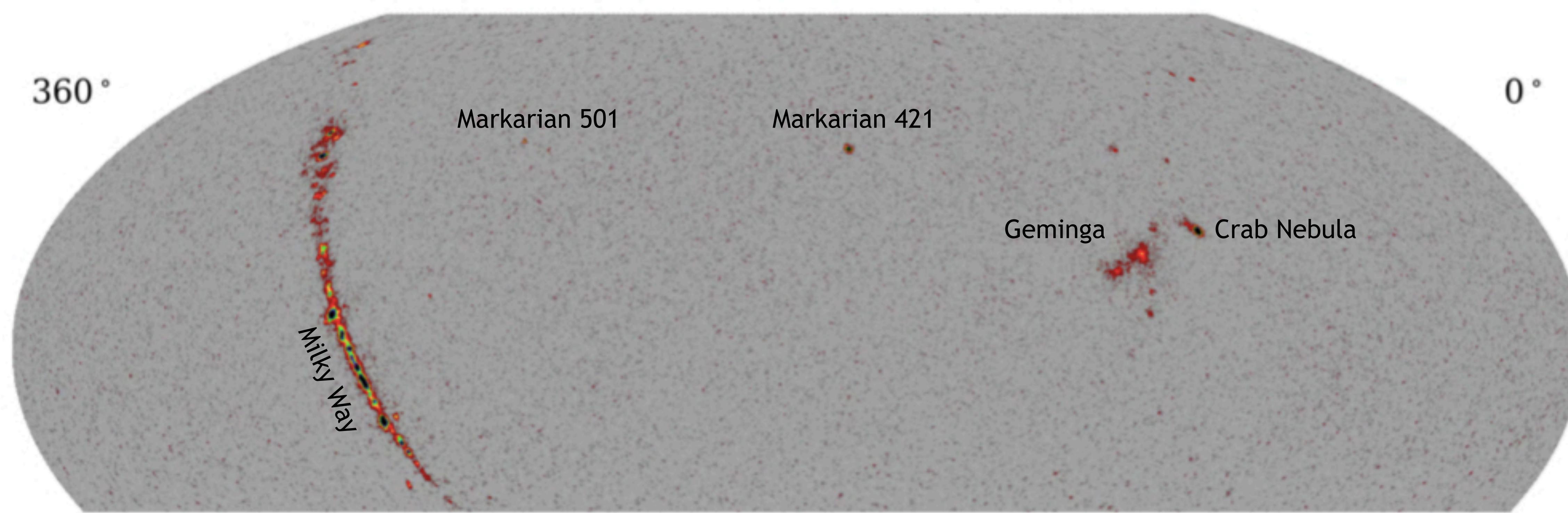


[Z. Hampel-Aris' PhD thesis, 2017]

- Templates are built with MC simulations for different intervals of
 - Energy ($\Delta \log E/\text{GeV} = 0.1$)
 - Zenital angle (θ : $[0^\circ, 17^\circ]$, $(17^\circ, 35^\circ]$, $(35^\circ, 60^\circ]$)

2) The HAWC observatory

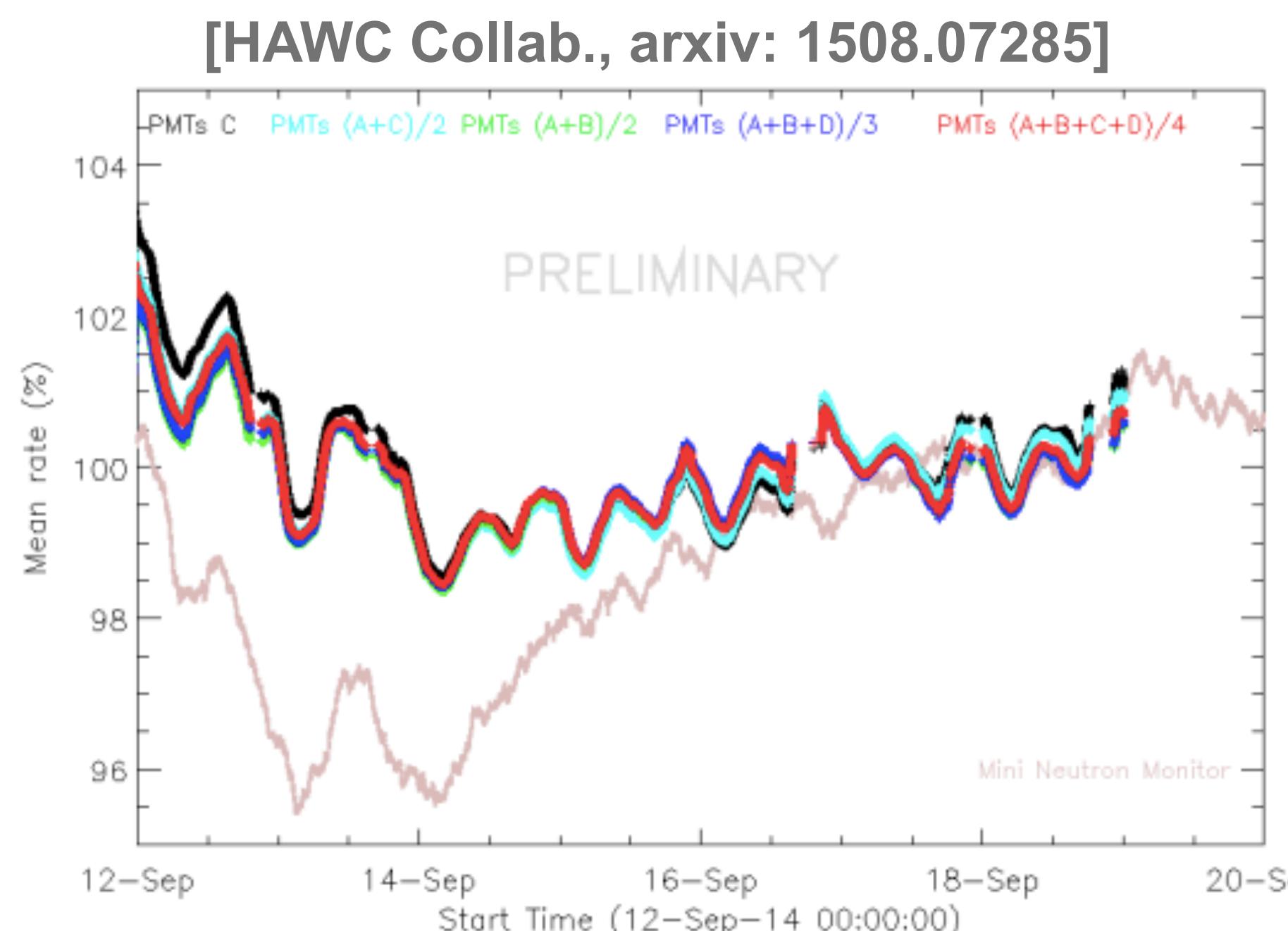
HAWC skymap at gamma-rays with 2090 days of data - Pass5



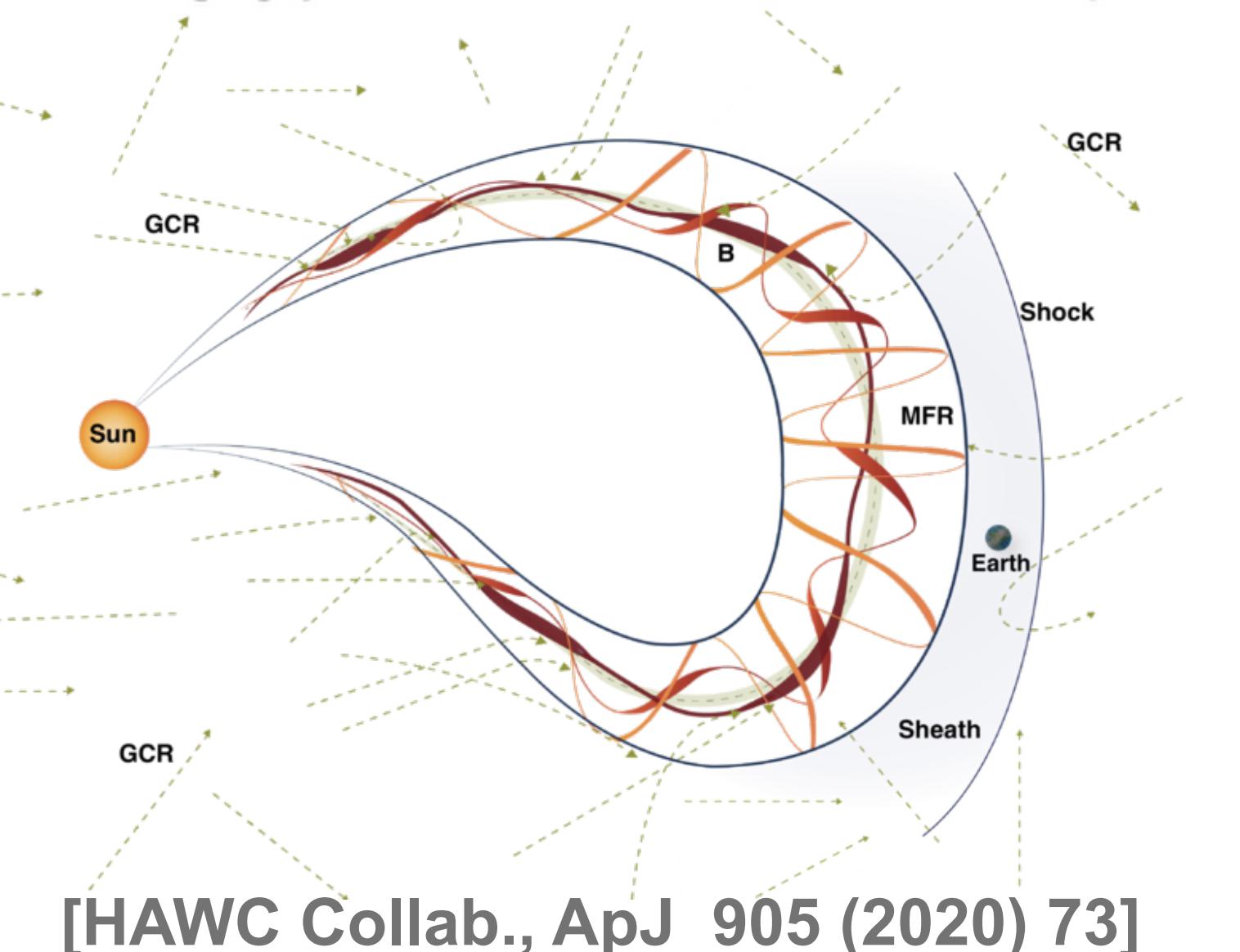
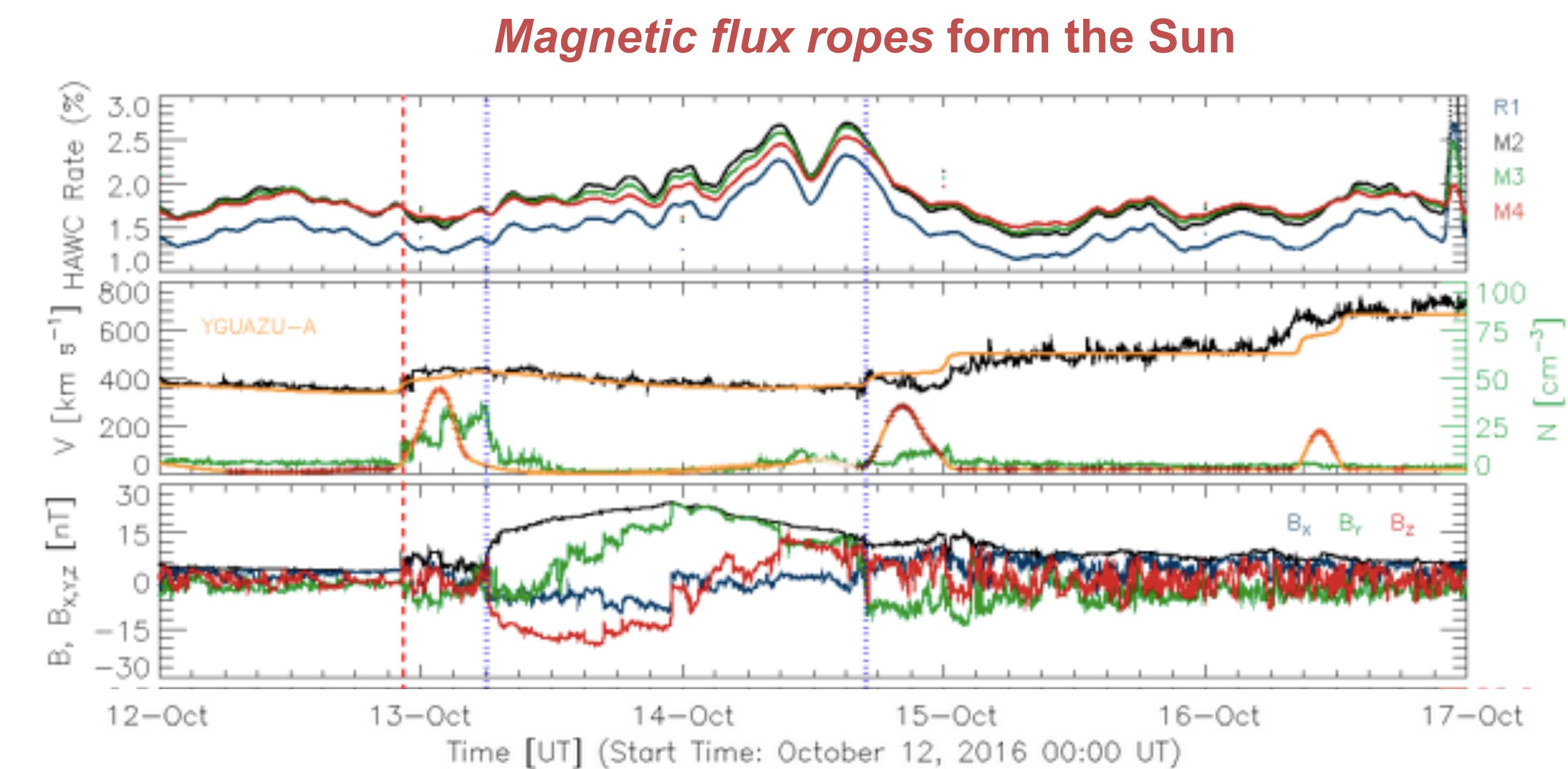
2) The HAWC observatory

Cosmic rays at HAWC

- ▶ Solar physics and interplanetary medium
- ▶ Sun and moon shadows
- ▶ Energy spectrum
- ▶ Composition.
- ▶ Anisotropies in arrival directions.
- ▶ Limits to antiprotons in cosmic ray intensity.
- ▶ Particle rates and Hunga Volcano pressure wave

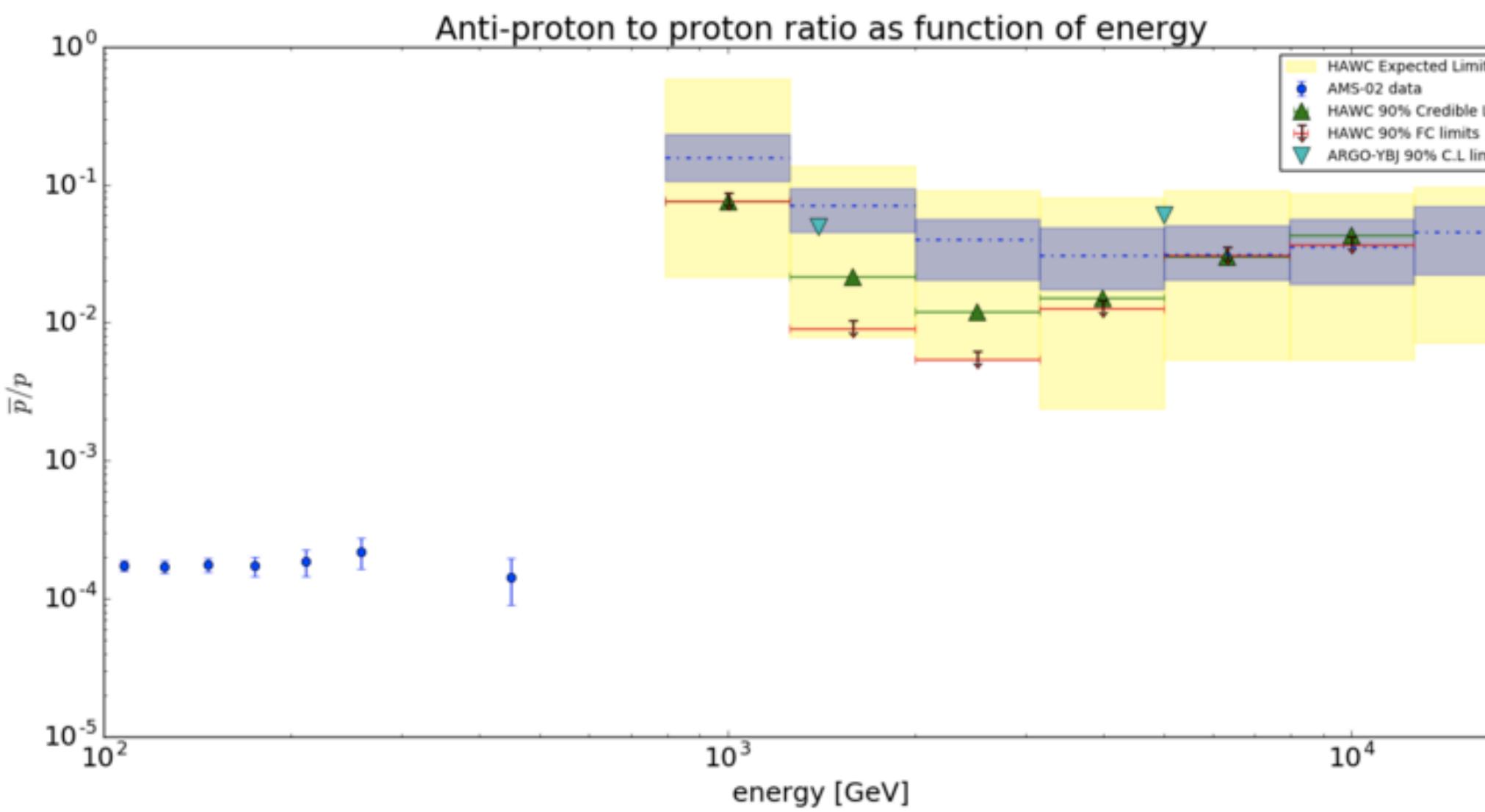


Reduction in rate of galactic CR (Forbush decrease by Sun)

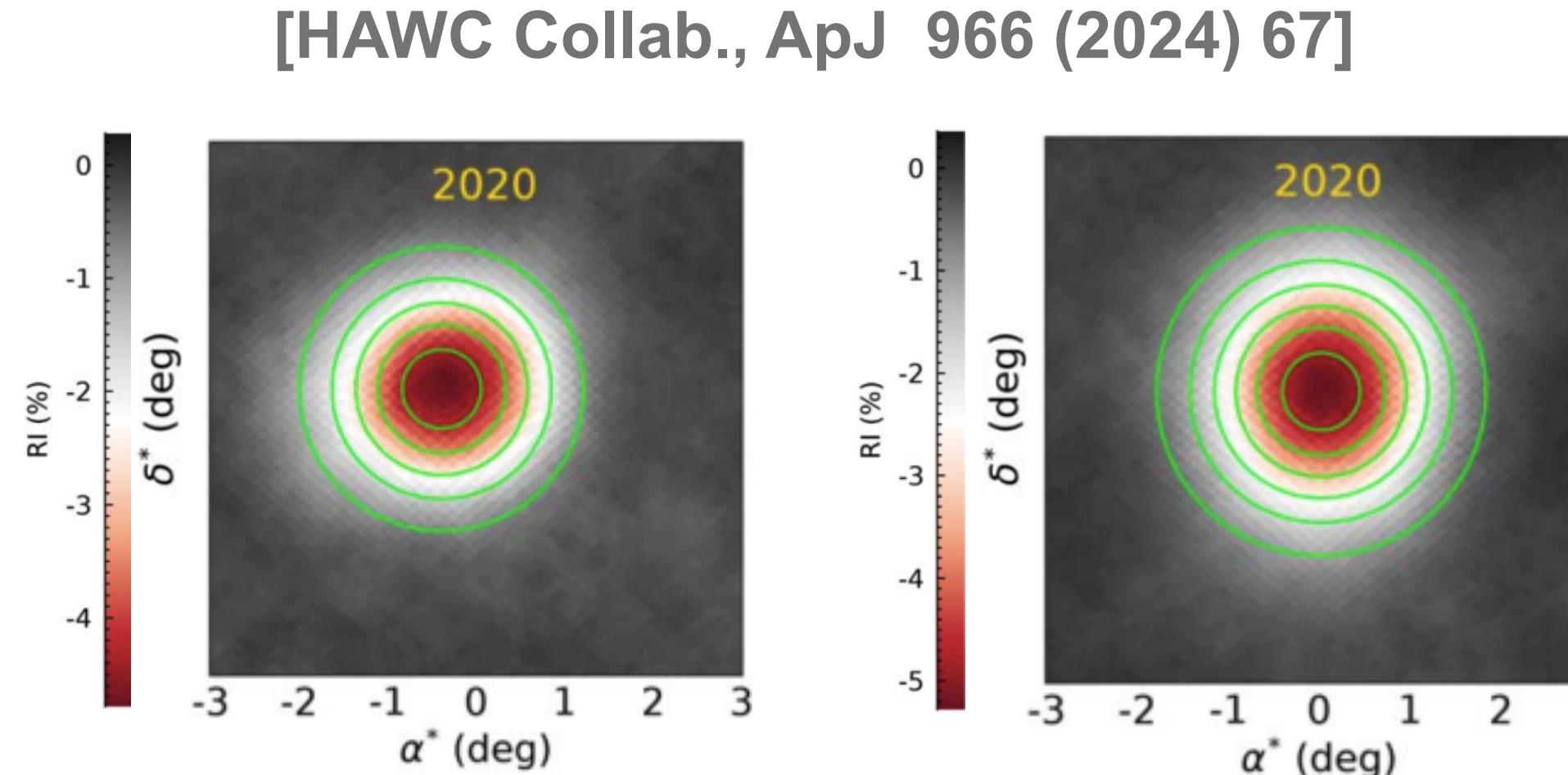


2) The HAWC observatory

[HAWC Collab., PRD 97 (2017) 102005]

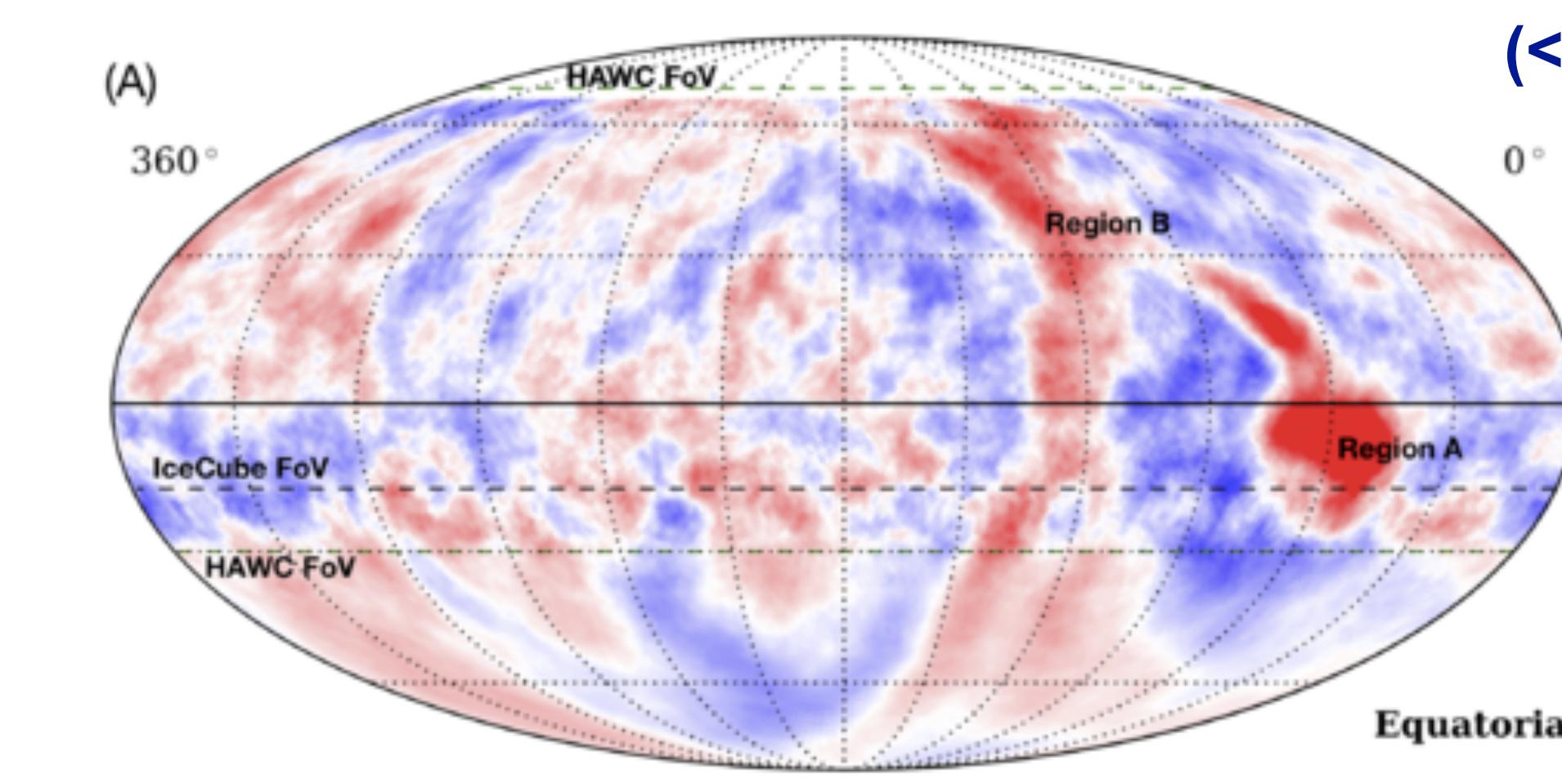
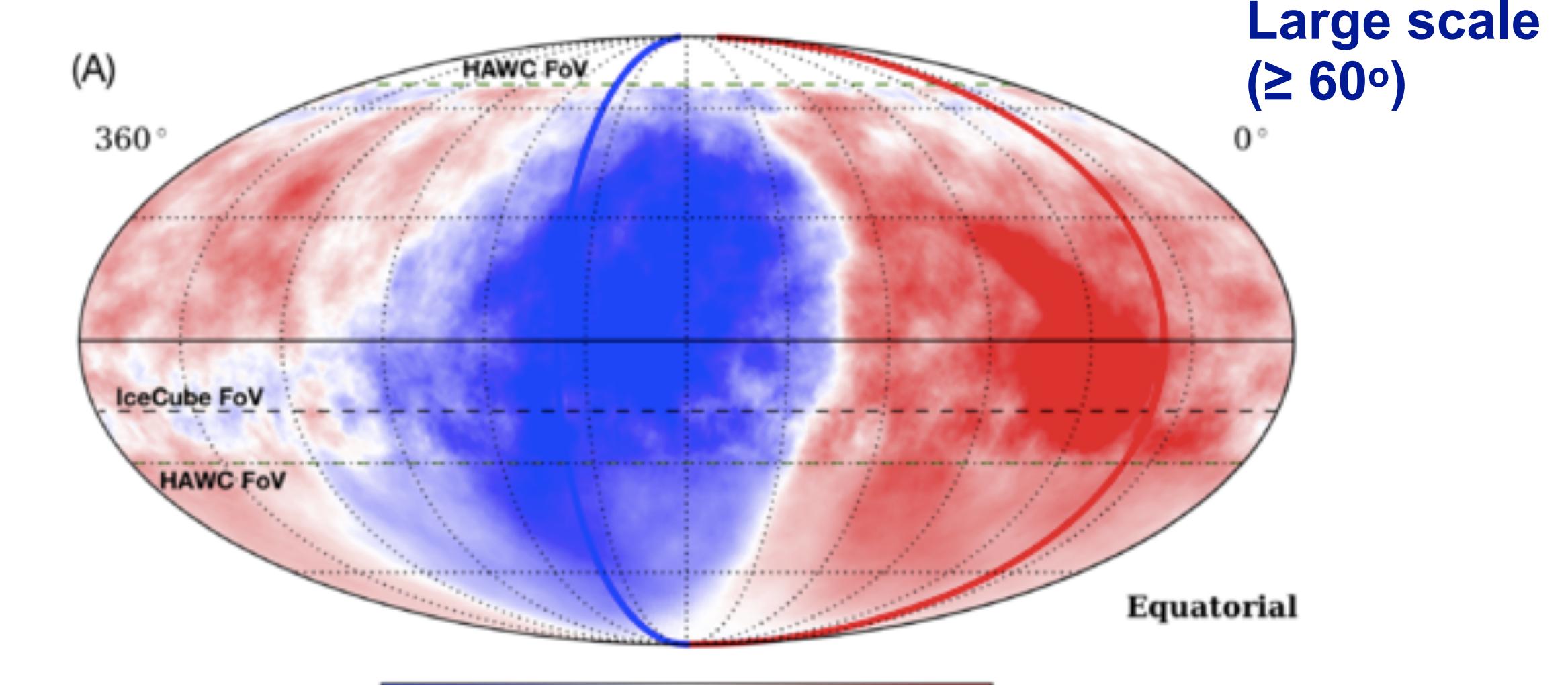


Antiproton/proton ratio in TeV CRs



Shadows of the Moon and the Sun in CRs

Anisotropies in arrival direction of cosmic rays

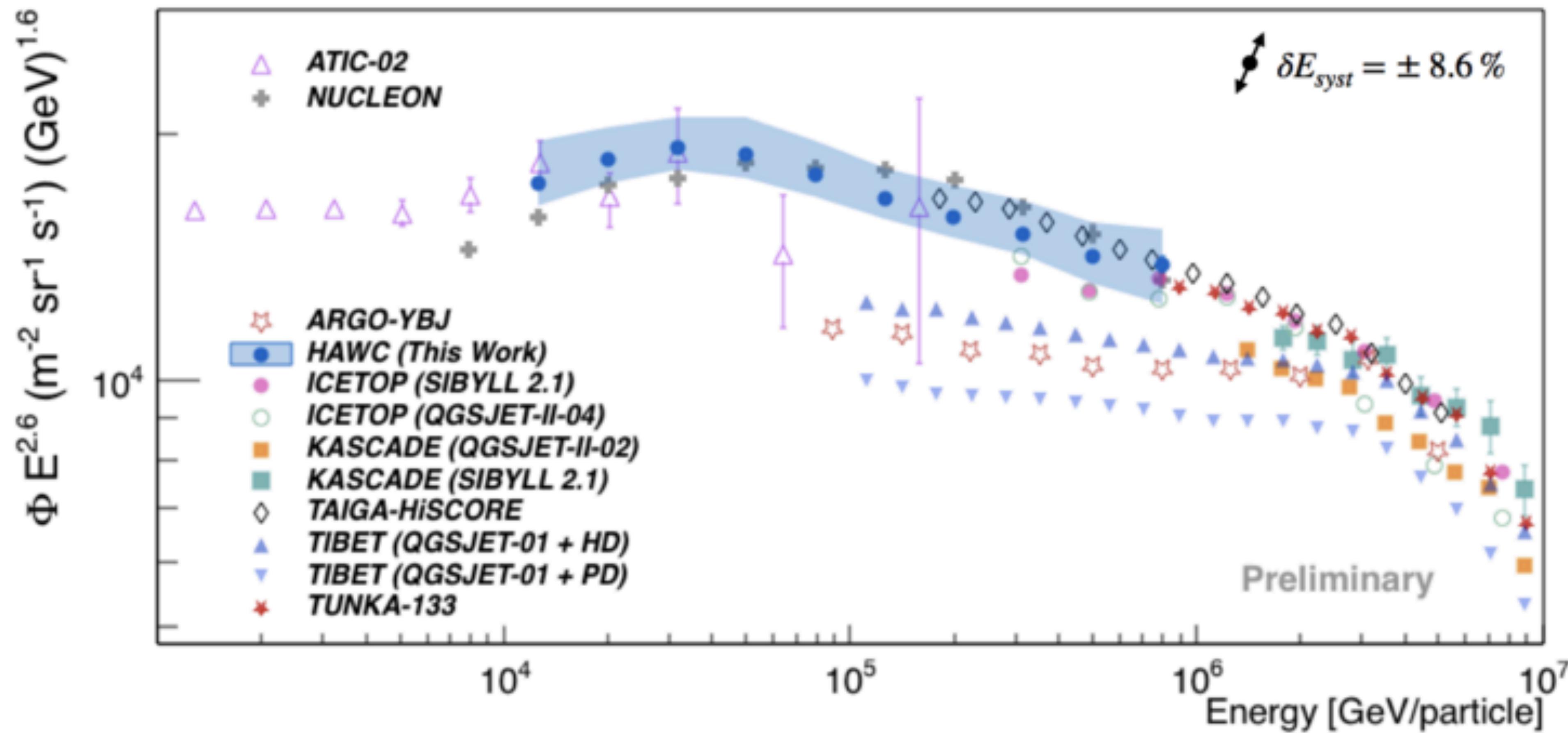


[HAWC and ICECUBE Collab., ApJ 871 (2019) 96]

2) The HAWC observatory

All-particle cosmic ray energy spectrum

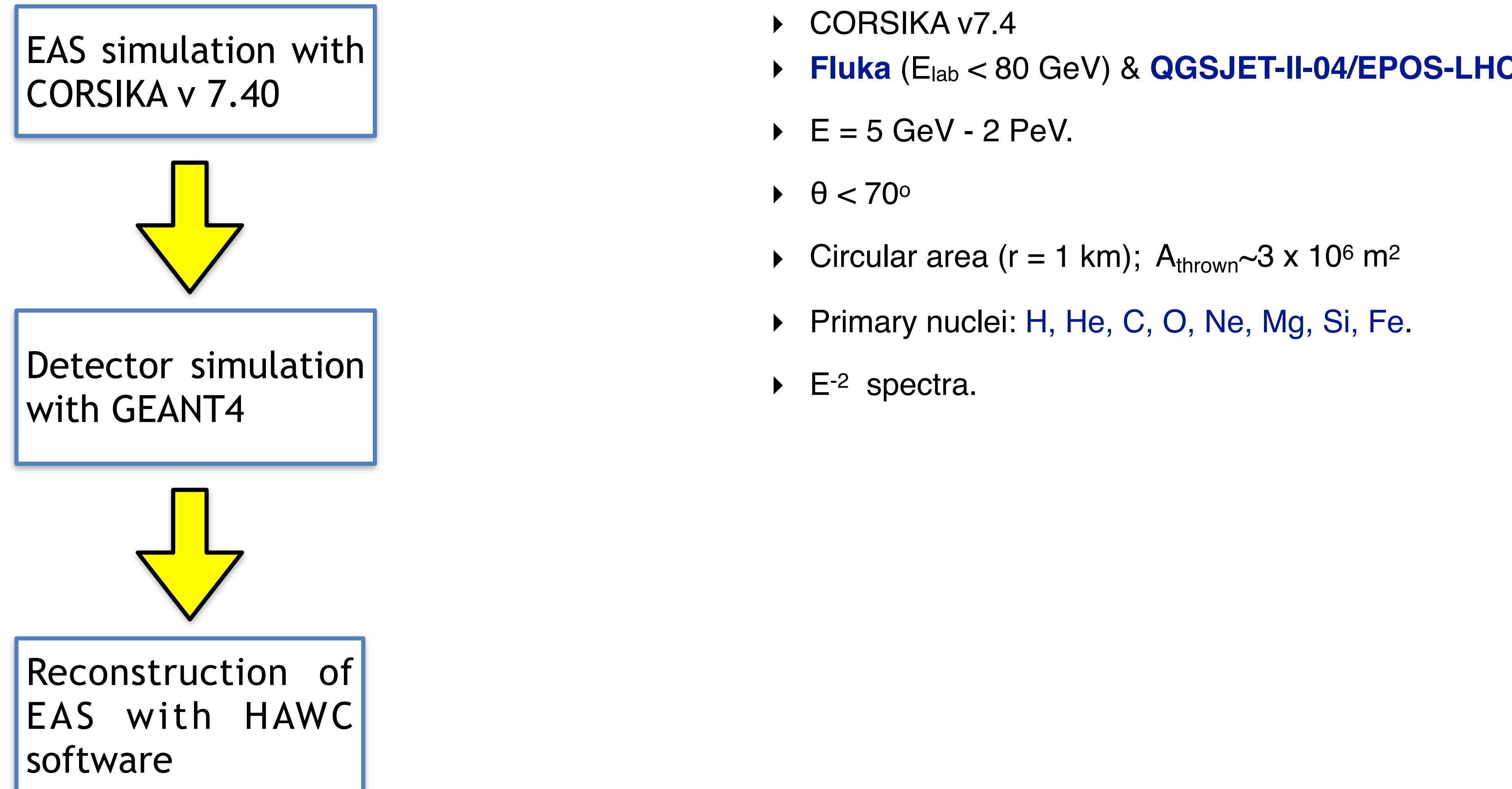
- ▶ Data from January 2017 to December 2020
- ▶ Energy calibration with QGSJET-II-04
- ▶ Confirmation of a break at $28.1 +1.3/-1.2$ (TeV) at 4.2σ



[HAWC Collab., PoS(ICRC2023)364]

3) MC simulations

Monte Carlo simulations

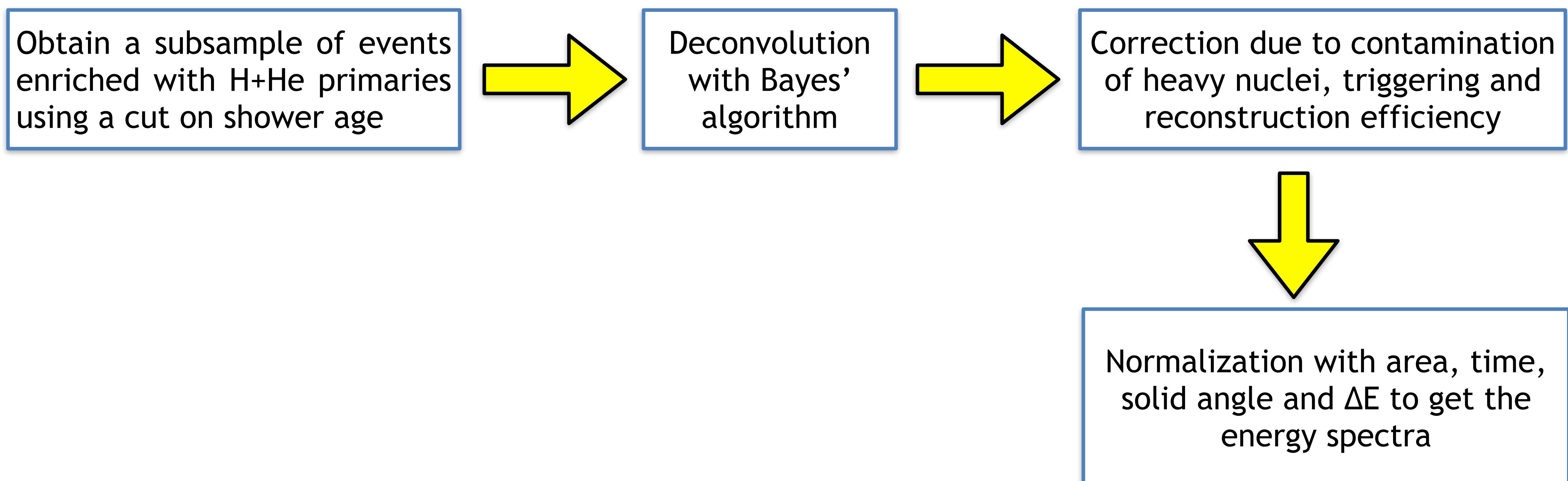


4) The spectrum of H+He nuclei

Objective:

- Reconstruction of the energy spectrum of the light mass group (H+He) of cosmic rays at tens of TeV.

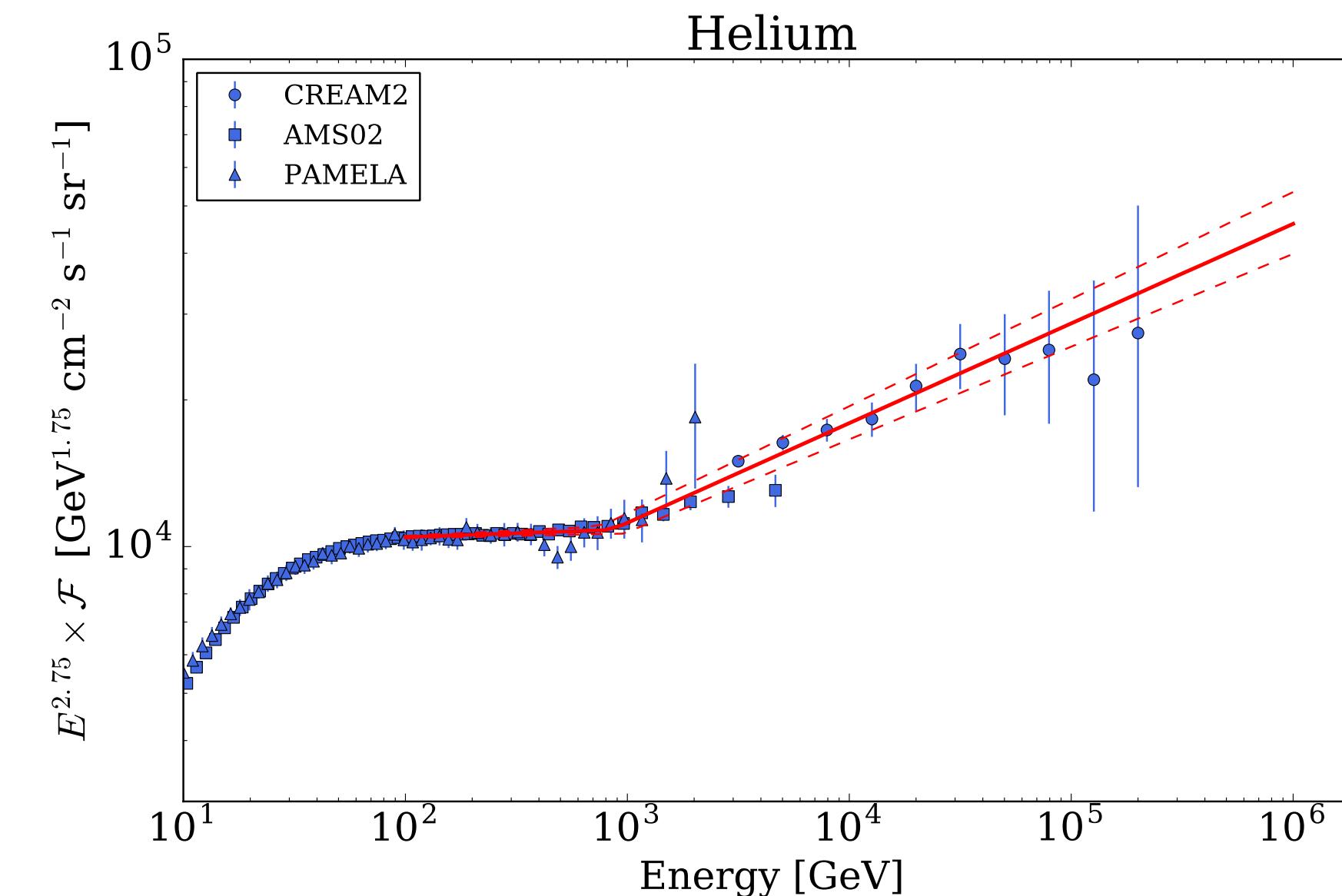
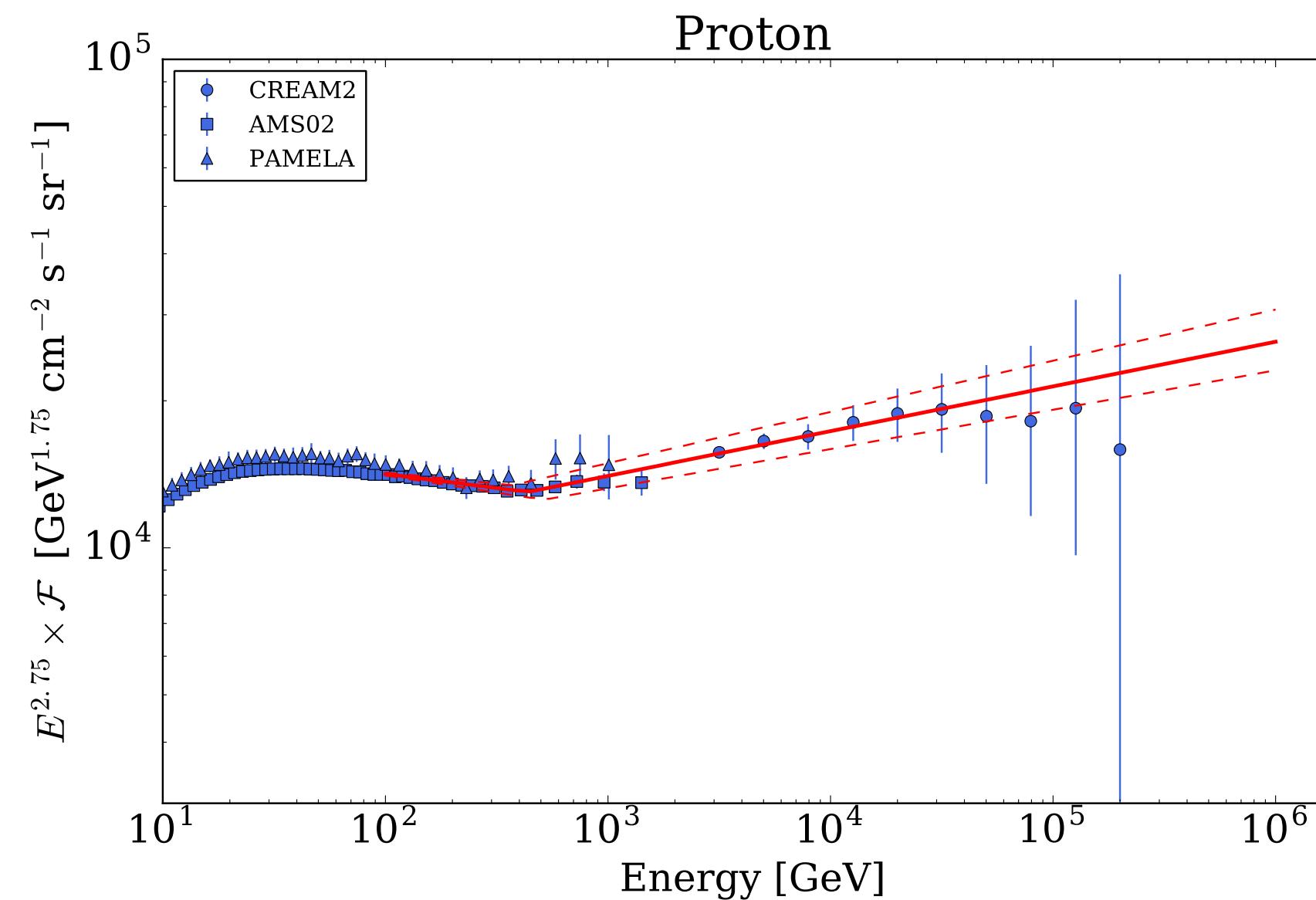
Method:



4) The spectrum of H+He nuclei

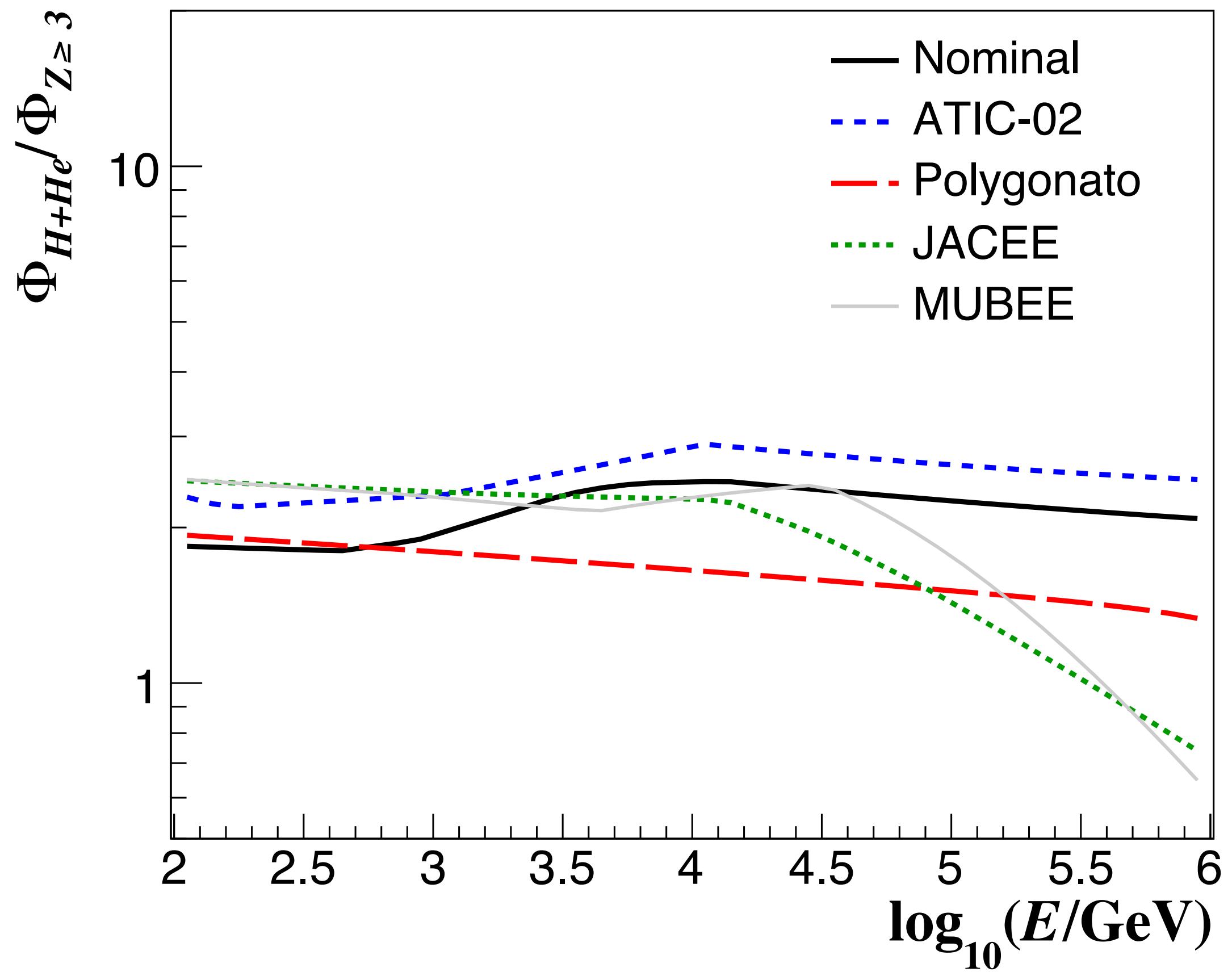
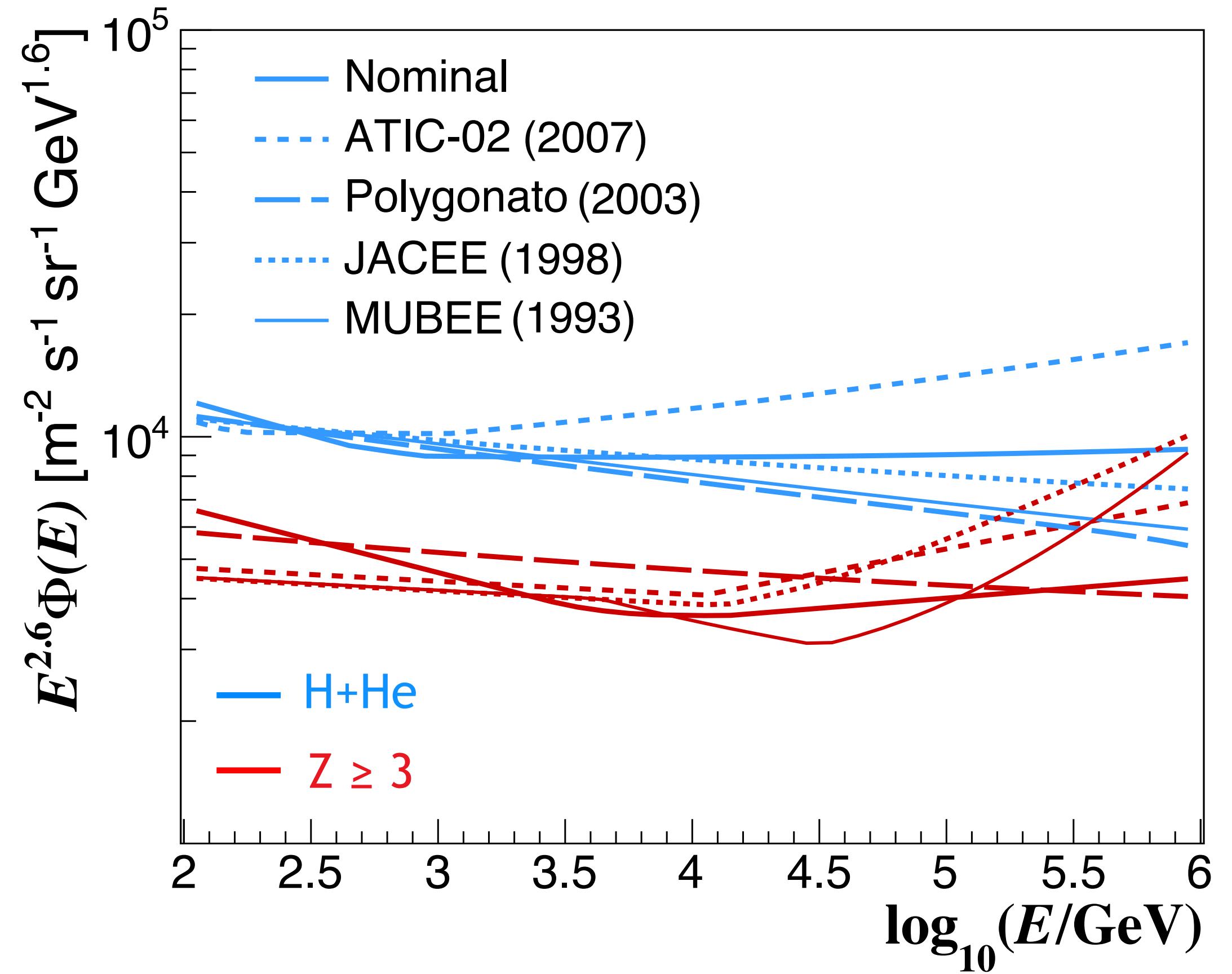
Cosmic-ray nominal composition model

- MC data is weighted to follow broken power-laws derived from fits to **AMS02** (2015), **CREAM-II** (2009 & 2011) and **PAMELA** (2011) [HAWC Collab., PRD 96 (2017)]



4) The spectrum of H+He nuclei

Cosmic ray composition models for systematic studies



4) The spectrum of H+He nuclei

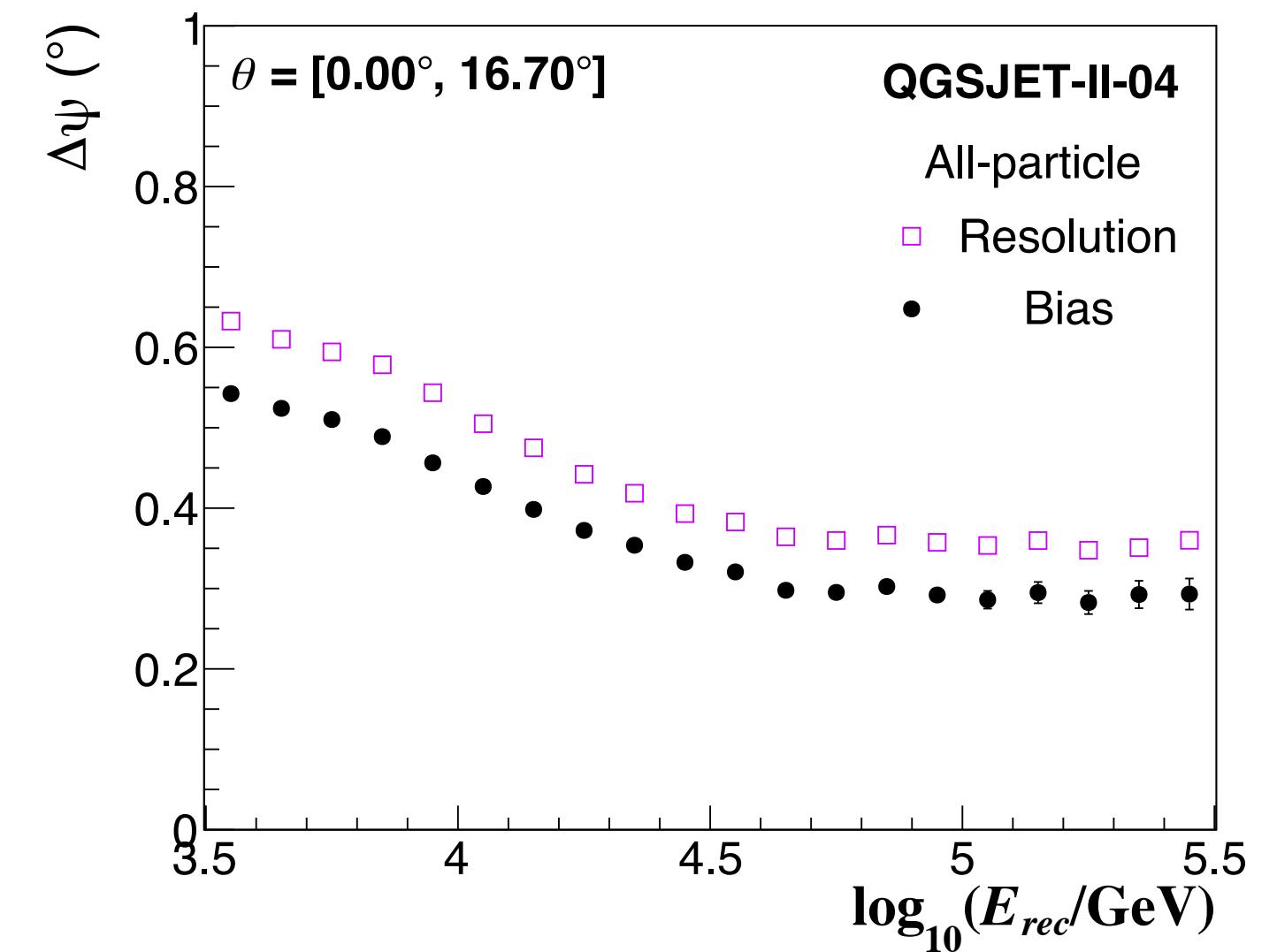
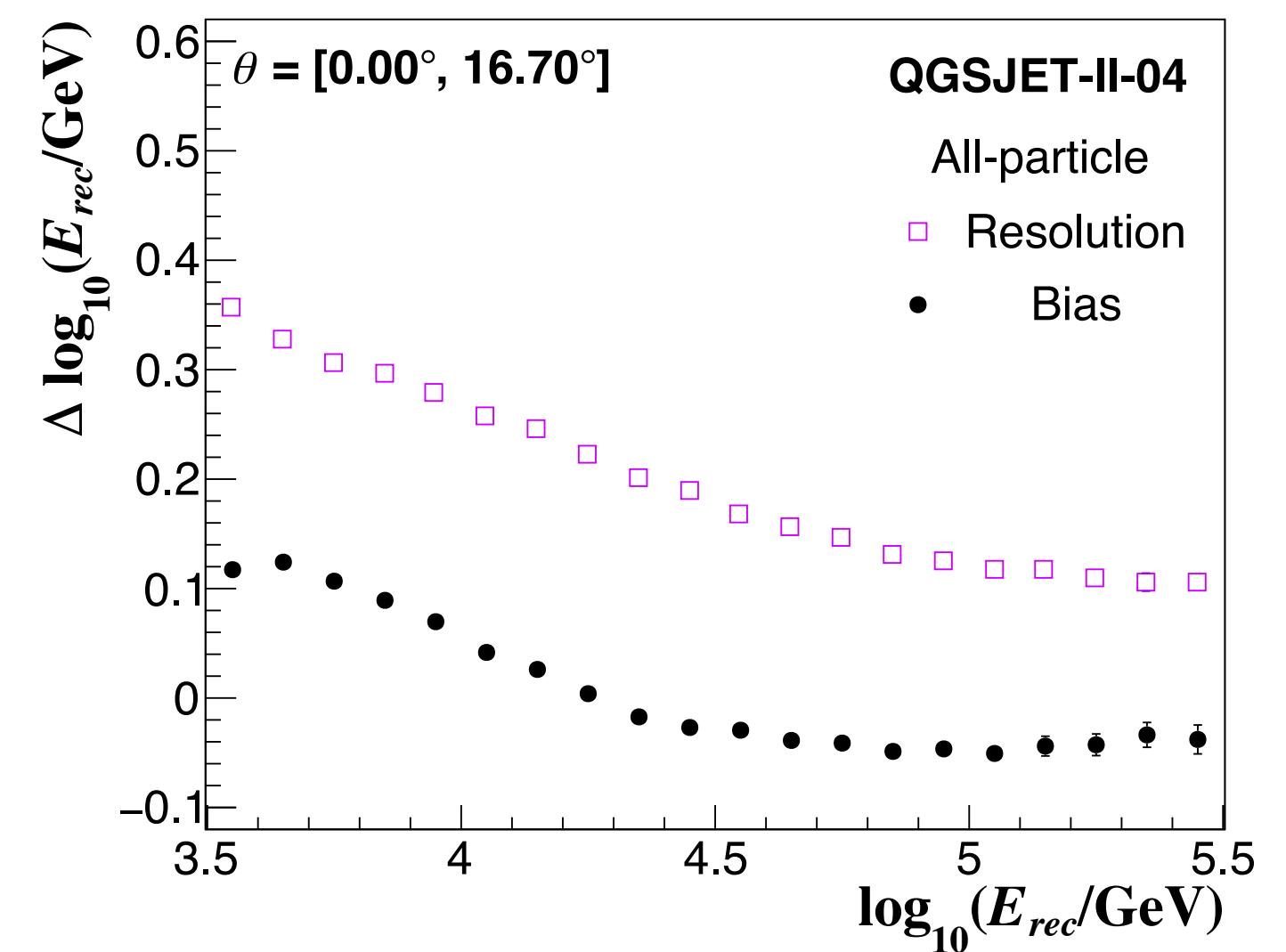
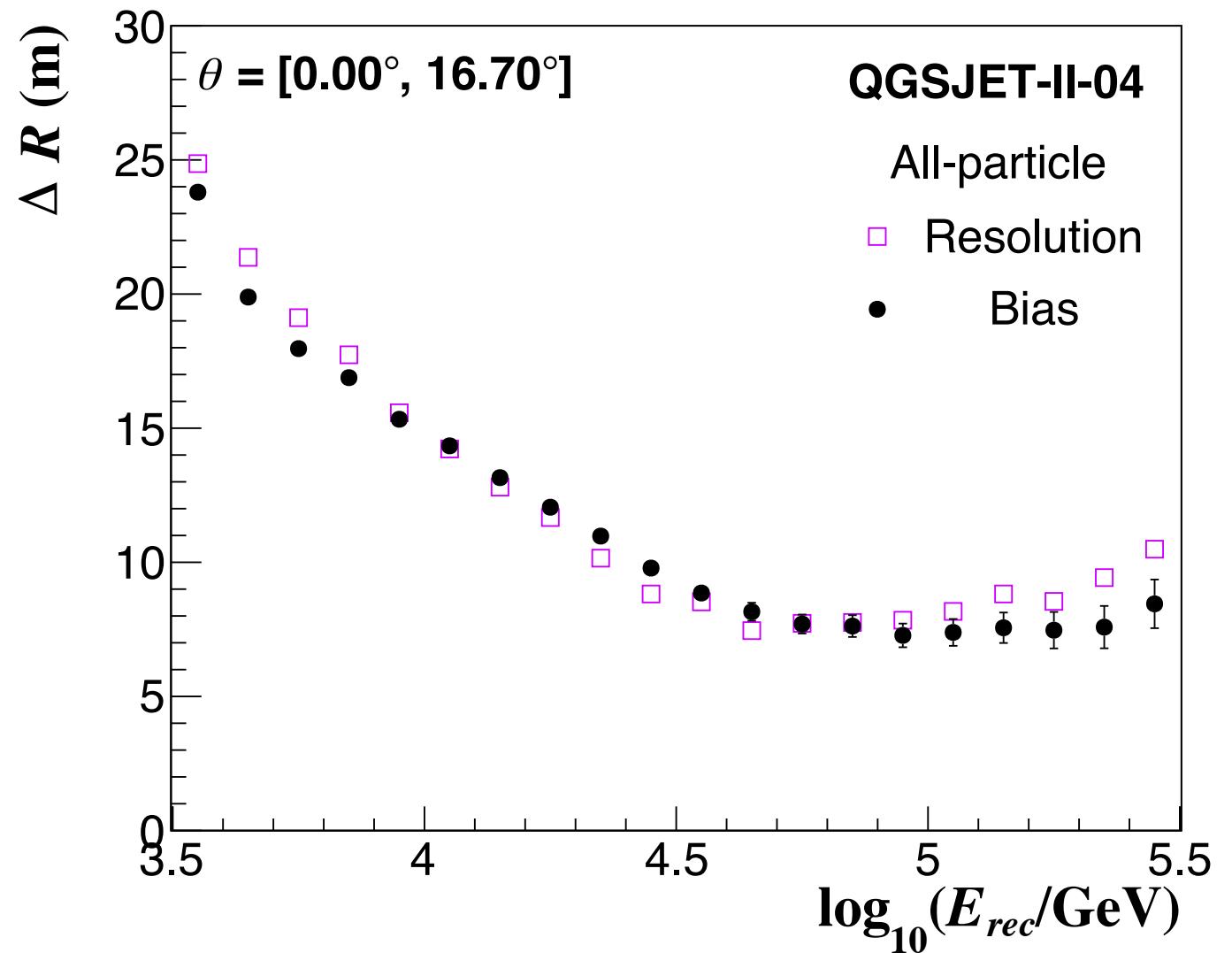
Selection cuts

- Important to reduce systematic effects on results:
 - ▶ $\theta < 16.7^\circ$
 - ▶ Successful core and arrival direction reconstruction
 - ▶ Activate at least 40 PMTs within 40 m from core
 - ▶ Fraction hit (# of hit PMT's/# available channels) ≥ 0.2
 - ▶ $\log_{10}(E/\text{GeV}) = [3.5, 5.5]$

- Resolution:

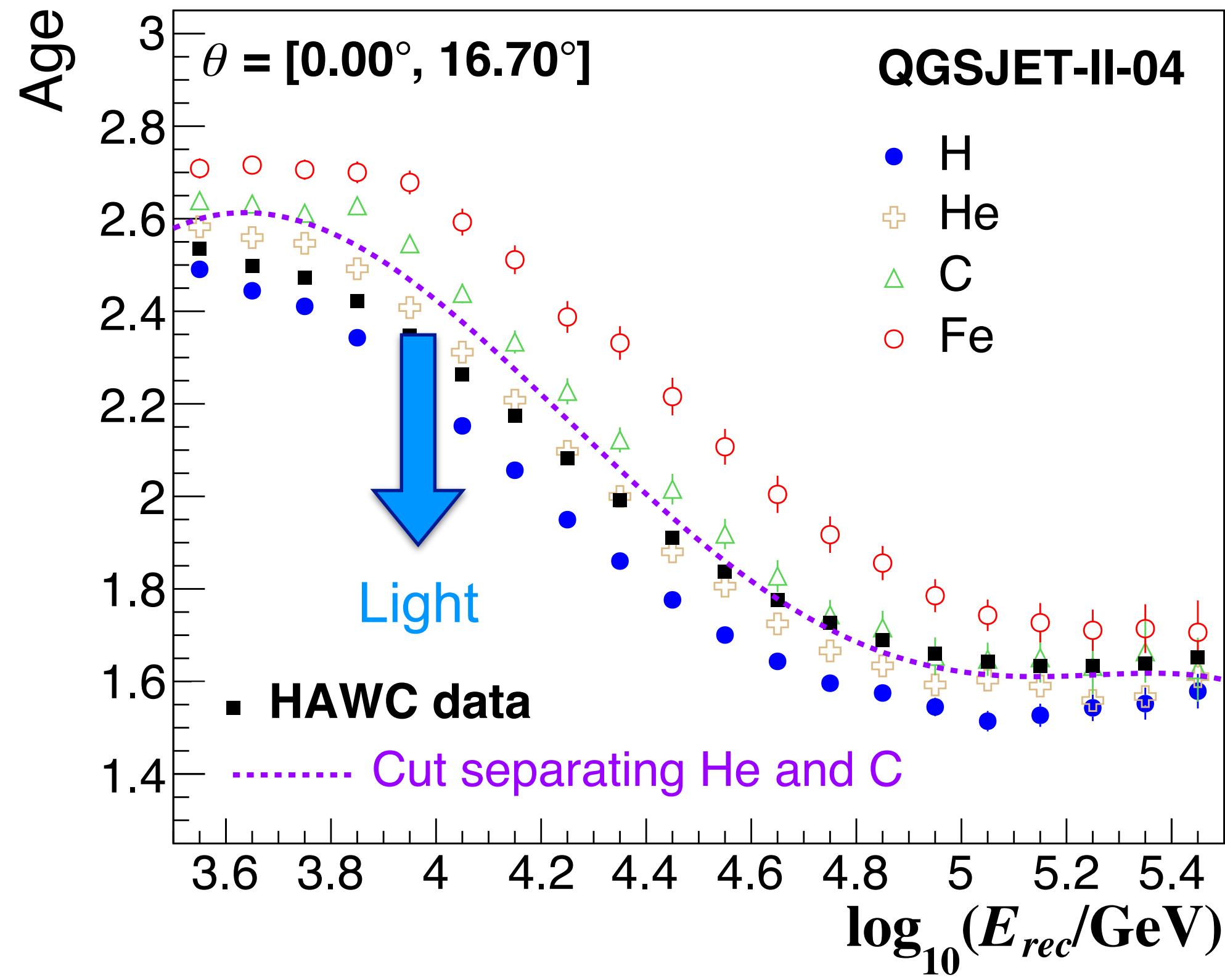
$E \geq 10 \text{ TeV}:$

Δcore	$\leq 15 \text{ m}$
$ \Delta\log_{10}(E/\text{GeV}) $	≤ 0.26
$\Delta\Psi$	$\leq 0.55^\circ$

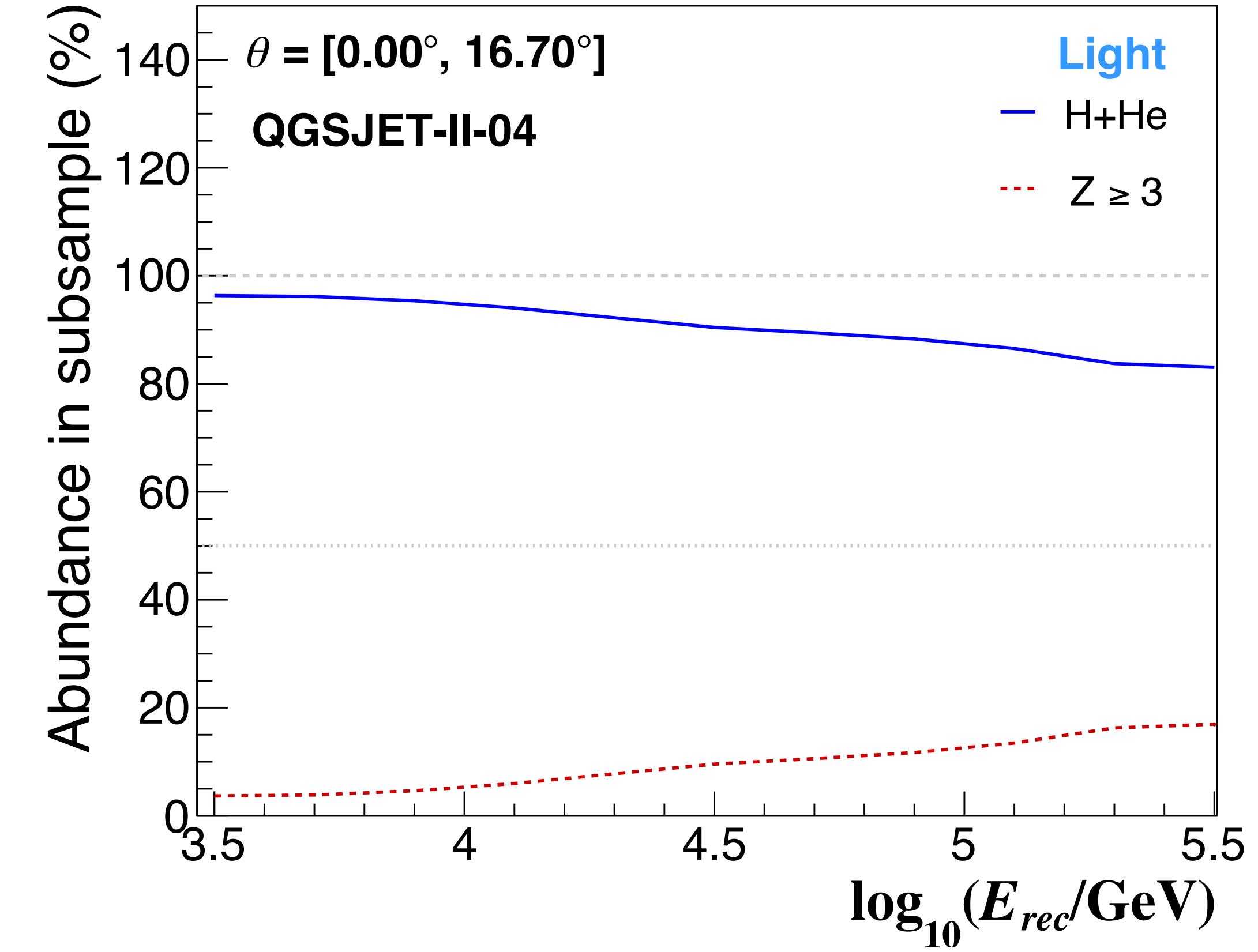


4) The spectrum of H+He nuclei

Analysis: Select a sample enriched with light nuclei



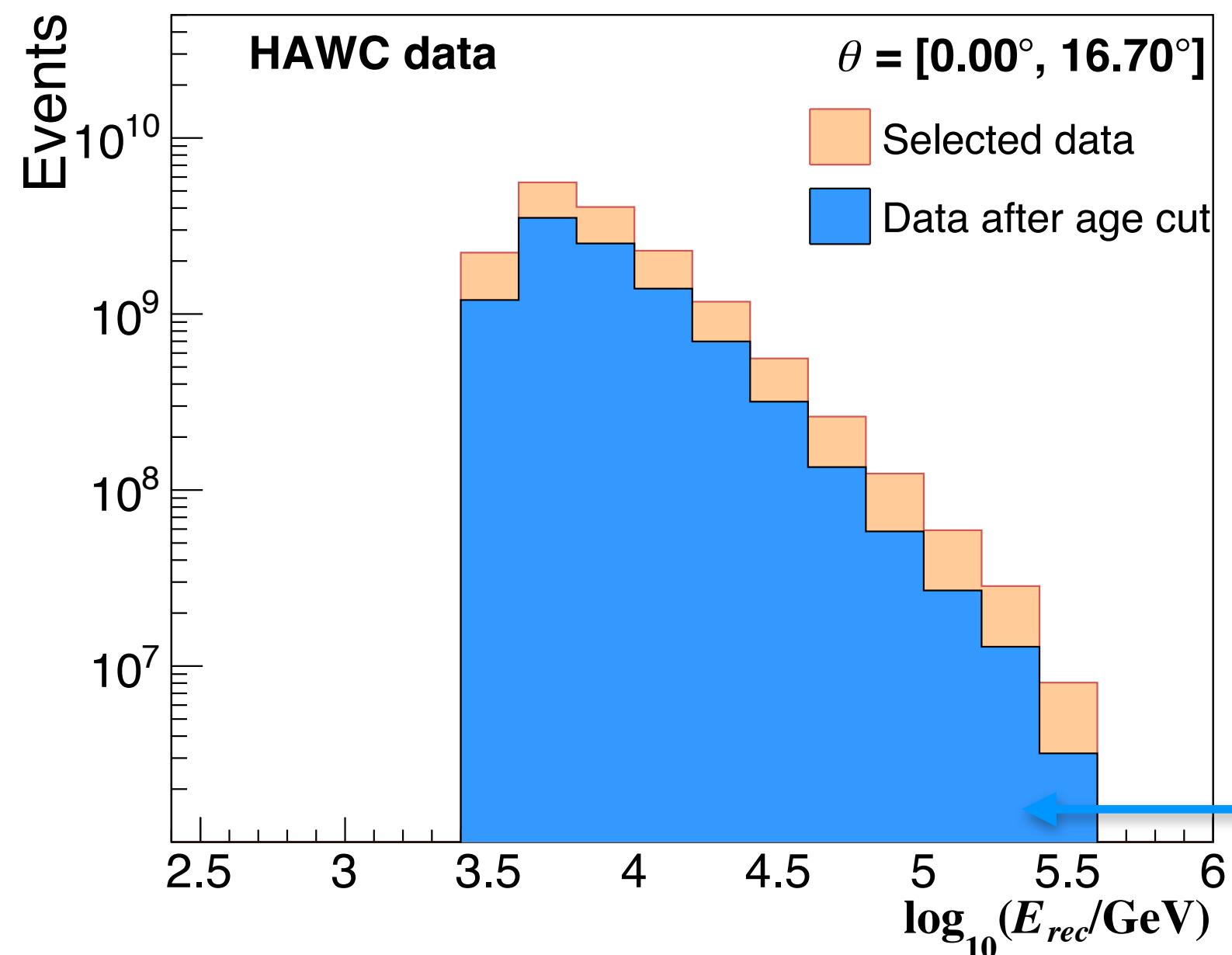
- Age parameter is sensitive to composition
- Select a subsample using a cut on the age
 - Subsample must have a large relative abundance of H and He.



- Content of H + He in subsample
 - More than 82% of H and He in subsample

4) The spectrum of H+He nuclei

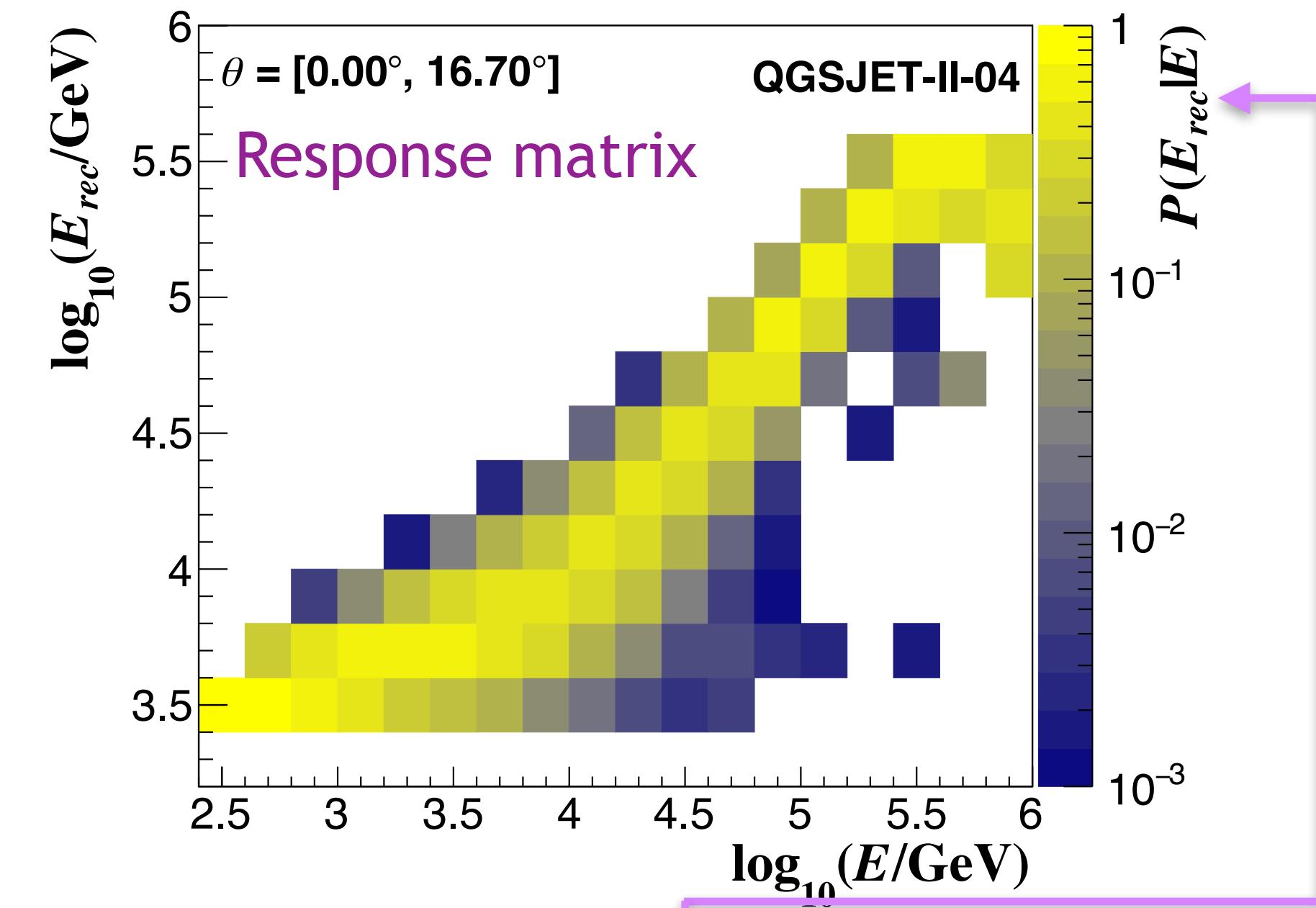
Build raw energy spectrum of subsample: $N_{\text{raw}}(E_{\text{rec}})$



- Experimental data used for analysis:
 $\Delta t_{\text{eff}} = 3.74 \text{ years (94\% livetime)}$
 June/11/15-June/03/19
 $\Delta \Omega = 0.27 \text{ sr}$

Total events	: $2.9 \times 10^{12} \text{ EAS}$
+ selection cuts:	$1.6 \times 10^{10} \text{ EAS}$
+ age cut:	$9.9 \times 10^9 \text{ EAS}$

Correct $N_{\text{raw}}(E_{\text{rec}})$ for migration effects



$$N_{\text{Raw}}(E_{\text{rec}, j}) = \sum_i P(E_{\text{rec}, j} | E_i) N^{\text{Unf}}(E_i)$$

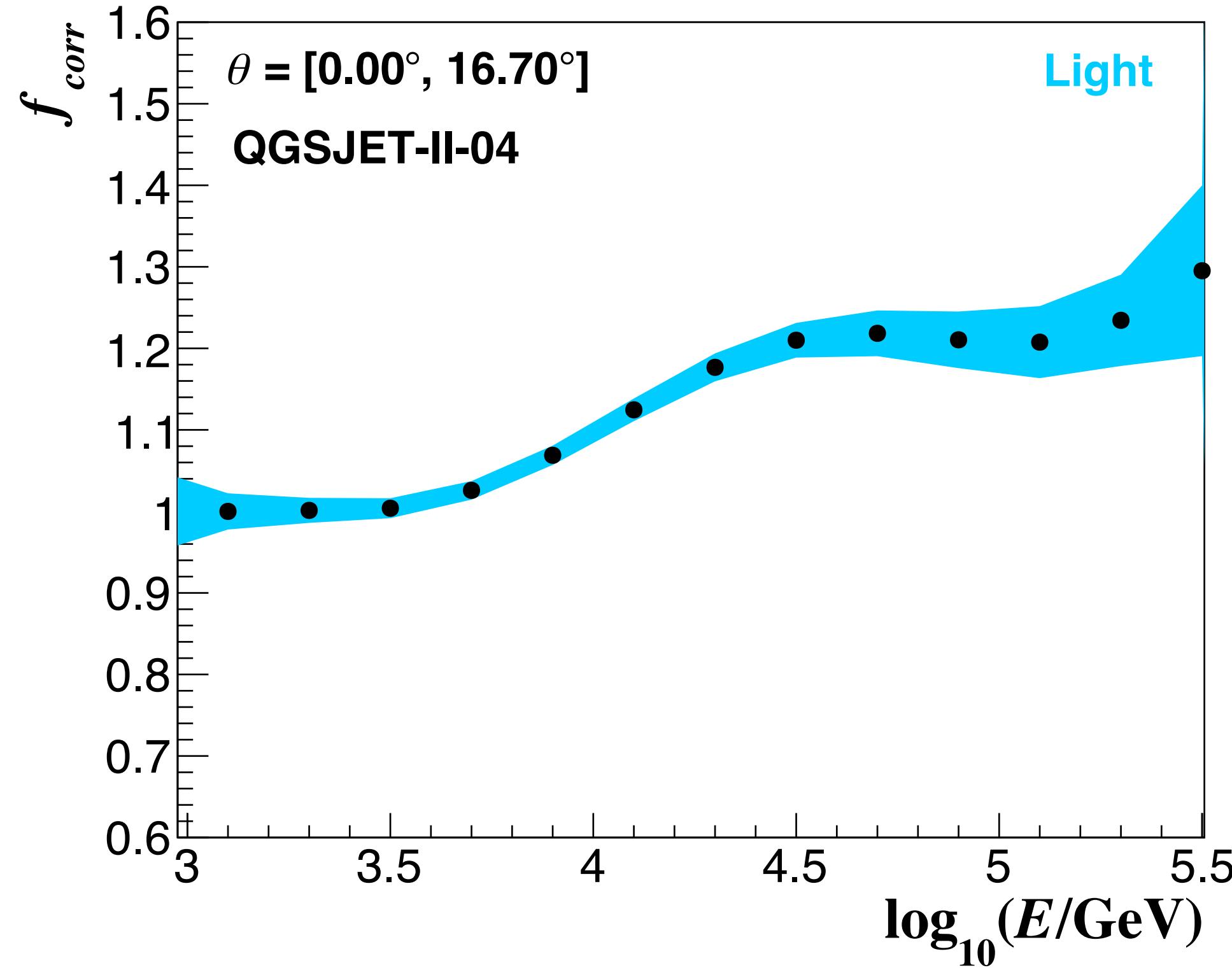
- Solve for $N^{\text{Unf}}(E_i)$ using Bayesian unfolding
 [G. D' Agostini, DESY 94-099]
- Stopping criterium: Minimum of weighted mean squared error

[G. Cowan, Stat. Data analysis, Oxford Press. 1998]

$$\text{WMSE} = \frac{1}{N_{\text{points}}} \sum_i \frac{\text{stat}_i^2 + \text{sys}_i^2}{n_i}$$

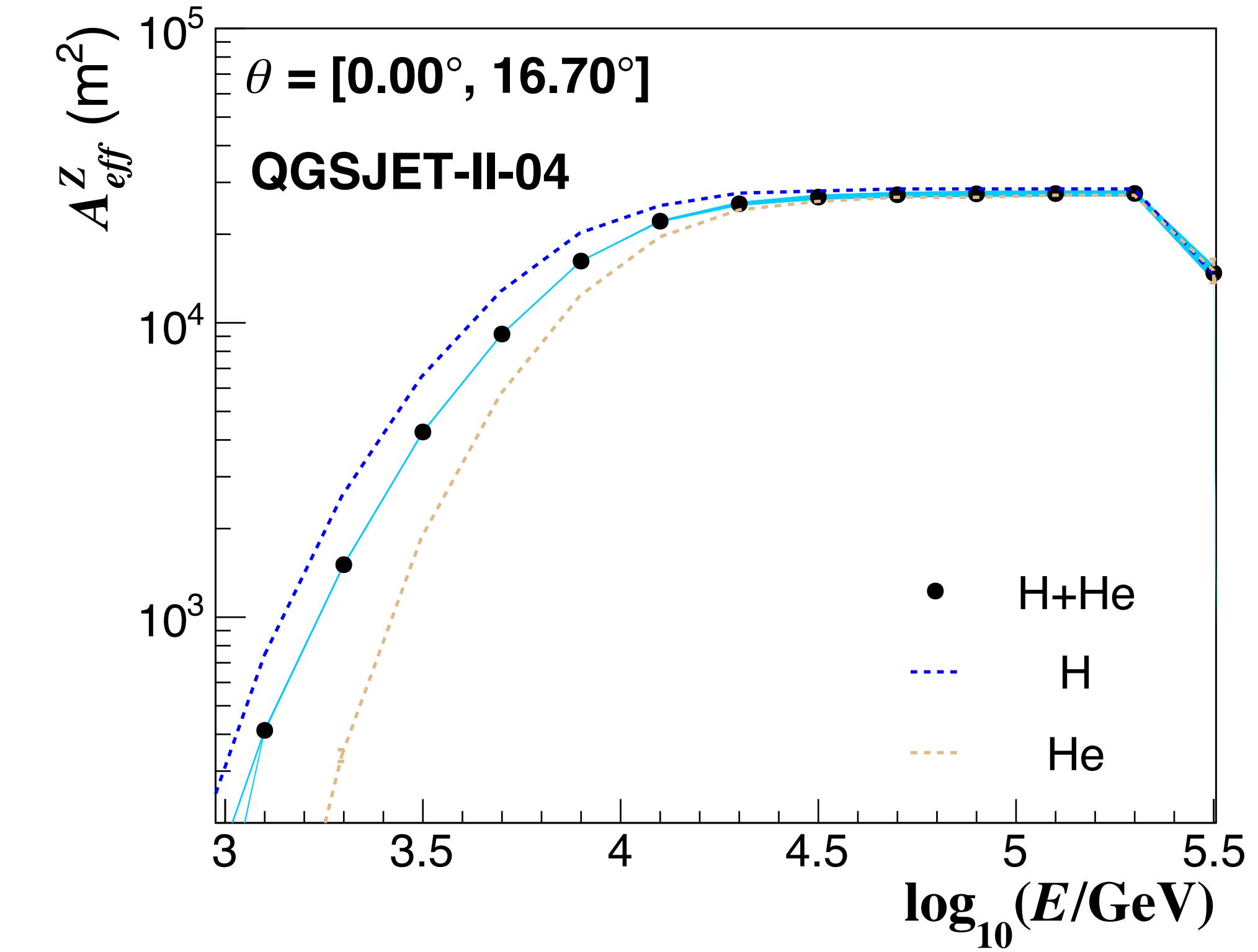
4) The spectrum of H+He nuclei

Obtain effective area from MC simulations



- Correction factor due to contamination of heavy events

$$f_{corr} = (\mathcal{N}_{\text{light}} / \mathcal{N}_{\text{light}^{\text{H+He}}})$$

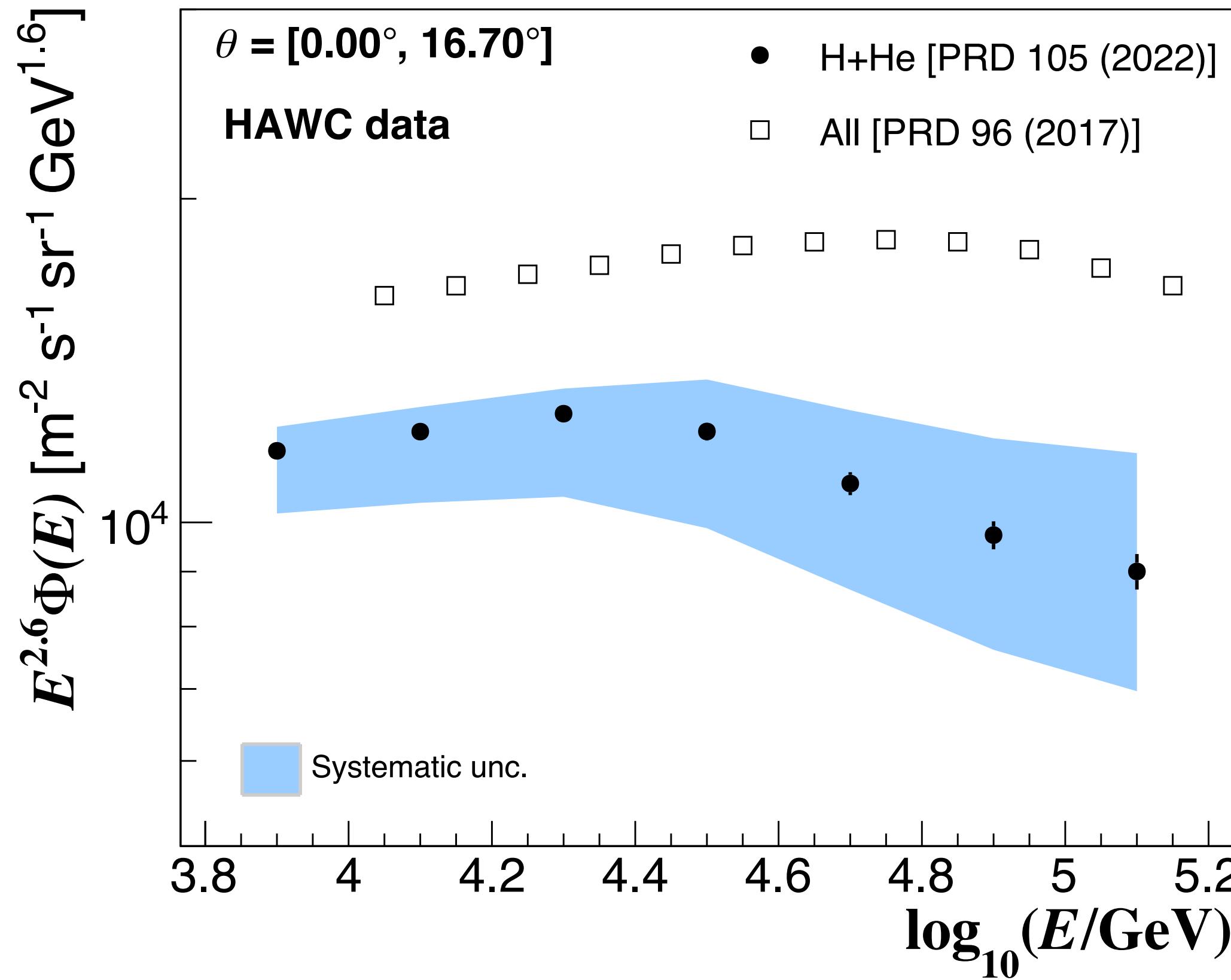


- Effective area of H+He in subsample

$$A_{eff}^{H+He}(E_i) = A_{\text{thrown}} \varepsilon^{H+He}(E_i) \frac{\cos\theta_{\max} + \cos\theta_{\min}}{2}$$

4) The spectrum of H+He nuclei

Get energy spectrum from N^{Unf} and effective area



- Energy spectrum was calculated as:

$$\Phi = N^{\text{Unf}}(E) / [\Delta E \cdot \Delta t_{\text{eff}} \cdot \Delta \Omega \cdot f_{\text{corr}}(E) \cdot A_{\text{eff}}^{\text{H+He}}(E)]$$

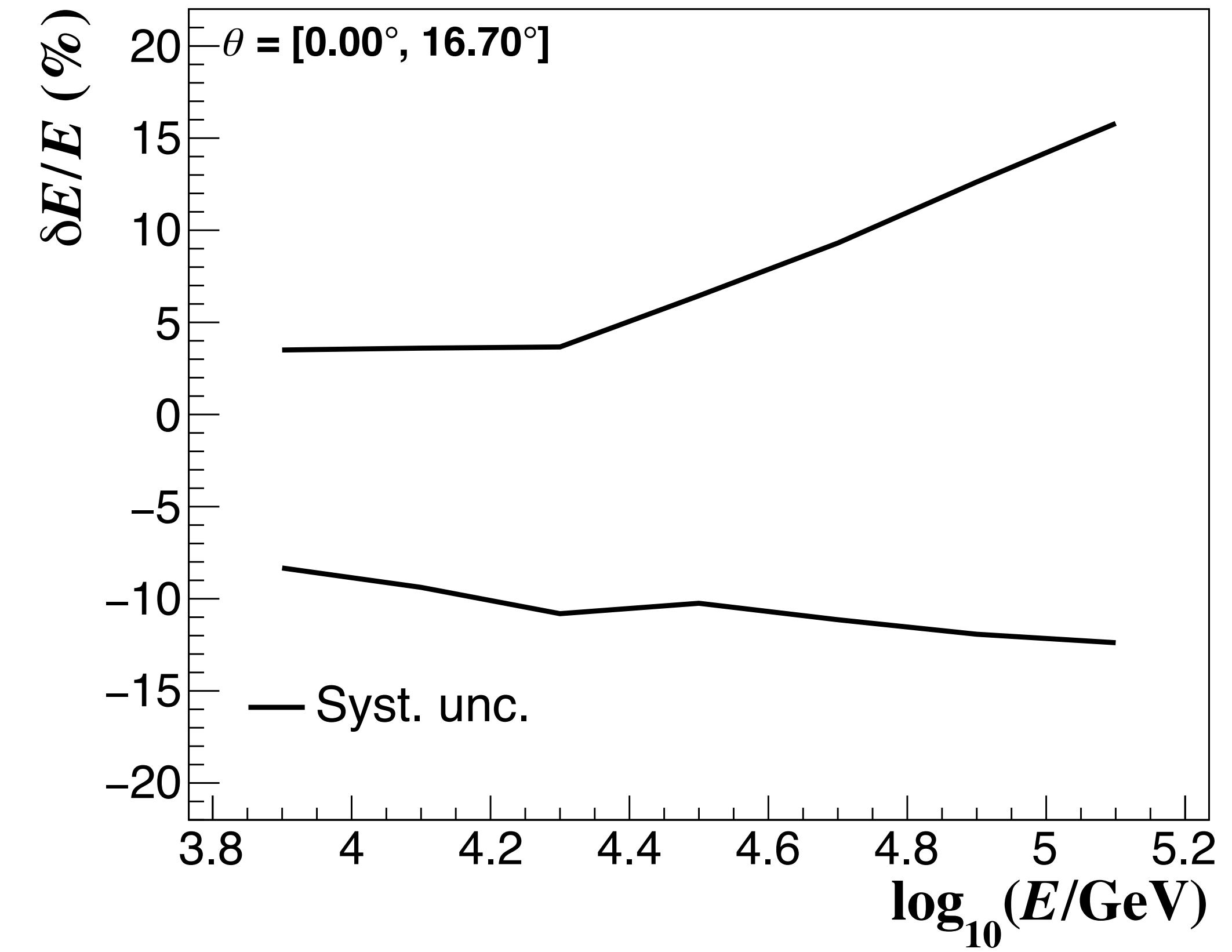
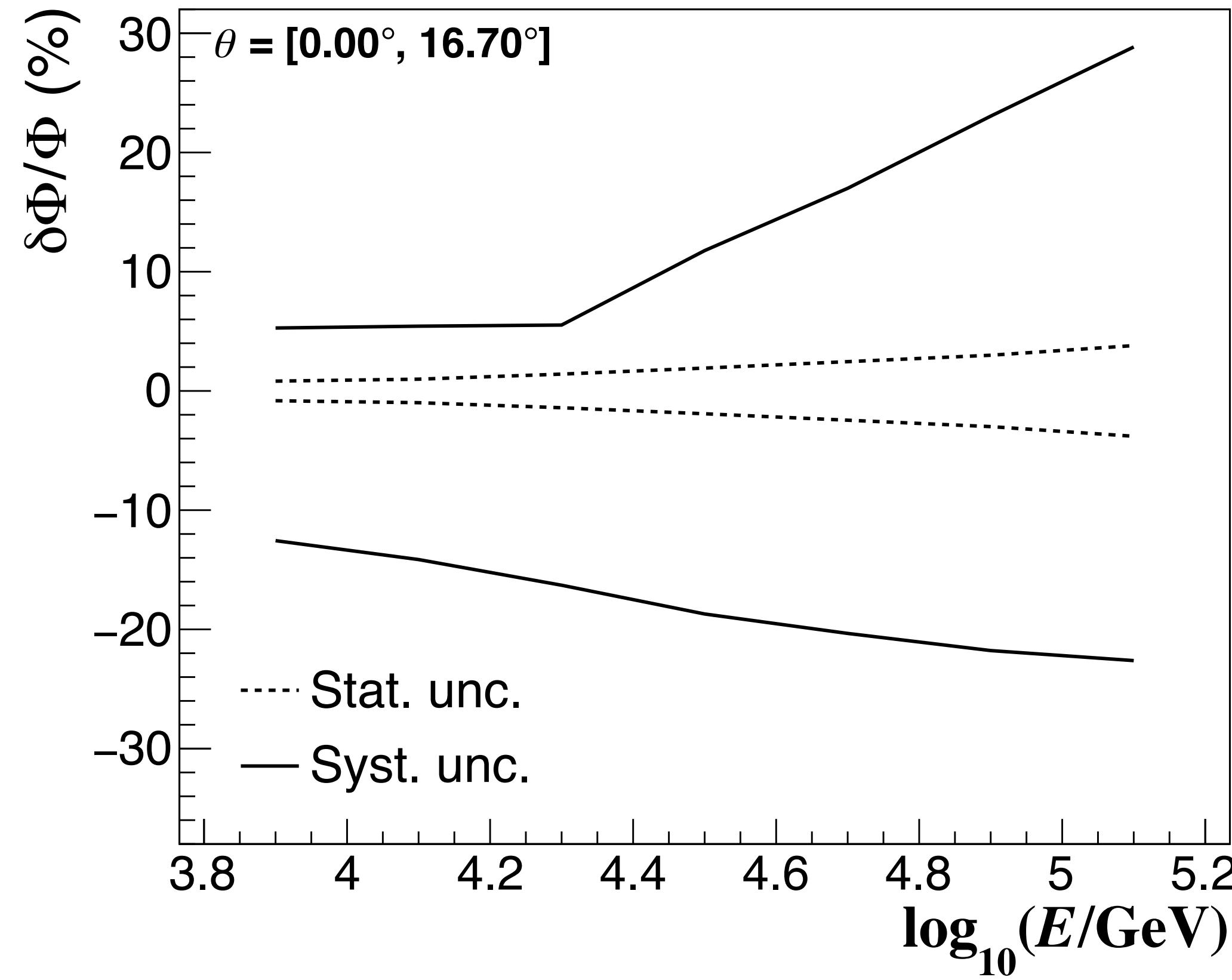
Statistical and systematic uncertainties

$\log_{10}(E/\text{GeV}) = 4.5$ (32 TeV)

	Relative error Φ (%)
Statistical	+/- 1.92
Exp. Data	+/- 0.01
Response matrix	+/- 1.92
Systematic	+11.77/-18.71
Composition	+0.86/-17.25
A _{eff}	+1.85/-2.04
Cut at He or C	+2.87/-0.75
Gold unfolding	+1.23
Seed unfolding	-1.42
Smoothing unfold.	+3.73/-1.32
PMT efficiency	+5.00
PMT threshold	+2.33/-1.53
PMT charge	+1.83
PMT late light	+8.77/-0.14
Hadronic model	-6.47
Total	+11.93/-18.81

4) The spectrum of H+He nuclei

Statistical and systematic uncertainties



4) The spectrum of H+He nuclei

Fit of spectrum

H+He

1. Use following functions:

—> Single power law:

$$d\Phi(E)/dE = \Phi_0 E^{\gamma_1}$$

—> Broken power law:

$$d\Phi(E)/dE = \Phi_0 E^{\gamma_1} [1 + (E/E_0)^{\varepsilon}]^{(\gamma_2 - \gamma_1)/\varepsilon}$$

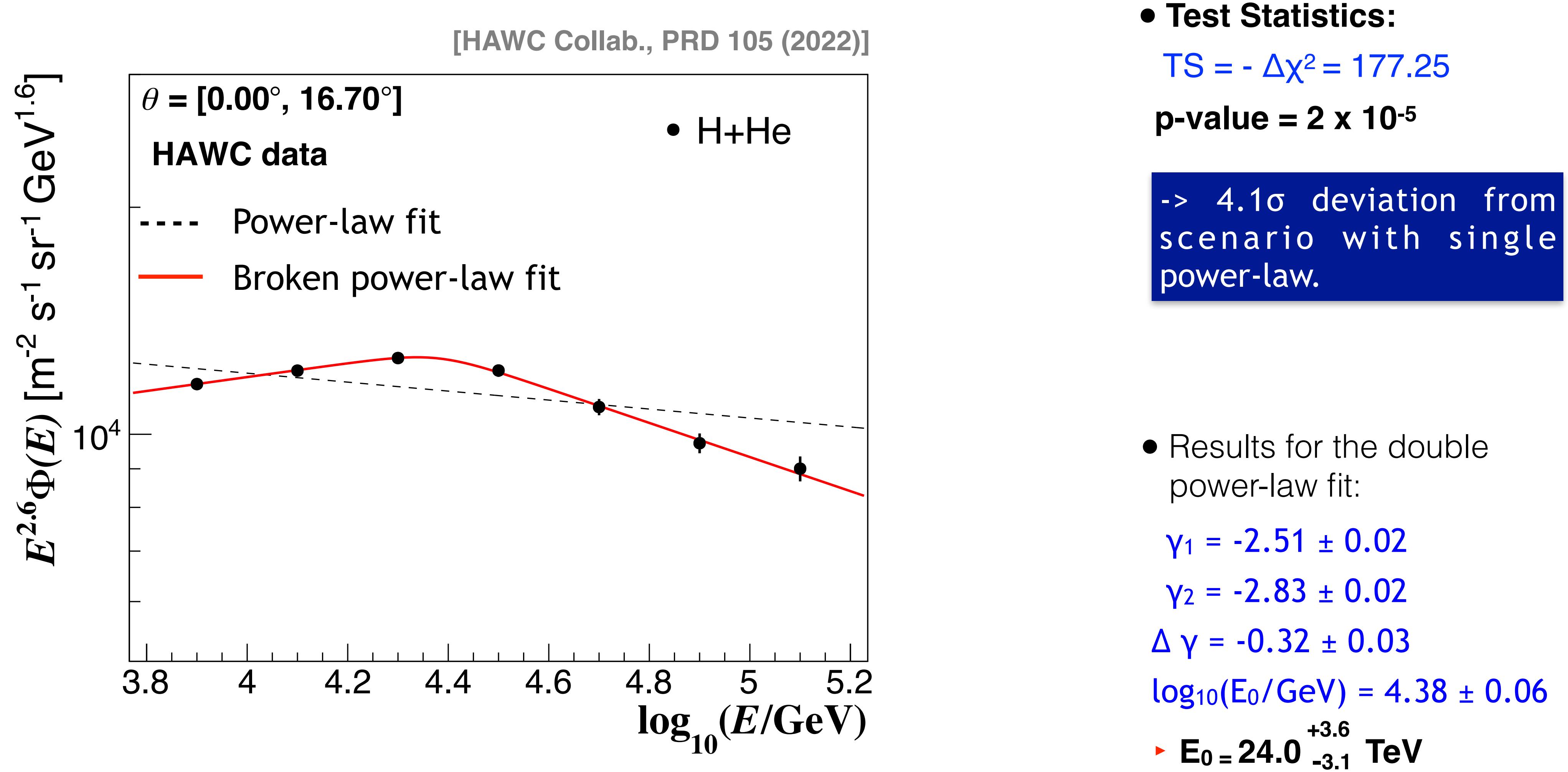
2. Minimize χ^2 with MINUIT and take into account correlation between points:

$$\chi^2 = \sum_{i,j} [\Phi_i^{\text{data}} - \Phi^{\text{fit}}(E_i)] [V_{\text{stat}}^{\text{Tot}}]^{-1}_{ij} [\Phi_j^{\text{data}} - \Phi^{\text{fit}}(E_j)]$$

[C. Patrignani et al. (PDG), Chin. Phys. C, 40 (2016) and (2017) update]

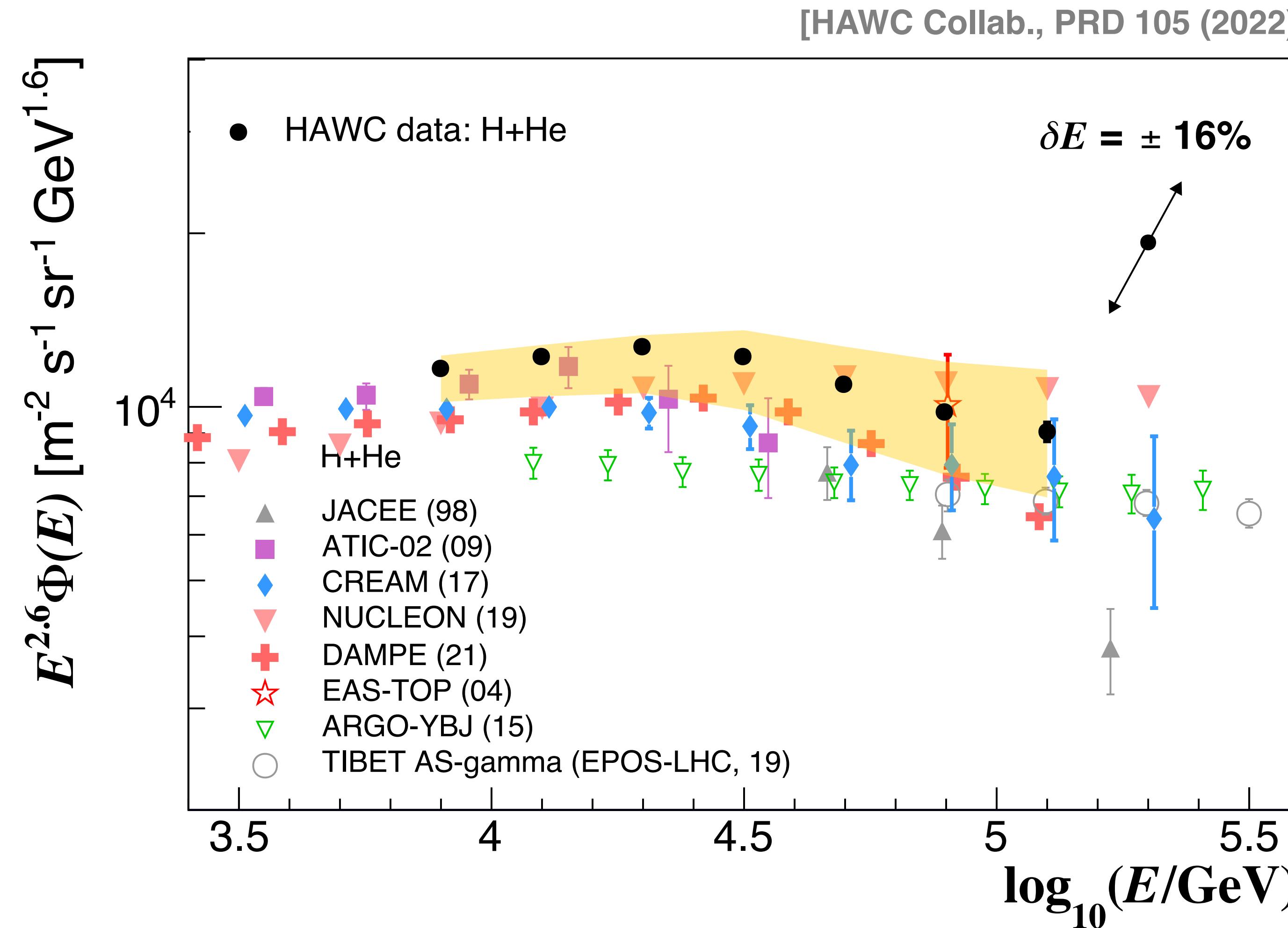
4) The spectrum of H+He nuclei

Fit of spectrum



4) The spectrum of H+He nuclei

Comparison with measurements from other experiments



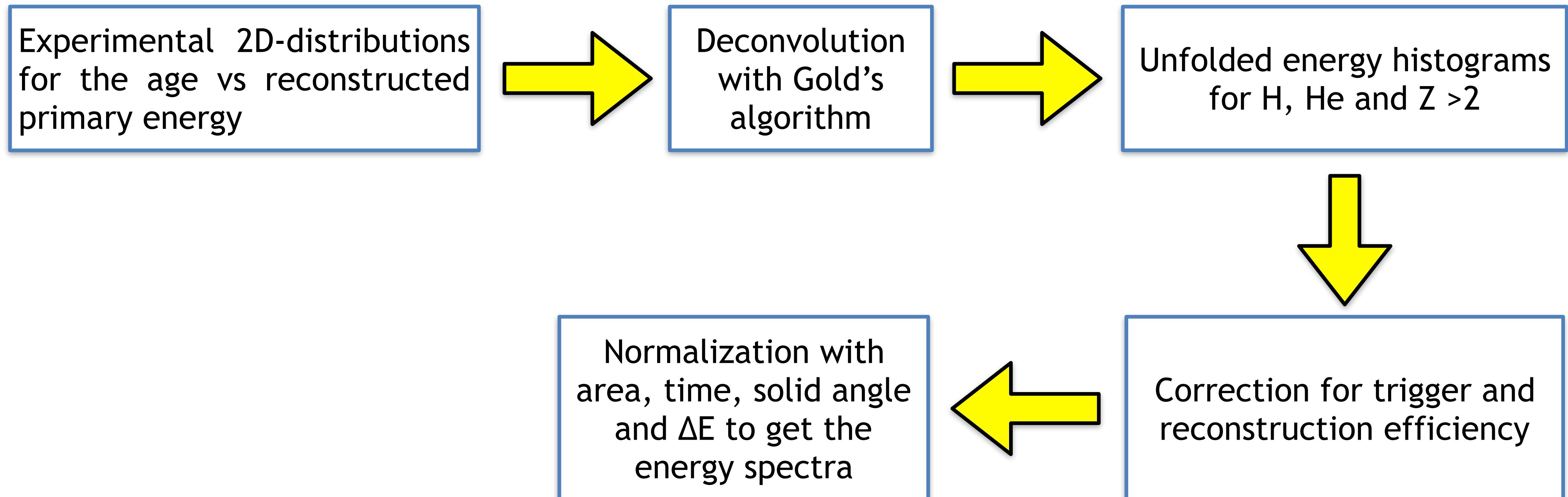
- **HAWC** data confirm previous hints from **ATIC-2**, **CREAM I-III** and **NUCLEON** about the existence of a break in the spectrum of the light component of cosmic rays in the $10^4 - 10^5$ GeV range.
- **HAWC** result is strengthened by the **DAMPE** data.
- **HAWC** data is in agreement with **ATIC-2** close to 10^4 GeV.

5) Unfolding of elemental mass groups

Objective:

- To estimate the CR energy spectra of H y He and heavy primaries ($Z > 2$) for $E = [10 \text{ TeV}, 1 \text{ PeV}]$ eusing HAWC data.

Method:

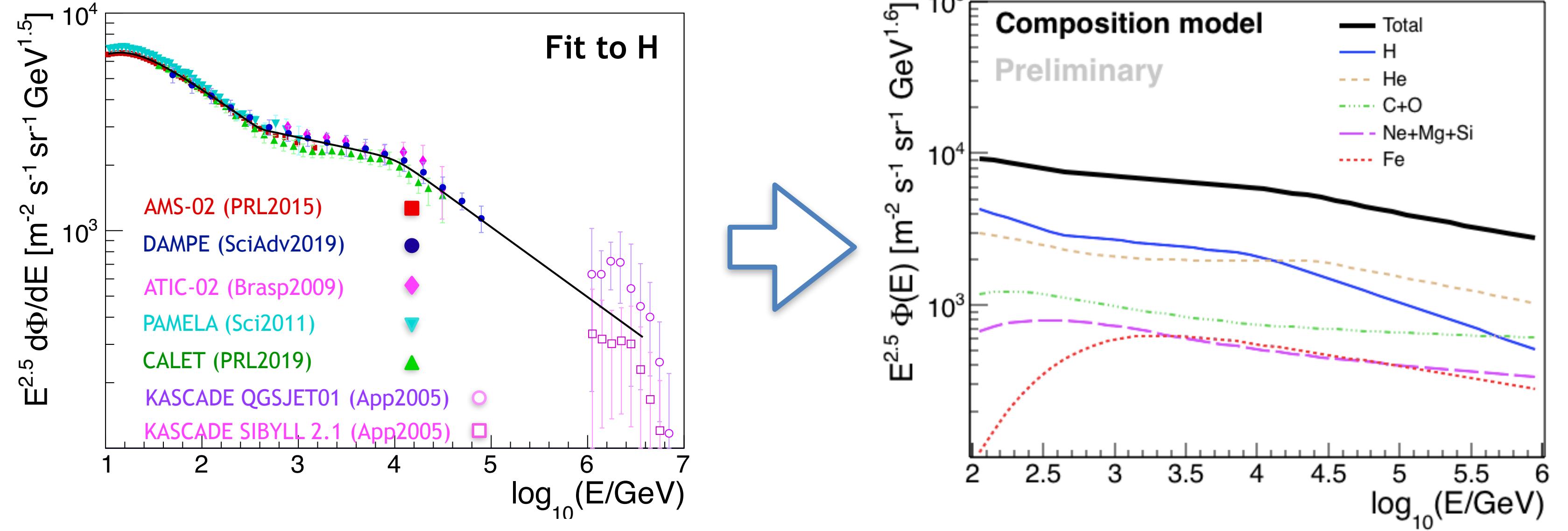


5) Unfolding of elemental mass groups

Cosmic-ray nominal composition model

- MC data is weighted to follow broken power-laws derived from fits to data from direct and indirect experiments.

- AMS-2 (2015, 2017, 2020 & 2021),
- CREAM I-II (2009 & 2011),
- PAMELA (2011 & 2014),
- ATIC-2 (2009),
- NUCLEON (2019),
- CALET (2019),
- DAMPE (2019 & 2021) y
- KASCADE (2005) .



5) Unfolding of elemental mass groups

Selection cuts

- ▶ $\theta = [20^\circ, 45^\circ]$ To reduce effect of PMT saturation at High Energies
- ▶ Successfully reconstructed
- ▶ Hit PMT's within radius of 40 m > 40
- ▶ Fraction of hit PMTs ≥ 0.2
- ▶ $\log_{10}(A) \leq 2.5$
- ▶ $\log_{10}(E_{\text{rec}}/\text{GeV}) = [3.8, 6.5]$
- ▶ LDFAge (S) $= [1.0, 3.8]$

Measured data

- ▶ January, 2016 - December, 2020
- ▶ $T_{\text{eff}} = 4.7$ years
- ▶ $\Delta\Omega = 1.46$ sr

HAWC data after selection cuts

37.5×10^9 events

Resolution $E_{\text{rec}} > 10$ TeV

- ▶ $\Delta R < 20$ m
- ▶ $\Delta\Psi < 0.7^\circ$
- ▶ $\Delta\log_{10}(E) < 0.4$

5) Unfolding of elemental mass groups

Analysis: Apply **Gold's unfolding method** on **shower age vs $\log_{10}(E)$** data to find the **elemental spectra for H, He and heavy nuclei ($Z > 2$)**.

[T. Antoni et al., KASCADE Collab., Astrop. Phys. 24 (2005) 1-25]

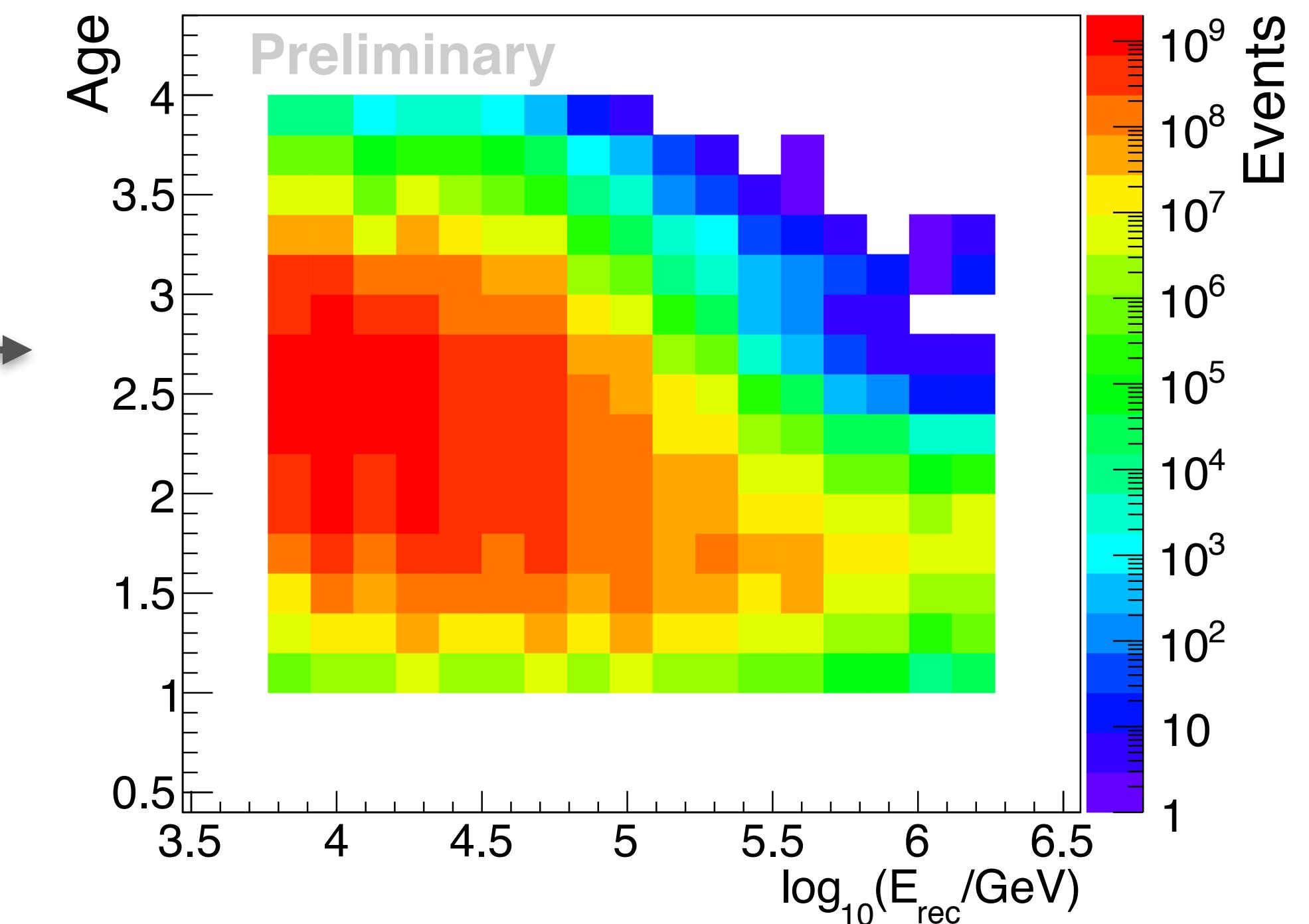
[R.Gold, Argonne National Laboratory Report ANL-6984, Argonne, 1964]

$$n(s, \lg E_{\text{rec}}) = \sum_A \sum_j P_A(s, \lg E_{\text{rec}} | \lg E_j) N_A(E_j)$$

↑
 $n(s, \lg E_{\text{rec}})$
HAWC 2D distribution of S vs E_{rec} .

$P_A(S, \lg E_{\text{rec}} | \lg E)$
MC response matrix of **mass group A.**

$N_A(E)$
True energy histogram of mass group A.

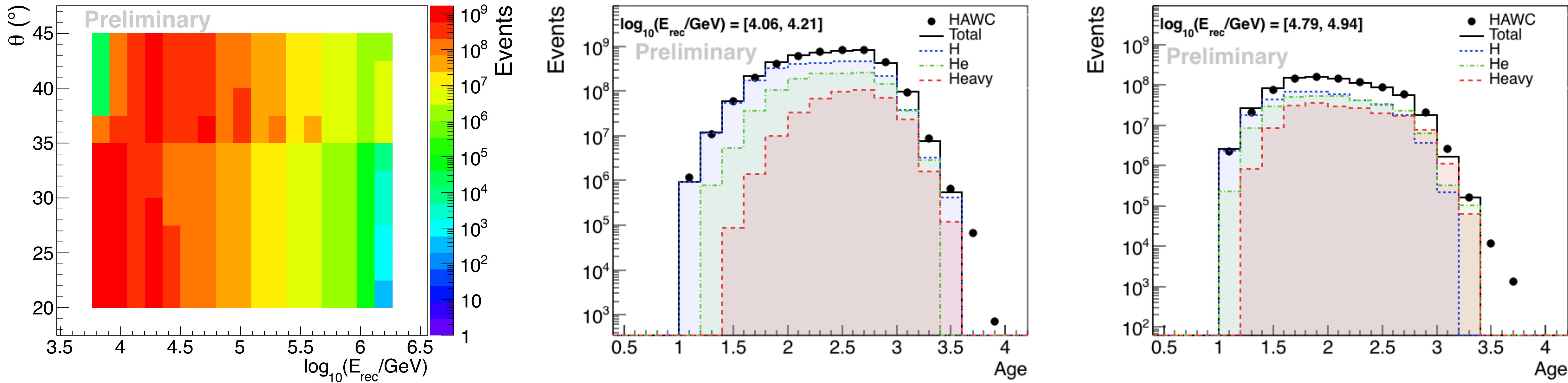


- $\Delta \log_{10}(E/\text{GeV}) = 0.15$
- $\Delta S = 0.2$

5) Unfolding of elemental mass groups

- ▶ Priors given by nominal composition model.
- ▶ Smoothing intermediate spectra with 353HQ-twice algorithm (ROOT-CERN libraries) and with a fit using a broken power law.
- ▶ **Stopping criterium:** Reduce the Chi2 for **Age vs Erec** and minimize Chi2 for **θ vs Erec** distributions.

$$\chi^2 = \frac{\sum_j (n_{\text{data}, j} - n_{\text{forward}, j})^2}{\sigma_{\text{stat}, j}^2 + \sigma_{\text{MC}, j}^2}$$



5) Unfolding of elemental mass groups

- Using the unfolded energy histograms, the elemental spectra are calculated as

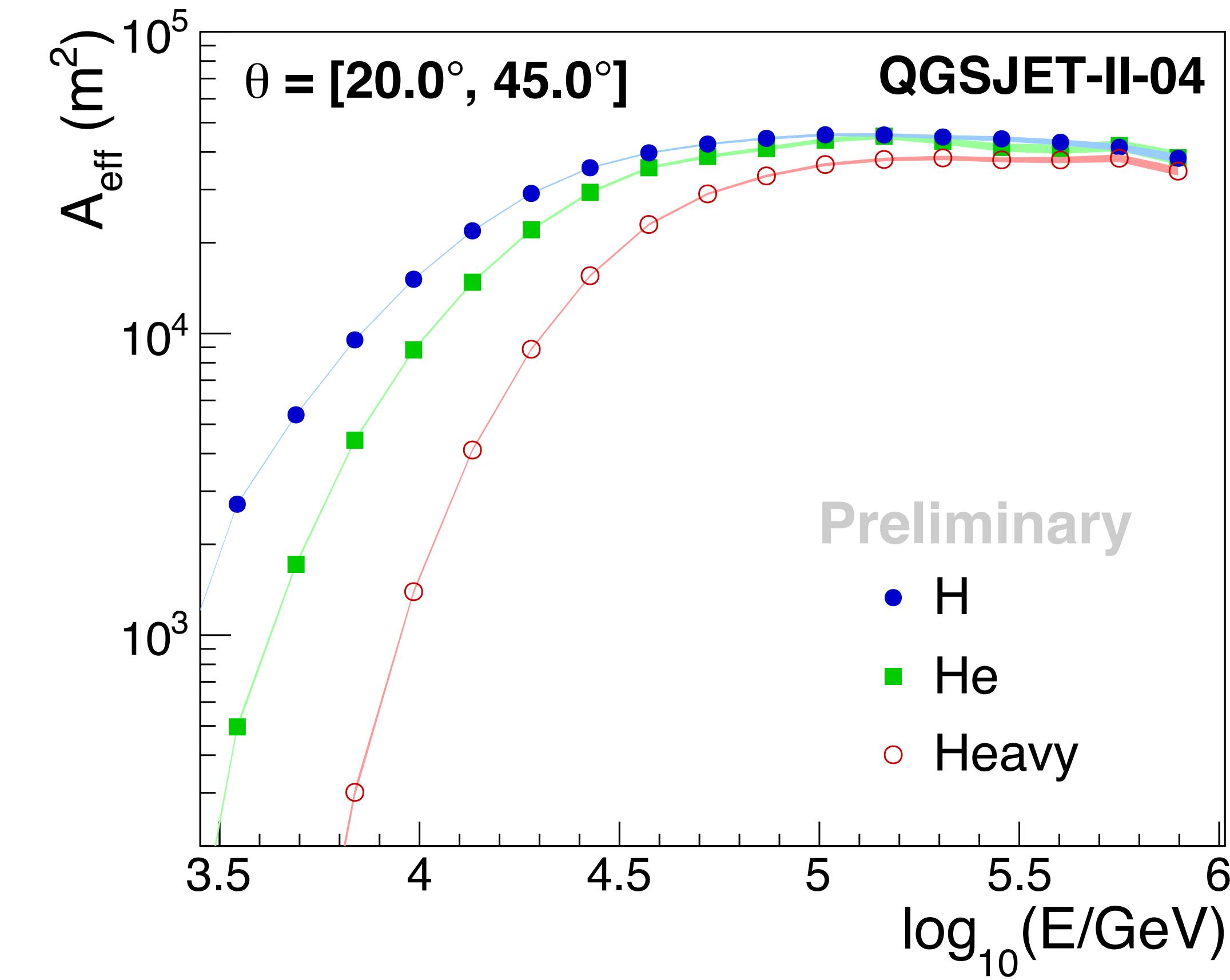
$$\Phi_A(E) = \frac{N_A(E)}{A_{\text{eff}} \Delta E T_{\text{eff}} \Delta \Omega}$$

A_{eff} = Effective area

ΔE = Energy bin

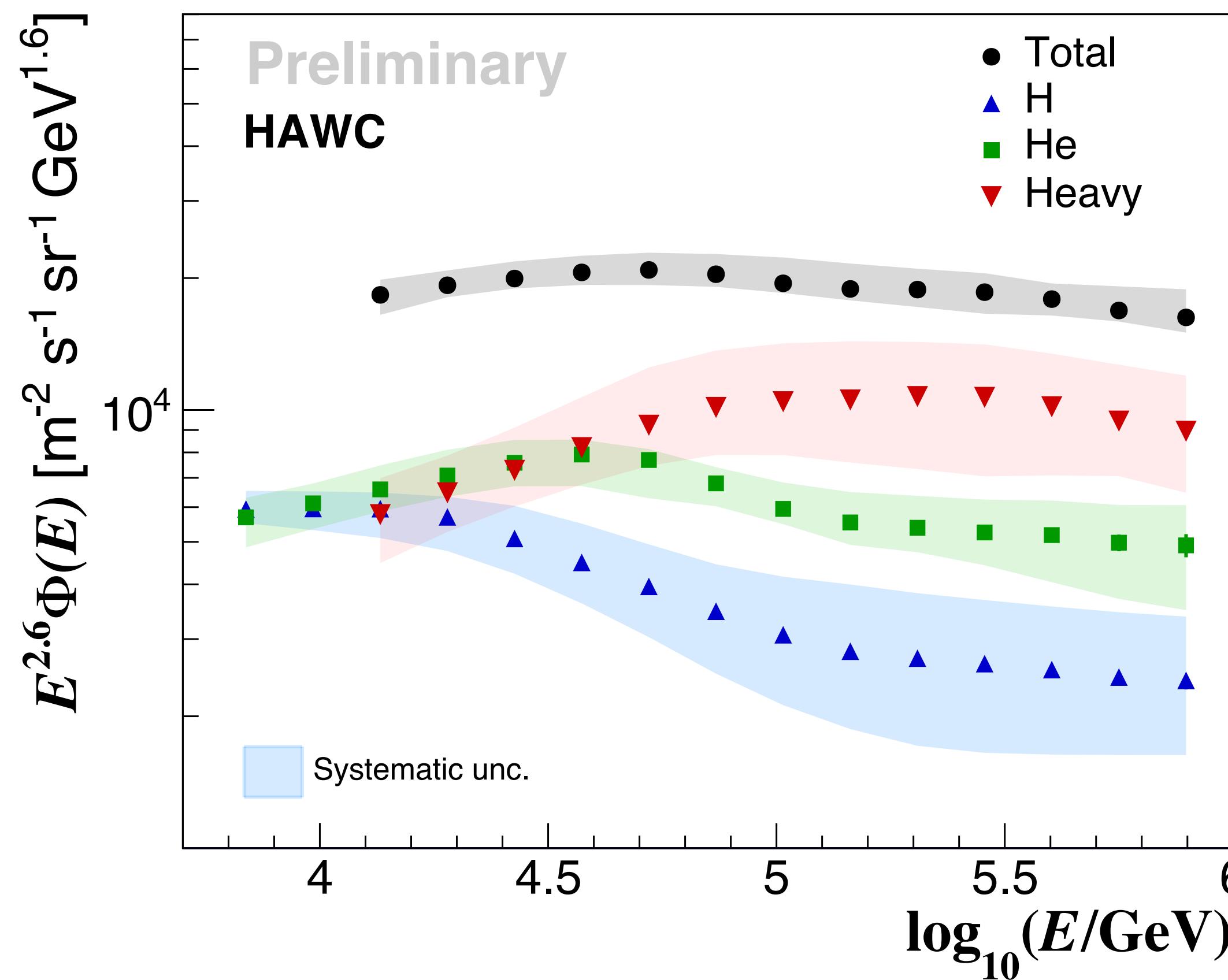
T_{eff} = Effective time

$\Delta \Omega$ = Solid angle



5) Unfolding of elemental mass groups

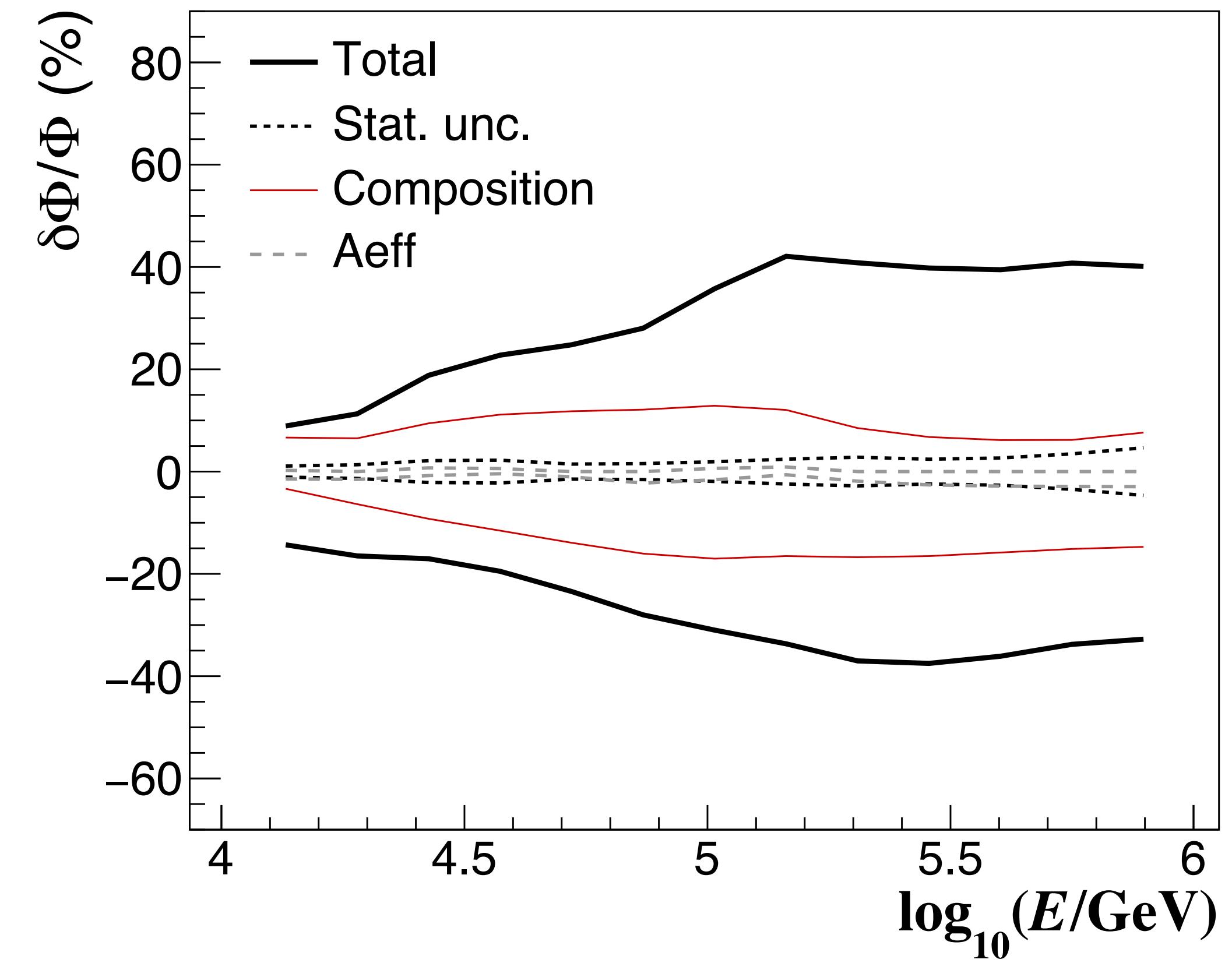
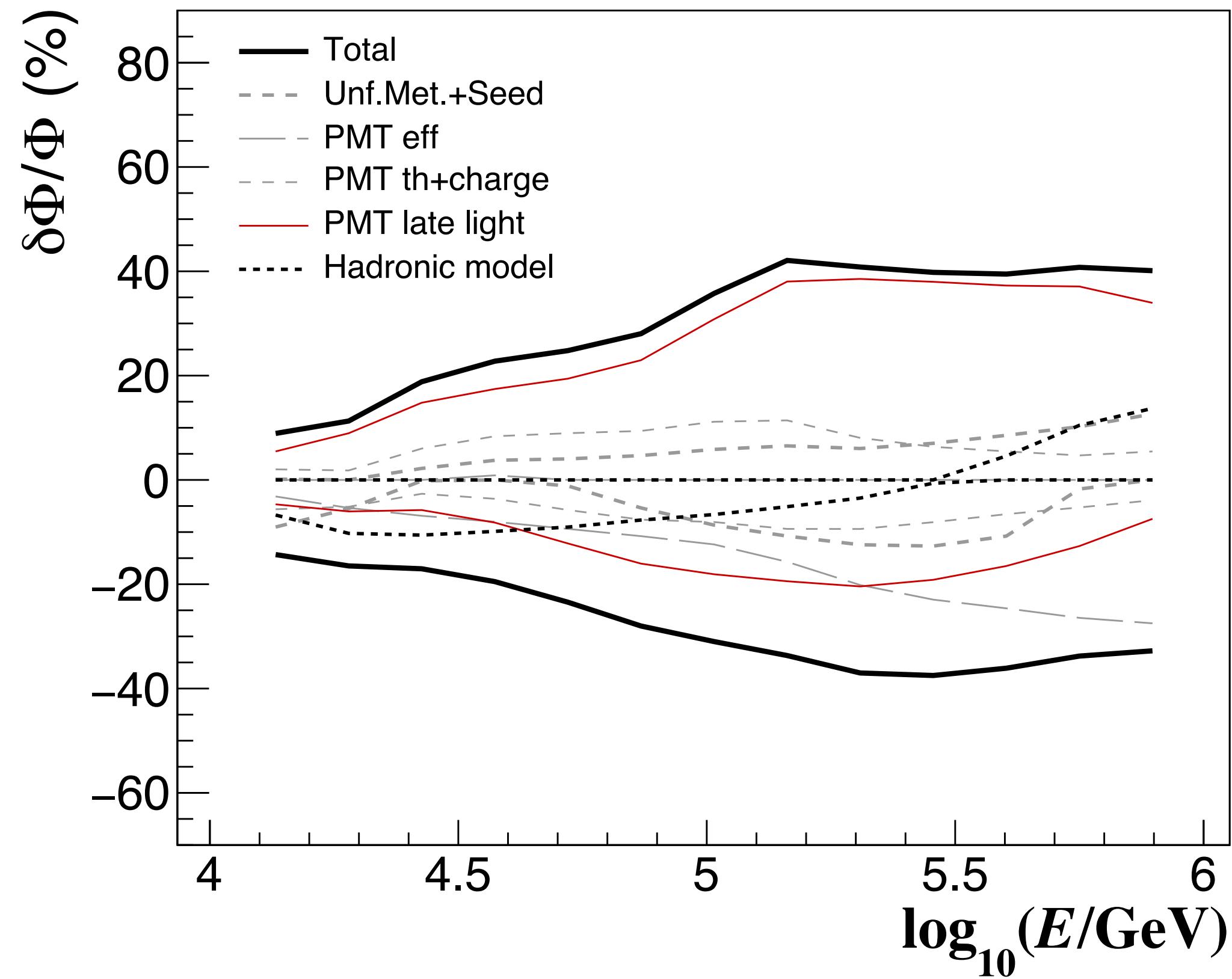
Results



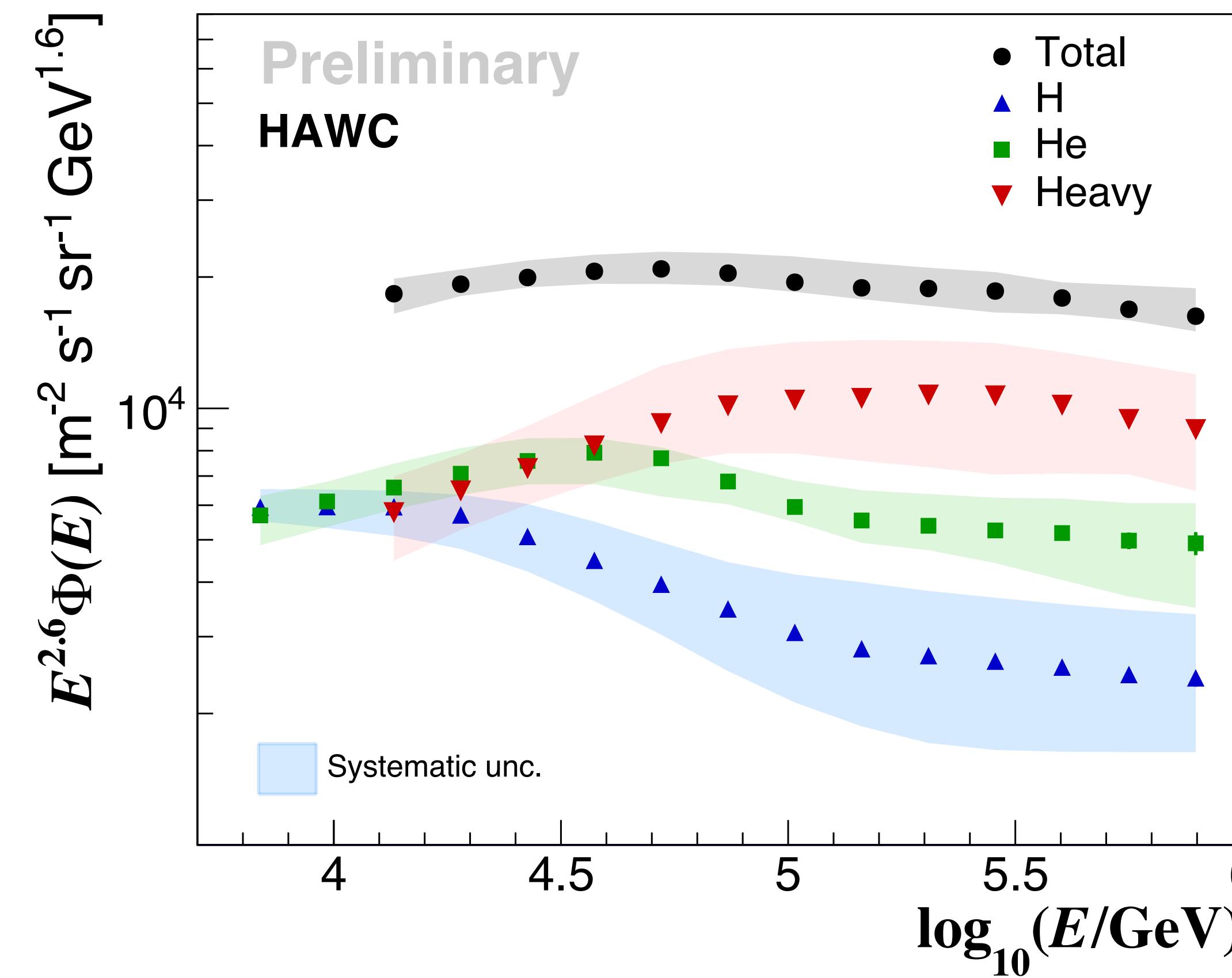
- **Statistical errors** < 6%.
 - Size of the experimental and MC data samples.
- **Systematic errors** < 42%
 - Uncertainties due to the PMT modeling (< 40%).
 - Hadronic interaction model: EPOS-LHC.
 - Unfolding procedure: *Priors* (Polygonato and GSF models), alternative unfolding algorithm (Bayesian method).
 - Effective area.
 - CR composition model: GSF, Polygonato, H3a, uncertainties in the nominal model

5) Unfolding of elemental mass groups

- Statistical and systematic uncertainties: H



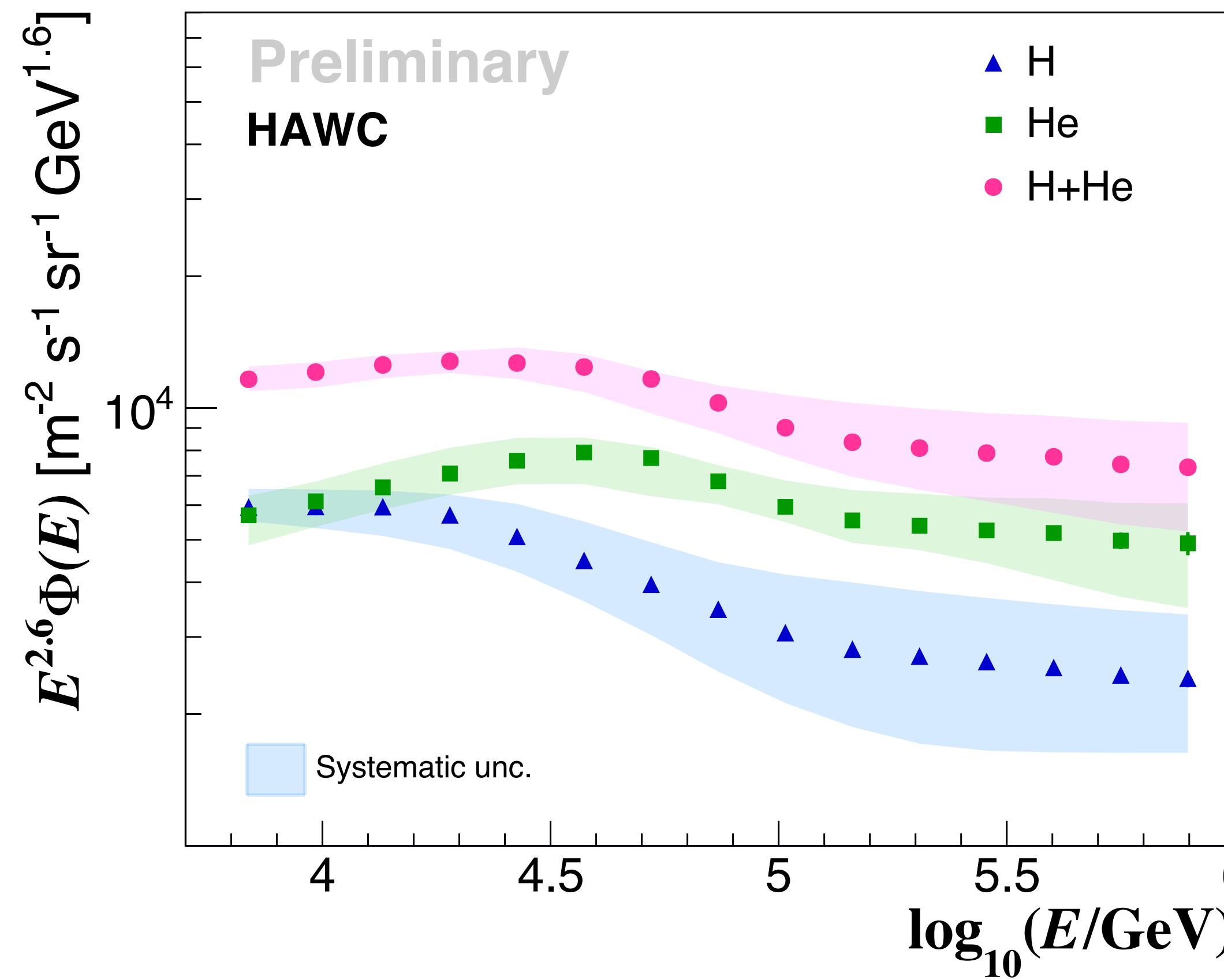
5) Unfolding of elemental mass groups



- The elemental spectra do not follow a power-law function.
- HAWC data show fine structure ($3\sigma, \text{stat}$) between 10 TeV and 1 PeV:
 - ▶ **Softenings** at $\mathcal{O}(10 \text{ TeV})$ for H, He and $Z > 2$.
 - ▶ **Break** at $\mathcal{O}(100 \text{ TeV})$ for $Z > 2$.
 - ▶ **Hardenings** close to 100 TeV for H and He.
- Composition becomes heavier from 10 TeV to 100 TeV.
- Bump in the all-particle spectrum at 46 TeV reported by HAWC in 2017 is due to the superposition of individual softenings in the spectra of light and heavy mass groups.

[HAWC Collab., PRD 96 (2017) 122001]

5) Unfolding of elemental mass groups



- We observe that the knee-like feature at 24 TeV in the spectrum of H+He observed by HAWC in 2022 comes from individual cuts in spectra for H and He.
[HAWC Collab., PRD 105 (2022) 063021]
- New feature in the spectrum of H+He nuclei: Hardening close to 100 TeV.
- $\Phi_H(E)/\Phi_{He}(E) < 1$ for $E = [10 \text{ TeV}, 1 \text{ PeV}]$.

5) Unfolding of elemental mass groups

Fit with a broken power-law (BL):

$$\Phi(E) = \Phi_0 E^{\gamma_0} \left[1 + \left(\frac{E}{E_0} \right)^{\varepsilon_0} \right]^{(\gamma_1 - \gamma_0)/\varepsilon_0} \left[1 + \left(\frac{E}{E_1} \right)^{\varepsilon_1} \right]^{(\gamma_2 - \gamma_1)/\varepsilon_1}$$

Sharpness of 1st break (fixed)

$$\varepsilon_0^H = 5$$

$$\varepsilon_0^{He} = 5$$

E_0 : Energy 1st break.

E_1 : Energy 2nd break.

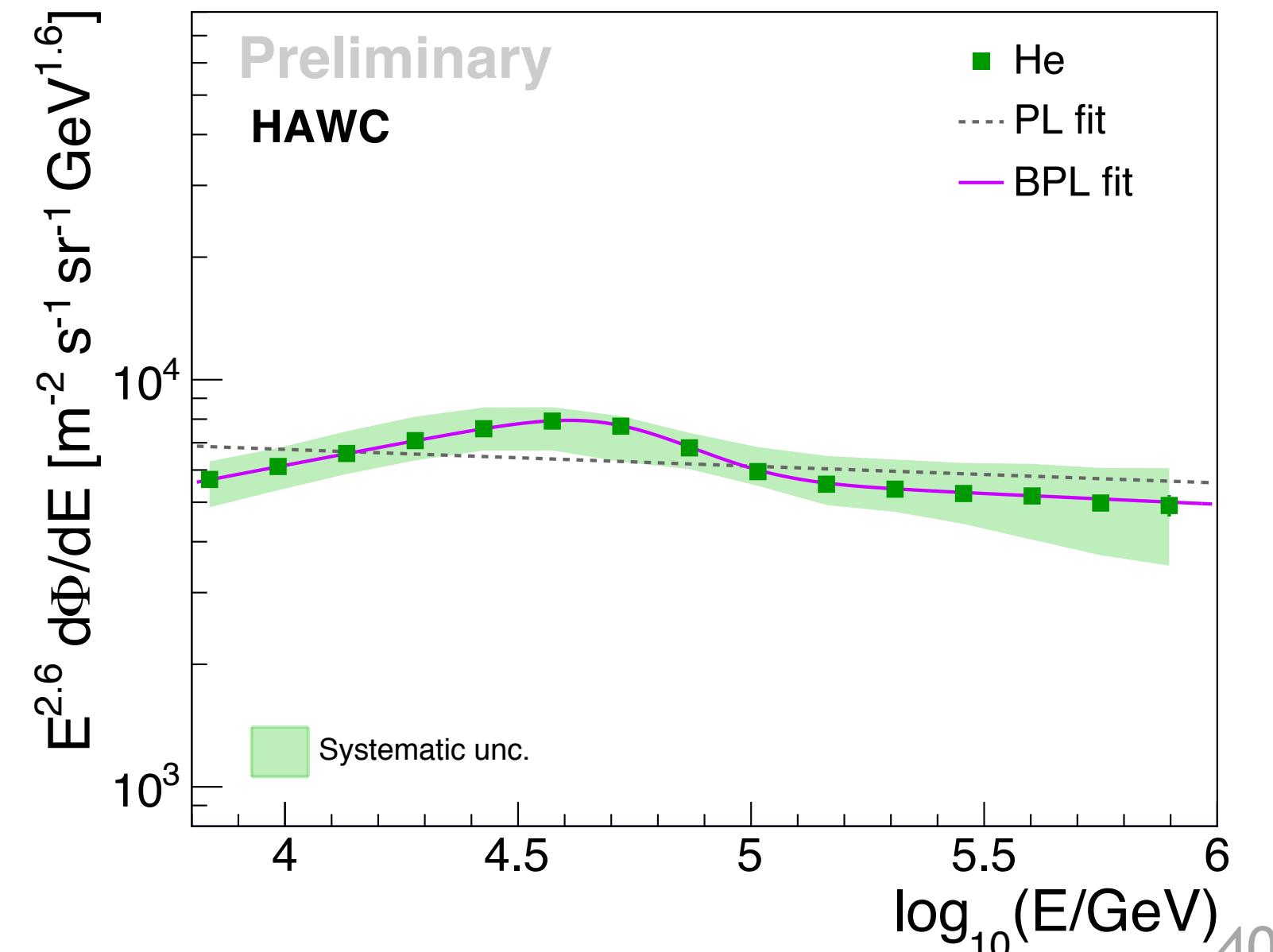
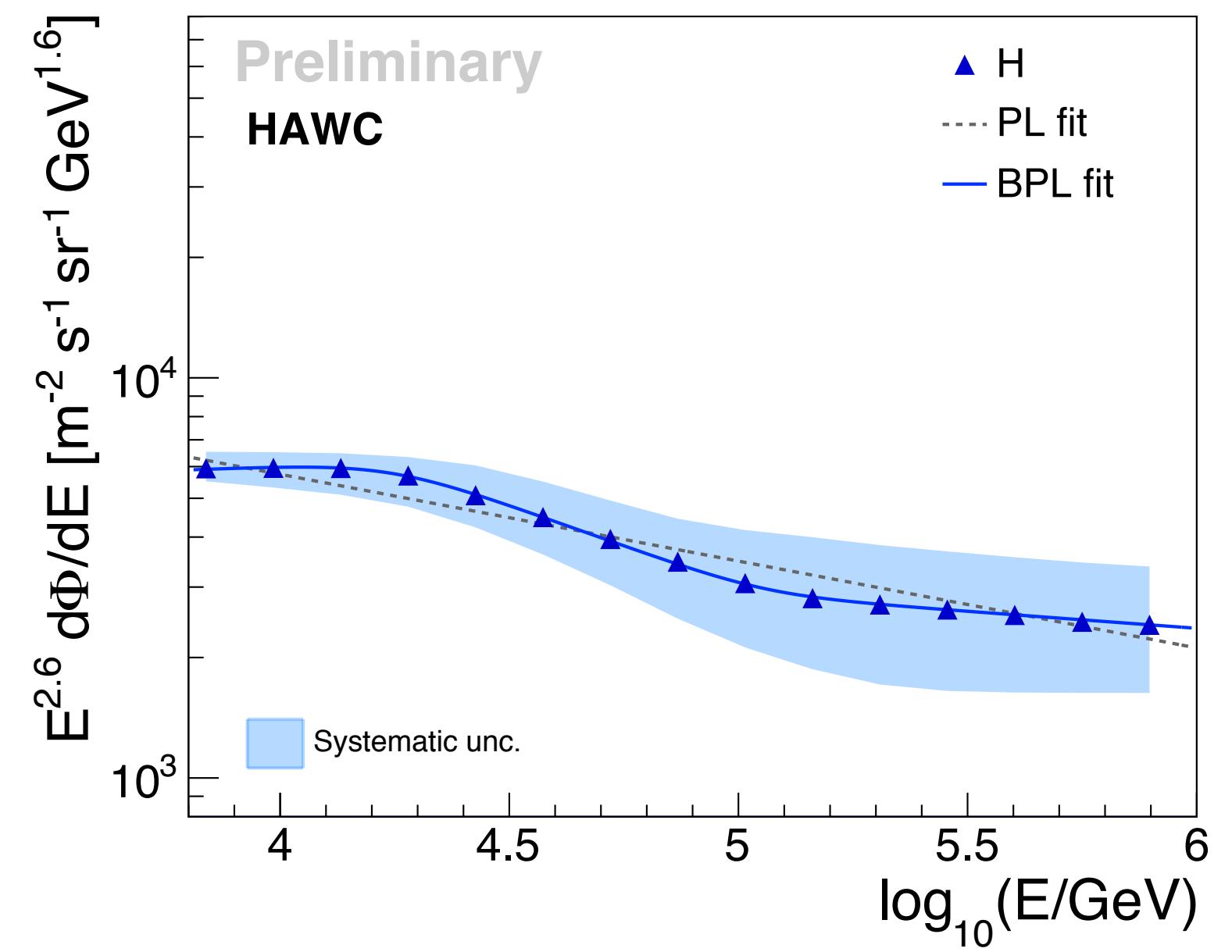
γ_1 : spectral index before E_0 .

γ_2 : spectral index after E_0 .

Best fit parameters

Mass group	E_0 (TeV)	E_1 (TeV)	γ_0	$\Delta\gamma_0 = \gamma_1 - \gamma_0$	$\Delta\gamma_1 = \gamma_2 - \gamma_1$
H	17.4 ± 0.6	$113.5^{+7.4}_{-7.0}$	-2.56 ± 0.01	-0.46 ± 0.01	0.33 ± 0.03
He	49.5 ± 1.3	$104.2^{+2.2}_{-2.1}$	-2.382 ± 0.004	-0.82 ± 0.03	0.55 ± 0.03

*PL: Power-law



5) Unfolding of elemental mass groups

Fit with a broken power-law (BL):

$$\Phi(E) = \Phi_0 E^{\gamma_0} \left[1 + \left(\frac{E}{E_0} \right)^{\varepsilon_0} \right]^{(\gamma_1 - \gamma_0)/\varepsilon_0} \left[1 + \left(\frac{E}{E_1} \right)^{\varepsilon_1} \right]^{(\gamma_2 - \gamma_1)/\varepsilon_1}$$

Sharpness of 1st break (fixed)

$$\varepsilon_0^H = 5$$

$$\varepsilon_0^{He} = 5$$

$$\varepsilon_0^{H+He} = 3$$

$$\varepsilon_0^{Heavy} = 10$$

E_0 : Energy 1st break.

E_1 : Energy 2nd break.

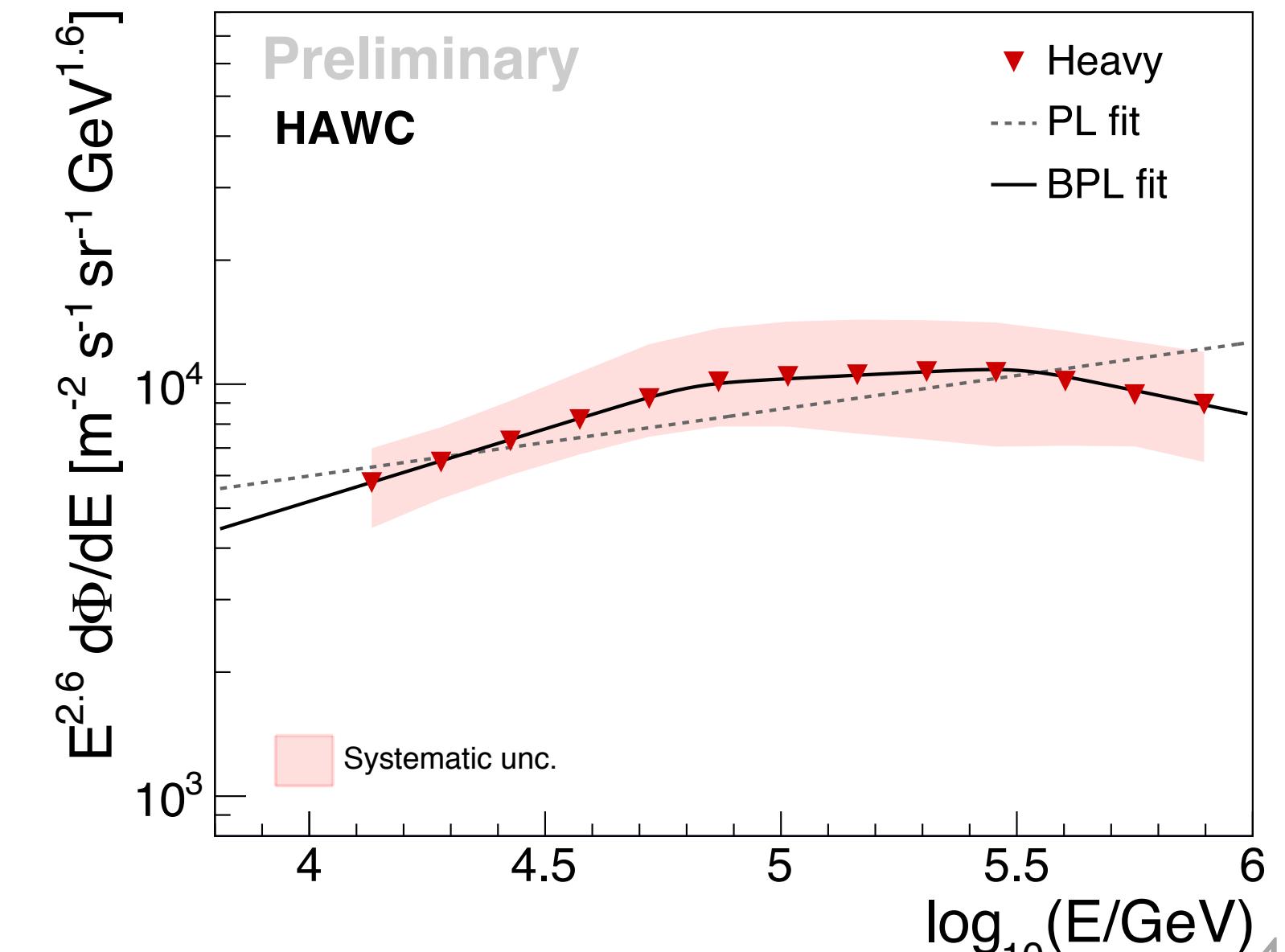
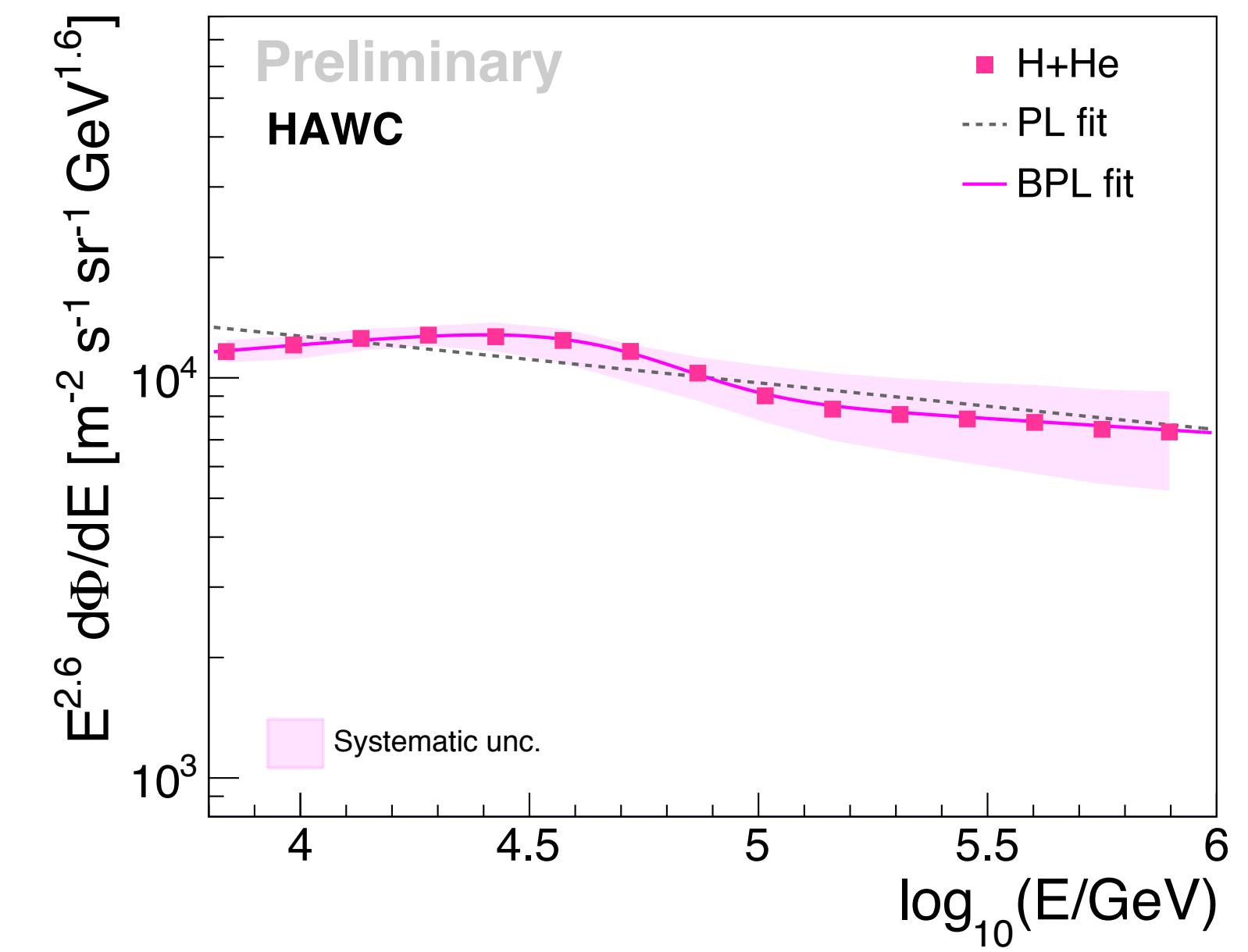
γ_1 : spectral index before E_0 .

γ_2 : spectral index after E_0 .

Best fit parameters

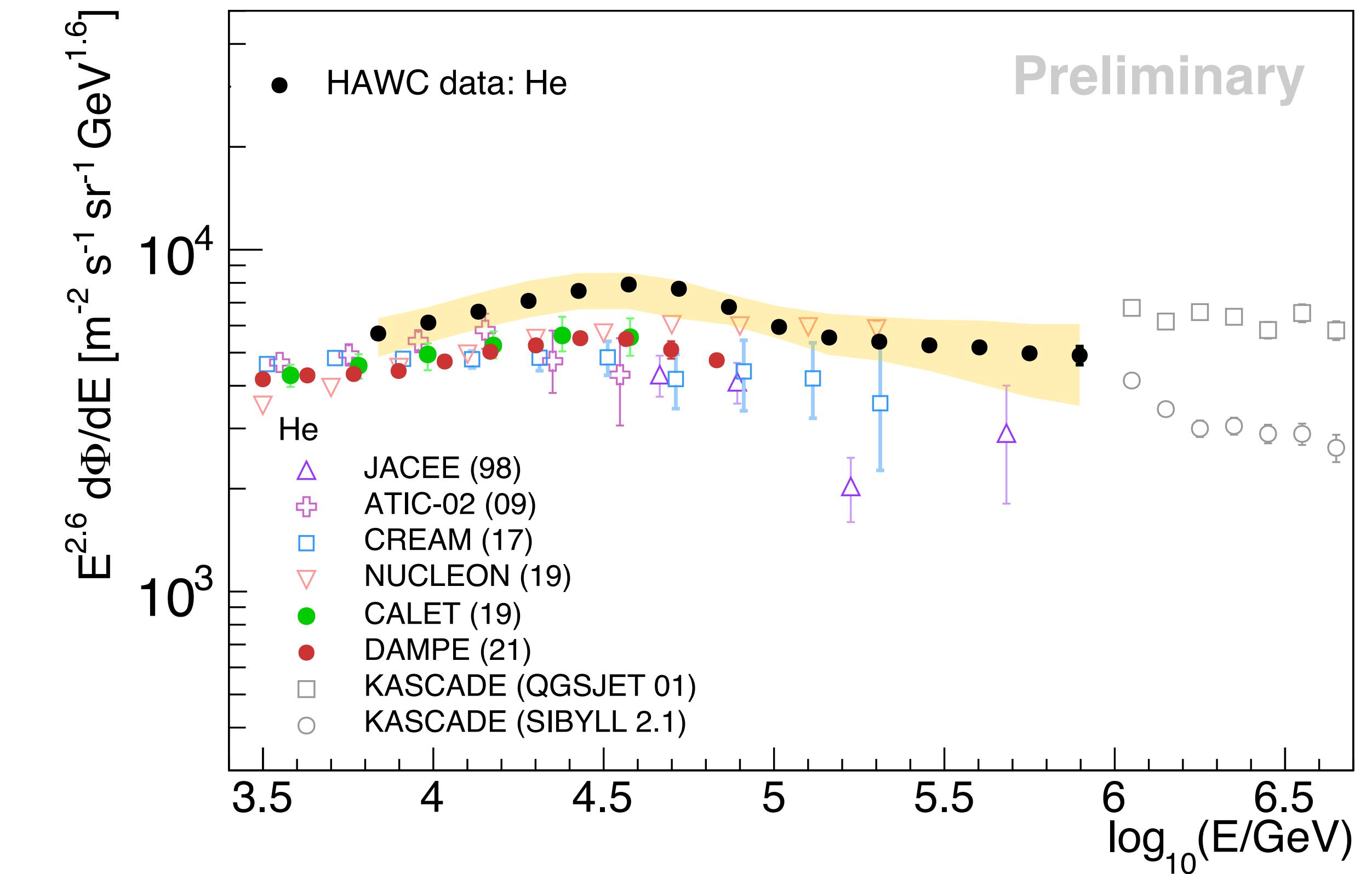
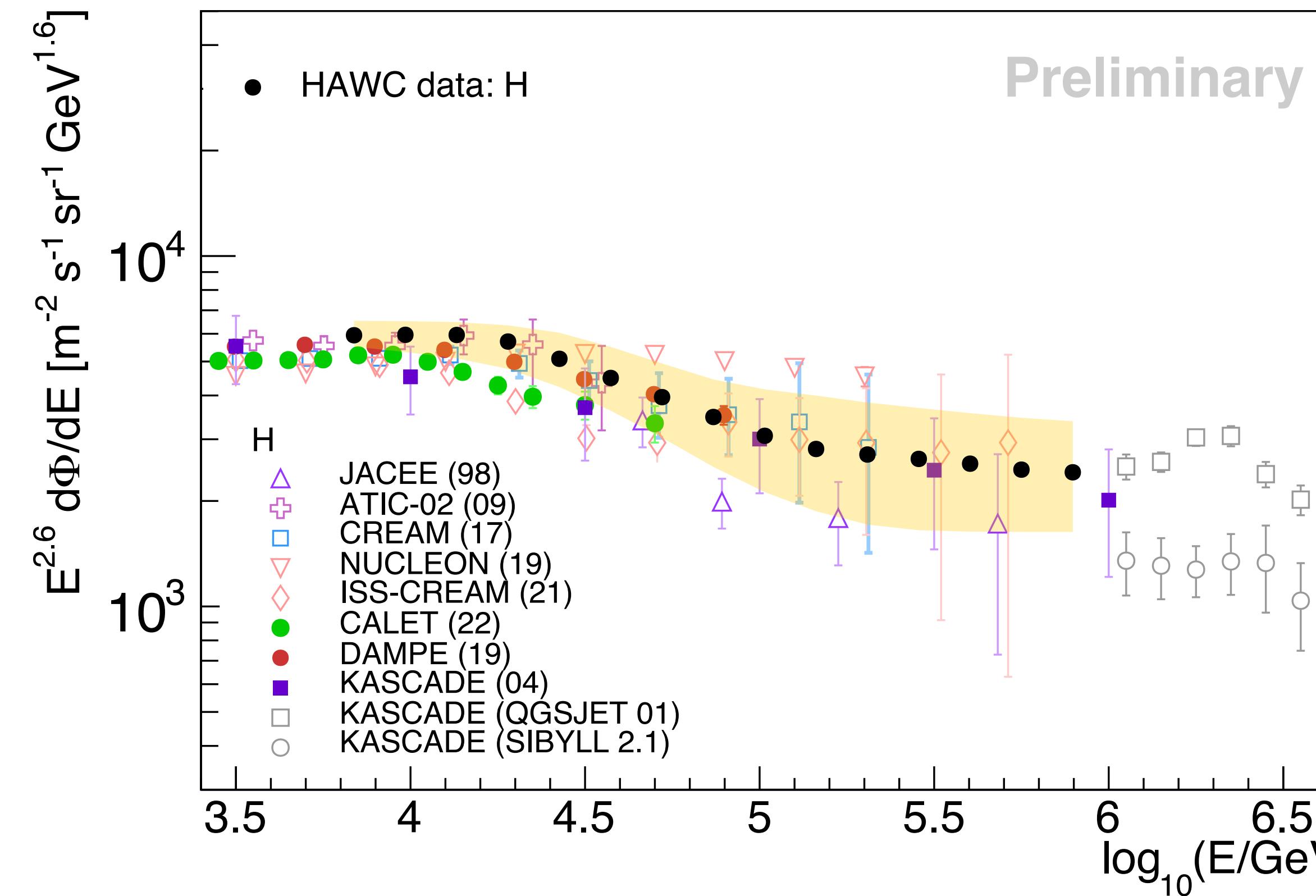
Mass group	E_0 (TeV)	E_1 (TeV)	γ_0	$\Delta\gamma_0 = \gamma_1 - \gamma_0$	$\Delta\gamma_1 = \gamma_2 - \gamma_1$
H	17.4 ± 0.6	$113.5^{+7.4}_{-7.0}$	-2.56 ± 0.01	-0.46 ± 0.01	0.33 ± 0.03
He	49.5 ± 1.3	$104.2^{+2.2}_{-2.1}$	-2.382 ± 0.004	-0.82 ± 0.03	0.55 ± 0.03
H+He	$51.2^{+3.8}_{-3.6}$	$90.3^{+9.3}_{-8.4}$	-2.506 ± 0.002	-0.79 ± 0.15	0.56 ± 0.16
Heavy	65.1 ± 1.3	$323.1^{+22.7}_{-21.2}$	-2.25 ± 0.01	-0.30 ± 0.01	-0.29 ± 0.02

*PL: Power-law



5) Unfolding of elemental mass groups

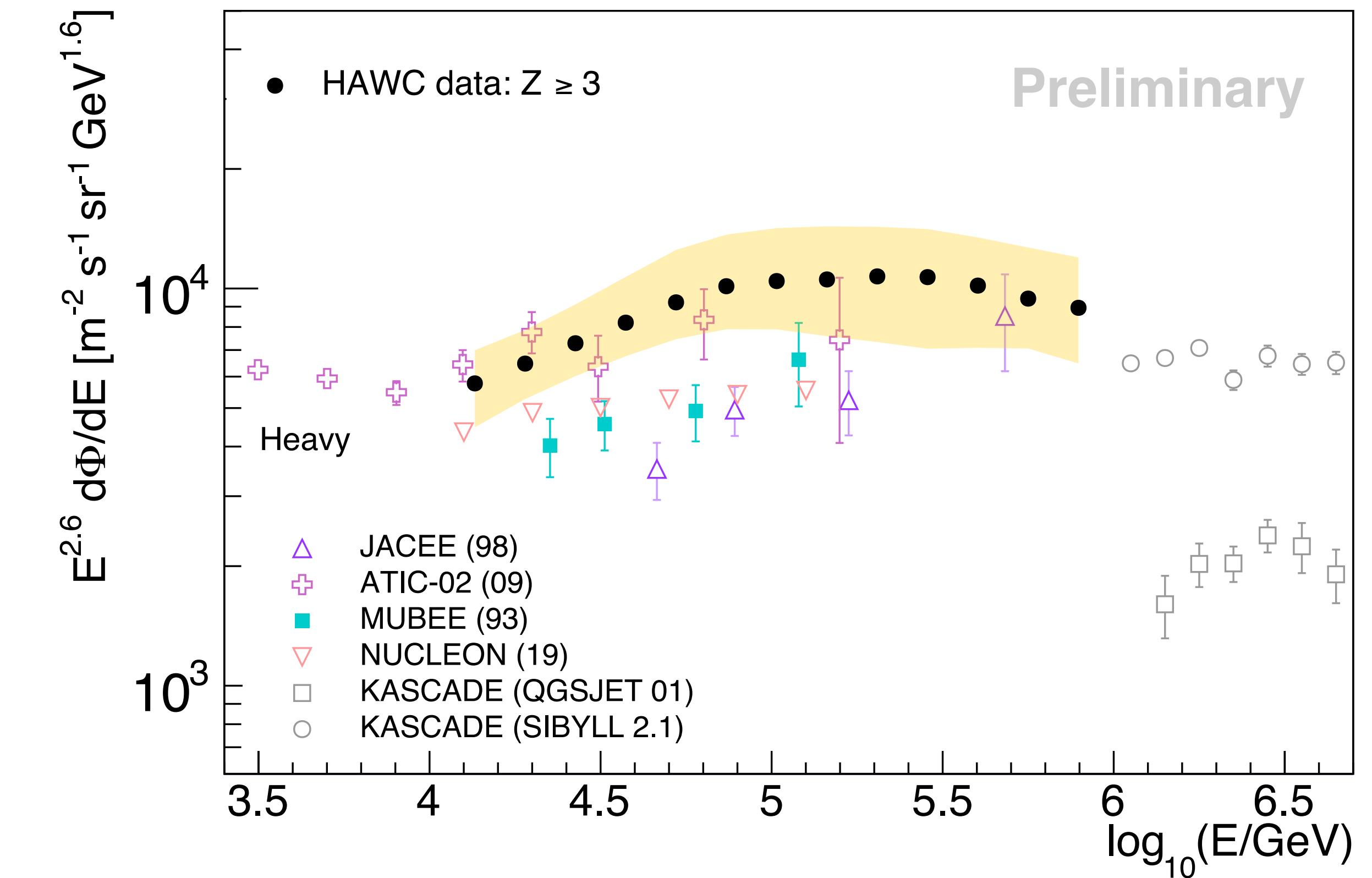
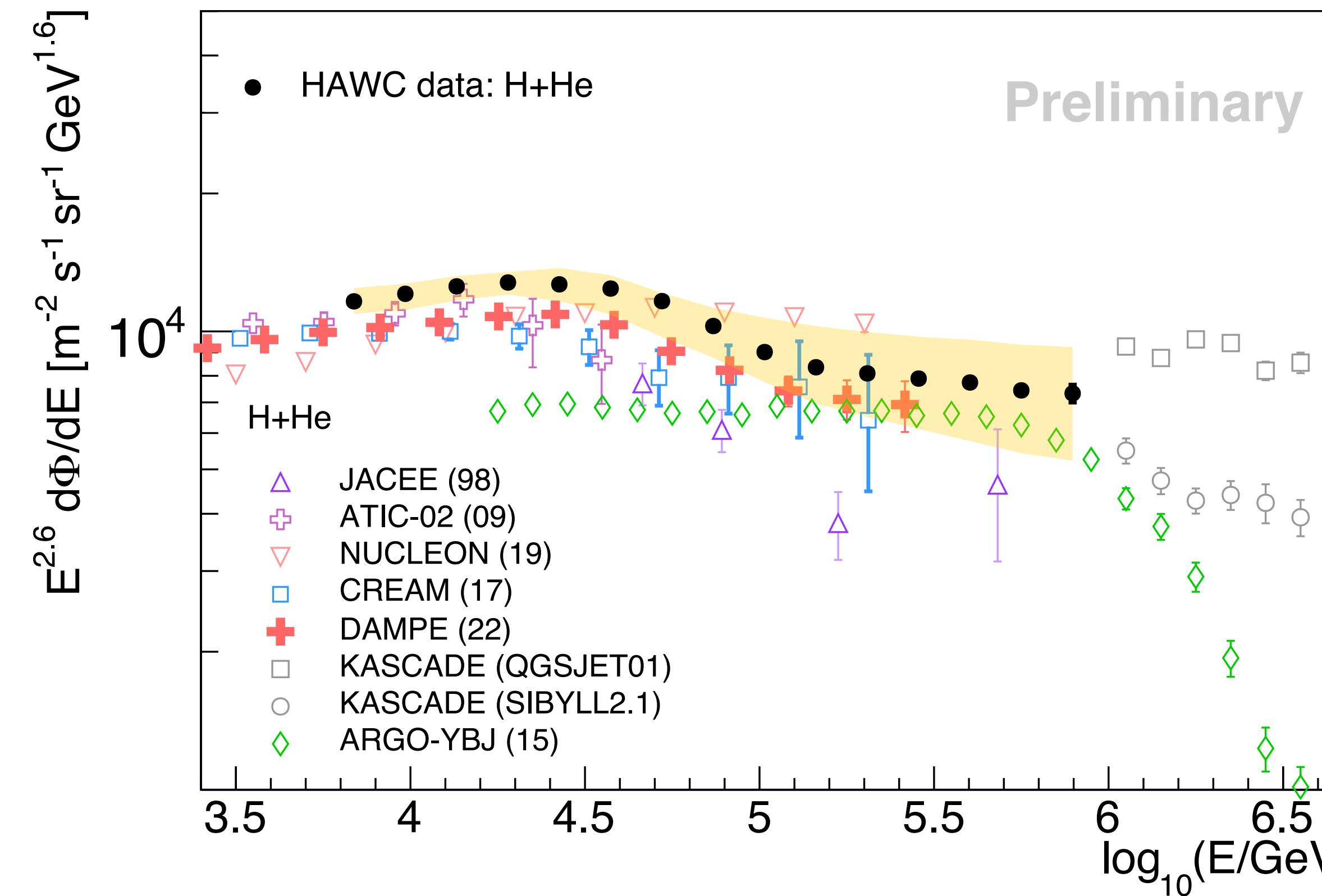
H and He spectra: Comparison with other experiments



- **Protons:** Good agreement of **HAWC** with direct data from **DAMPE**, **ATIC-02** and **CREAM I-III** within systematic errors.
- **Helium:** Good agreement of **HAWC** with **ATIC-02** at 10 TeV, but above other direct data.
- **HAWC** confirms softenings at tens of TeV observed by **DAMPE** and **CALET**, first hinted by **ATIC-02**, **CREAM** and **NUCLEON**.

5) Unfolding of elemental mass groups

Light (H + He) and Heavy ($Z > 2$) spectra: Comparison with other experiments



- Good agreement of **HAWC** with **ATIC-02** at low energies.
- ARGO-YBJ** disagrees with **HAWC** data for $E < 100$ TeV.

- Agreement of **HAWC** with **ATIC-02** within systematic errors.
- HAWC** data is above **NUCLEON**, **MUBEE** and **JACEE** observations.

6) Summary

- We have estimated the elemental energy spectra for H, He and heavy nuclei ($Z > 2$) with HAWC for $E = [10 \text{ TeV}, 1 \text{ PeV}]$.
- HAWC results reveal individual softenings at tens of TeV in the spectra of the different mass groups of cosmic rays, whose positions seem to move to higher energies for heavy primaries.
- HAWC shows hardenings close to 100 TeV in the spectra of H and He and a softening at 323 TeV in the heavy component of cosmic rays.
- HAWC confirms the TeV knee-like features observed by DAMPE and CALET for the spectra of H and He.
- Cosmic ray composition becomes heavier at high energies within the range 10 - 100 TeV.
- In an independent way, we have estimated the spectrum of H+He CR nuclei, which confirms the presence of a softening at tens of TeV.

Backup

Gold's unfolding method

Steps:

1) Apply a Van Cittert's transformation: To obtain a positive definite solution

That is done by multiplying the equation system by P^T from the right hand side

$$P^T N = P^T P \Phi$$

2) Introduce statistical errors in the equation using an extended response matrix instead of P

$$\tilde{P} = CP$$

by using an error matrix $C(m \times m)$ defined as

$$C_{ij} = \delta_{ij}/\sigma_i$$

$$\sigma_i = 1/\sqrt{n_i}$$

and replacing N by

$$\tilde{N} = CN$$

Poisson distribution

3) After applying the above substitutions

$$N_{mod} = R_{mod} \Phi$$

With

$$N_{mod} = (CP)^T CN$$

$$R_{mod} = (CP)^T (CP)$$

Gold's unfolding method

Steps:

4) Φ is found iteratively using the set of equations:

$$\Phi^{k+1}_i = \frac{\Phi^k_i n_{\text{mod}, i}}{\sum_{j=1} R_{\text{mod}, ij} \Phi^k_j}$$

Where

k: iteration

Note:

- As in reference R. Alfaro., PRD 96 (2017), 122001, we have smoothed the Φ^k solution in each iterative step, but at the last iteration.