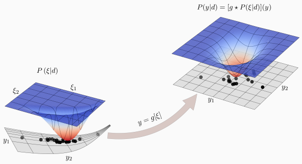
Signal reconstruction for fields

WITH PROBABILISTIC FORWARD MODELING



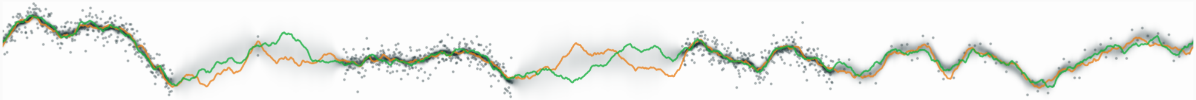
Philipp Frank[1]

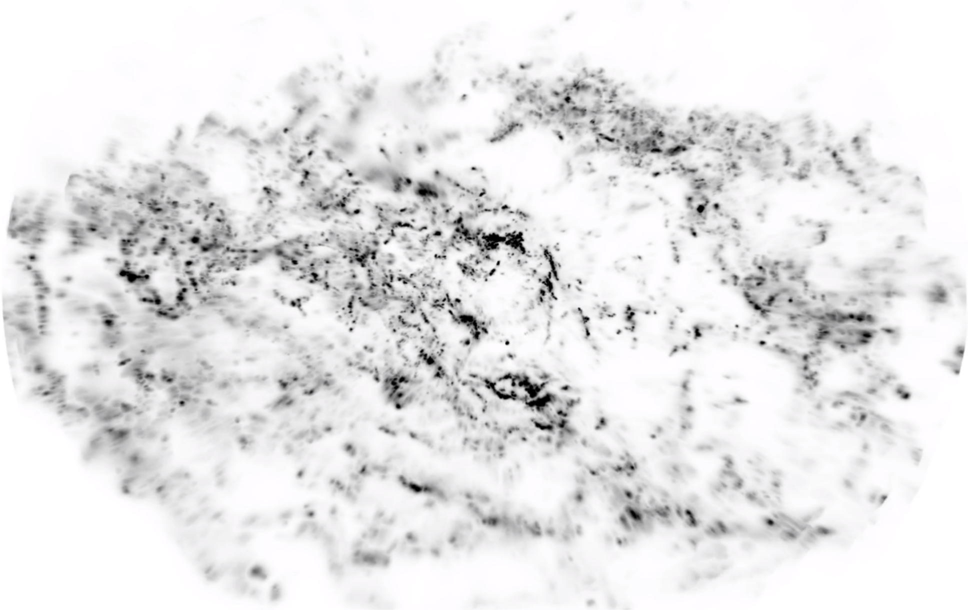
Institute for Astroparticle Physics, Karlsruhe Institute of Technology, Karlsruhe, Germany

June 27, 2024

Mail: philipp@mpa-garching.mpg.de, Web: www.ph-frank.de

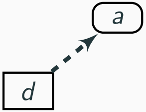
(1) Max-Planck Institute for Astrophysics MPA, Garching, Germany





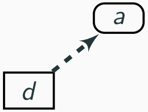
Probabilistic Estimators

Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic (Bayesian) Estimators



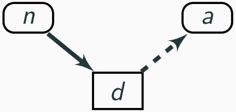
Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\,(d; M) \qquad .$$

With: $d =$ Data, $\quad M =$ Model.

Probabilistic (Bayesian) Estimators



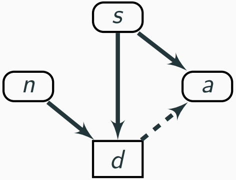
Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) = \int a\, \mathcal{P}\left(a|d, M\right) \mathrm{d}a \qquad .$$

With: d = Data, M = Model.

Probabilistic (Bayesian) Estimators



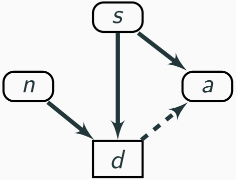
Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) = \int a\,\mathcal{P}\left(a|d, M\right)\mathrm{d}a = \int a\left(s\right)\ \mathcal{P}\left(s|d, M\right)\mathrm{d}s\ .$$

With: $d =$ Data, $s =$ Signal, $M =$ Model.

Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

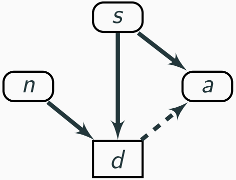
$$\hat{a} = E\,(d; M) \; = \int a\,\mathcal{P}\,(a|d, M)\,\mathrm{d}a = \int a\,(s)\;\; \mathcal{P}\,(s|d, M)\,\mathrm{d}s\ .$$

With: $d =$ Data, $s =$ Signal, $M =$ Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}\,(s|d, M) = \frac{\mathcal{P}\,(s, d|M)}{\int \mathcal{P}\,(s, d|M)\ \mathrm{d}s} \qquad .$$

Probabilistic (Bayesian) Estimators



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Probabilistic estimator

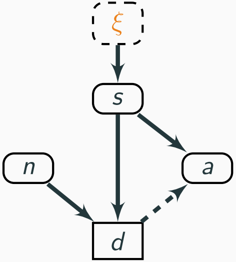
$$\hat{a} = E\left(d; M\right) = \int a\, \mathcal{P}\left(a|d, M\right) \mathrm{d}a = \int a\left(s\right)\ \mathcal{P}\left(s|d, M\right) \mathrm{d}s\ .$$

With: $d =$ Data, $s =$ Signal, $M =$ Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}\left(s|d, M\right) = \frac{\mathcal{P}\left(s, d|M\right)}{\int \mathcal{P}\left(s, d|M\right)\ \mathrm{d}s} = \frac{\overbrace{\mathcal{P}\left(d|s\qquad, M\right)}^{\text{Likelihood}}\ \overbrace{\mathcal{P}\left(s|M\right)}^{\text{Prior}}}{\int \mathcal{P}\left(s, d|M\right)\ \mathrm{d}s}\ .$$

Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) = \int a\, \mathcal{P}\left(a|d, M\right) \mathrm{d}a = \int a\left(s\right)\; \mathcal{P}\left(s|d, M\right) \mathrm{d}s\ .$$

With: d = Data, s = Signal, M = Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}\left(\xi|d, M\right) = \frac{\mathcal{P}\left(\xi, d|M\right)}{\int \mathcal{P}\left(\xi, d|M\right)\ \mathrm{d}\xi} = \frac{\overbrace{\mathcal{P}\left(d|s\left(\xi\right), M\right)}^{\text{Likelihood}}\ \overbrace{\mathcal{N}\left(\xi; 0, \mathbb{1}\right)}^{\text{Prior}}}{\int \mathcal{P}\left(\xi, d|M\right)\ \mathrm{d}\xi}\ .$$

With: ξ = Parameters.

Priors - Gaussian & generative processes

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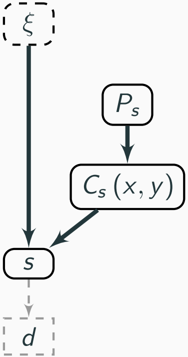
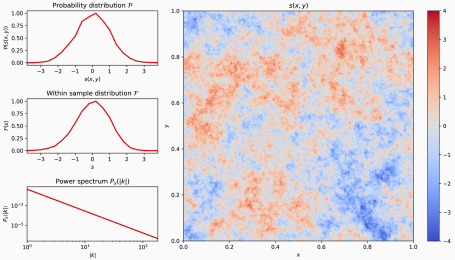
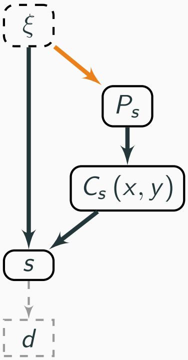
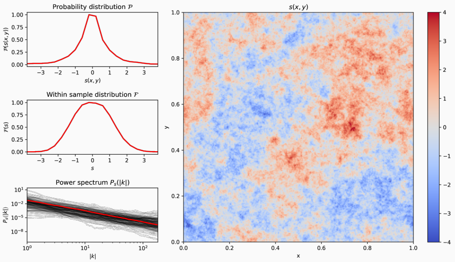


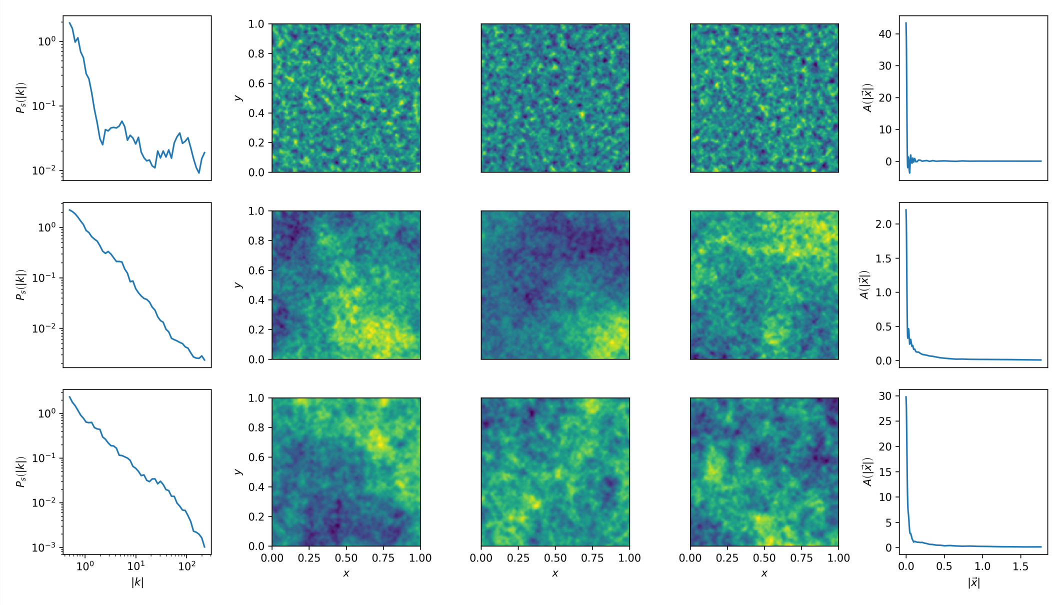


Priors - Gaussian & generative processes

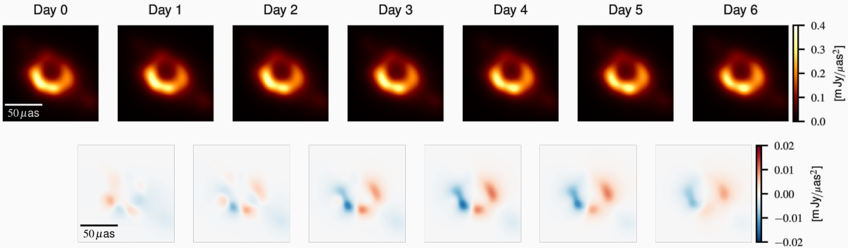


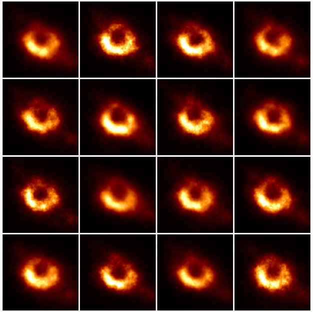


Priors - Gaussian & generative processes



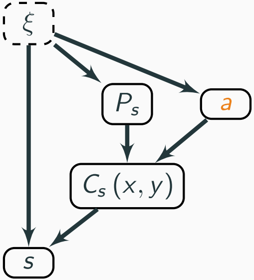
Priors - VLBI imaging of M87[1]

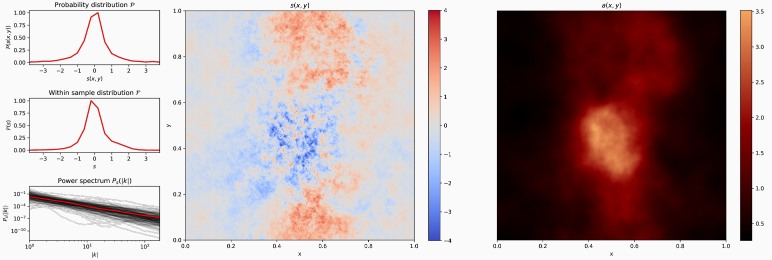




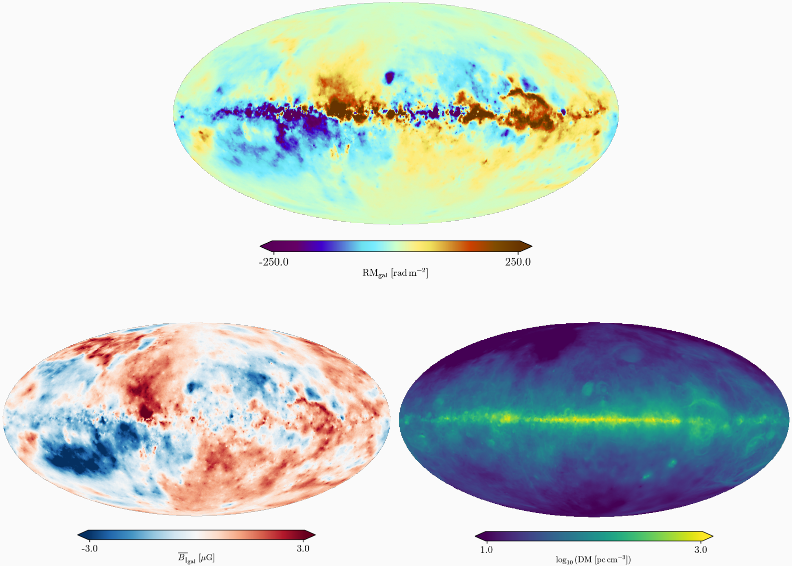
[1]Arras, Philipp Frank, Haim, et al. 2022.

Priors - Gaussian & generative processes



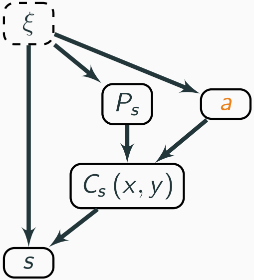


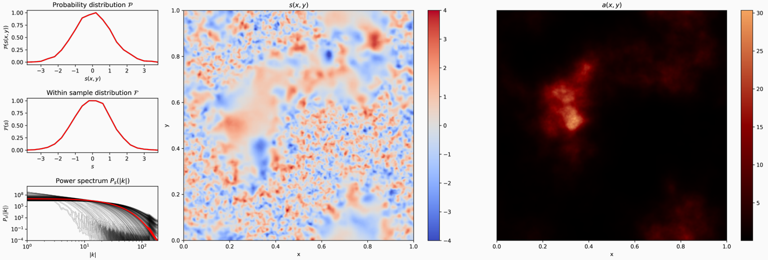
Priors - Faraday tomography[2]



[2]Hutschenreuter, Haverkorn, Philipp Frank, et al. 2023.

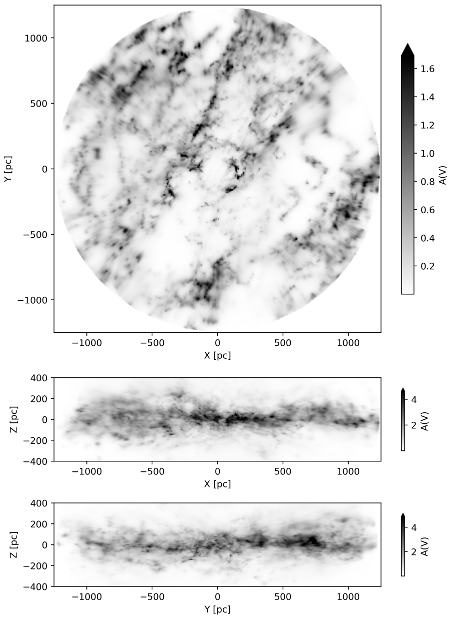
Priors - Gaussian & generative processes





Priors - GAIA 3D dust tomography[3]

- HEALPix angular + Log-radial grid
- Log-Normal process on $\sim 0.66 \times 10^9$ voxels
- $\sim 80 \times 10^6$ measurements



[3] Edenhofer, Zucker, Philipp Frank, et al. 2024.

Likelihood - Instrument description

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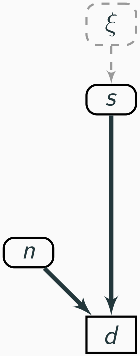
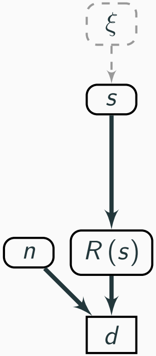
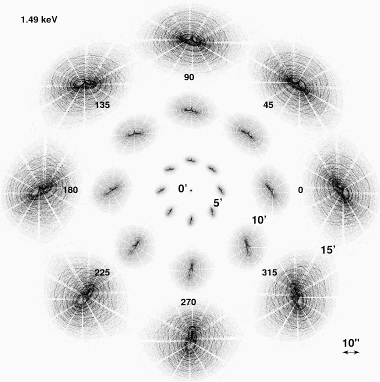


[4] Credit: https://cxc.harvard.edu/

Likelihood - Instrument description

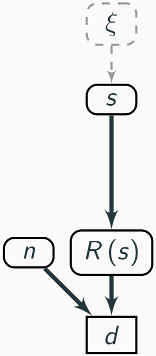


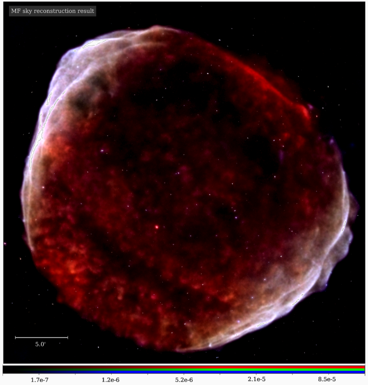


Simulated Chandra PSF[4]

[4] Credit: https://cxc.harvard.edu/

Likelihood - Instrument description

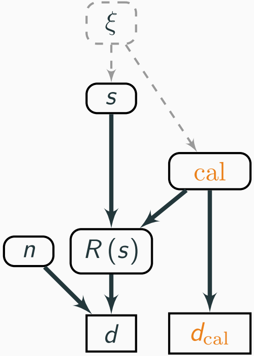


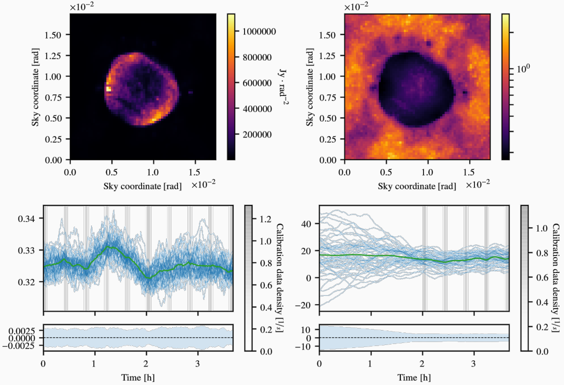


SN1006 from Chandra data[5]

[5] Westerkamp, Eberle, M. Guardiani, et al. 2024.

Likelihood - Instrument description

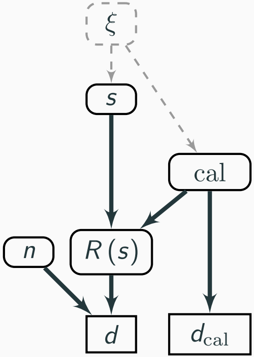


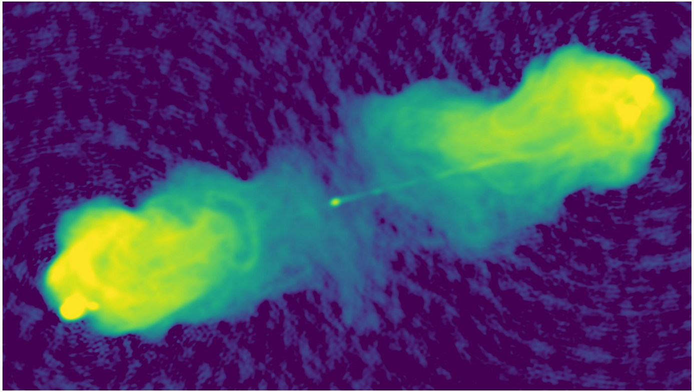


SN1006 from VLA data[6]

[6]Arras, Philipp Frank, Leike, et al. 2019.

Likelihood - Instrument description

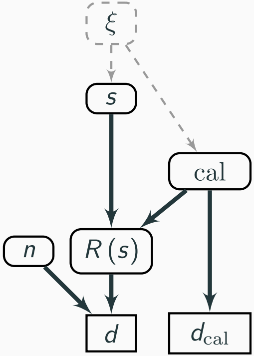


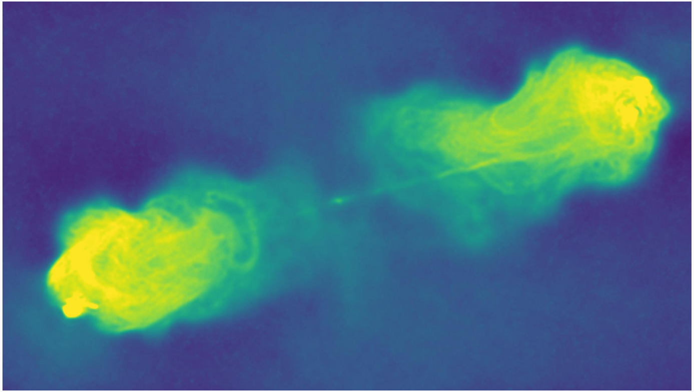


CygnusA with MSClean[7]

[7] Roth, Arras, Reinecke, et al. 2023.

Likelihood - Instrument description

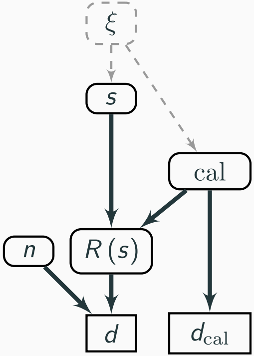


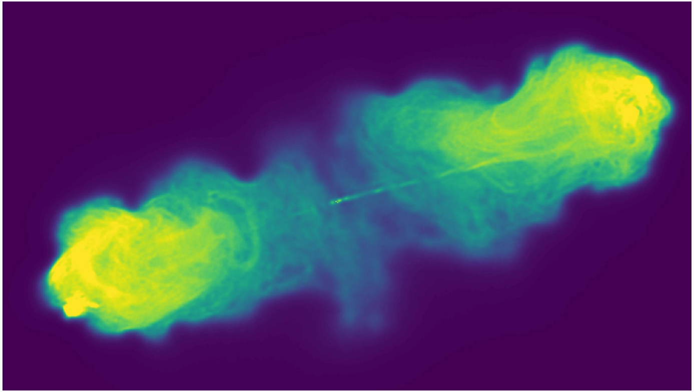


CygnusA with Resolve[7]

[7] Roth, Arras, Reinecke, et al. 2023.

Likelihood - Instrument description

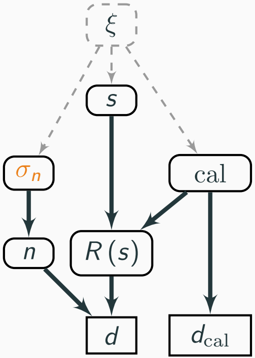


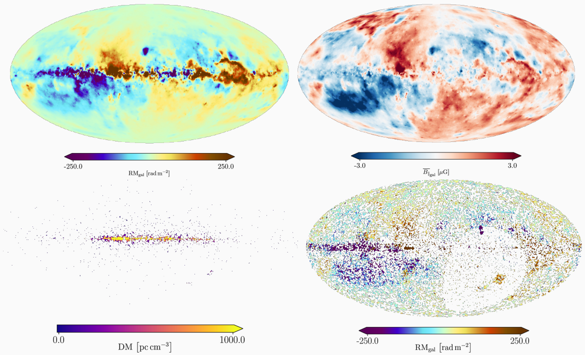


CygnusA with DDE-Resolve[7]

[7] Roth, Arras, Reinecke, et al. 2023.

Likelihood - Instrument description

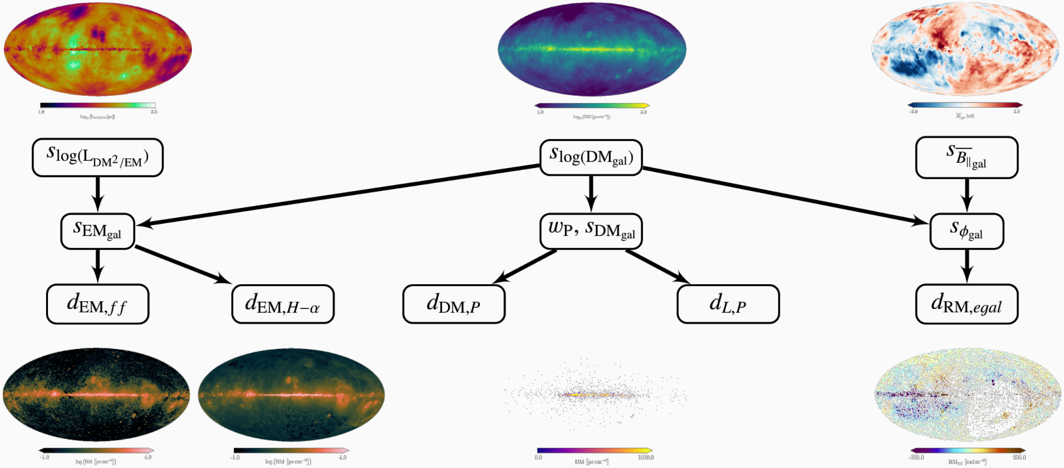




Faraday sky using rotation and dispersion measure data[8]

[8] Hutschenreuter, Haverkorn, Philipp Frank, et al. 2023.

Likelihood - Faraday tomography[8]



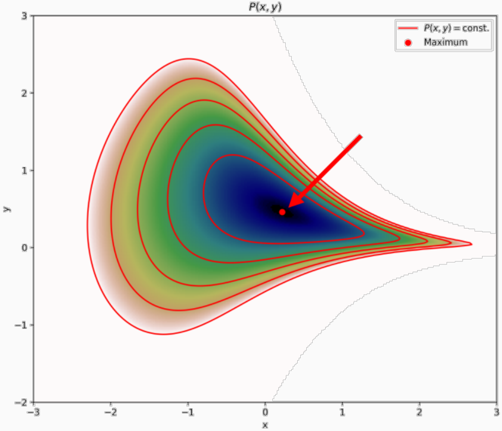
[8] Hutschenreuter, Haverkorn, Philipp Frank, et al. 2023.

Approximate Inference

Posterior expectation

$$\langle f(\xi)\rangle_{\mathcal{P}(\xi|d)} = \int f \, \mathrm{d}\mathcal{P} = \int f(\xi)\, \mathcal{P}(\xi|d)\, \mathrm{d}\xi$$

Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.

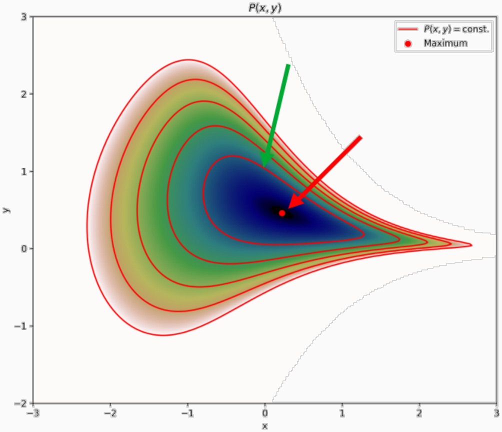


Approximate Inference

Posterior expectation

$$\langle f(\xi)\rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi)\; \mathcal{P}(\xi|d)\; \mathrm{d}\xi$$

Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.

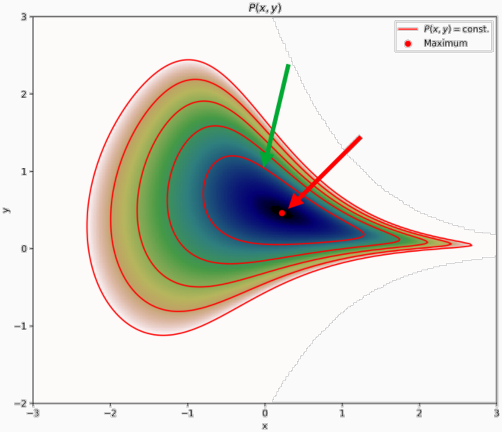


Approximate Inference

Posterior expectation

$$\langle f(\xi) \rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi) \; \mathcal{P}(\xi|d) \; \mathrm{d}\xi \approx \int f(\xi) \; \mathcal{Q}(\xi|d) \; \mathrm{d}\xi$$

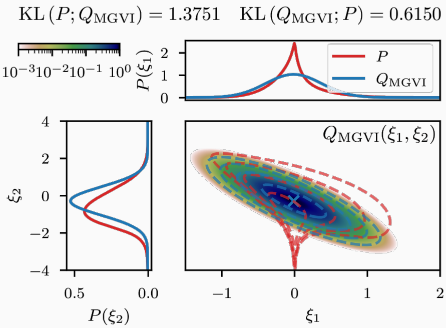
Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.



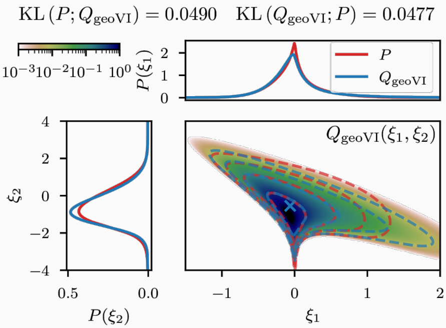
Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right) \mathcal{Q}_\sigma(\xi)\, \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.



[9]

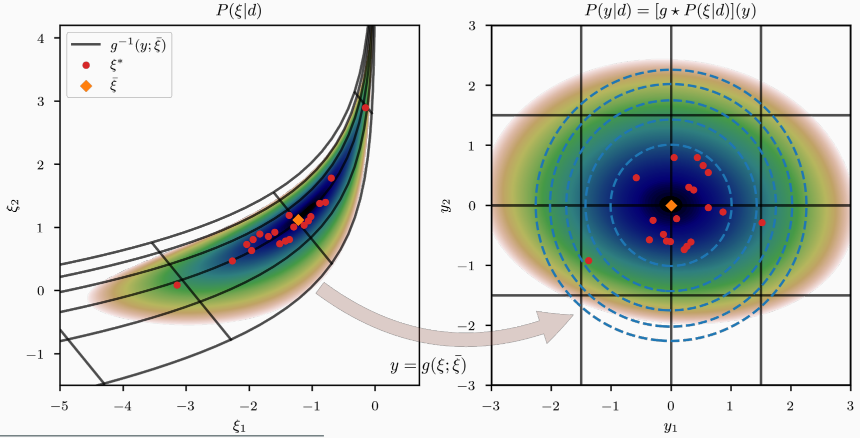


[10]

[9] Jakob Knollmüller and T. A. Enßlin 2019.
[10] Philipp Frank, Leike, and Enßlin 2021.

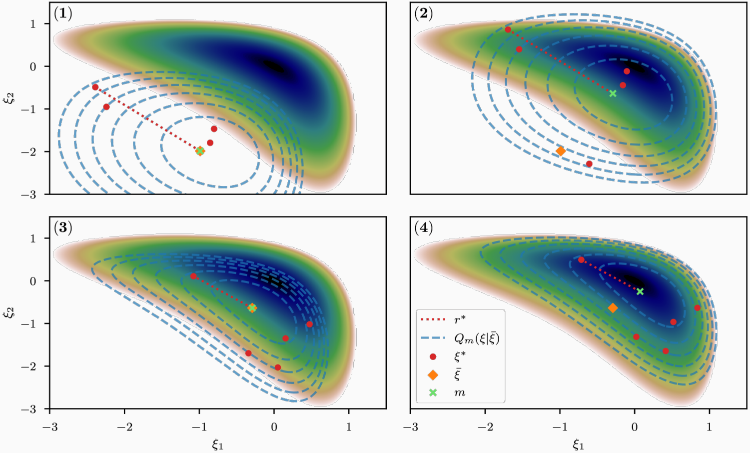
Geometric Variational Inference

Normalizing coordinate transformation $y = g_\sigma(\xi)$ with $\sigma = \bar{\xi}$.
Approximate distribution $\mathcal{Q}(y) = \mathcal{N}(y; 0, \mathbb{1})$



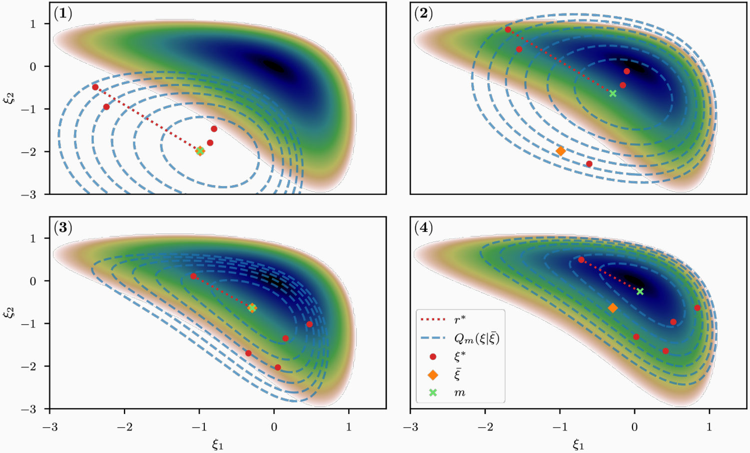
[10] Philipp Frank, Leike, and Enßlin 2021.

Approximate Inference - geoVI[10]



[10]Philipp Frank, Leike, and Enßlin 2021.

Approximate Inference - geoVI[10]



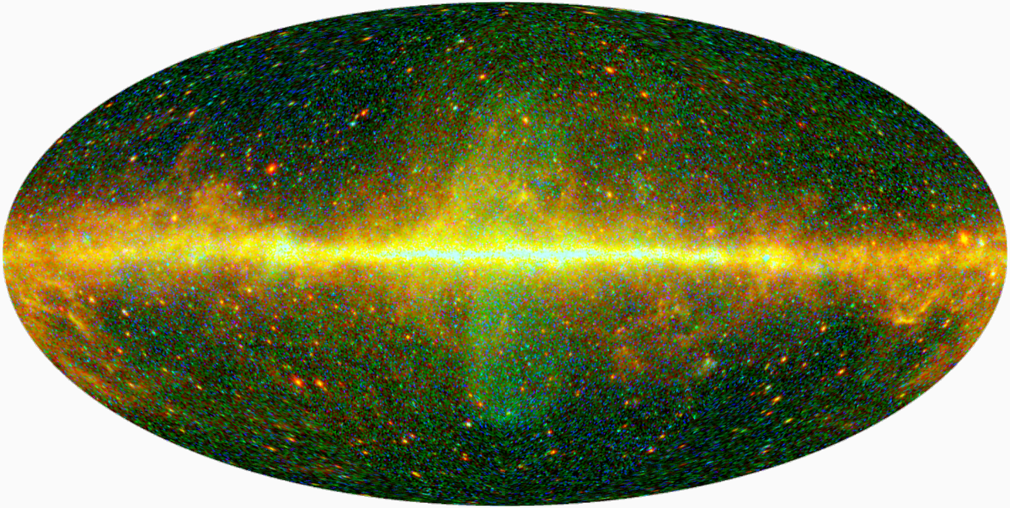
Challenge: Not in 2 but millions of dimensions!

[10] Philipp Frank, Leike, and Enßlin 2021.

Applications

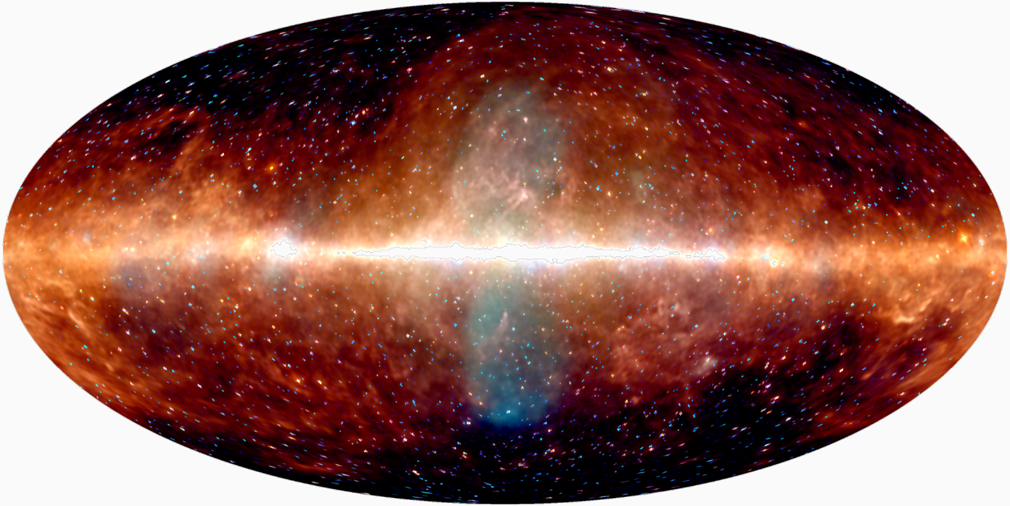
Fermi

Fermi γ-ray sky[11]



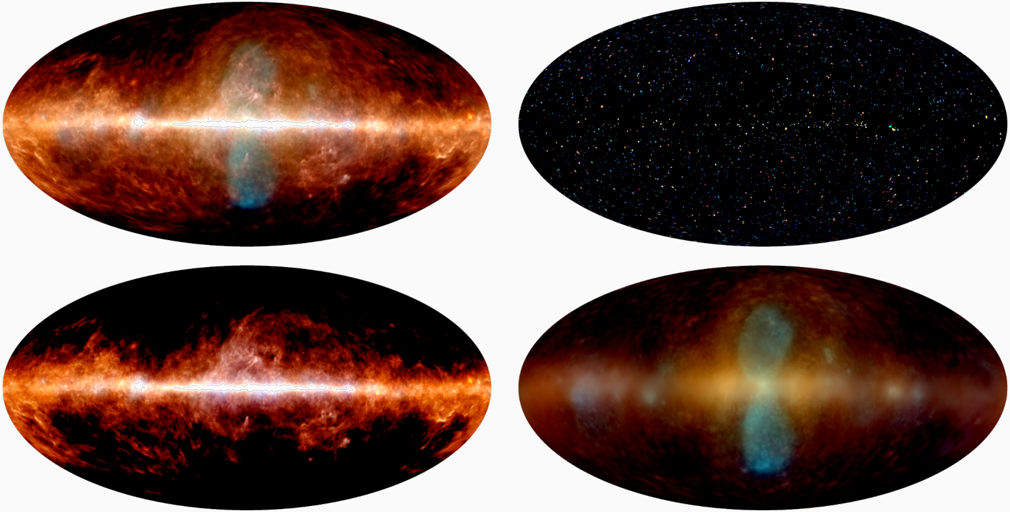
[11] Scheel-Platz, J. Knollmüller, P. Arras, et al. 2023.

Fermi γ-ray sky[11]



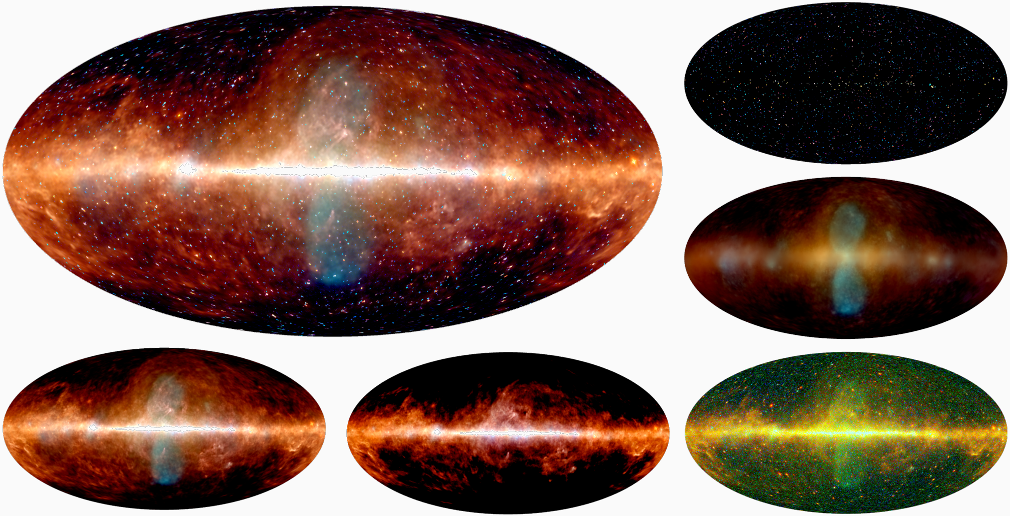
[11] Scheel-Platz, J. Knollmüller, P. Arras, et al. 2023.

Fermi γ-ray sky[11]



[11] Scheel-Platz, J. Knollmüller, P. Arras, et al. 2023.

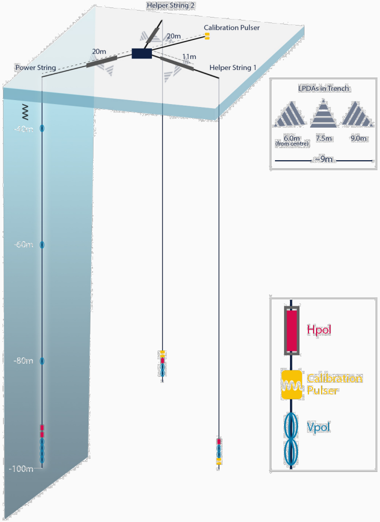
Fermi γ-ray sky[11]

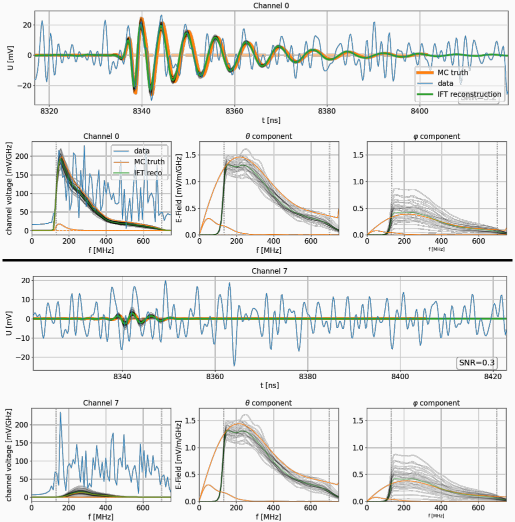


[11] Scheel-Platz, J. Knollmüller, P. Arras, et al. 2023.

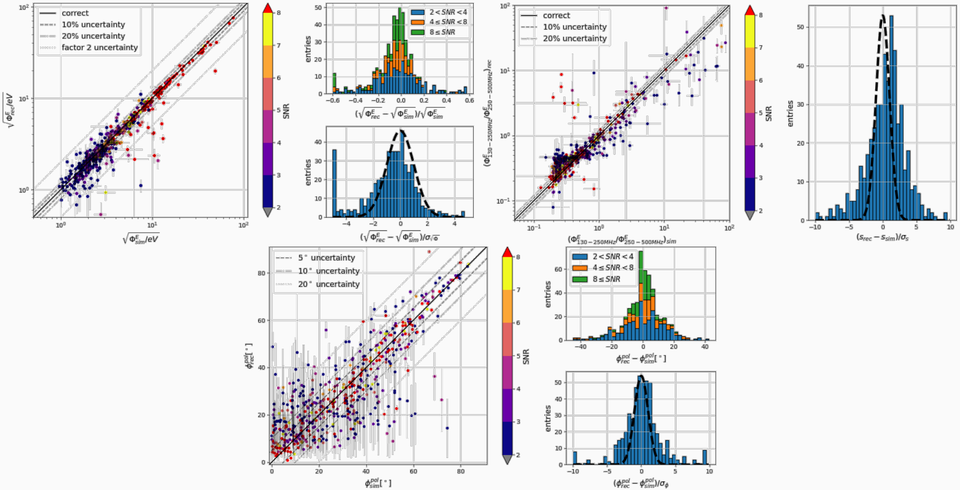
Radio pulse from CR air shower

Radio pulse from CR air shower[12]





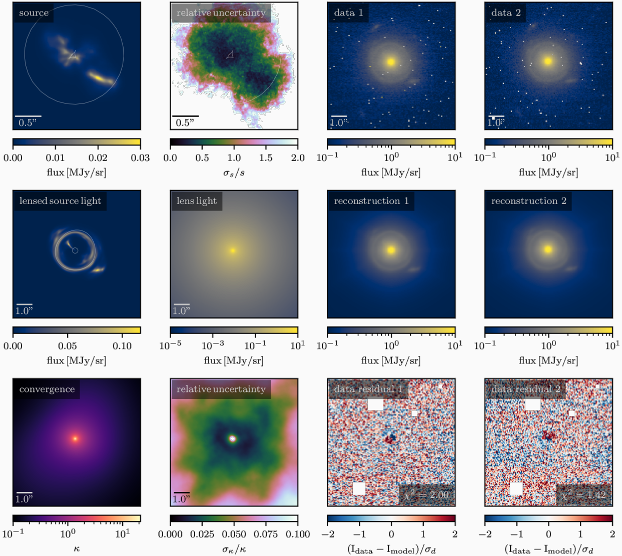
Radio pulse from CR air shower[12]



[12]Welling, P. Frank, T. Enßlin, et al. 2021.

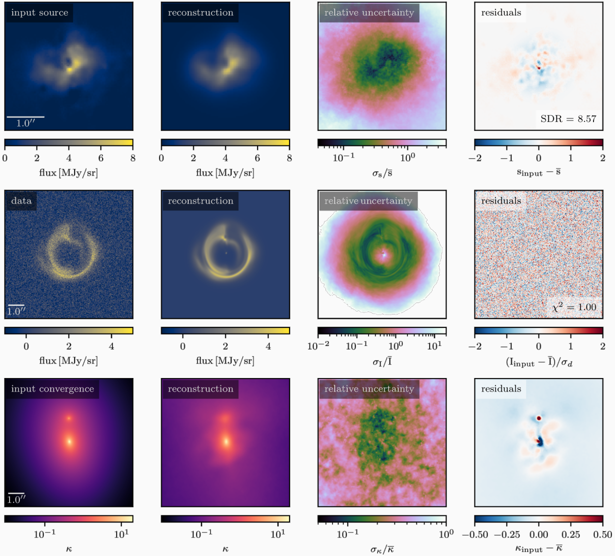
Strong lensing with JWST

Strong lensing with JWST[13]



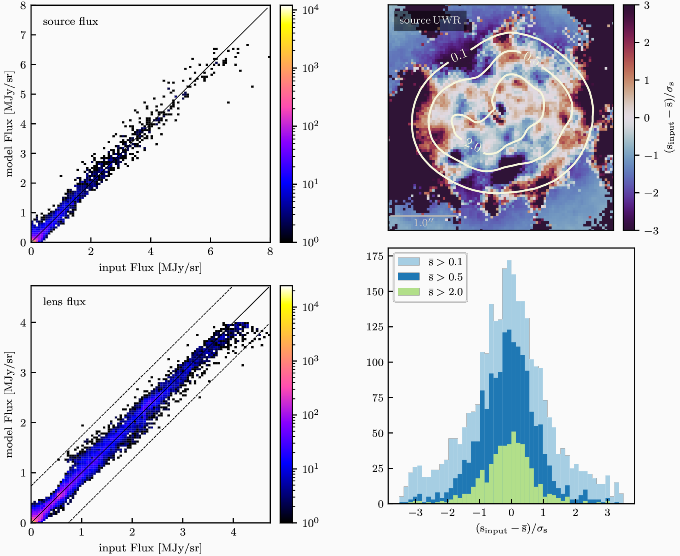
[13] Rüstig, Matteo Guardiani, Roth, et al. 2024.

Strong lensing with JWST[13]



[13] Rüstig, Matteo Guardiani, Roth, et al. 2024.

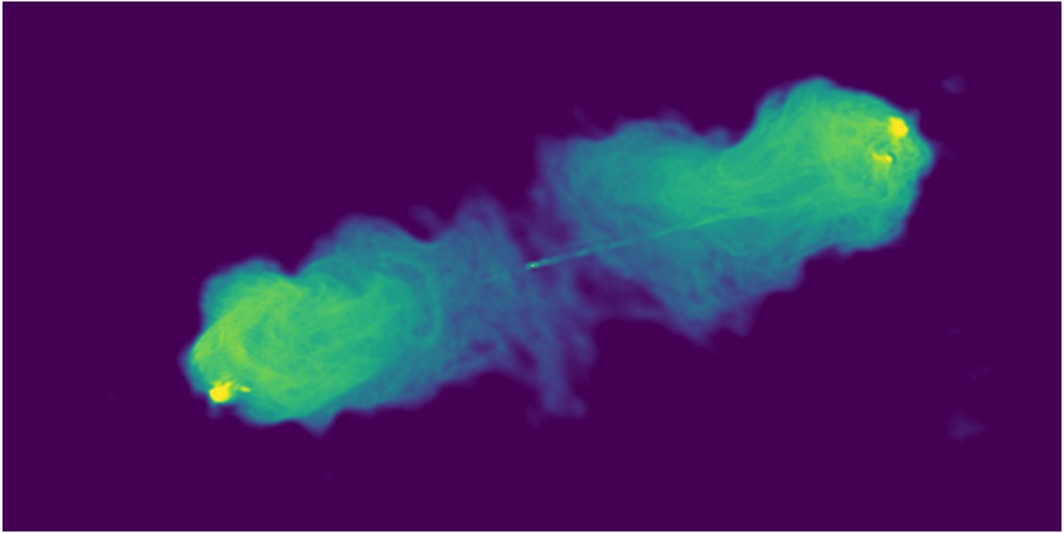
Strong lensing with JWST[13]



[13] Rüstig, Matteo Guardiani, Roth, et al. 2024.

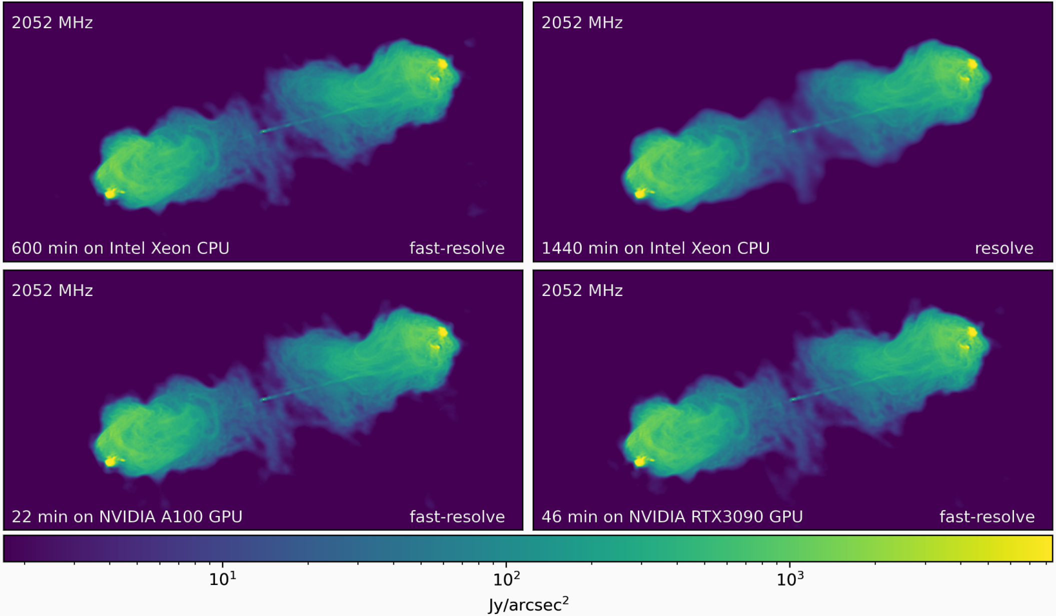
Radio interferometry

Radio interferometry - FastResolve[14]



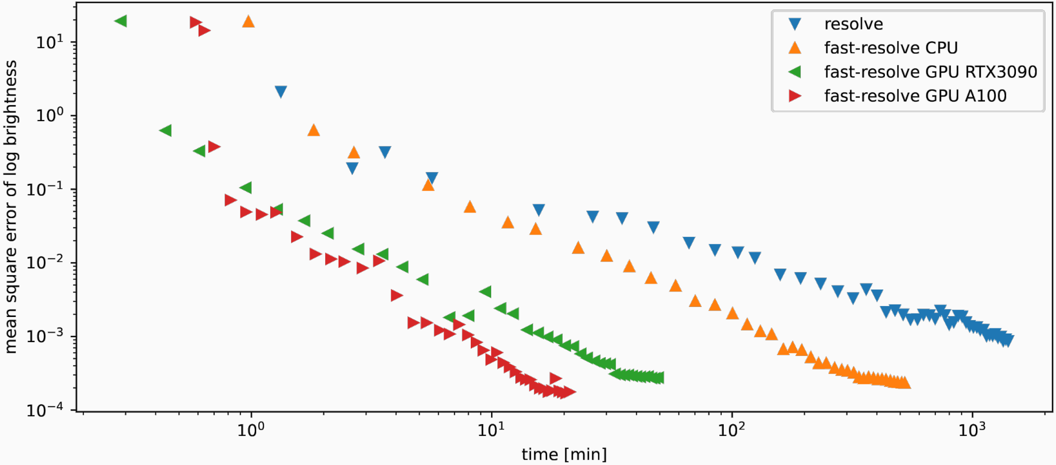
[14] Roth, Philipp Frank, Bester, et al. 2024.

Radio interferometry - FastResolve[14]



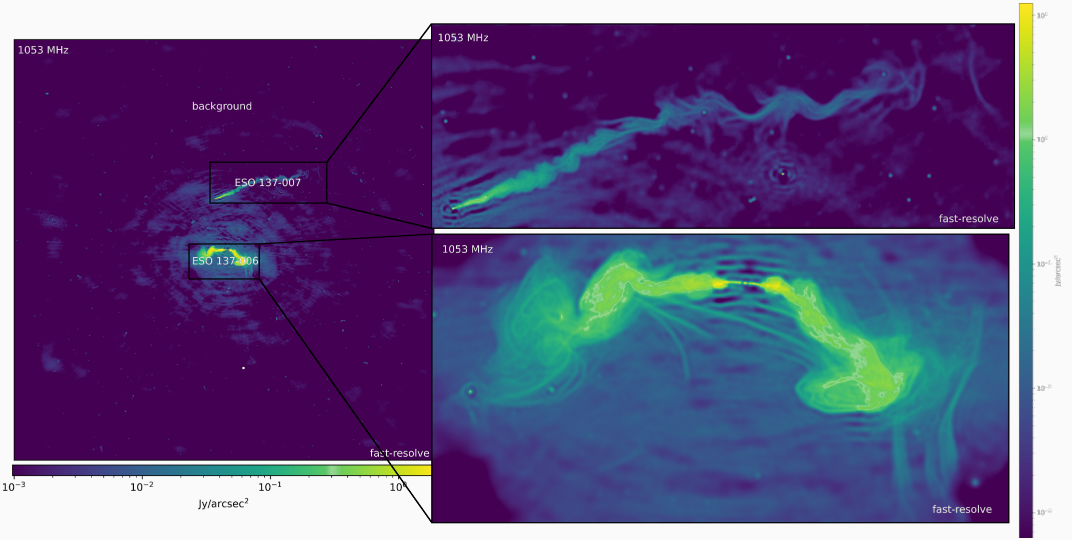
[14] Roth, Philipp Frank, Bester, et al. 2024.

Radio interferometry - FastResolve[14]



[14] Roth, Philipp Frank, Bester, et al. 2024.

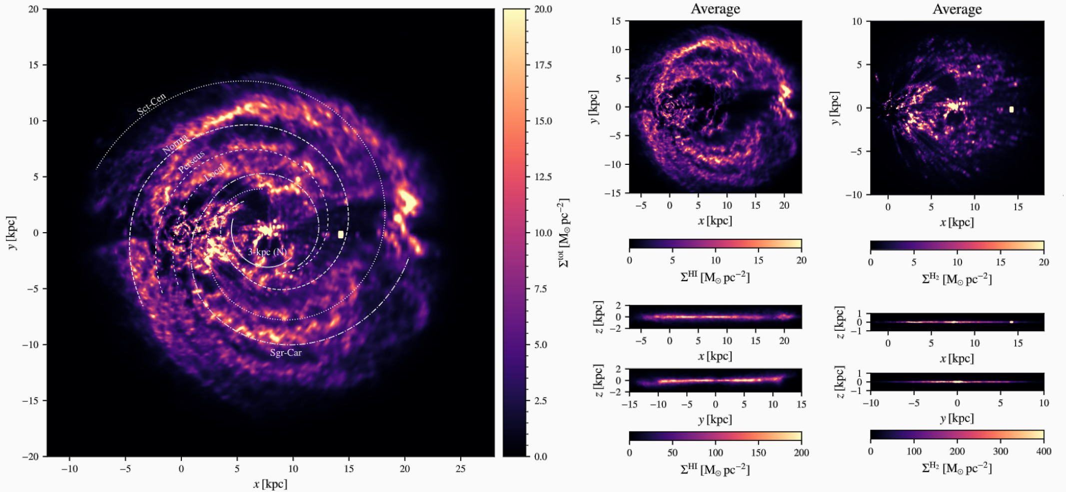
Radio interferometry - FastResolve[14]



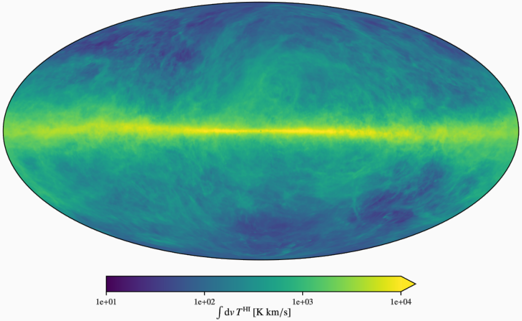
[14]Roth, Philipp Frank, Bester, et al. 2024.

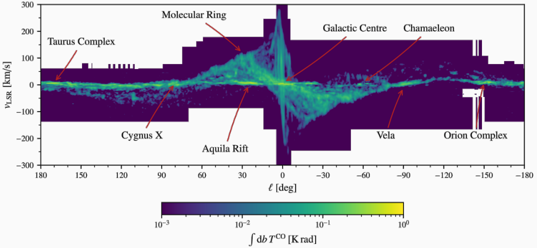
HI and CO in the Milky Way (preliminary)

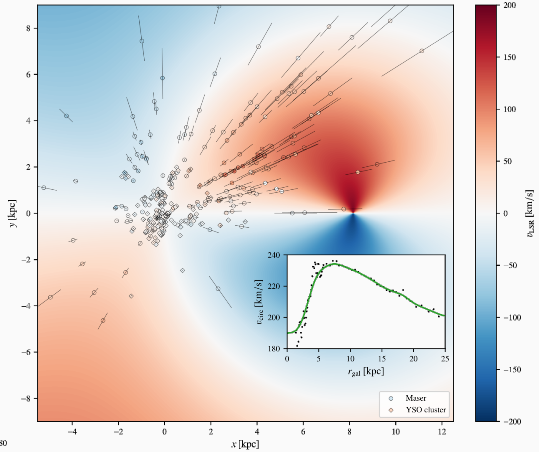
3D Distributions of HI and CO in the Milky Way (preliminary)



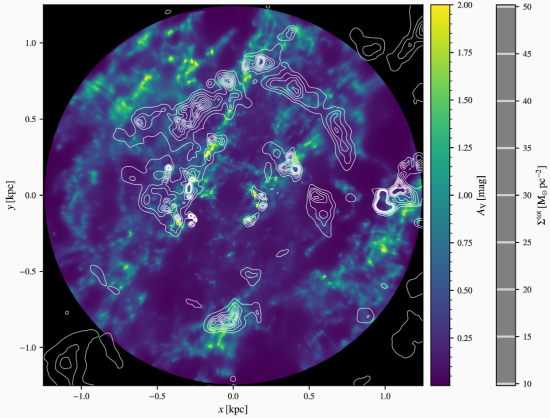
3D Distributions of HI and CO in the Milky Way (preliminary)

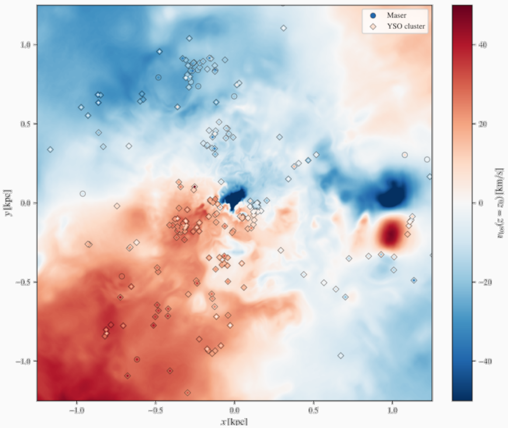






3D Distributions of HI and CO in the Milky Way (preliminary)





Software - NIFTy

NIFTy

- adaptive Gaussian Processes
- Common Likelihoods + designated Libraries
  - Resolve (gitlab.mpcdf.mpg.de/ift/resolve)
  - X-UBIK
  - LensCharm (gitlab.mpcdf.mpg.de/ift/lenscharm)
- (Geo-)metric Variational Inference

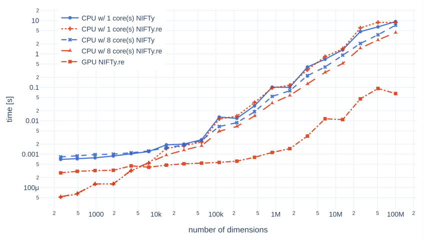
- NIFTy Code: gitlab.mpcdf.mpg.de/ift/nifty
- NIFTy Docs: ift.pages.mpcdf.de/nifty

[15] Edenhofer, Philipp Frank, Roth, et al. 2024.

Software - NIFTy → NIFTy.re[15]

- adaptive Gaussian Processes
- Common Likelihoods + designated Libraries
  - Resolve (gitlab.mpcdf.mpg.de/ift/resolve)
  - X-UBIK
  - LensCharm (gitlab.mpcdf.mpg.de/ift/lenscharm)
- (Geo-)metric Variational Inference
- Jax re-implementation → NIFTy.re
  - GPU support
  - Just-in-time compilation
  - Full integration to JaxML ecosystem

- NIFTy Code: gitlab.mpcdf.mpg.de/ift/nifty
- NIFTy Docs: ift.pages.mpcdf.de/nifty



[15] Edenhofer, Philipp Frank, Roth, et al. 2024.

References

- Arras, P et al. (July 2019). **"Unified radio interferometric calibration and imaging with joint uncertainty quantification".** In: *A&A* 627, A134, A134. DOI: 10.1051/0004-6361/201935555. arXiv: 1903.11169 [astro-ph.IM].
- Arras, P et al. (2022). **"Variable structures in M87* from space, time and frequency resolved interferometry".** In: *Nature Astronomy* 6.2, pp. 259–269.
- Edenhofer, Gordian et al. (May 2024). **"A parsec-scale Galactic 3D dust map out to 1.25 kpc from the Sun".** In: *A&A* 685, A82, A82. DOI: 10.1051/0004-6361/202347628. arXiv: 2308.01295 [astro-ph.GA].
- Edenhofer, Gordian et al. (Feb. 2024). **"Re-Envisioning Numerical Information Field Theory (NIFTy.re): A Library for Gaussian Processes and Variational Inference".** In: *arXiv e-prints*, arXiv:2402.16683, arXiv:2402.16683. arXiv: 2402.16683 [astro-ph.IM].

- Frank, Philipp, Reimar Leike, and Torsten A. Enßlin (2021). **"Geometric Variational Inference"**. In: *Entropy* 23.7. ISSN: 1099-4300. DOI: 10.3390/e23070853. URL: https://www.mdpi.com/1099-4300/23/7/853.
- Hutschenreuter, Sebastian et al. (Apr. 2023). **"Disentangling the Faraday rotation sky"**. In: *arXiv e-prints*, arXiv:2304.12350, arXiv:2304.12350. DOI: 10.48550/arXiv.2304.12350. arXiv: 2304.12350 [astro-ph.GA].
- Knollmüller, Jakob and Torsten A. Enßlin (Jan. 2019). **"Metric Gaussian Variational Inference"**. In: *arXiv e-prints*, arXiv:1901.11033, arXiv:1901.11033. DOI: 10.48550/arXiv.1901.11033. arXiv: 1901.11033 [stat.ML].
- Roth, Jakob et al. (Oct. 2023). **"Bayesian radio interferometric imaging with direction-dependent calibration"**. In: *A&A* 678, A177, A177. DOI: 10.1051/0004-6361/202346851. arXiv: 2305.05489 [astro-ph.IM].
- Roth, Jakob et al. (June 2024). **"fast-resolve: Fast Bayesian Radio Interferometric Imaging"**. In: *arXiv e-prints*, arXiv:2406.09144, arXiv:2406.09144. arXiv: 2406.09144 [astro-ph.IM].

Rüstig, Julian et al. (Feb. 2024). **"Introducing LensCharm. A charming Bayesian strong lensing reconstruction framework".** In: *A&A* 682, A146, A146. DOI: 10.1051/0004-6361/202348256.

Scheel-Platz, L. I. et al. (Dec. 2023). **"Multicomponent imaging of the Fermi gamma-ray sky in the spatio-spectral domain".** In: *A&A* 680, A2, A2. DOI: 10.1051/0004-6361/202243819. arXiv: 2204.09360 [astro-ph.HE].

Welling, C. et al. (Apr. 2021). **"Reconstructing non-repeating radio pulses with Information Field Theory".** In: *Journal of Cosmology and Astroparticle Physics* 2021.4, 071, p. 071. DOI: 10.1088/1475-7516/2021/04/071. arXiv: 2102.00258 [astro-ph.HE].

Westerkamp, M. et al. (Apr. 2024). **"The first spatio-spectral Bayesian imaging of SN1006 in X-rays".** In: *A&A* 684, A155, A155. DOI: 10.1051/0004-6361/202347750. arXiv: 2308.09176 [astro-ph.HE].