New physics and primordial neutrinos decoupling: a DSMC approach

Based on 2409.15129 and 2409.07378

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Introduction I

- Measurements of CMB and BBN $\Rightarrow$ learning the state of the Universe at times as small as $t \simeq 10^{-2}$ s
- Opportunity to **constrain/discover new physics** that may be present at that epoch
- **Neutrinos** are essential components of the primordial plasma
- To study their dynamics, we need to solve the Boltzmann equation

$$\partial_t f_{\nu_\alpha}(p,t) - Hp\partial_p f_{\nu_\alpha} = \mathcal{I}_{\text{coll}}[f_{\nu_\alpha}, p] \quad (1)$$

Introduction II

Collision integral $\mathcal{I}_{\text{coll}}$:

$$\boldsymbol{\mathcal{I}_{\text{coll}} = \frac{1}{2E_{\nu_\alpha}} \sum \int \prod_{i=2} \frac{d^3p_i}{(2\pi)^3 2E_i} \prod_{f=1} \frac{d^3p_f}{(2\pi)^3 2E_f} |\mathcal{M}|^2 F[f] (2\pi)^4 \delta^{(4)} \left(\sum_{i=1} p_i - \sum_{f=1} p_f\right)} \quad (2)$$

$\boldsymbol{\mathcal{I}}_{\text{coll}}$ includes $\boldsymbol{\nu - \nu}$, $\boldsymbol{\nu -}$ EM interactions and (probably) **new physics:**

Lepton asymmetry	**Decaying LLPs**	**NSI**
AD mechanism Decays of asymmetric HNLs ...	ALPs HNLs Majorons ...	$B - L_\alpha$ mediators neutrinophilic particles ...

The scope of the talk:

Model-independent and efficient approach to solve Eq. (1)

Existing approaches to solve neutrino Boltzmann equation I

Complexities of the problem:

1. Stiffness (fast EM/decay rates, slow weak rates)
2. Importance of high-energy neutrino tail (as $|\mathcal{M}|^2 \propto s^2$)
3. Complexity of matrix elements $\mathcal{M}$
   Example: decays into jets, $2 \to 3$ scatterings/specific non-standard neutrino interactions, many-body exclusive decays
4. Decays of heavy LLPs:
   - Inject high-energy neutrinos with $E_\nu \gg 3T$
   - Inject mesons/muons that evolve non-trivial evolution

Existing approaches to solve neutrino Boltzmann equation II

1. Integrated approach [2001.04466]:

1. *Assumption* – non-thermal distortions are always small:

$$f_{\nu_\alpha}(p,t) \approx \frac{1}{\exp[p/T_{\nu_\alpha}(t)] + 1} \tag{3}$$

2. Integrate Eq. (1) over $d^3\mathbf{p}$, replace Eq. (2) with semi-analytic expression in terms of $T_{\nu_\alpha}, T_{\text{EM}}$
3. Solve the resulting semi-analytic system in terms of $T_{\nu_\alpha}, T_{\text{EM}}$

Strong sides:

- Transparency and performance

Limitations:

- Works only if the effect of non-thermal distortions is small

Existing approaches to solve neutrino Boltzmann equation III

What if non-thermal distortions are large?

E.g., decays of LLPs with $m \gg 3T$

2. Momentum discretization approach [9506015]:

1. Reduce the integration in collision integral (2) as much as possible:

$$\mathcal{I}_{\text{coll}} = \int G(x) \prod_{j=1}^{l} dx_j, \quad l \geq 2 \tag{4}$$

2. Discretize the comoving momentum: $y(t) = pa(t) \to \{y_i\}, i = \overline{1, n}$
3. Convert Eq. (1) into a system of n ODEs on modes $f^i_{\nu_\alpha}(t) \equiv f_{\nu_\alpha}(y_i, t)$. Eq. (4) is now a product of l sums, generically having n^l terms
4. Solve the system in terms of $f^i_{\nu_\alpha}(t)$

Existing approaches to solve neutrino Boltzmann equation IV

Strong sides:

- Trace the neutrino distribution shape important for N_{eff}, BBN, cosmological ν mass bound

Limitations:

- Requires the presence of analytic matrix elements
- Computational complexity: $\boldsymbol{t_{\text{computation}} \propto \mathcal{O}(E_{\nu,\text{max}}^{l+1})}$. Assuming the ΛCDM case, for which $\boldsymbol{E_{\nu,\text{max}}^{\Lambda\text{CDM}} \simeq 20-50}$ MeV, one gets

$$\frac{\boldsymbol{t_{\text{computation}}(E_{\nu,\text{max}})}}{\boldsymbol{t_{\text{computation}}(\Lambda\text{CDM})}} \sim \left(\frac{E_{\nu,\text{max}}}{E_{\nu,\text{max}}^{\Lambda\text{CDM}}}\right)^{l+1} \tag{5}$$

  Weeks to compute the impact of 1 GeV neutrinos

  Typical situation if GeV-scale particles are present in the plasma

- Implementation complexity: solver stability, energy conservation, grid adjustment, and this is all model-dependent

Existing approaches to solve neutrino Boltzmann equation V

There are no methods to solve the neutrino Boltzmann equation that would deliver:

- **Model independence**
- **Performance**
- **Transparency**

In light of ongoing CMB observations, such an approach is needed

Direct simulation Monte Carlo I

- Idea: instead of solving Eqs. (1) explicitly, start with $N \gg 1$ particles – neutrinos, EM particles, new physics – and simulate their interactions
- In the physics of rarefied gases, the approach is known as **Direct Simulation Monte Carlo**, or **DSMC** [Physics of Fluids 31, 067104 (2019)]
  - It is automatically free from $E_{\nu,\text{max}}$ dependence
  - We already have efficient approaches to simulate the phase space from accelerator particle physics
    MadGraph, pythia, SensCalc
  - In particular, analytic matrix elements in case of decays are not required
    May use tabular data produced externally by, e.g., pythia
  - Existing approaches allow for very high performance and efficient parallelization even in the case of huge $N \sim 10^8$

Direct simulation Monte Carlo II

Vanilla DSMC (utilizing so-called **No-Time-Counter method** [Prog.Astron.Aeron. 117, 211–226 (1989)]):

0. Update the coordinates and velocities of particles due to external forces
1. Split the system of volume V into cells containing N_{cell} particles
2. For each cell, per timestep Δt, sample

$$N_{\text{sample}} = \frac{1}{2} N_{\text{cell}}(N_{\text{cell}} - 1) \overbrace{\frac{(\sigma v)_{\text{max}}}{V_{\text{cell}}} \Delta t}^{\omega_{\text{cell}}^{\text{max}} \cdot \Delta t} \tag{6}$$

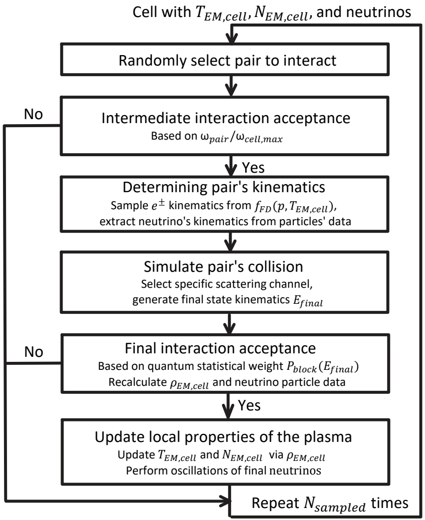
   pairs of particles to interact

3. Iteratively repeat the procedure: for each sampled pair, accept the interaction with the probability $P_{\text{acc}} = (\sigma v)_{\text{pair}}/(\sigma v)_{\text{max}}$, generate the kinematics and final state in case it is accepted

Neutrino DSMC I

To apply it to neutrinos, DSMC requires fundamental modifications:

1. **Expansion of the Universe**: redshift particles' momenta and system volume
2. **EM plasma properties**: represent the EM particles locally and globally just by one quantity – temperature $T_{\rm EM}$, and update it if any interaction involving EM particles occurs
3. **Quantum statistics**: add additional decision on whether to approve the interaction based on the blocking factors $1 - f_{\rm final}(E_{\rm final})$ for the final states
4. **Decaying particles**: introduce m LLPs defined by the energy density, decay $\Delta m = m(t)\Delta t/\tau$ particles per each timestep Δt, simulate decays e.g., in `SensCalc`/`PYTHIA8`



Neutrino DSMC II

- We have developed a **neutrino DSMC prototype** written in `Mathematica+C++`
- **Cross-checks:** comparing with the integrated and unintegrated approaches in the case of a few well-defined setups

Already delivered	**To be done**
1) Includes the ΛCDM dynamics of neutrinos plus decaying LLPs/SM particles 2) Performs as fast as discretization approach for injections of νs with $\mathcal{O}(50\text{ MeV})$ 3) Independent of ν energy	1) Add finite electron mass 2) Improve performance by $\mathcal{O}(10)$ 3) Add NSI

Code may be provided upon request

Case studies I

Toy scenarios with initial conditions intimately related to real cases:

Instant injection of neutrinos with $E_\nu \gg 3T$	Instant injection of decaying mesons	Equilibrium shape of $f_{\nu_\alpha}, T_{\nu_\alpha} \neq T_{\text{EM}}$
Decays of HNLs, majorons, νphilic, ...	Decays of LLPs with $m \gtrsim m_\pi$	Heating of EM plasma "Equilibrium" ν heating

To characterize the evolution of νs, consider the quantity

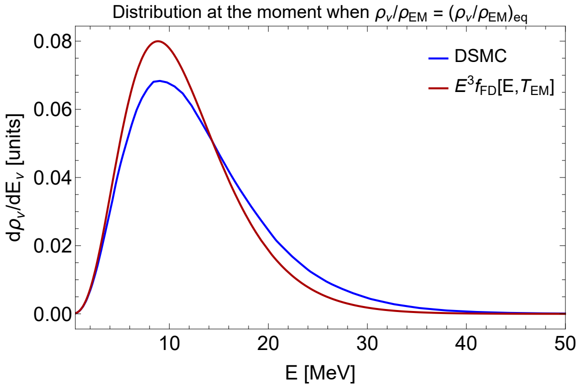
$$\delta\rho_\nu = \left(\frac{\rho_{\text{EM}}}{\rho_\nu}\right)_{\Lambda\text{CDM}} \left(\frac{\rho_\nu}{\rho_{\text{EM}}}\right) - 1 \tag{7}$$

as well as the shape of the neutrino distribution

$\delta\rho_\nu > (<)0$: N_{eff} likely increases (decreases)

Case studies II

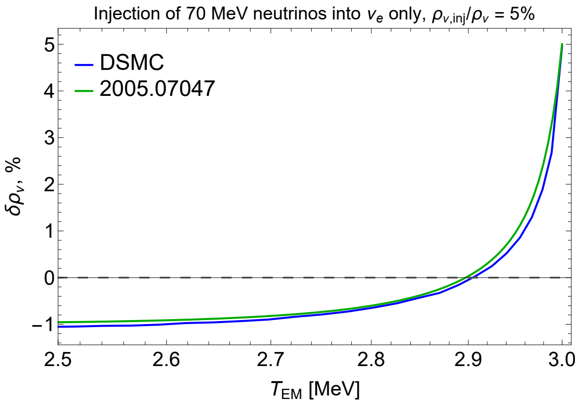
- What happens if we inject νs with energies $E_\nu \gg 3T$?
- Their interactions will be much faster than thermal interactions ($\sigma_{\text{int}} \sim s$)
- They will either pump the energy to the EM plasma (where it thermalizes instantly) or interact with thermal neutrinos
- It leaves characteristic imprint on the f_ν compared to thermal:
  - **Overrepresented** at high p
  - **Underrepresented** at low p

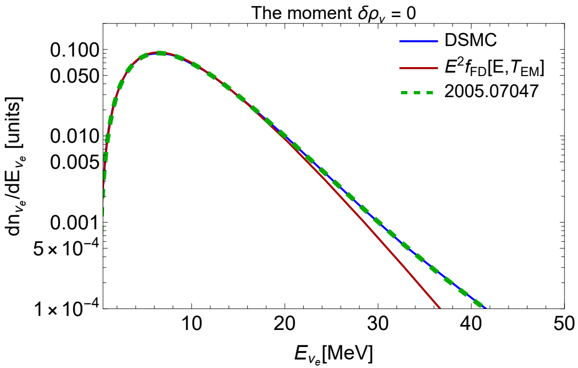


The spectral distortions may cause $\delta\rho_\nu$ drop below 0 $\Rightarrow$ decrease in N_{eff}?

High-energy neutrinos: decrease or increase N_{eff}? Open question ([2103.09831] and refs. therein)

Case studies III

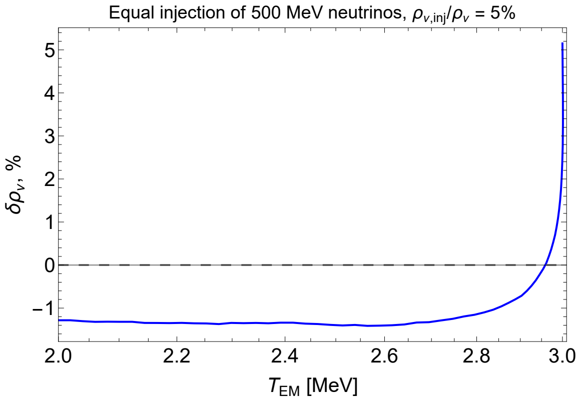




- Our simulations on high-energy neutrino injections agree with the discretization approaches from [2103.09831], [2005.07047]
- Distortions are important for, e.g., n/p ratio evolution

Case studies IV

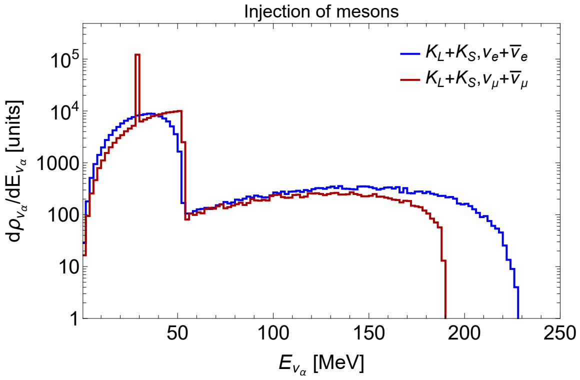
– The same is qualitatively true for any higher neutrino energy
– With the same running time, we have simulated the injection of 500 MeV neutrinos

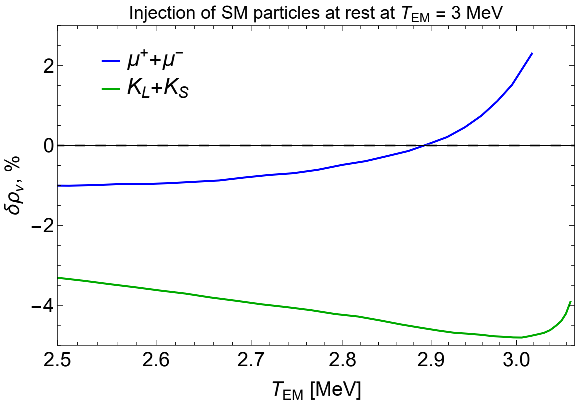


In presence of high-energy neutrinos, $\Delta N_{\mathrm{eff}} < 0$

Exact threshold in E_ν depends on the injection temperature

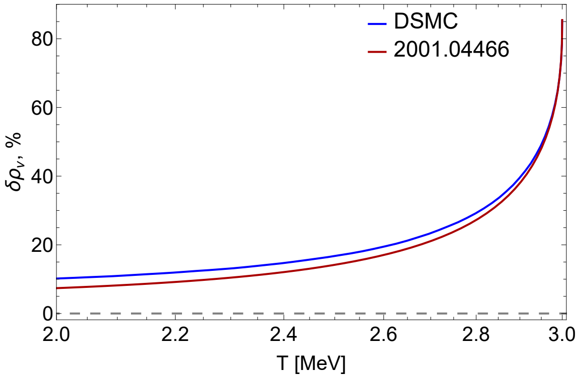
Case studies V





– The same conclusion holds if injecting the mesons and muons, assuming that they all decay while being thermalized

Case studies VI



- Even if we just start with equilibrium shape of the neutrino spectrum but with $\boldsymbol{T_{\nu_\alpha}} \neq \boldsymbol{T}_{\text{EM}}$, distortions will develop
- These are the result of the energy-dependent thermalization rate

Conclusions

- We have developed a novel approach to solve the neutrino Boltzmann equation that may handle a broad range of new physics scenarios
- Immediate application: injections of neutrinos with $\boldsymbol{E_\nu \gg 3T}$ at MeV temperatures always decrease $\boldsymbol{N}_{\text{eff}}$
- Under improvement, the approach may become a universal robust tool linking models of new physics to the impact on CMB and BBN
- It may also be used to study other systems: dense medium of supernova, production of DM in the Early Universe

Backup slides

What is DSMC [paper] I

– Consider the Liouville equation for the N-particle probability distribution density $F_N(\mathcal{R}, \mathcal{V}, t)$ with a short-range potential $\Phi_{i,j}$ of binary interactions:

$$\frac{\partial F_N}{\partial t} + \sum_{i=1}^{N} \mathbf{v}_i \frac{\partial F_N}{\partial \mathbf{r}_i} + \sum_{1\leq i<j\leq N} \Phi_{i,j} F_N = 0 \tag{8}$$

– DSMC approach approximately solves it using the following scheme:

1. Apply the $N-1$ space variable reduction $F_N \to \tilde{F}_N = \int F_N \prod_{s=2}^{N} d\mathbf{r}_s$
2. Switch to the iteration scheme by considering the equation on the time intervals $(t; t+\Delta t)$
3. Decompose the space domain $\mathcal{D}$ onto disconnected sub-domains $\mathcal{D} = \cup_{l=1}^{M} \mathcal{D}^{(l)}$ ("cells"), populated by fixed amounts of particles during Δt
4. Split the evolution: evolve the system subsequently due to "ballistic motion" (free-streaming in the absence of collisions), binary collisions within $\mathcal{D}^{(l)}$, and then interchanging the particles between cells as a result of the first two steps

What is DSMC [paper] II

– Under an assumption that the system obeys ergodic conditions, the DSMC approach may be converted to an analog of the BGGKY system for $3 + 3N$ phase space, which **reduces to the Boltzmann equation** in the limit $N \to \infty$ and assuming the molecular chaos

No-time-counter scheme I

- The central part of the DSMC approach is to evolve particles' interaction inside the cell $\mathcal{D}^{(l)}$
- Below, we describe the so-called **No-Time-Counter** (NTC) method often used for this purpose
  1. One defines the timestep of the simulation, defined by

$$\boldsymbol{\Delta t \sim \left(\chi_{\text{particle}} \cdot \sigma v \cdot \frac{N}{V_{\text{system}}}\right)^{-1}_{\text{max}}} \tag{9}$$

     $\boldsymbol{\chi}_{\text{particle}}$ is the particles' weight (how many physical particles are represented by each particle), $\boldsymbol{N}$ is the total number of interacting particles, $\boldsymbol{V}_{\text{system}}$ is the system's volume, $\boldsymbol{\sigma}$ is the interaction cross-section, $\boldsymbol{v}$ is the relative velocity. $\boldsymbol{\Delta t}$ must be sufficiently small to resolve the equilibration

No-time-counter scheme II

2. Within a cell (containing n_{cell} particles), one samples randomly

$$\boldsymbol{N}_{\text{sampled}} = \chi_{\text{particle}} \frac{\boldsymbol{N}_{\text{cell}}(\boldsymbol{N}_{\text{cell}} - \mathbf{1})}{\mathbf{2}} \frac{\boldsymbol{\omega}_{\text{cell,max}} \boldsymbol{\Delta t}}{\boldsymbol{V}_{\text{cell}}} \tag{10}$$

   pairs, where N_{cell} is the number of particles in cell, $\boldsymbol{\omega}_{\text{cell,max}} = (\boldsymbol{\sigma v})_{\text{max}}$ is the maximum interaction cross-section within the cell, and $\boldsymbol{V}_{\text{cell}} = \boldsymbol{V}_{\text{system}}/\boldsymbol{n}_{\text{cells}}$ is cell's volume
3. Once the pair is chosen, one accepts its interaction with the probability

$$\boldsymbol{P}_{\text{acc}} = \boldsymbol{\omega}/\boldsymbol{\omega}_{\text{cell,max}}, \quad \boldsymbol{\omega} = (\boldsymbol{\sigma v})_{\text{pair}} \tag{11}$$

4. If the interaction is accepted, one simulates the possible final states for the given pair and its scattering kinematics

– In standard DSMC applications, a very low number of particles per cell is typically enough to simulate the interactions adequately – $\mathcal{O}(10 - 20)$
– Has been tested for various systems, including relativistic ones [paper], [0902.1762]
– Detailed discussions and comparison with other DSMC methods may be found in [paper]

Details of neutrino DSMC I

– **Particles species**:
  - 3 types of neutrinos $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$
  - EM particles – electrons and photons

– **Interactions**:
  - Interaction processes (plus charge conjugated ones):

$$\nu_\alpha + \nu_\beta \to \nu_\alpha + \nu_\beta, \quad \nu_\alpha + \bar{\nu}_\alpha \leftrightarrow e^+ + e^-, \tag{12}$$

$$\nu_\alpha + e^\pm \to \nu_\alpha + e^\pm, \quad \nu_\alpha + \bar{\nu}_\beta \leftrightarrow \nu_{\alpha'} + \bar{\nu}_{\beta'} \tag{13}$$

  - The cross-sections and collision kinematics are calculated according to the matrix elements (see, e.g., [2006.07387])
  - Photons are not involved in any direct interaction but share the same temperature with electrons

– **Neutrino oscillations**:
  - In each neutrino interaction, each neutrino ν_β may interact as ν_α with the weight $P_{\text{osc}}(\beta \to \alpha)$, where P_{osc} is the oscillation probability
  - At the end of each iteration, the flavors of neutrinos are re-sampled according to P_{osc}

Details of neutrino DSMC II

- **Aspects of simulating the interactions:**
  - Due to full isotropy and homogeneity in the system, the spatial degrees of freedom are dropped – the interactions are considered in 0D space
  - To speed up the calculations, however, particles are split into fictitious cells; interactions are only allowed between the particles of the same cell (so no boundary effects, etc.)
  - The expansion of the Universe is accounted for by redshifting all energies/momenta and volume
  - *No-time-counter* (NTC) method is used to simulate the pairs' interaction in each cell; the local thermalization of the EM plasma, neutrino oscillations, and the Pauli principle are accounted for
  - After simulating interactions in each cell, the properties of the EM plasma and neutrinos are averaged over cells (instant EM thermalization and neutrino oscillations)

NTC method: modifications for studying neutrinos I

Our system is much more complicated than a typical system simulated with NTC. A few key changes are required

– **Feature 1: EM plasma equilibration**

- Electrons and photons immediately acquire thermal distribution parametrized by some temperature T
- Once one of the particles forming the pair is an electron, one samples its energy by using the FD distribution with the local cell's temperature T_{EM}
- After each interaction in the cell involving electrons:
  1. One computes the change in the total energy of the EM plasma in the cell, $E_{\text{EM,tot}}$
  2. One recomputes the local EM plasma temperature T_{EM} using the relation

$$\rho_{\text{EM,cell}} \equiv \frac{E_{\text{EM,tot}}}{V_{\text{cell}}} = 4\rho_{\text{Fermi-Dirac}}(T_{\text{EM}}) + 2\rho_{\text{Bose-Einstein}}(T_{\text{EM}}), \tag{14}$$

     where 4 accounts for helicities and particle-antiparticle and 2 for helicities
  3. Having T_{EM}, one samples the number of electrons N_e by using the relation

$$n_e \equiv \frac{N_e}{V_{\text{cell}}} = 4n_{\text{Fermi-Dirac}}(T_{\text{EM}}) \tag{15}$$

- After simulating interactions within cells, we use the relations (14), (15) to rebuild the global population of EM particles

NTC method: modifications for studying neutrinos II

– **Feature 2: Expansion of the Universe**:

- At each timestep, one first calculates the value of the Hubble factor via the relation

$$H = \sqrt{\frac{8\pi}{3m_{\text{Pl}}^2}\rho}, \quad \rho = \frac{E_{\text{total}}}{V_{\text{system}}} \tag{16}$$

- The timestep Δt for the iteration (determined by the maximal collision rate) must be $\ll H^{-1}$:

$$\Delta t = \min\left[0.01H^{-1}, \left(\chi_{\text{particle}}\cdot(\sigma v)_{\text{max}}\cdot\frac{N}{V_{\text{system}}}\right)^{-1}_{\text{max}}\right] \tag{17}$$

- The system's and cell's volumes and all energies/momenta are redshifted:

$$V \to V(1+3H\Delta t), \quad E \to E(1-H\Delta t) \tag{18}$$

NTC method: modifications for studying neutrinos III

- **Feature 3: Quantum statistics**:
  - After interaction has been accepted and kinematics generated, there is the final decision to accept the final states based on the Pauli blocking factors $\boldsymbol{P}_{\text{block}}$
  - The blocking factors are approximated by the relations

$$P_{\text{block}}(E_{\text{final}}) \approx 1 - f_{\text{FD}}(E_{\text{final}}, T), \tag{19}$$

where $\boldsymbol{T}$ is the local temperature of the given species in the cell, and $\boldsymbol{E}_{\text{final}}$ the final energy of the given particle

NTC method: modifications for studying neutrinos IV

- **Feature 4: number of particles per cell**
  - In the plasma we consider, there are distinct 4 species
  - We describe some of their aspects by temperatures, which makes sense only if there is a significant amount of particles
  - Therefore, we take $\mathcal{O}(100-1000)$ particles per cell, which highly exceeds the amounts taken in the traditional NTC simulations
- **Feature 5: decaying LLPs**
  - Start with the number $N_{\text{LLP},0}$ of LLPs defined by the initial number density
  - Assuming the exponential decay law $N_{\text{LLP}}(t) = \exp[-t/\tau]N_{\text{LLP},0}$, per each timestep, decay $\boldsymbol{\Delta N_{\text{LLP}} = N_{\text{LLP}}(t) - N_{\text{LLP}}(t+\Delta t)}$ LLPs

Cross-checks I

- We have performed a number of cross-checks: in short,
  1. The system tends to the equilibrium state defined by the relations

$$\left(\frac{\rho_\nu}{\rho_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu,\bar{\nu}} \frac{7}{8}}{\sum_{s_e, e^\pm} \frac{7}{8} + \sum_{s_\gamma}} = \frac{6 \cdot \frac{7}{8}}{4 \cdot \frac{7}{8} + 2}, \tag{20}$$

$$\left(\frac{n_\nu}{n_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu,\bar{\nu}}}{\sum_{s_e, e^\pm} + \sum_{s_\gamma} \frac{4}{3}} = \frac{6}{4 + \frac{8}{3}}, \tag{21}$$

$$f_{\nu,\text{eq}}(E) = f_{\text{Fermi-Dirac}}(E,T) = \frac{1}{\exp\left[\frac{E}{T}\right] + 1} \tag{22}$$

  2. Assuming the thermal-like initial conditions

$$f_{\nu_\alpha,\text{ini}} = f_{\text{Fermi-Dirac}}(E, T_{\nu_\alpha}), \quad T_{\nu_\alpha} \neq T_{\text{EM}}, \tag{23}$$

  the equilibration of the system approximately follows the theoretical approach from [2001.04466], with and without expansion of the Universe

Injection of monochromatic neutrinos I

- Let us consider the injection of monochromatic neutrinos with energy E_{inj}
- From the qualitative grounds (supplemented by Boltzmann code studies) discussed in [2103.09831], we expect very different behavior of the ratio $\rho_\nu/\rho_{\text{EM}}$ during the thermalization depending on E_{inj}:
  - If $E_{\text{inj}} \simeq T$, the ratio would behave similarly to the one under the assumptions of simply increasing the neutrino temperature $T_\nu > T_\gamma$. In particular, when tending to the equilibrium value (27), it will always be higher than it
  - If $E_{\text{inj}} \gg T$, the ratio would behave in a different fashion: it will fastly drop below the equilibrium value and then slowly tend to it from below. If the injection occurs at $T \sim$ MeV scale, it may result in a decrease of N_{eff}
- Appendix A of [2103.09831] discusses the evolution of $\delta\rho_\nu$ under the injection of 70 MeV neutrinos, and it is shown that $\delta\rho_\nu$ falls below zero
- The same effect is observed for heavy HNLs, which decay into high-energy neutrinos: although they inject most of their energy into neutrinos, the resulting N_{eff} would decrease

Injection of monochromatic neutrinos II

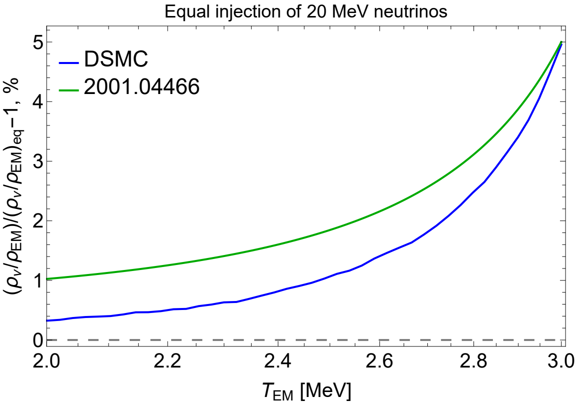
- The findings of [2103.09831] are qualitatively reproduced in [2109.11176], which also studied the effects of heavy HNLs into $N_{\rm eff}$
- However, other works [0008138], [2104.11752] that studied HNLs have found no drop $\delta\rho_\nu < 0$
- The results of all these papers are based on independent Boltzmann codes. It would be complicated to analyze them: in addition to the complexity of the scheme, only the one from [2103.09831] is public
- As the DSMC is a fully independent and transparent approach, it is suited well to resolve the discrepancy

Injection of monochromatic neutrinos III

- We consider the SM plasma at $T = 3$ MeV; for easier reproducibility, we turn off neutrino oscillations
- We consider several injection setups:
  - Injection of neutrinos with $E_{\text{inj}} = 20$ MeV equally into all flavors, $\rho_{\nu,\text{inj}}/\rho_{\nu,\text{thermal}} = 5\%$. The evolution for this setup should not show the drop of $\delta\rho_\nu < 0$, as neutrinos have too low energy. However, already for this setup, there should be significant deviations from the temperature-based description of the thermalization from [2001.04466]
  - Injection of neutrinos with $E_{\text{inj}} = 70$ MeV, considering various injection patterns and amounts
    (A particular realization of this setup is discussed in [2103.09831])
  - Injection of high-energy neutrinos with $E_{\text{inj}} = 500$ MeV, $\rho_{\nu,\text{inj}}/\rho_{\nu,\text{thermal}} = 5\%$, to study how the performance of the code depends on E_{inj}
- Apart from [2103.09831], for comparison for the injection of 70 MeV neutrinos, we will use results of an independent Boltzmann solver developed by Kensuke Akita based on [2005.07047]

Injection of monochromatic neutrinos IV

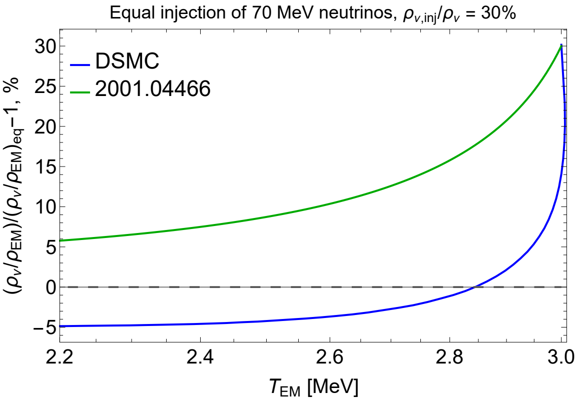
- As expected, the injection of 20 MeV neutrinos would not result in a drop $\delta\rho_\nu < 0$
- However, there is already a sizable deviation from the temperature-based approach from [2001.04466]



- It is caused by the energy dependence of the weak interaction cross-section, $\sigma \propto s$, which enhances the energy transfer rate for this setup: $E_{\nu,\text{inj}} > \langle E_\nu \rangle = 3.15T$

Injection of monochromatic neutrinos V

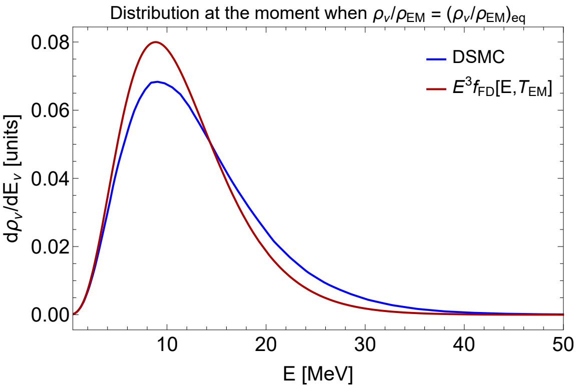
- Let us now consider the injection of 70 MeV neutrinos. We will first start with a large injection fraction $\rho_{\nu_{\rm inj}}/\rho_{\nu,\rm eq} = 30\%$
- Compared to the 20 MeV case, the situation changes: the ratio $\delta\rho_\nu$ drops below 0 and freezes there due to the expansion of the Universe



Injection of monochromatic neutrinos VI

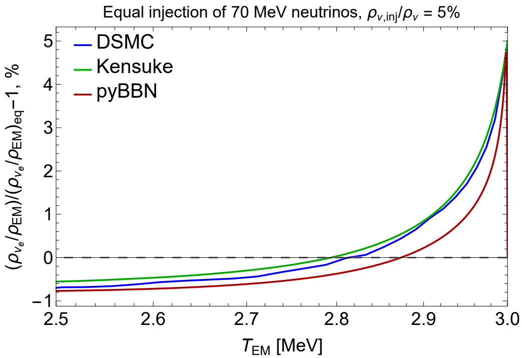
Mechanism:

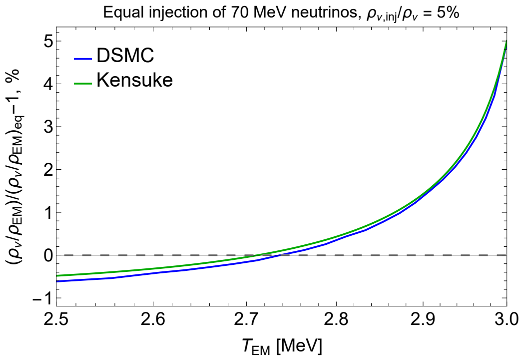
- The main thermalization mechanism is annihilations $\nu\bar{\nu} \to e^+e^-$ or $\nu\nu \to \nu\nu$, where one of the νs is high-energy, and another one may be from the initial thermal population
- Annihilations knock out thermal neutrinos to the EM plasma and to the high-energy neutrino tail



- At the moment where $\rho_\nu = \rho_{\text{EM}}$, the neutrino distribution is still non-thermal: 1) its high-energy tail is hotter than the EM plasma, 2) its low-energy part is underrepresented
- Since weak interactions prefer high-energy interacting particles, the first feature still leads to the shift of the $\nu \leftrightarrow$ EM energy exchange to the EM sector

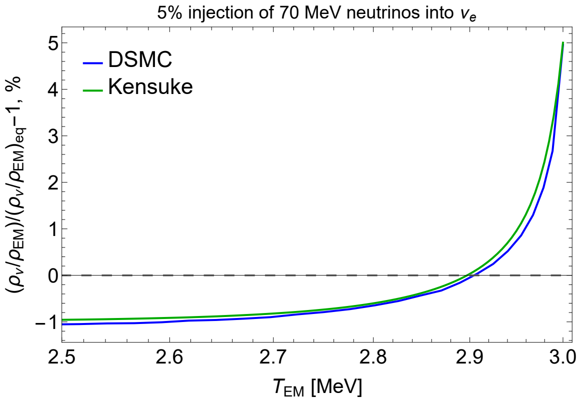
Injection of monochromatic neutrinos VII

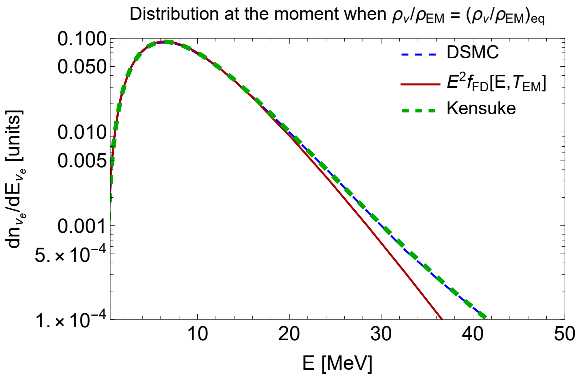




- We will now consider the injection $\rho_{\nu_\alpha,\text{inj}}/\rho_{\nu,\text{eq}} = 5/3\%$, similar to the one studied in [2103.09831]
- The qualitative effect of $\delta\rho_\nu < 0$ remains
- We obtain a close agreement with Kensuke's code, but there is a quantitative disagreement with [2103.09831] (possibly a different setup has been used there?)

Injection of monochromatic neutrinos VIII





- The case of the injection $\rho_{\nu_e,\text{inj}}/\rho_{\nu,\text{eq}} = 5\%$, $\rho_{\nu_{\mu,\tau},\text{inj}} = 0$ shows a similar behavior $\delta\rho_\nu < 0$
- Again, the behavior closely agrees with Kensuke's code;

Injection of monochromatic neutrinos IX

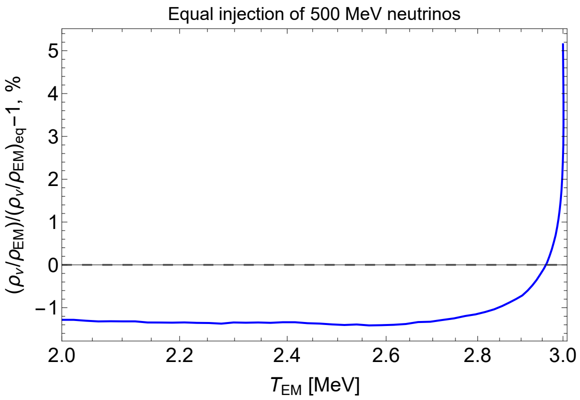
- Let us now proceed to the case of the injection of 500 MeV neutrinos
- The Boltzmann codes discussed above are not suitable for such cases, as the computation time would grow enormously. For instance, if keeping the linearity of the momentum grid, such that the number of grid points $N_{\text{grid}} \sim E_{\text{inj}}$, the computation time would increase (compared to the 70 MeV case) by a large factor

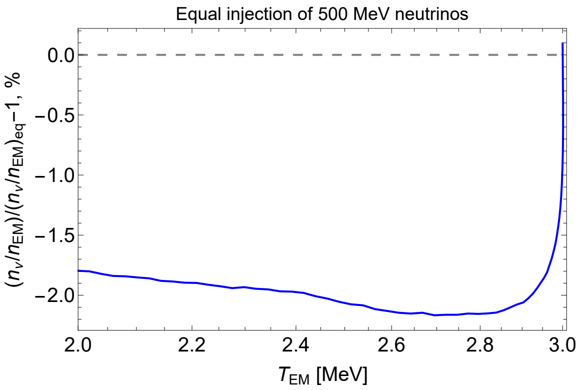
$$\frac{t_{500\text{ MeV}}}{t_{70\text{ MeV}}} \sim \left(\frac{500}{70}\right)^3 \times \chi_{\text{timestep}} \approx 360 \times \chi_{\text{timestep}}, \tag{24}$$

  where χ_{timestep} is a slowdown due to a decrease in the timestep required to resolve the equilibration

  Switching to the logarithmic grid may somewhat improve the timing, but additional studies would require whether it may be made accurate enough
- In contrast, with DSMC, the dropdown in the computation time would be caused almost solely by χ_{timestep}

Injection of monochromatic neutrinos X





– To summarize, an independent study of the injection of high-energy neutrinos with DSMC has confirmed that the injection of high-energy neutrinos around their decoupling would lead to a decrease in $N_{\rm eff}$

Indirect injection of neutrinos: decays of SM particles I

- In many new physics scenarios, neutrinos are produced secondarily – by decays of unstable SM particles m: muons, pions, kaons, etc.
- In this case, they have a continuous spectrum determined by the properties of the decaying particles
- Depending on the type of m, it may either lose all the kinetic energy before decaying ($\pi^{\pm}, \mu^{\pm}, K^{\pm}$), or decay in-flight (the case of K^0)
- The phase space of decaying particles must be carefully implemented to obtain the neutrinos kinematics. Fortunately, we do not need to do this from scratch: we may use one of the modules of `SensCalc` [2305.13383]

  modifications: 1) depending on the decaying particle type, remove its kinetic energy and add it directly to the EM sector, 2) allow $\pi^{\pm}, \mu^{\pm}, K^{\pm}, K_L$ *to decay*
- The same module simulates decays of various new physics particles: HNLs, vector mediators, dark scalars, ALPs, etc. We will study these physics cases in future works

Indirect injection of neutrinos: decays of SM particles II

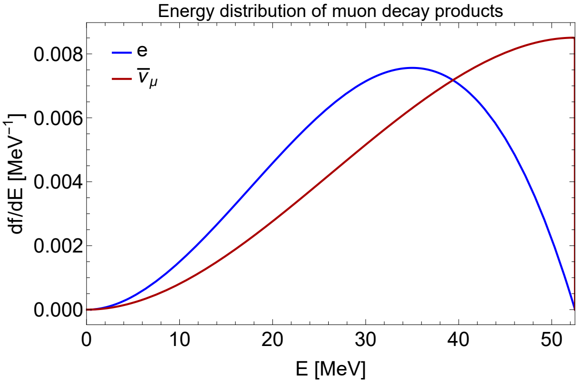
- Consider neutrinos appearing from decays of charged pions and muons
- Independently of the initial kinetic energies of π, μ, E_{kin}, they will thermalize before decaying
- The past study [1706.01920] of the impact of neutrinos from decays of muons and pions (in the context of Higgs-like scalars) in the limit $E_{\text{kin}} \ll m_{\mu/\pi}$ came to the conclusion that it would increase N_{eff}. It did not account for the impact of $E_{\nu,\text{inj}} \gg \langle E_{\nu,\text{thermal}} \rangle$
- Let us study this with DSMC

Indirect injection of neutrinos: decays of SM particles III

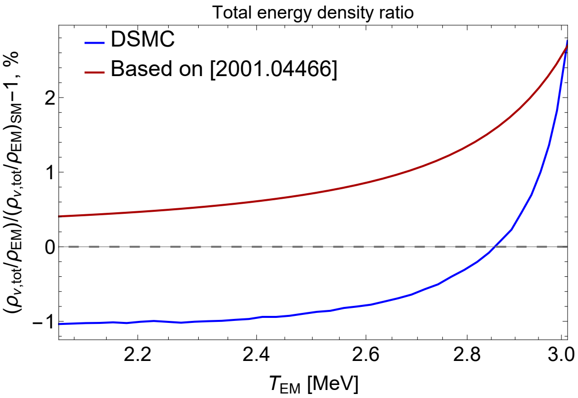
– Consider for simplicity here decaying muons

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \tag{25}$$

– The spectrum of neutrinos from decays of μ is continuous, with $E_{\nu,\max} \approx m_\mu/2 \approx 52.5$ MeV
– On average, only 30% of μ's energy goes directly to the EM plasma; for $\pi^\pm$, the ratio is even smaller



Indirect injection of neutrinos: decays of SM particles IV



– Since $E_{\nu,\mathrm{inj}} \gg \langle E_{\nu,\mathrm{thermal}}\rangle$ around neutrino decoupling, we see the same feature $\delta\rho_\nu < 0$

Injection of neutrinos and large lepton asymmetry I

- Another interesting case is when, in addition to injections of neutrinos, the thermal plasma is characterized by a large lepton asymmetry stored in the neutrino sector
- We parametrize the asymmetry in terms of the neutrino chemical potential μ_ν. The correction to N_{eff} caused by $\mu_\nu \neq 0$ is always positive independently of the sign of μ_ν
- However, it may be possible to diminish this correction by injections of high-energy neutrinos

Cross-check 1: Dynamical equilibrium I

– **Cross-check 1: Dynamical equilibrium.** Independently of the initial conditions, the system of neutrinos and EM particles tends to the equilibrium state having the following properties:

- The ratios of energy and number densities agree with the simple estimates

$$\left(\frac{\rho_\nu}{\rho_{\rm EM}}\right)_{\rm eq} = \frac{\sum_{\nu,\bar{\nu}} \frac{7}{8}}{\sum_{s_e,e^{\pm}} \frac{7}{8} + \sum_{s_\gamma}} = \frac{6\cdot\frac{7}{8}}{4\cdot\frac{7}{8}+2}, \quad (26)$$

$$\left(\frac{n_\nu}{n_{\rm EM}}\right)_{\rm eq} = \frac{\sum_{\nu,\bar{\nu}}}{\sum_{s_e,e^{\pm}} + \sum_{s_\gamma} \frac{4}{3}} = \frac{6}{4+\frac{8}{3}}, \quad (27)$$

  where s is the polarization

- In particular, the shape of the neutrino energy distribution function tends to the FD distribution (times the phase space factor) with the temperature $T_\nu = T_{\rm EM}$:

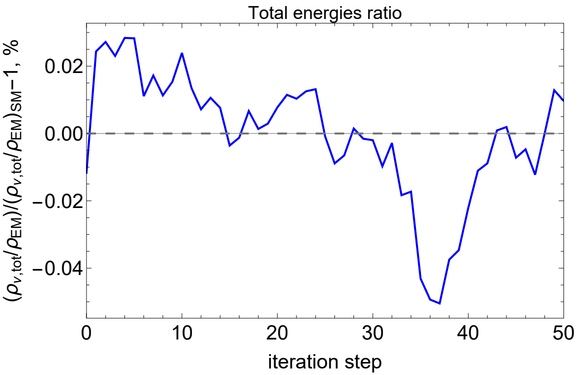
$$f_{\rm eq}(E) \sim \frac{E^2}{\exp\left[\frac{E}{T}\right]+1} \quad (28)$$

Cross-check 1: Dynamical equilibrium II

– Here and below, we will consider the quantities of the type

$$\delta\rho = \left(\frac{\rho_{\mathrm{EM}}}{\rho_\nu}\right)_{\mathrm{eq}} \frac{\rho_\nu}{\rho_{\mathrm{EM}}} - 1 \qquad (29)$$

to characterize the macroscopic equilibration

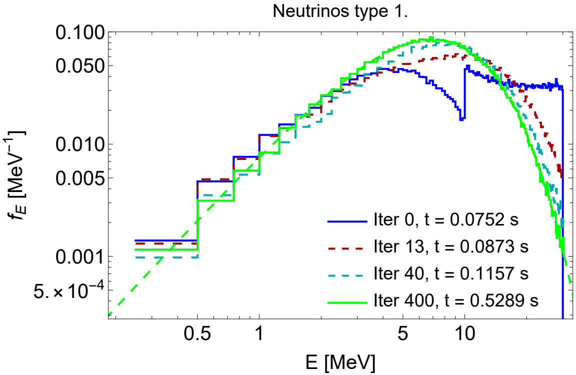


– The first check: if the initial condition is full equilibrium, the system must stay in it
– The DSMC simulation reproduces this behavior
The plot shows the evolution of the system with the initial condition chosen such that neutrinos have the Fermi-Dirac distributions with the temperatures $T_{\nu_\alpha} = T_{EM}$

Cross-check 1: Dynamical equilibrium III

– The second test: we start with an arbitrary initial distribution of neutrinos and study its relaxation

– In the absence of the expansion of the Universe, the final state must be described by Eqs. (27), (28)



– We reproduce this behavior:
- The blue curve: the initial neutrino distribution
- The green solid curve: the distribution after reaching the equilibration as obtained by the simulation. It matches the theoretical distribution (28) with the temperature defined by $\rho_{\nu,\text{sim}} \propto T^4$ (dashed green)

Cross-check 2: initial conditions parametrized by temperatures I

– **Cross-check 2: "Equilibrium" rates.** Assuming the initial conditions parametrized solely by the neutrino and EM temperatures $T_{\nu_\alpha}, T_{\mathrm{EM}}$, the energy and number transfer rates *at the first iterations* must agree with the theoretical predictions obtained in [2001.04466]:

$$\frac{\delta \rho_{\nu_e}}{\delta t} = \frac{G_F^2}{\pi^5}\left[4\left(g_{eL}^2 + g_{eR}^2\right) F(T_\gamma, T_{\nu_e}) + F(T_{\nu_\mu}, T_{\nu_e}) + F(T_{\nu_\tau}, T_{\nu_e})\right], \tag{30}$$

$$\frac{\delta \rho_{\nu_\mu}}{\delta t} = \frac{G_F^2}{\pi^5}\left[4\left(g_{\mu L}^2 + g_{\mu R}^2\right) F(T_\gamma, T_{\nu_\mu}) - F(T_{\nu_\mu}, T_{\nu_e}) + F(T_{\nu_\tau}, T_{\nu_\mu})\right], \tag{31}$$

$$\frac{\delta \rho_{\nu_\tau}}{\delta t} = \frac{G_F^2}{\pi^5}\left[4\left(g_{\mu L}^2 + g_{\mu R}^2\right) F(T_\gamma, T_{\nu_\tau}) - F(T_{\nu_\tau}, T_{\nu_e}) - F(T_{\nu_\mu}, T_{\nu_\tau})\right], \tag{32}$$

$$F(T_1, T_2) = 32 \cdot 0.884 \cdot \left(T_1^9 - T_2^9\right) + 56 \cdot 0.829 \cdot T_1^4 T_2^4 \left(T_1 - T_2\right). \tag{33}$$

Cross-check 2: initial conditions parametrized by temperatures II

– **Cross-check 2: "Equilibrium" rates**. Assuming the initial conditions parametrized solely by the neutrino and EM temperatures $T_{\nu_\alpha}, T_{\text{EM}}$, the energy and number transfer rates *at the first iteration* agree with the theoretical predictions obtained in [2001.04466]:

$$\frac{\delta n_{\nu_e}}{\delta t} = 0.852 \cdot 8\frac{G_F^2}{\pi^5}\left[4\left(g_{eL}^2+g_{eR}^2\right)\left(T_\gamma^8 - T_{\nu_e}^8\right) + \left(T_{\nu_\mu}^8 - T_{\nu_e}^8\right) + \left(T_{\nu_\tau}^8 - T_{\nu_e}^8\right)\right], \quad (34)$$

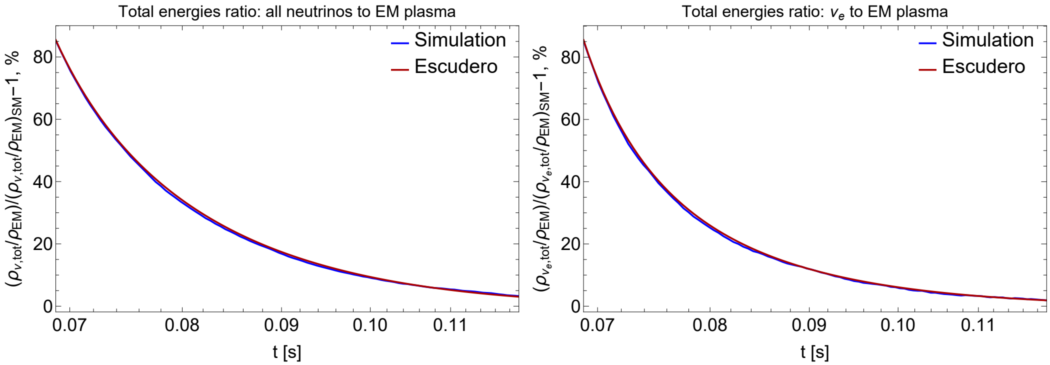
$$\frac{\delta n_{\nu_\mu}}{\delta t} = 0.852 \cdot 8\frac{G_F^2}{\pi^5}\left[4\left(g_{\mu L}^2+g_{\mu R}^2\right)\left(T_\gamma^8 - T_{\nu_\mu}^8\right) - \left(T_{\nu_\mu}^8 - T_{\nu_e}^8\right) + \left(T_{\nu_\tau}^8 - T_{\nu_\mu}^8\right)\right], \quad (35)$$

$$\frac{\delta n_{\nu_\tau}}{\delta t} = 0.852 \cdot 8\frac{G_F^2}{\pi^5}\left[4\left(g_{\mu L}^2+g_{\mu R}^2\right)\left(T_\gamma^8 - T_{\nu_\tau}^8\right) - \left(T_{\nu_\tau}^8 - T_{\nu_e}^8\right) - \left(T_{\nu_\tau}^8 - T_{\nu_e}^8\right)\right], \quad (36)$$

Cross-check 2: initial conditions parametrized by temperatures III

- We split this cross-check into two stages
- In the first one, we have made a simplified simulation where one fully characterizes neutrinos by their temperatures T_{ν_α}. The simulation, therefore, determines only the temperature exchange rate
- At each iteration, it must agree perfectly with the approach described in [2001.04466]
- To check this, we have implemented the approach from [2001.04466] as well. Here, we turn off the expansion of the Universe

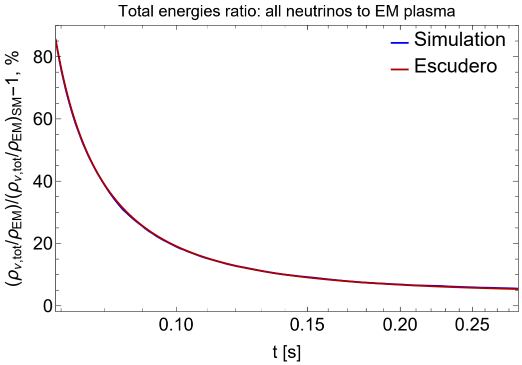
Cross-check 2: initial conditions parametrized by temperatures IV

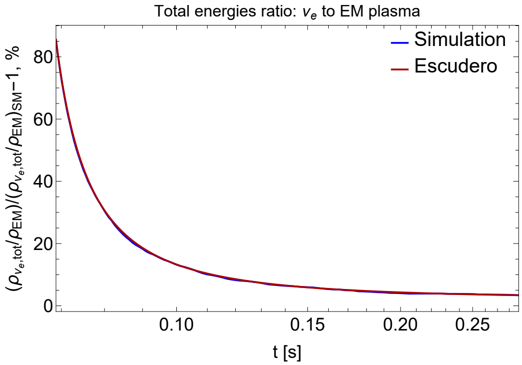


- The agreement is perfect if we include a factor of 2 in Eq. (10) for the number of sampled pairs N_{sampled}
- This factor is universal; we are studying its origin

The simulation assumes the initial conditions according to $T_{\nu_\alpha} = 3.5$ *MeV*, $T_{EM} = 3$ *MeV*

Cross-check 3: including the Hubble expansion I





- **Cross-check 3: Hubble expansion**. In the presence of the expansion, the thermalization must respect the neutrino decoupling
- To make this cross-check, we again consider the two simulations, simplified "equilibrium" and realistic ones, and compare their predictions with the approach from [2001.04466]. Results agree in the same fashion as in the case $H = 0$