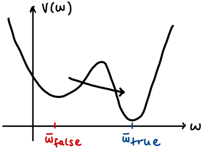
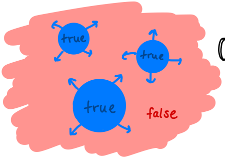
Phase Transitions and Gravitational Waves with BSMPTv3

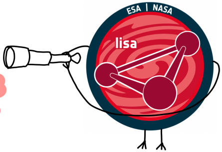
Lisa Biermann (`lisa.biermann@psi.ch`)

talk based on

[Basler, LB, Mühlleitner, Müller, Santos, Viana, ’24]







BLV and Phase Transitions

→ Necessary conditions for baryogenesis: [Sakharov, ’67]

+ $\cancel{C}$ and $\cancel{CP}$ + departure from thermal equilibrium

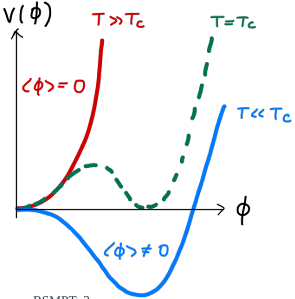
BLV and Phase Transitions

→ Necessary conditions for baryogenesis: [Sakharov, '67]

+ $\not{C}$ and $\cancel{CP}$ + departure from thermal equilibrium

via

first-order (EW) phase transition



BLV and Phase Transitions

→ Necessary conditions for baryogenesis: [Sakharov, ’67]

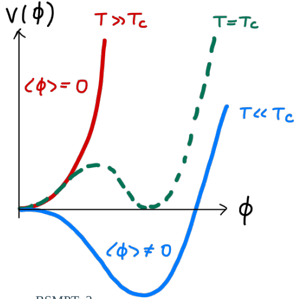
\+ NOT enough in SM

$\cancel{C}$ and $\cancel{CP}$ + departure from thermal equilibrium

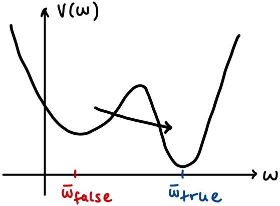
via

first-order (EW) phase transition

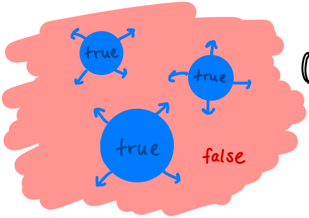
NOT in SM



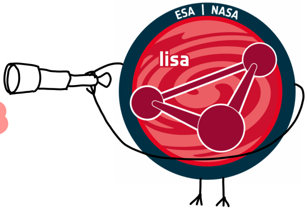
A Glimpse of the Early Universe through Phase Transitions



First-order phase transition (FOPT) in the early universe



True-vacuum bubble nucleation and expansion into hot plasma



Gravitational waves (GWs) in range of LISA (mHz)

- Models *beyond* the SM can undergo a first-order electroweak phase transition (FOEWPT) (between barrier-separated false vacuum and true vacuum)
  → Strong FOEWPT is necessary ingredient for *electroweak baryogenesis*
  ↪ talks by Bahaa Ilyas and Marieke Postma
- FOPT leads to formation of true vacuum bubbles which expand into surrounding plasma
- Interactions with plasma (and collisions of bubbles) source GWs *within* sensitivity of LISA

⇒ `BSMPTv3`: First public code that provides whole chain from particle physics model to GWs (model independent)!

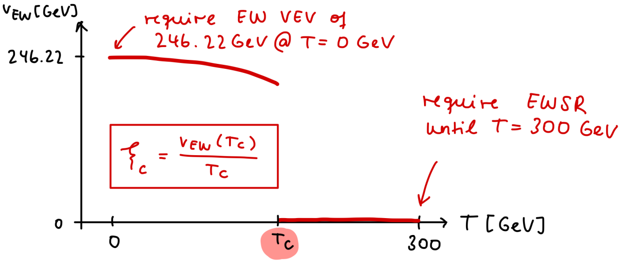
BSMPTv1/v2

[Basler, Mühlleitner, Müller, ’18/’20]

- Open-source code `BSMPT` for computation of EWPTs

“Beyond the Standard Model Phase Transitions”
`https://github.com/phbasler/BSMPT`

- Implementation of the one-loop resummed effective potential at finite temperature
- *On-shell* renormalization scheme
- `v1/v2`: Derivation of **critical temperature** via discontinuity in global EW minimum in $T \in [0,\, 300]$ GeV



$$\boxed{\xi_c = \frac{v_{EW}(T_c)}{T_c}}$$

→ `v1/v2`: Search for *possible* **one-step (strong) first-order EWPTs**!

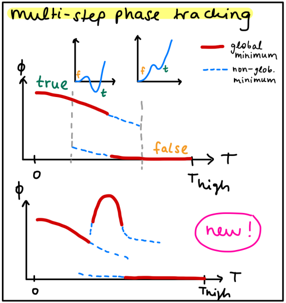
→ Input for `BSMPT`: theor. and exp. valid parameters points generated with `ScannerS` [Coimbra et al., ’13; Mühlleitner et al., ’20] (e.g. using `HiggsTools` [Bahl et al., ’22] to require agreement with Higgs data)

BSMPTv3

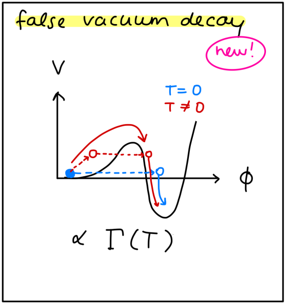
[Basler, LB, Mühlleitner, Müller, Santos, Viana, ’24]

→ Existence of critical temperature is only a prerequisite for an EWPT!

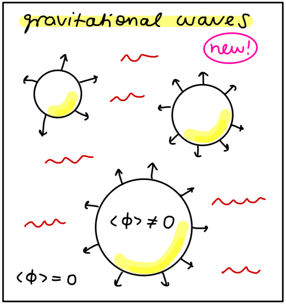
⇒ `BSMPTv3`: Self-contained open-source tool that implements the whole chain from a particle physics model to the calculation of EWPTs and the sourced GW spectrum



- Local minimum tracking
- Identification of phase overlaps and multi-step PT histories
- Handle discrete symmetries, flat directions and check for EWSR

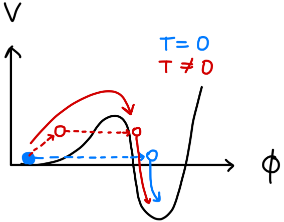


False vacuum is metastable for $T < T_c$ and *can* decay via tunnelling and thermal fluctuations



First-order PTs can source gravitational waves!

False Vacuum Decay



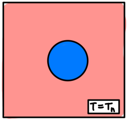
$$\boxed{\Gamma(T \neq 0) \simeq T^4 \left(\frac{\boldsymbol{S_3}}{2\pi T}\right)^{3/2} \mathrm{e}^{-\boldsymbol{S_3}/T}}$$

bounce action S_3 determined numerically in `BSMPTv3`

- False vacuum decay leads to bubble nucleation and expansion
- Bubble expansion races against expansion of the universe parametrized by Hubble rate $H(T)$
- **Characteristic temperatures**:

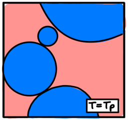
Nucleation temperature:

$$\frac{\Gamma(T_n)}{H^4(T_n)} = 1$$



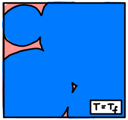
Percolation temperature:

$$P_f(T_p) = 0.71$$



Completion temperature:

$$P_f(T_f) = 0.01$$



P_f: false vacuum fraction (optional user input in `BSMPTv3`)

Gravitational Waves sourced by First-Order EWPTs

- Bubble expansion through hot thermal plasma
- `BSMPTv3` assumes a *terminal* bubble wall velocity v_w due to friction between plasma and bubble wall
  - $\Rightarrow$ Vacuum energy is transferred to plasma (parametrized by efficiency factor κ [Espinosa et al., '10])
  - $\Rightarrow$ Plasma interactions break spherical symmetry $\Rightarrow$ source of isotropic stochastic GW background
  - $\rightarrow$ GWs are sourced as plasma sound waves [Giblin et al., '13/14; Hindmarsh et al., '14/15] and plasma turbulence [Caprini et al., '06; Kahniashvili et al., '08/10]

- GW spectrum determined by: PT strength $\boldsymbol{\alpha}$, inverse PT duration $\boldsymbol{\beta/H}$, wall velocity $\boldsymbol{v_w}$, transition temperature $\boldsymbol{T_*}$

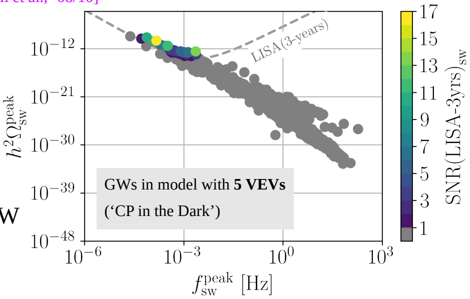
calculated optional user input

$$\boxed{h^2\Omega_{\text{GW}} = h^2\Omega_{\text{GW}}(\boldsymbol{\alpha}, \boldsymbol{\beta/H}, \boldsymbol{v_w}, \boldsymbol{T_*})}$$

- Signal-to-noise ratio at future space-bound GW observatory LISA [Caprini et al., '19]

$$\text{SNR}(\mathcal{T}) = \sqrt{\mathcal{T}\int_{f_{\text{min}}}^{f_{\text{max}}} \mathrm{d}f \left[\frac{h^2\Omega_{\text{GW}}(f)}{h^2\Omega_{\text{Sens}}(f)}\right]^2}$$

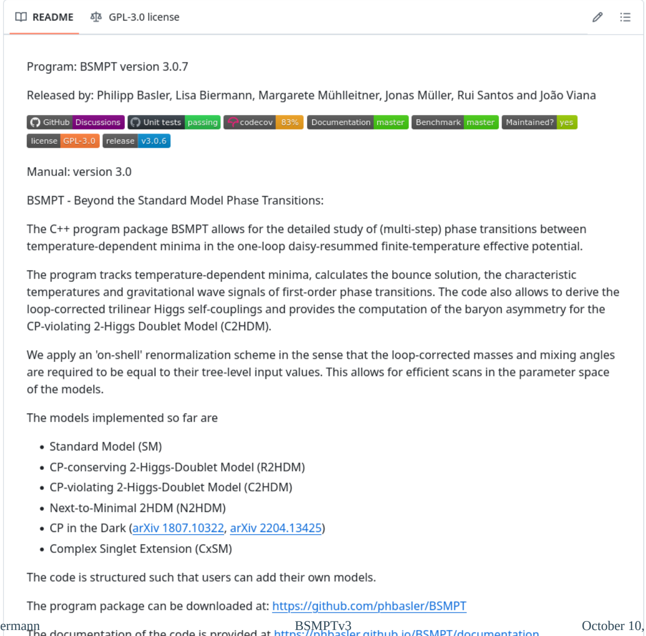
$\rightarrow$ SNR > 1 is within sensitivity of LISA



$h^2\Omega_{\text{Sens}}$	nominal LISA sensitivity
$\mathcal{T}$	exp. acquisition time
f_{min}, f_{max}	LISA sensitivity range

Installation and Usage

- BSMPT is open source: `https://github.com/phbasler/BSMPT` (documentation)



Program: BSMPT version 3.0.7

Released by: Philipp Basler, Lisa Biermann, Margarete Mühlleitner, Jonas Müller, Rui Santos and João Viana

Manual: version 3.0

BSMPT - Beyond the Standard Model Phase Transitions:

The C++ program package BSMPT allows for the detailed study of (multi-step) phase transitions between temperature-dependent minima in the one-loop daisy-resummed finite-temperature effective potential.

The program tracks temperature-dependent minima, calculates the bounce solution, the characteristic temperatures and gravitational wave signals of first-order phase transitions. The code also allows to derive the loop-corrected trilinear Higgs self-couplings and provides the computation of the baryon asymmetry for the CP-violating 2-Higgs Doublet Model (C2HDM).

We apply an 'on-shell' renormalization scheme in the sense that the loop-corrected masses and mixing angles are required to be equal to their tree-level input values. This allows for efficient scans in the parameter space of the models.

The models implemented so far are

- Standard Model (SM)
- CP-conserving 2-Higgs-Doublet Model (R2HDM)
- CP-violating 2-Higgs-Doublet Model (C2HDM)
- Next-to-Minimal 2HDM (N2HDM)
- CP in the Dark (arXiv 1807.10322, arXiv 2204.13425)
- Complex Singlet Extension (CxSM)

The code is structured such that users can add their own models.

The program package can be downloaded at: https://github.com/phbasler/BSMPT

The documentation of the code is provided at https://phbasler.github.io/BSMPT/documentation

Installation and Usage

- Installation:

```
[lisa@pc: ~]$ pip3 install cmake conan
[lisa@pc: ~]$ git clone git@github.com:phbasler/BSMPT.git
[lisa@pc: ~]$ cd BSMPT
[lisa@pc: BSMPT]$ python3 Build.py
======== Input profiles ========
Profile host:
[settings]
arch=x86_64
build_type=Release
compiler=gcc
compiler.cppstd=gnu17
compiler.libcxx=libstdc++11
compiler.version=11
os=Linux
[...]
[lisa@pc: BSMPT]$ ls build/linux-x86_64-release/bin/
benchmarks  BSMPT  CalcCT  CalcGW  CalcTemps  GenericTests
MinimaTracer  NLOVEV  PotPlotter  standalone  Test
TripleHiggsCouplingsNLO  VEVEVO
[lisa@pc: BSMPT]$
```

get required packages

clone the repository

run installation script

available executables

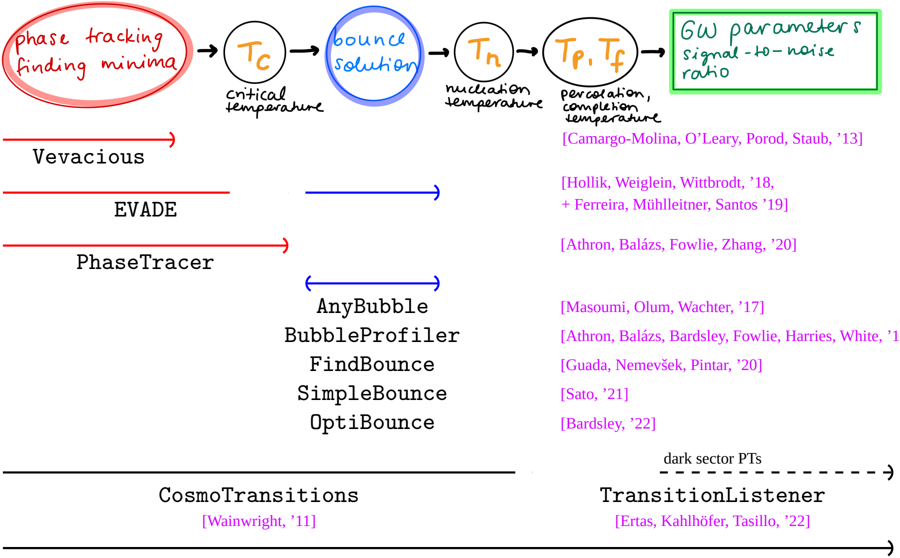
Installation and Usage

- New executables of `BSMPTv3`:
  - `MinimaTracer`: tracing of minima as function of temperature
  - `CalcTemps`: calculation of characteristic temperatures for all found FOPTs
  - `CalcGW`: calculation of GW spectrum + SNR for all found FOPTs
  - `PotPlotter`: visualization of multi-dimensional potential contours

---

```
[lisa@pc: BSMPT/build/linux-x86_64-release]$ ./bin/CalcGW --help
CalcGW calculates the gravitational wave signal
it is called by

        ./bin/CalcGW model input output firstline lastline

or with arguments

        ./bin/CalcGW [arguments]

with the following arguments, ([*] are required arguments, others are optional):

argument                  default   description
--help                              shows this menu
--model=                            [*] model name
--input=                            [*] input file (in tsv format)
--output=                           [*] output file (in tsv format)
--firstline=                        [*] line number of first line in input file
                                        (expects line 1 to be a legend)
--lastline=                         [*] line number of last line in input file
--thigh=                  300       high temperature [GeV]
[...]
[lisa@pc: BSMPT/build/linux-x86_64-release]$
```

Public Codes on the Market



Vevacious [Camargo-Molina, O'Leary, Porod, Staub, '13]

EVADE [Hollik, Weiglein, Wittbrodt, '18, + Ferreira, Mühlleitner, Santos '19]

PhaseTracer [Athron, Balázs, Fowlie, Zhang, '20]

AnyBubble [Masoumi, Olum, Wachter, '17]

BubbleProfiler [Athron, Balázs, Bardsley, Fowlie, Harries, White, '19]

FindBounce [Guada, Nemevšek, Pintar, '20]

SimpleBounce [Sato, '21]

OptiBounce [Bardsley, '22]

CosmoTransitions [Wainwright, '11]

TransitionListener [Ertas, Kahlhöfer, Tasillo, '22]

BSMPTv3

[Basler, Mühlleitner, Müller, '18/'20]
[Basler, LB, Mühlleitner, Müller, Santos, Viana, '24]

Comparison

	`CosmoTransitions`	`BSMPTv3`
	`python` library	`C++` package
models	user-defined	already implemented: SM, SM + singlet, SM + doublet (CP-conserving and CP-violating), SM + doublet + singlet; user-defined
usage	write own python code using routines, write model implementation	run executables with theor./exp. valid parameter points (generated via `ScannerS`)
renormalization	$\overline{\text{MS}}$	OS-scheme
input for model implementation	tree-level potential, full-field-dependent boson and fermion masses	scalar-coupling tensors, finite CTs for OS-scheme → automatized with `SymPy` and `Maple` model generation interface ☛details new stand-alone features*

* new stand-alone features of `BSMPTv3`: minima tracking, bounce solution, temperatures, GW spectrum directly for user-defined potential function (no model implementation needed!)

Runtime

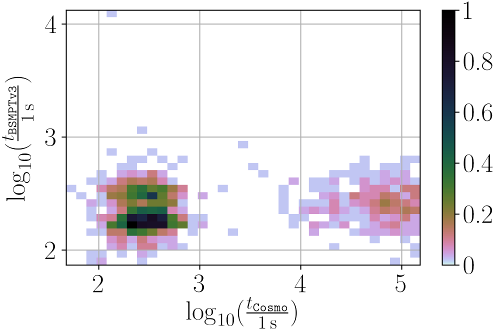
m_{H_a} [GeV]	m_{H_b} [GeV]	m_A [GeV]	$m_{H^\pm}$ [GeV]	c_{H_bVV}	$\tan\beta$	m_{12}^2 [GeV^2]
125.09	[30, 1500]	[30, 1500]	[150, 1500]	[−0.3, 0.3]	[0.8, 25]	$[1\times10^{-3}, 5\times10^{5}]$

Table 3: Scan ranges for the CP-conserving 2HDM type 1 in the input parameters used by `ScannerS`.

- CP-conserving Two-Higgs Doublet Model (type 1) with **four** VEV directions

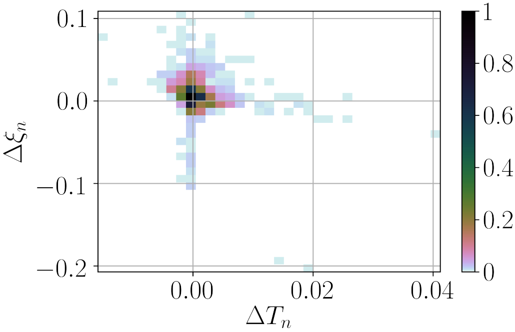
$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_1 + i\eta_1 \\ \zeta_1 + \boldsymbol{\omega_1} + i\psi_1\end{pmatrix},\ \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_2 + \boldsymbol{\omega}_{\mathrm{CB}} + i\eta_2 \\ \zeta_2 + \boldsymbol{\omega_2} + i(\psi_2 + \boldsymbol{\omega}_{\mathrm{CP}})\end{pmatrix}$$

- Broad parameter scan with `ScannerS`, `HiggsTools`
- Comparison between `BSMPTv3` and `CosmoTransitions` (for same-transitions subset)



- `BSMPTv3`: mean (median) runtime of 4.2 min (3.5 min)
- `CosmoTransitions`: mean (median) runtime of 41.5 min (5.6 min)
- `BSMPTv3` up to $\times 10^3$ faster than `CosmoTransitions`

Results



$$\Delta T_i = \frac{\left(T_i^{\texttt{BSMPTv3}} - T_i^{\texttt{Cosmo}}\right)}{T_i^{\texttt{BSMPTv3}}}$$

$$\Delta \xi_i = \frac{\left(\xi_i^{\texttt{BSMPTv3}} - \xi_i^{\texttt{Cosmo}}\right)}{\xi_i^{\texttt{BSMPTv3}}}$$

$$\text{with} \quad \xi_i = \frac{\sqrt{\sum_k \omega_k^2(T_i)}}{T_i}$$

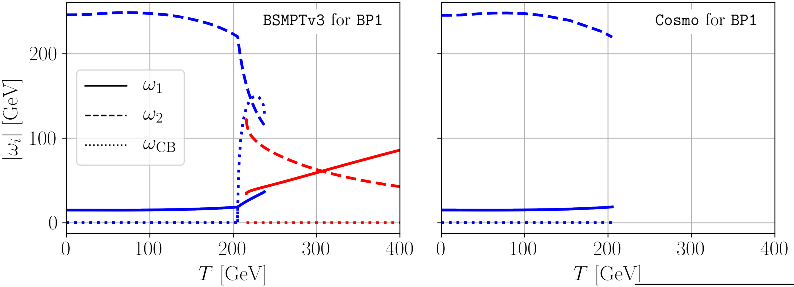
$$\text{and} \quad \omega_k \in \{\omega_{\text{CB}},\, \omega_1,\, \omega_2,\, \omega_{\text{CP}}\}$$

- Mean (median) relative differences:
  - $\Delta T_c < 0.1\,\%$ (critical temperature)
  - $\Delta T_n < 1\,\%$ (nucleation temperature)
- Outliers in $\Delta\xi_n$ correlated w/ rapidly changing potential in small T interval

Complicated Histories

point from: [Aoki, LB, Borschensky, Ivanov, Mühlleitner, Shibuya, ’23]

$$\texttt{BP1:}\quad \text{type} = 1\,,\ \lambda_1 = 6.931\,,\ \lambda_2 = 0.2631\,,\ \lambda_3 = 1.287\,,\ \lambda_4 = 4.772\,,\ \lambda_5 = 4.728\,,$$
$$m_{12}^2 = 1.893 \times 10^4\,\text{GeV}^2\,,\ \tan\beta = 16.578\,.$$



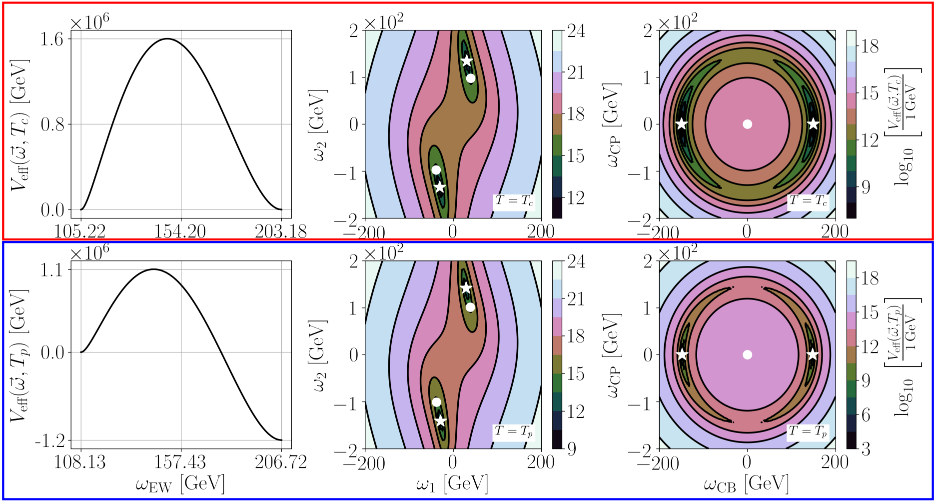
- 2HDM-point for which three field directions develop a non-zero VEV: $\{\omega_1, \omega_2, \omega_{\text{CB}}\}$ (intermediate charge-breaking (CB) phase!)
- First-order PT from neutral to CB phase
- Second-order PT to neutral, EW minimum
- `BSMPTv3` tracks phases, calculates temperatures in < 7 min
- `CosmoTransitions` fails to trace phases for $T > 206$ GeV

	BP1
phases$_{\texttt{BSMPT}}$	0: $\{216, 400\}$ 1: $\{0, 237\}$
pairs$_{\texttt{BSMPT}}$	0: $[0 \to 1]\,\{216, 237\}$
$t_{\texttt{MinimaTracer}}$	41.47 s
T_c	226.3
T_n	$\{222.9, 222.9\}$
T_p	222.6
T_f	222.6
$t_{\texttt{CalcTemps}}$	6.87 min
history	$0 - (0) \to 1$
phases$_{\texttt{Cosmo}}$	$\{0, 206\}$
$T_{\text{crit},\texttt{Cosmo}}$	–
$T^{\text{approx}}_{\text{nucl},\texttt{Cosmo}}$	–
$t_{\texttt{Cosmo}}$	3.95 s

Complicated Histories - `BP1` visualized with `PotPlotter`

$$\omega_{\text{EW}} = \sqrt{\sum_{i=1,2,\text{CB},\text{CP}} \omega_i^2}$$

critical temperature T_c



percolation temperature T_p

slice from *false*
to *true* vacuum

ω_1-ω_2-contour

ω_{CB}-ω_{CP}-contour

Why `BSMPTv3`?

→ The first public (open-source) code that implements the full chain from particle physics model to gravitational waves in a model-independent way

→ Optimized phase tracking over any temperature interval

→ Numerical derivation of bounce solution for any number of field dimensions

→ Besides critical and nucleation, calculation of percolation and completion temperatures

→ Able to treat multi-step PTs, discrete symmetries, flat directions, check for EWSR, report of transition history

→ Calculation of PT parameters and peak frequency/amplitude for (acoustic and turbulence) GW spectrum

→ Computation of signal-to-noise-ratio at LISA

→ For all implemented models (CxSM, R2HDM, C2HDM, N2HDM, CP in the Dark) and *beyond*: (stand-alone features [new in `v3`] + model implementation interface [unchanged from `v1/v2`])

→ Embedded in the existing `BSMPT` code (triple Higgs couplings, EWBG calculation for C2HDM, can use `ScannerS` input)

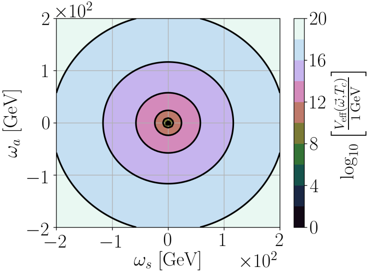
→ On average faster than `CosmoTransitions` (with overall agreement) and can deal (better) with higher dimensional potentials/complicated PT histories

Thanks!

☞ `https://github.com/phbasler/BSMPT`
ℹ `https://arxiv.org/abs/2404.19037`
☺ `https://github.com/phbasler/BSMPT/discussions`
✉ `mailto:bsmpt@lists.kit.edu`

Phase Tracking with Discrete Symmetries and Flat Directions in `BSMPTv3`

`BP3:` $v = 246.22\,\text{GeV}\,,\ v_s = 0\,\text{GeV}\,,\ v_a = 0\,\text{GeV}\,,\ m^2 = -15\,650\,\text{GeV}^2\,,$
$b_2 = -8859\,\text{GeV}^2\,,\ \lambda = 0.52\,,\ \delta_2 = 0.55\,,\ d_2 = 0.5\,,$
$a_1 = 0\,\text{GeV}^3\,,\ b_1 = 0\,\text{GeV}^2\,.$



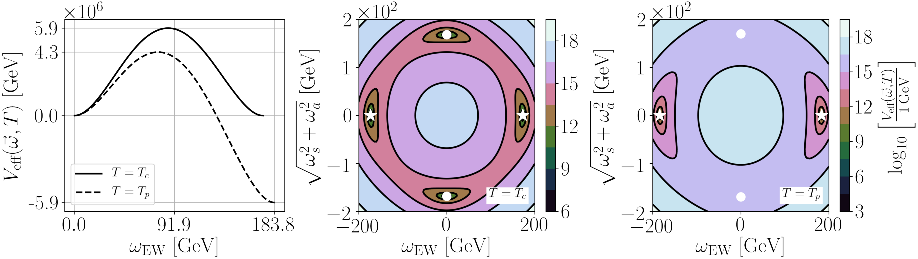
$$V = \frac{m^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 + \frac{\delta_2}{2}\Phi^\dagger\Phi|\mathbb{S}|^2 + \frac{b_2}{2}|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4 + \left(\frac{b_1}{4}\mathbb{S}^2 + a_1\mathbb{S} + c.c.\right)$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ \omega_{\text{EW}} + h + iG^0 \end{pmatrix}\,,$$

$$\mathbb{S} = \frac{1}{\sqrt{2}}\left(s + \omega_s + i\left(a + \omega_a\right)\right)\,,$$

$$V \propto (\omega_s^2 + \omega_a^2)^2 \equiv \omega_s^2$$

$$V(\omega_{\text{EW}}, \omega_s) = V(-\omega_{\text{EW}}, \omega_s) = V(\omega_{\text{EW}}, -\omega_s)$$



Wall velocity in `BSMPTv3`

see [Athron et al., '23] for a review and references within

By default $v_w = 0.95$, or set to user input or one of the following estimates:

- Estimate by [Lewicki et al., '22] (assuming steady-state ($\dot{v}_b = 0$) and local thermal equilibrium):

$$v_b \simeq \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_\gamma}} & \text{if} \quad \sqrt{\frac{\Delta V}{\alpha \rho_r(T_*)}} < v_{\text{CJ}} \\ 1 & \text{if} \quad \sqrt{\frac{\Delta V}{\alpha \rho_r(T_*)}} > v_{\text{CJ}} \end{cases}$$

- Estimate by [Laurent et al., '23] (assuming local thermal equilibrium):

$$v_b = \left(\left| \frac{3\alpha + \Psi - 1}{2\left(2 - 3\Psi + \Psi^3\right)} \right|^{\frac{p}{2}} + \left| v_{\text{CJ}} \left(1 - a \frac{(1-\Psi)^b}{\alpha} \right) \right|^{\frac{p}{2}} \right)^{\frac{1}{p}}$$

with Chapman-Jouguet velocity $v_{\text{CJ}} = \frac{1}{1+\alpha} \left(c_s + \sqrt{\alpha^2 + \frac{2}{3}\alpha} \right)$

$\rho_r(T_*) = \frac{\pi^2}{30} g^*(T_*) T_*^4$

rel. matter density

$\Psi = \frac{\omega_t}{\omega_f}$ enthalpy ratio

$a = 0.2233$ num. fit result

$b = 1.704$ num. fit result

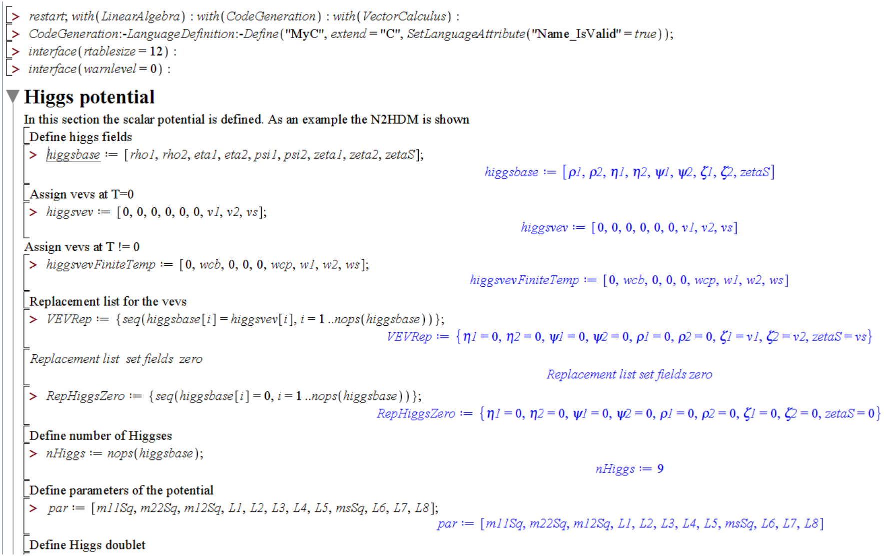
$p = -3.433$ num. fit result

$c_s = \frac{1}{\sqrt{3}}$ sound speed

- Estimates of v_b in *local thermal equilibrium* serve as **upper bound** as v_b gets reduced by non-equilibrium effects!

Model Implementation with `BSMPT` — `Maple`

- model-worksheet: `BSMPT/tools/ModelGeneration/Maple/CreateModel.mw`
- *plug-and-play*:



```
> restart; with(LinearAlgebra) : with(CodeGeneration) : with(VectorCalculus) :
> CodeGeneration:-LanguageDefinition:-Define("MyC", extend = "C", SetLanguageAttribute("Name_IsValid" = true));
> interface(rtablesize = 12) :
> interface(warnlevel = 0) :
```

Higgs potential

In this section the scalar potential is defined. As an example the N2HDM is shown

Define higgs fields

```
> higgsbase := [rho1, rho2, eta1, eta2, psi1, psi2, zeta1, zeta2, zetaS];
                                    higgsbase := [ρ1, ρ2, η1, η2, ψ1, ψ2, ζ1, ζ2, zetaS]
```

Assign vevs at T=0

```
> higgsvev := [0, 0, 0, 0, 0, 0, v1, v2, vs];
                                    higgsvev := [0, 0, 0, 0, 0, 0, v1, v2, vs]
```

Assign vevs at T != 0

```
> higgsvevFiniteTemp := [0, wcb, 0, 0, 0, wcp, w1, w2, ws];
                                    higgsvevFiniteTemp := [0, wcb, 0, 0, 0, wcp, w1, w2, ws]
```

Replacement list for the vevs

```
> VEVRep := {seq(higgsbase[i] = higgsvev[i], i = 1 ..nops(higgsbase))};
          VEVRep := {η1 = 0, η2 = 0, ψ1 = 0, ψ2 = 0, ρ1 = 0, ρ2 = 0, ζ1 = v1, ζ2 = v2, zetaS = vs}
```

Replacement list set fields zero

Replacement list set fields zero

```
> RepHiggsZero := {seq(higgsbase[i] = 0, i = 1 ..nops(higgsbase))};
          RepHiggsZero := {η1 = 0, η2 = 0, ψ1 = 0, ψ2 = 0, ρ1 = 0, ρ2 = 0, ζ1 = 0, ζ2 = 0, zetaS = 0}
```

Define number of Higgses

```
> nHiggs := nops(higgsbase);
                                    nHiggs := 9
```

Define parameters of the potential

```
> par := [m11Sq, m22Sq, m12Sq, L1, L2, L3, L4, L5, msSq, L6, L7, L8];
                 par := [m11Sq, m22Sq, m12Sq, L1, L2, L3, L4, L5, msSq, L6, L7, L8]
```

Define Higgs doublet

…

Model Implementation with `BSMPT` — `python`

- SymPy toolkit in: `BSMPT/tools/ModelGeneration/sympy/`
- Need to write `MODEL.py` (provided for reference: `SM.py` and `G2HDM.py` (generic 2HDM))
- Excerpt from `SM.py`:

```python
[...]
# parameters
msq, la = symbols('msq lambda', real=True)
params=[msq,la]
# fields
rho,eta,zeta,psi = symbols('rho eta zeta psi', real=True)
# VHiggs
phi = Matrix([[rho+I*eta], [zeta+I*psi]]) * 1/sqrt(2)
phiSq = simplify((Dagger(phi)*phi)[0])
VHiggs = msq/2 * phiSq + la/factorial(4) * phiSq**2
# VGauge
W1, W2, W3, B0 = symbols('W1 W2 W3 B0',real=True)
Dmu = -I*Cg/2 * (sigma1*W1 + sigma2 * W2 + sigma3*W3) -I*Cgs/2 * sigma0 * B0
VGauge = simplify(Dagger(Dmu*phi)*(Dmu*phi))[0,0]
[...]
# Generate the model
toyModel = ModelGenerator.ModelGenerator(params,dparams,CTTadpoles,Higgsfields,VHiggs,\
                                         zeroTempVEV,finiteTempVEV)
toyModel.setGauge([W1,W2,W3,B0],VGauge)
toyModel.setLepton(LepBase, VFLep)
toyModel.setQuark(QuarkBase, VQuark)
```

- Get scalar-coupling tensors and finite counterterms:

```
# display tensors
[lisa@pc: ~]$ python3 MODEL.py --show tensors
# show finite counterterms
[lisa@pc: ~]$ python3 MODEL.py --show ct
```

Stand-alone Features of `BSMPTv3`

- Exemplary shown here: `BSMPT/standalone/CalculateAction.cpp`
- Calculation of Euclidean action for user-defined potential and initial guess path
- Calculation using analytical derivatives possible, if gradient of potential is provided

---

```cpp
// Define the potential
std::function<double(std::vector<double>)> V = [&](std::vector<double> x)
{
  double c  = 5;
  double fx = 0;
  double fy = 80;

  double r1 = x[0] * x[0] + c * x[1] * x[1];
  double r2 = c * pow(x[0] - 1, 2) + pow(x[1] - 1, 2);
  double r3 = fx * (0.25 * pow(x[0], 4) - pow(x[0], 3) / 3.);
  r3 += fy * (0.25 * pow(x[1], 4) - pow(x[1], 3) / 3.);

  return (r1 * r2 + r3);
};

// Define the false and true vacuum
std::vector<double> FalseVacuum = {0, 0};
std::vector<double> TrueVacuum  = {1, 1};

// Your best guess for the path
std::vector<std::vector<double>> path = {TrueVacuum, FalseVacuum};

 // Calculate the action
BounceActionInt bc(path, TrueVacuum, FalseVacuum, V, 0, 6);
bc.CalculateAction();

std::cout << "Action calculated using numerical derivatives is " << bc.Action
          << "\n";
```

CP-conserving Two-Higgs Doublet Model

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] \; .$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\,\eta_1 \\ \zeta_1 + \omega_1 + i\,\psi_1 \end{pmatrix} , \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \omega_{\text{CB}} + i\,\eta_2 \\ \zeta_2 + \omega_2 + i\,(\psi_2 + \omega_{\text{CP}}) \end{pmatrix}$$

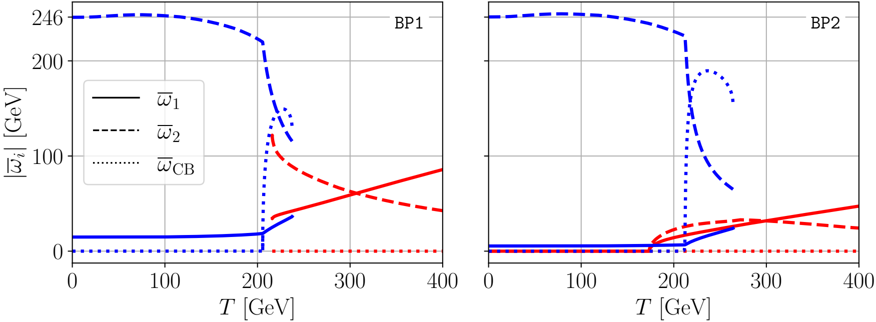
$$\{\omega_{\text{CB}},\, \omega_1,\, \omega_2,\, \omega_{\text{CP}}\}|_{T=0} = \{0, v_1, v_2, 0\}\,, \text{ with}$$

$$\omega_{\text{EW}}|_{T=0} \equiv \sqrt{\omega_1^2 + \omega_2^2 + \omega_{\text{CB}}^2 + \omega_{\text{CP}}^2}\Bigg|_{T=0} = \sqrt{v_1^2 + v_2^2} \equiv v = 246 \text{ GeV}$$

Charge-Breaking EWPTs in a 2HDM

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \overline{\omega}_1\end{pmatrix}, \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\overline{\omega}_{\mathrm{CB}}\\ \overline{\omega}_2 + i\overline{\omega}_{\mathrm{CP}}\end{pmatrix}$$

- `BSMPTv2`: Intermediate **global minima** with $|\overline{\omega}_{\mathrm{CB}}| > 0$ possible
  [Aoki, LB, Borschensky, Ivanov, Mühlleitner, Shibuya, '23]
- Analysis with `BSMPTv3` shows two PT histories: [Basler, LB, Mühlleitner, Müller, Santos, Viana, '24]



- First-order EWPT from neutral to CB phase ($T_c = 226\,\mathrm{GeV}, T_n, T_p, T_f = 223\,\mathrm{GeV}$)

$\Rightarrow$ **CB phase populated by whole universe**

$\rightarrow$ $U(1)_{\mathrm{EM}}$ restoration around 200 GeV

- First-order EWPT **after** true phase has transitioned to neutral phase ($T_c = 231\,\mathrm{GeV}, T_n, T_p, T_f = 200\,\mathrm{GeV}$)

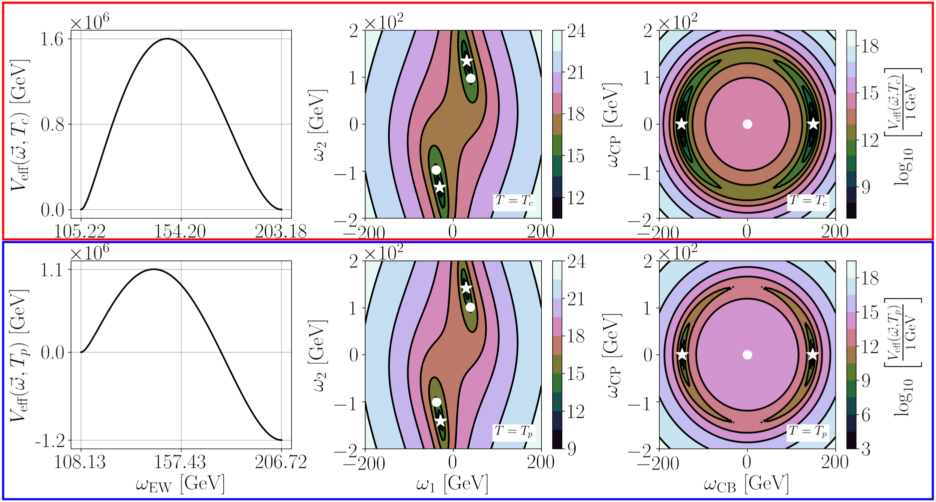
$\Rightarrow$ **CB phase not populated**

Complicated Histories - `BP1` visualized with `PotPlotter`

$\omega_{\text{EW}} = \sqrt{\sum_{i=1,2,\text{CB},\text{CP}} \omega_i^2}$

`BP1`: type $= 1\,,\ \lambda_1 = 6.931\,,\ \lambda_2 = 0.2631\,,\ \lambda_3 = 1.287\,,\ \lambda_4 = 4.772\,,\ \lambda_5 = 4.728\,,$
$m_{12}^2 = 1.893 \times 10^4\,\text{GeV}^2\,,\ \tan\beta = 16.578\,.$

critical temperature T_c



percolation temperature T_p

slice from *false*
to *true* vacuum

ω_1-ω_2-contour

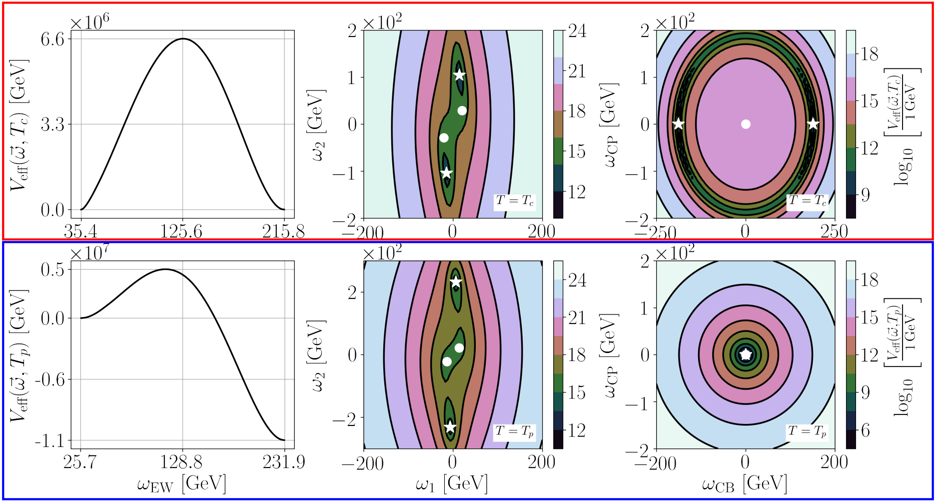
ω_{CB}-ω_{CP}-contour

Complicated Histories - `BP2` visualized with `PotPlotter`

$$\omega_{\text{EW}} = \sqrt{\sum_{i=1,2,\text{CB},\text{CP}} \omega_i^2}$$

`BP2`: type $= 1\,,\ \lambda_1 = 6.846\,,\ \lambda_2 = 0.2589\,,\ \lambda_3 = 1.466\,,\ \lambda_4 = 4.498\,,\ \lambda_5 = 4.450\,,$
$m_{12}^2 = 6.630 \times 10^3\,\text{GeV}^2\,,\ \tan\beta = 45.320\,,$

critical temperature T_c



percolation temperature T_p

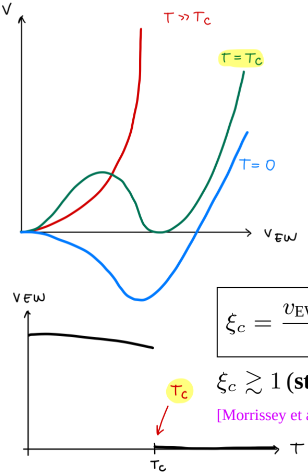
slice from *false*
to *true* vacuum

ω_1-ω_2-contour

ω_{CB}-ω_{CP}-contour

Electroweak Phase Transition

First-Order EWPT



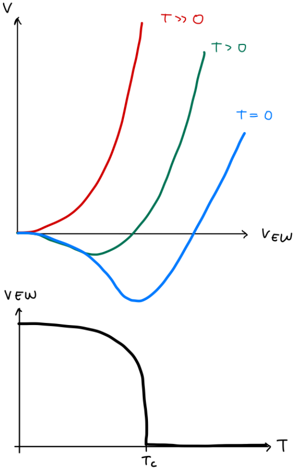
$$\boxed{\xi_c = \frac{v_{\mathrm{EW}}(T_c)}{T_c}}$$

$\xi_c \gtrsim 1$ **(strong)**

[Morrissey et al., ’12]

→ Discontinuity in v_{EW} of global minimum at critical temperature T_c

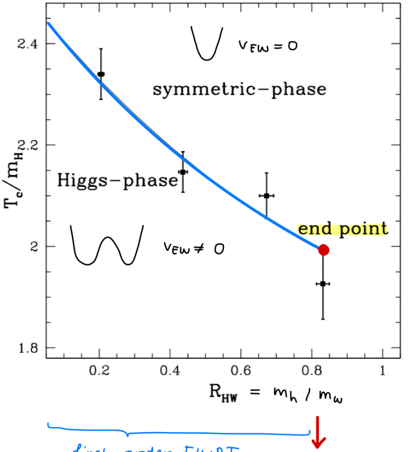
Second-Order EWPT



→ v_{EW} changes continuously

Phase Diagram of the Standard Model

[Kajantie et al., ’96]
[Csikor et al., ’99]



first-order EWPT

Second-order EWPT for $m_h = 72.4$ GeV

[Csikor et al., ’99]:
“Taking into account all errors
our end point value
excludes the possibility
of any EWPT in the SM.”

$\Downarrow$

For $m_h > 72.4\,\mathrm{GeV}$:
smooth cross-over

$\Rightarrow$ **EW symmetry restoration but no EWPT in the SM!**