On the Neutrino Mass Ordering

Renata Zukanovich Funchal - Instituto de Física, University of São Paulo

Based on S. J. Parke, RZF - arXiv:2404.0873

9th International Workshop on

BARYON and LEPTON NUMBER VIOLATION

BL 2024

Oct 8-11, 2024

Karlsruhe Institute of Technology

Neutrino Flavor Oscillations

Driven by Mass and Mixing

flavor eigenstates

mass eigenstates

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\, \nu_i \qquad \alpha = e, \mu, \tau$$

atmospheric, reactor, solar

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

mixing matrix

- two independent mass scales have been identified

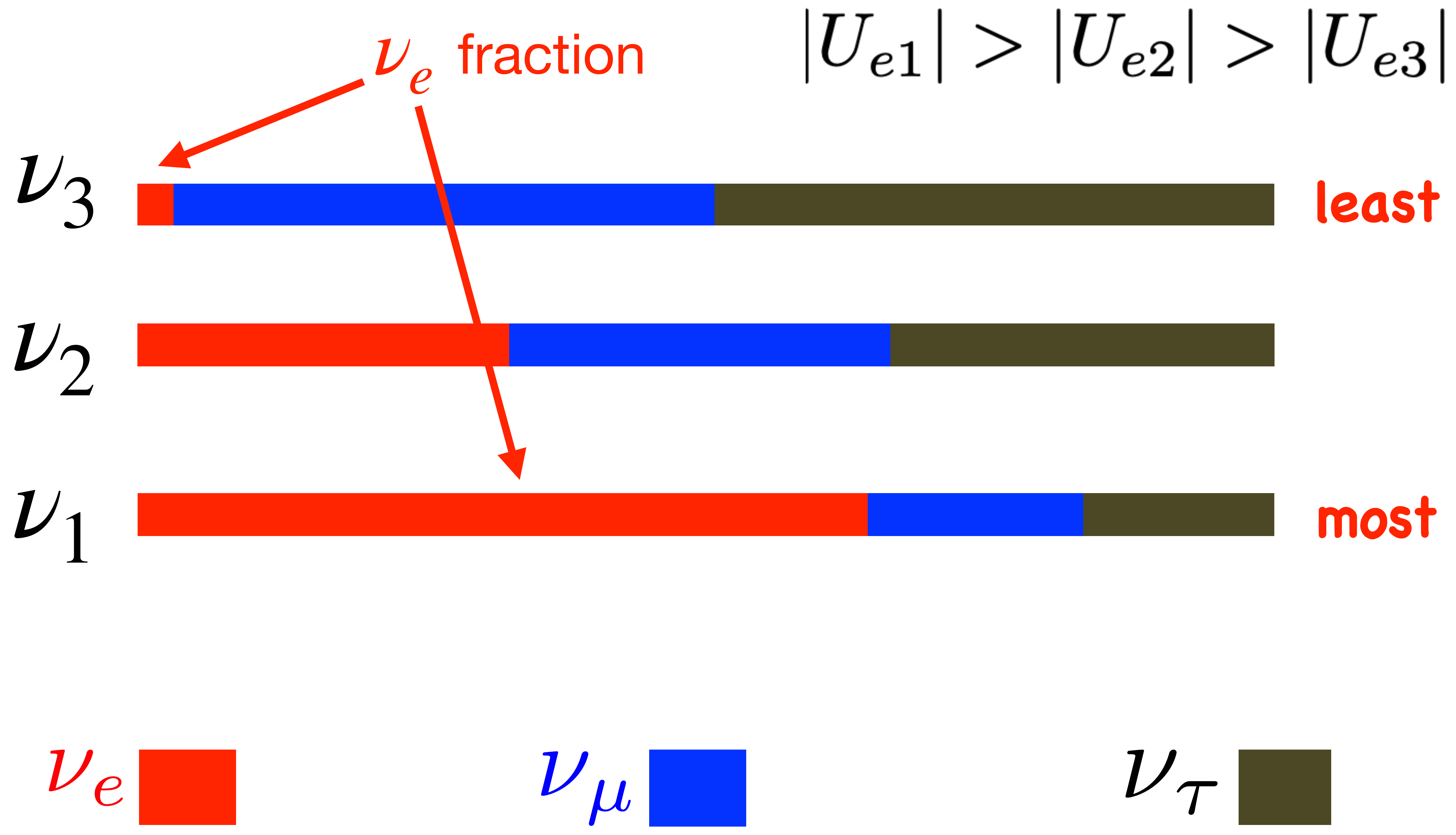
$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 \approx 7.4 \times 10^{-5}\, \mathrm{eV}^2$$ (so-called solar scale)

$$\Delta m^2_{31} \equiv m_3^2 - m_1^2 \approx \pm 2.5 \times 10^{-3}\, \mathrm{eV}^2$$ (so-called atm. scale)

$$\Delta m^2_{32} = \Delta m^2_{31} - \Delta m^2_{21} \qquad |\Delta m^2_{31}| \approx |\Delta m^2_{32}| = \Delta m^2_{\mathrm{atm}}$$

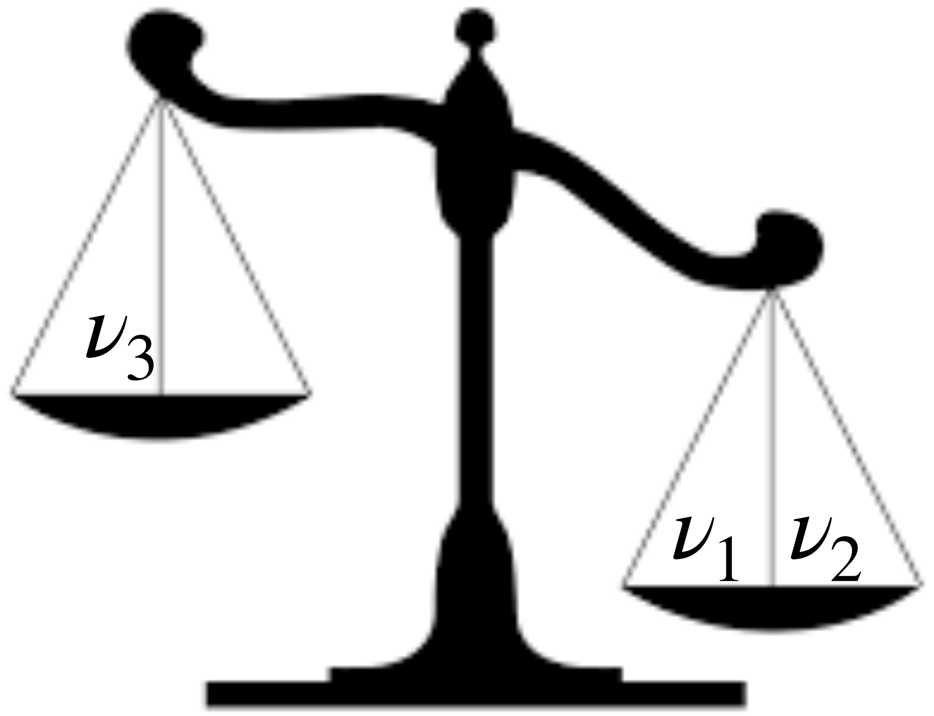
Mass Ordering

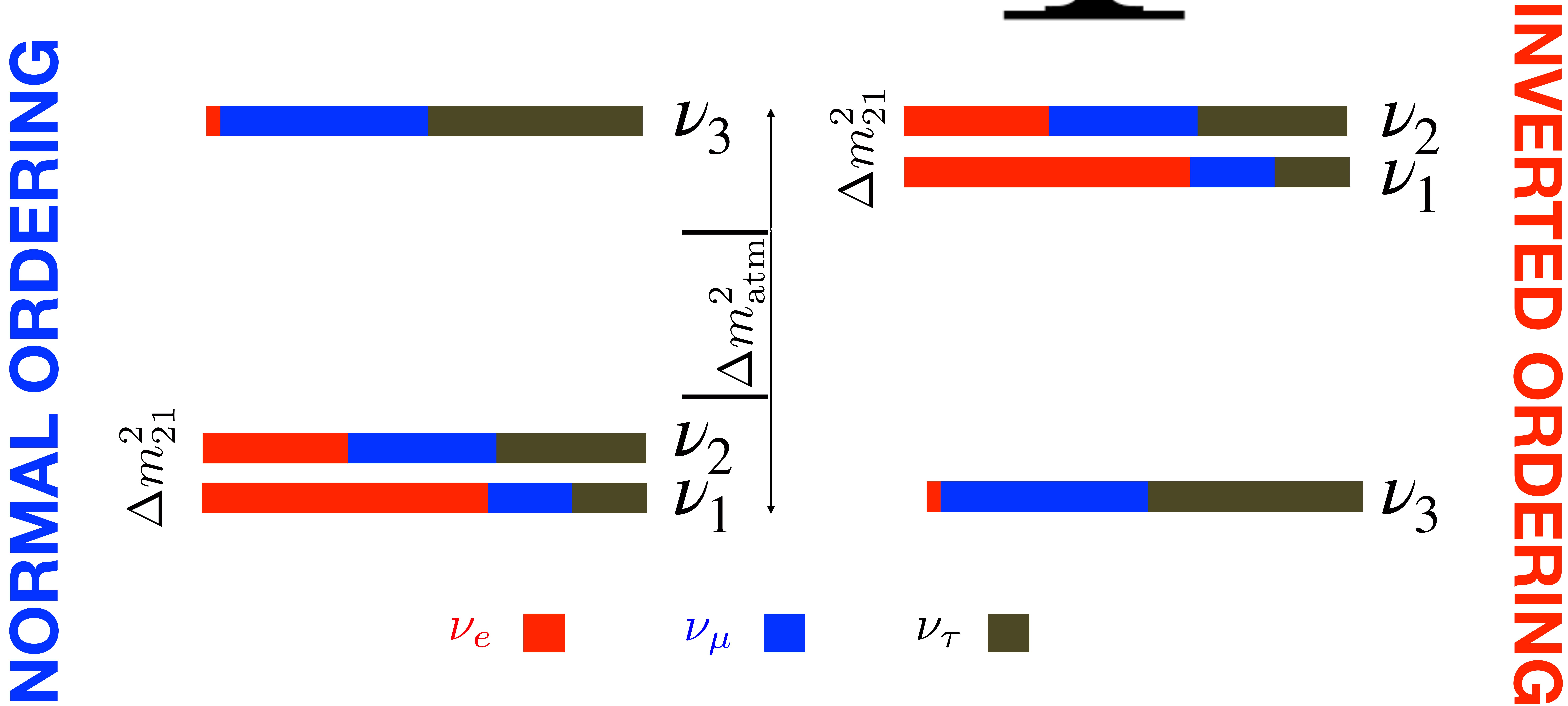
Normal or Inverted



Mass Ordering

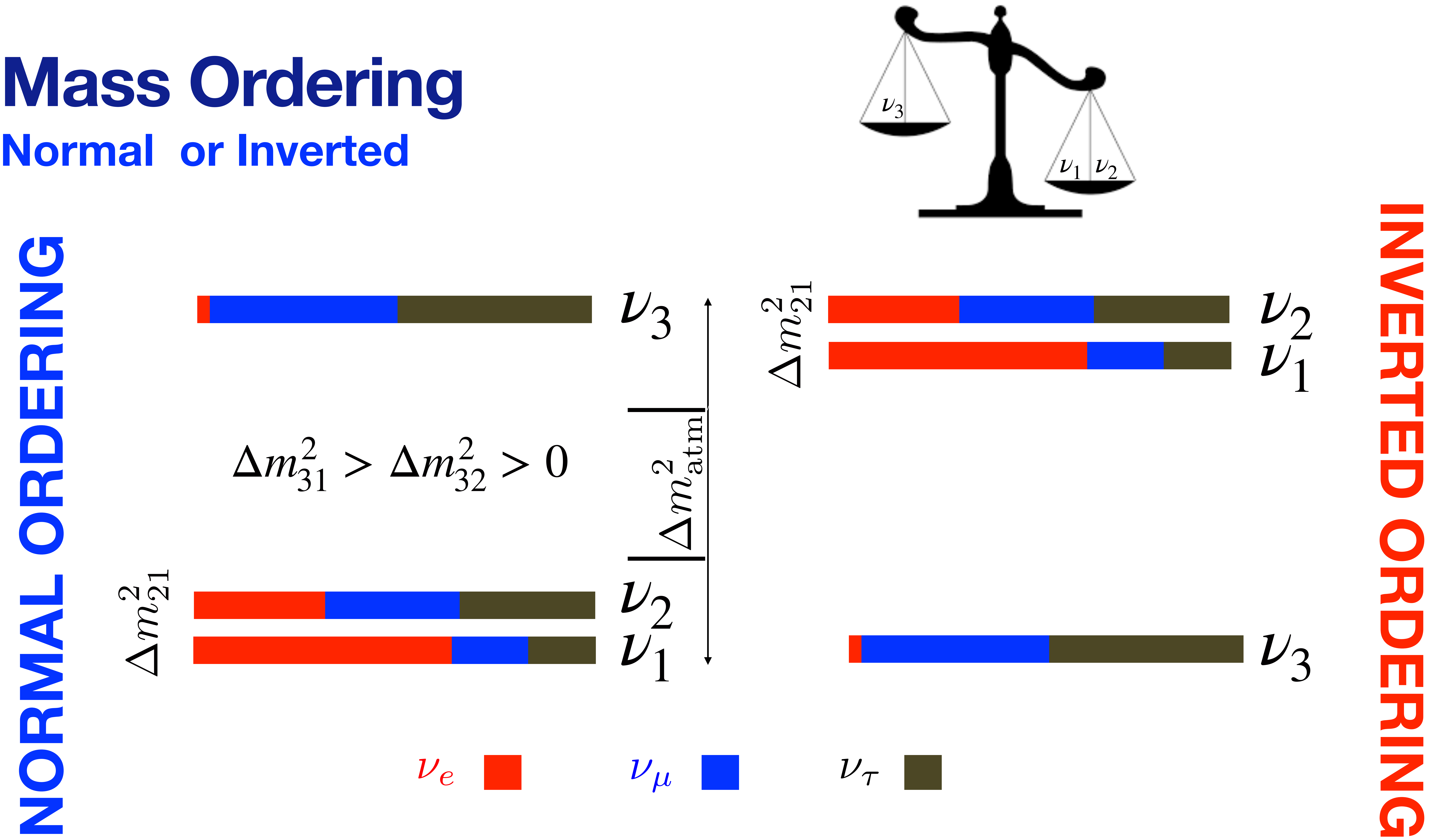
Normal or Inverted





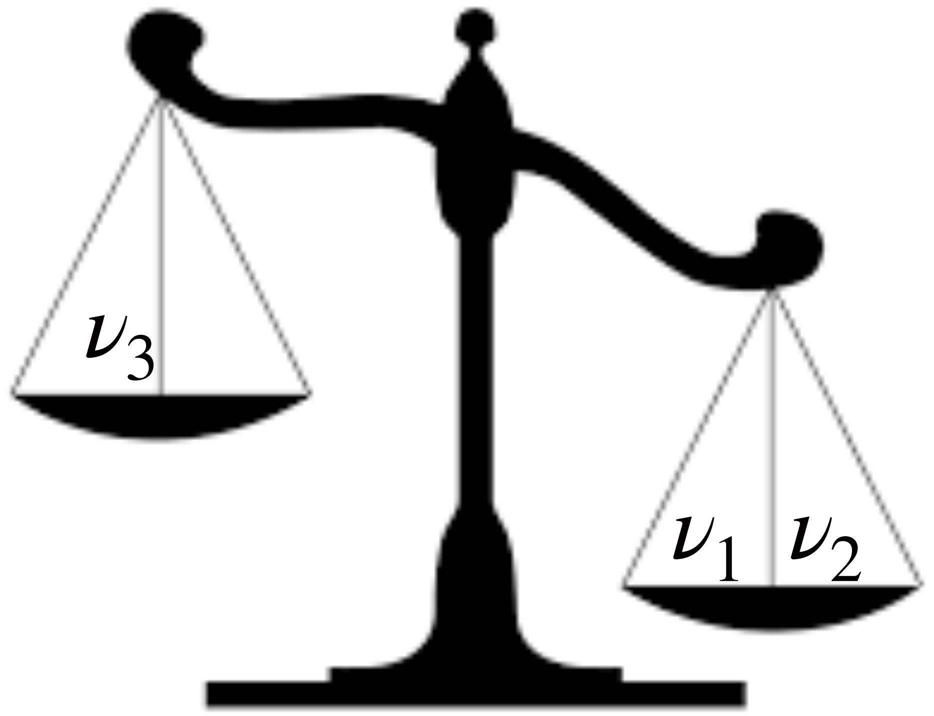
Mass Ordering

Normal or Inverted

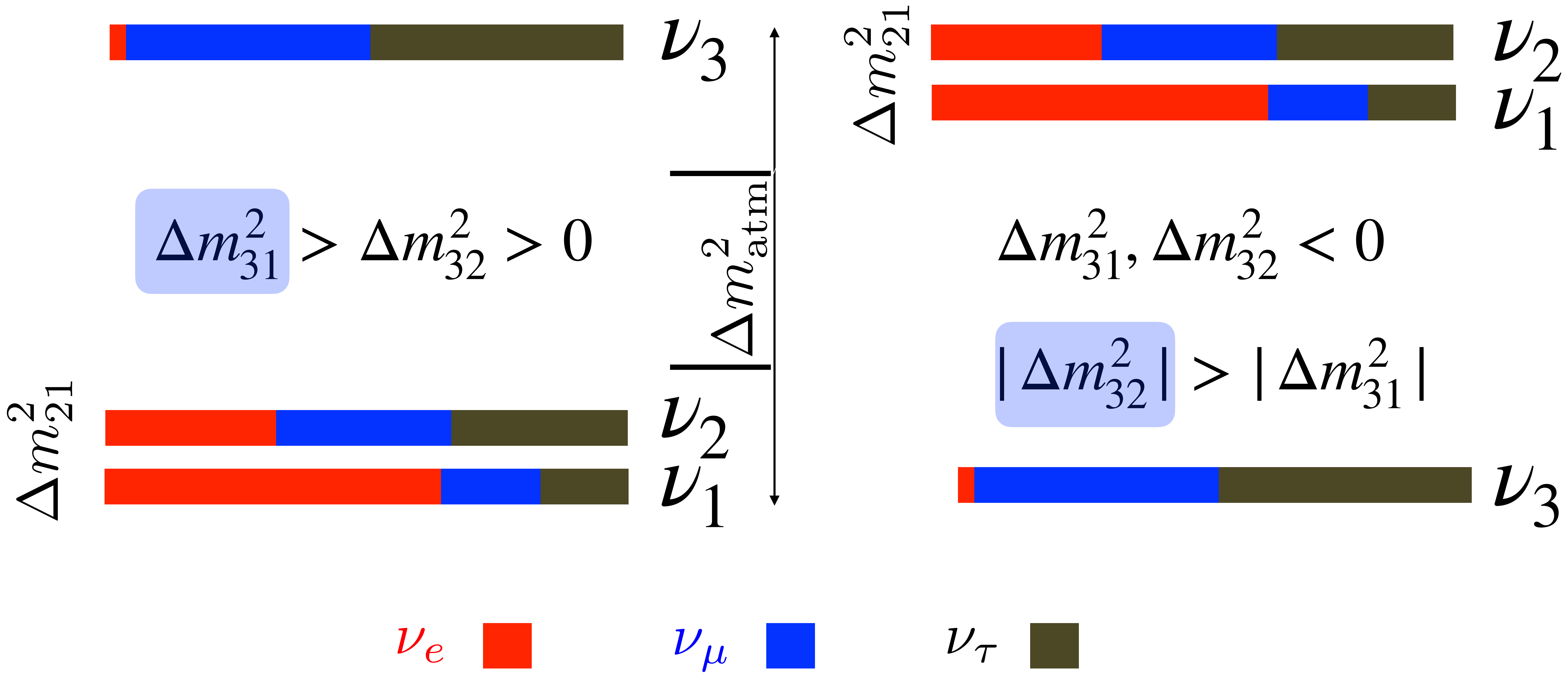


Mass Ordering

Normal or Inverted



NORMAL ORDERING



INVERTED ORDERING

Effect Δm^2_{atm} - scale

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where

the oscillation phase

$$\Delta_{21} \equiv \frac{\Delta m^2_{21} L}{4E} \ll 1$$

L = baseline

E = neutrino energy

survival probabilities

$$\nu_e \to \nu_e / \bar{\nu}_e \to \bar{\nu}_e$$

$$\nu_\mu \to \nu_\mu / \bar{\nu}_\mu \to \bar{\nu}_\mu$$

effectively described by a different Δm^2_{atm}

$\neq \Delta m^2_{31} \quad \neq \Delta m^2_{32}$

Effect $\Delta m^2_{\rm atm}$ - scale

$$\nu_e \to \nu_e/\bar{\nu}_e \to \bar{\nu}_e$$

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

effective scale for ν_e disappearance

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

Effect $\Delta m^2_{\rm atm}$ - scale

$$\nu_e \to \nu_e/\overline{\nu}_e \to \overline{\nu}_e$$

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

DayaBay/RENO

$\Delta_{21}(\langle E \rangle \sim 4\,\mathrm{MeV}, \mathrm{L} \sim 1\,\mathrm{km}) \sim 2\,\%$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

effective scale for ν_e disappearance

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

Effect $\Delta m^2_{\mathrm{atm}}$ - scale

$$\nu_e \to \nu_e / \bar{\nu}_e \to \bar{\nu}_e$$

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

DayaBay/RENO

$$\Delta_{21}(\langle E \rangle \sim 4\,\mathrm{MeV}, \mathrm{L} \sim 1\,\mathrm{km}) \sim 2\,\%$$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

effective scale for ν_e disappearance

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

$$\Delta m^2_{31} |^{\mathrm{NO}}_e = |\Delta m^2_{ee}| + \Delta m^2_{21} \sin^2 \theta_{12}$$

Effect $\Delta m^2_{\rm atm}$ - scale

For Disappearance Experiments

$$\nu_e \rightarrow \nu_e / \bar{\nu}_e \rightarrow \bar{\nu}_e$$

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

$$P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

DayaBay/RENO

$\Delta_{21}(\langle E \rangle \sim 4\,\mathrm{MeV}, \mathrm{L} \sim 1\,\mathrm{km}) \sim 2\,\%$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2\theta_{12} + \Delta m^2_{32} \sin^2\theta_{21}$$

effective scale for ν_e disappearance

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

$$\Delta m^2_{31}|^{\rm NO}_e = |\Delta m^2_{ee}| + \Delta m^2_{21} \sin^2\theta_{12}$$

$$|\Delta m^2_{32}|^{\rm IO}_e = |\Delta m^2_{ee}| + \Delta m^2_{21} \cos^2\theta_{12}$$

Effect $\Delta m^2_{\rm atm}$ - scale

$$\nu_e \to \nu_e / \overline{\nu}_e \to \overline{\nu}_e$$

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

DayaBay/RENO

$$\Delta_{21}(\langle E \rangle \sim 4\,\mathrm{MeV}, \mathrm{L} \sim 1\,\mathrm{km}) \sim 2\,\%$$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

effective scale for ν_e disappearance

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

$$\Delta m^2_{31} |^{\rm NO}_e = |\Delta m^2_{ee}| + \Delta m^2_{21} \sin^2 \theta_{12}$$

$$|\Delta m^2_{32}|^{\rm IO}_e = |\Delta m^2_{ee}| + \Delta m^2_{21} \cos^2 \theta_{12}$$

$$|\Delta m^2_{ee}|, \Delta m^2_{31}|^{\rm NO}_e, |\Delta m^2_{32}|^{\rm IO}_e$$

$$|\Delta m^2_{32}|^{\rm IO}_e - \Delta m^2_{31}|^{\rm NO}_e = \Delta m^2_{21} \cos 2\theta_{12}$$

Effect Δm^2_{atm} - scale

For Disappearance Experiments

$$\nu_\mu \to \nu_\mu / \overline{\nu}_\mu \to \overline{\nu}_\mu$$

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\text{eff}} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2 \theta_{12} + \Delta m^2_{32} \cos^2 \theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta \, \Delta m^2_{21}$$

effective scale for ν_μ disappearance

$$\Delta_{\mu\mu} \equiv \frac{\Delta m^2_{\mu\mu} L}{4E}$$

Effect $\Delta m^2_{\rm atm}$ - scale

For Disappearance Experiments

$$\nu_\mu \to \nu_\mu / \overline{\nu}_\mu \to \overline{\nu}_\mu$$

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

T2K

$\Delta_{21}(\langle E \rangle \sim 0.6\,\text{GeV}, \text{L} \sim 295\,\text{km}) \sim 5\,\%$

NOvA

$\Delta_{21}(\langle E \rangle \sim 2.0\,\text{GeV}, \text{L} \sim 810\,\text{km}) \sim 4\,\%$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\rm eff} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta\, \Delta m^2_{21}$$

effective scale for ν_μ disappearance

$$\Delta_{\mu\mu} \equiv \frac{\Delta m^2_{\mu\mu} L}{4E}$$

Effect $\Delta m^2_{\rm atm}$ - scale

$$\nu_\mu \to \nu_\mu / \overline{\nu}_\mu \to \overline{\nu}_\mu$$

For Disappearance Experiments

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\rm eff} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

T2K

$\Delta_{21}(\langle E \rangle \sim 0.6\,\mathrm{GeV}, \mathrm{L} \sim 295\,\mathrm{km}) \sim 5\,\%$

NOvA

$\Delta_{21}(\langle E \rangle \sim 2.0\,\mathrm{GeV}, \mathrm{L} \sim 810\,\mathrm{km}) \sim 4\,\%$

$$\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta\, \Delta m^2_{21}$$

$$\Delta m^2_{31}\,|^{\rm NO}_{\mu} = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\cos^2\theta_{12} - \sin\theta_{13} \cos\delta^{\rm NO})$$

effective scale for ν_μ disappearance

Effect Δm^2_{atm} - scale

For Disappearance Experiments

$$\nu_\mu \to \nu_\mu / \bar{\nu}_\mu \to \bar{\nu}_\mu$$

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

T2K

$\Delta_{21}(\langle E \rangle \sim 0.6\,\text{GeV}, \text{L} \sim 295\,\text{km}) \sim 5\,\%$

NOvA

$\Delta_{21}(\langle E \rangle \sim 2.0\,\text{GeV}, \text{L} \sim 810\,\text{km}) \sim 4\,\%$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\text{eff}} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta\, \Delta m^2_{21}$$

effective scale for ν_μ disappearance

$$\Delta m^2_{31}\,|^{\text{NO}}_{\mu} = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\cos^2\theta_{12} - \sin\theta_{13}\cos\delta^{\text{NO}})$$

$$|\,\Delta m^2_{32}\,|^{\text{IO}}_{\mu} = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\sin^2\theta_{12} + \sin\theta_{13}\cos\delta^{\text{IO}})$$

Effect $\Delta m^2_{\rm atm}$ - scale

For Disappearance Experiments

$$\nu_\mu \to \nu_\mu / \overline{\nu}_\mu \to \overline{\nu}_\mu$$

in vacuum

[H. Nunokawa, S. J. Parke, RZF (2005)]

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

T2K

$\Delta_{21}(\langle E \rangle \sim 0.6\,\text{GeV}, \text{L} \sim 295\,\text{km}) \sim 5\,\%$

NOvA

$\Delta_{21}(\langle E \rangle \sim 2.0\,\text{GeV}, \text{L} \sim 810\,\text{km}) \sim 4\,\%$

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\rm eff} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

$$\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta\, \Delta m^2_{21}$$

effective scale for ν_μ disappearance

$$\Delta m^2_{31}\,|^{\rm NO}_\mu = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\cos^2\theta_{12} - \sin\theta_{13}\cos\delta^{\rm NO})$$

$$|\,\Delta m^2_{32}\,|^{\rm IO}_\mu = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\sin^2\theta_{12} + \sin\theta_{13}\cos\delta^{\rm IO})$$

$$|\,\Delta m^2_{\mu\mu}\,|,\ \Delta m^2_{31}\,|^{\rm NO}_\mu,\ |\,\Delta m^2_{32}\,|^{\rm IO}_\mu$$

$$\Delta m^2_{31}\,|^{\rm NO}_\mu - |\,\Delta m^2_{32}\,|^{\rm IO}_\mu = \Delta m^2_{21}\,(\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})$$

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF arXiv:2404.0873]

$$(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})\,\Delta m^2_{21}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{atm}|$$

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF arXiv:2404.0873]

$$(\Delta m^2_{31}|^{\rm NO}_\mu - \Delta m^2_{31}|^{\rm NO}_e) + (|\Delta m^2_{32}|^{\rm IO}_e - |\Delta m^2_{32}|^{\rm IO}_\mu) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})\,\Delta m^2_{21}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{\rm atm}|$$

$$\Delta m^2_{31}|^{\rm NO}_\mu = \Delta m^2_{31}|^{\rm NO}_e$$

if NO is true

$$\Delta m^2_{31}|^{\rm NO}_{\rm T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3}{\rm eV}^2$$

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF arXiv:2404.0873]

$$(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})\,\Delta m^2_{21}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{\rm atm}|$$

$$\Delta m^2_{31}|^{NO}_{\mu} = \Delta m^2_{31}|^{NO}_{e}$$ **if NO is true**

$$\Delta m^2_{31}|^{NO}_{\rm T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3}{\rm eV}^2$$

$$|\Delta m^2_{32}|^{IO}_{e} = |\Delta m^2_{32}|^{IO}_{\mu}$$ **if IO is true**

$$|\Delta m^2_{32}|^{IO}_{\rm T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3}{\rm eV}^2$$

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF arXiv:2404.0873]

$$(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})\,\Delta m^2_{21}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{atm}|$$

$\Delta m^2_{31}|^{NO}_{\mu} = \Delta m^2_{31}|^{NO}_{e}$ **if NO is true** $\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3}\mathrm{eV}^2$

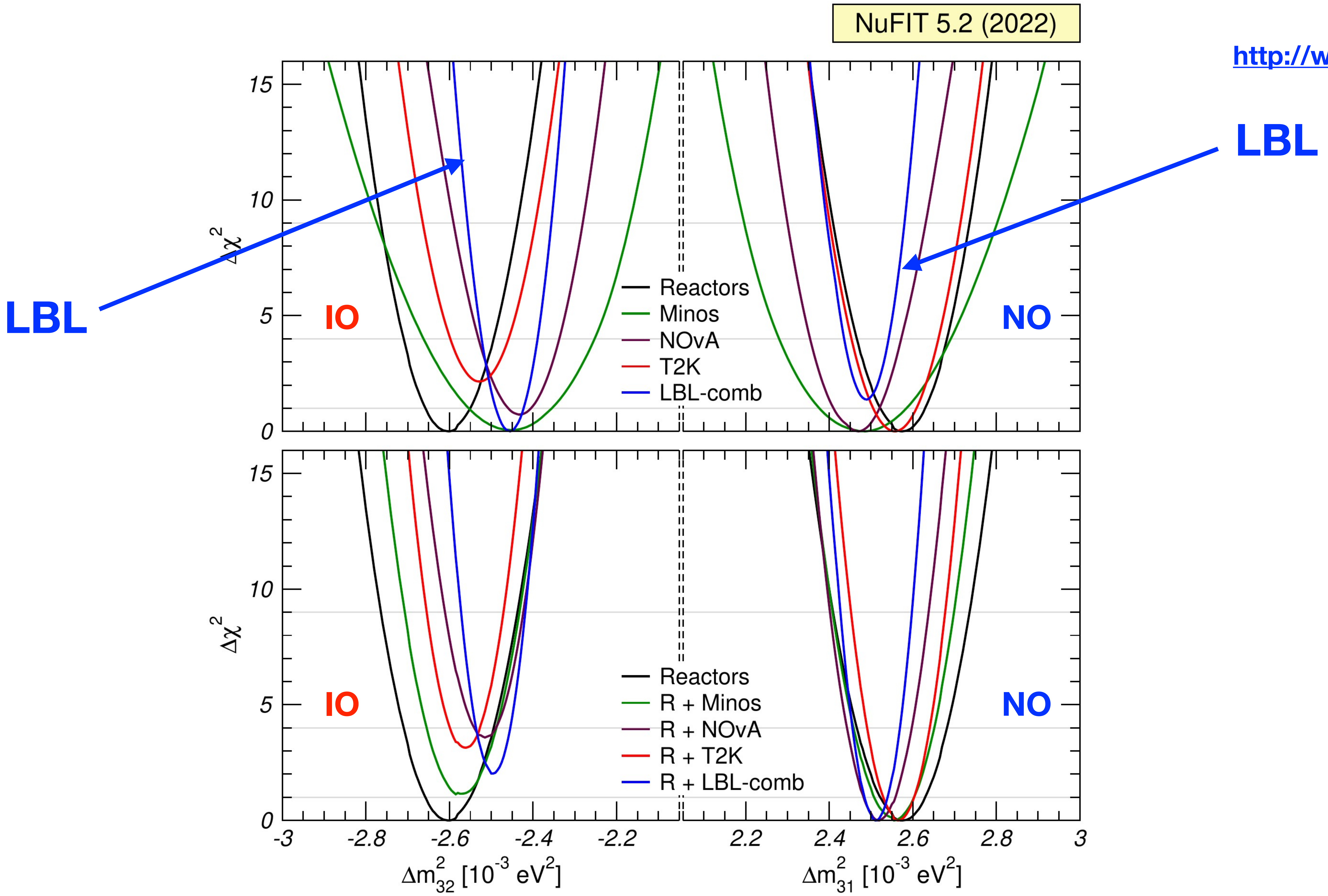
$$\sim 1.2\,\%$$

$|\Delta m^2_{32}|^{IO}_{e} = |\Delta m^2_{32}|^{IO}_{\mu}$ **if IO is true** $|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3}\mathrm{eV}^2$

[http://www.nufit.org - v52.fig-chisq]-dma.pdf]

NuFIT

We already see this effect in the current data

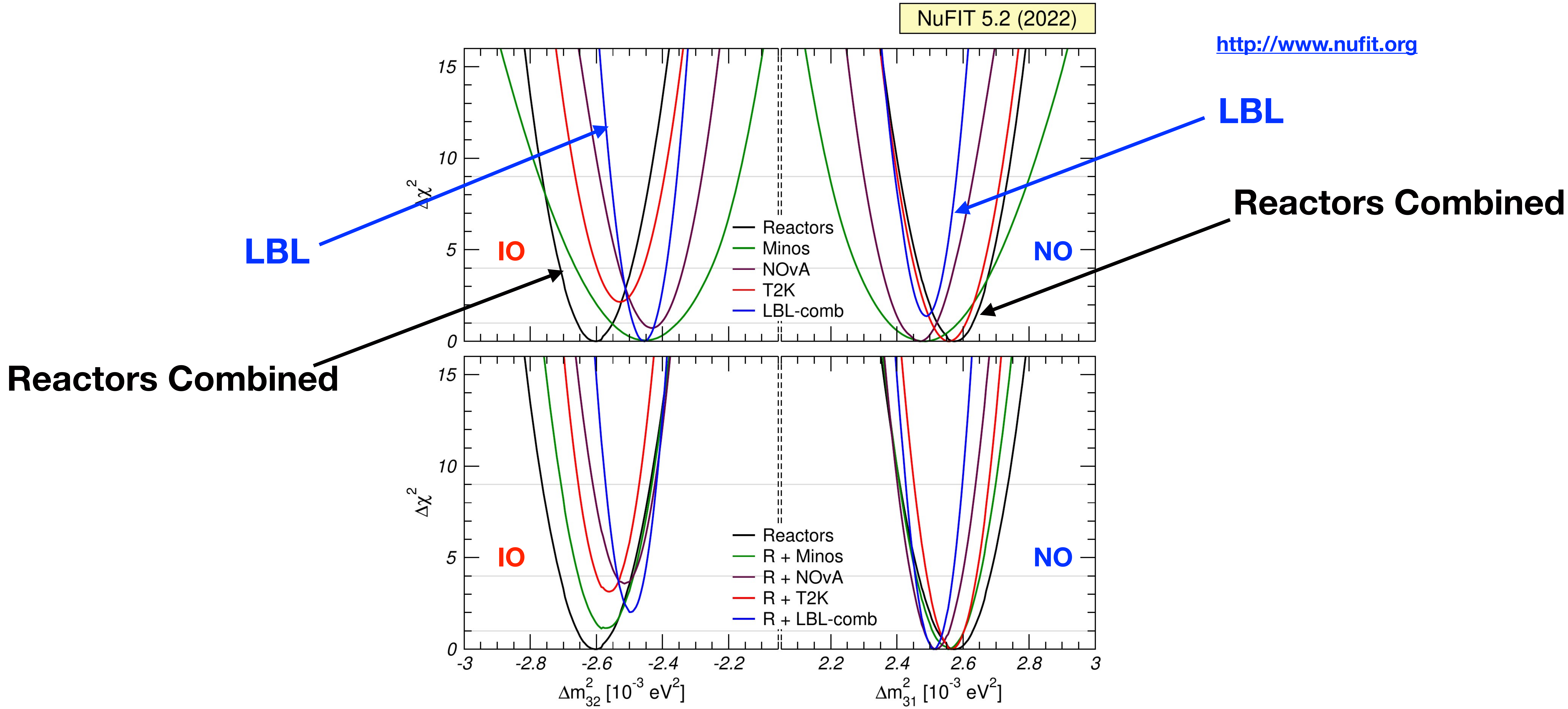


http://www.nufit.org

NuFIT

We already see this effect in the current data

http://www.nufit.org



NuFIT

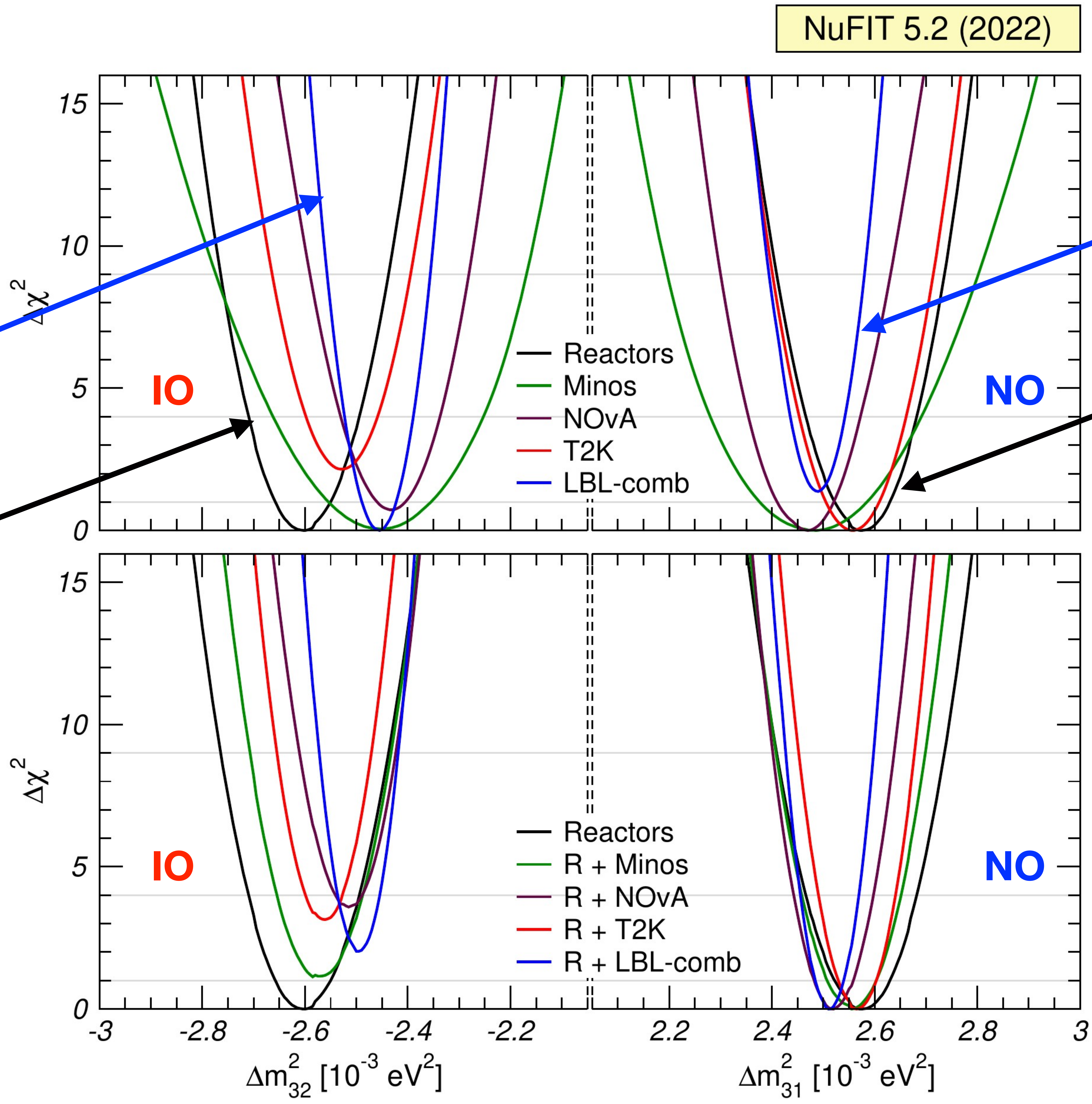
We already see this effect in the current data

http://www.nufit.org

$\Delta m^2_{31}\,|^{NO}_{\mu} > |\,\Delta m^2_{32}\,|^{IO}_{\mu}$

LBL

Reactors Combined



LBL

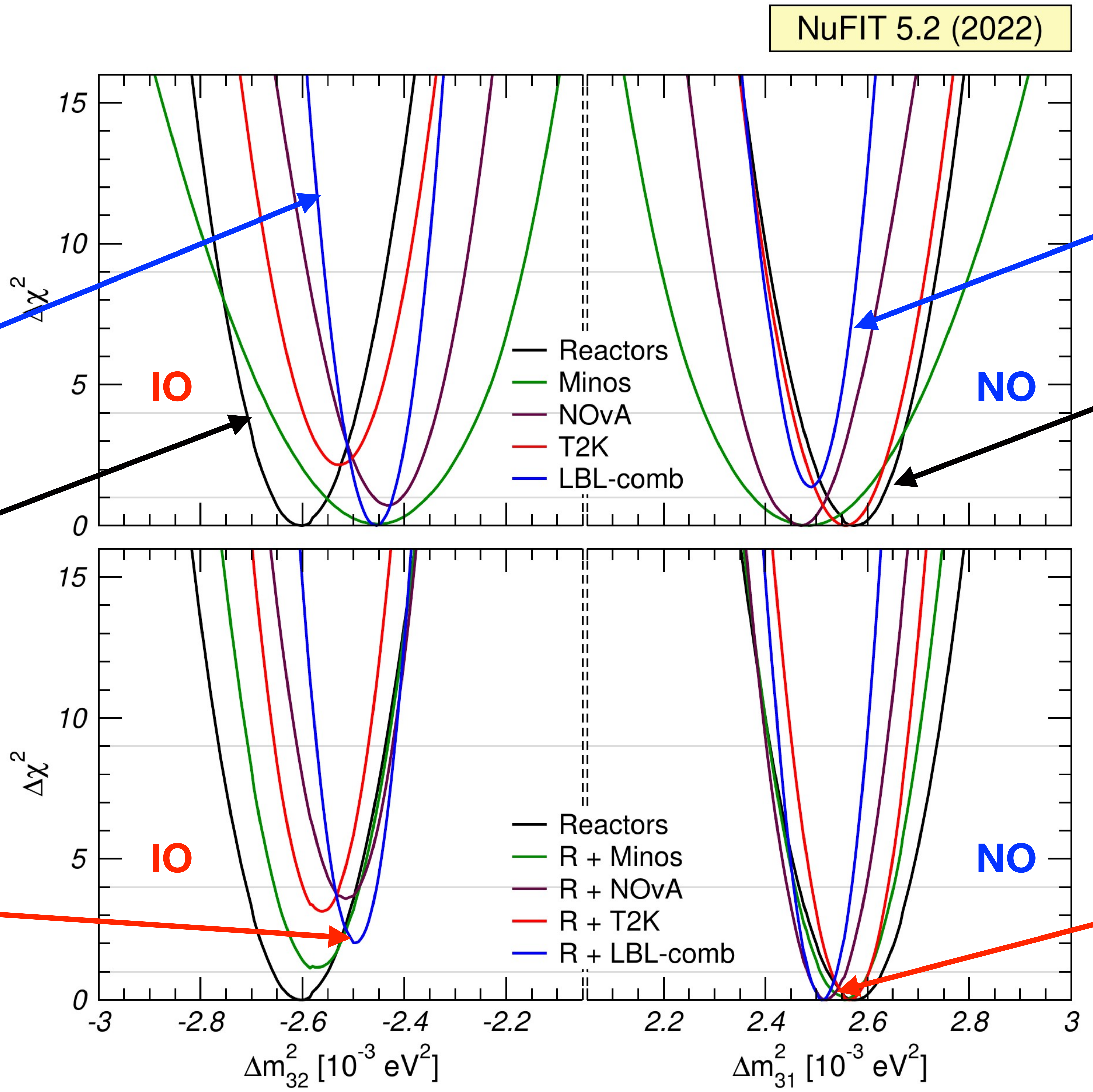
Reactors Combined

$|\,\Delta m^2_{32}\,|^{IO}_{e} > \Delta m^2_{31}\,|^{NO}_{e}$

NuFIT

We already see this effect in the current data

$\Delta m^2_{31}|^{NO}_{\mu} > |\Delta m^2_{32}|^{IO}_{\mu}$

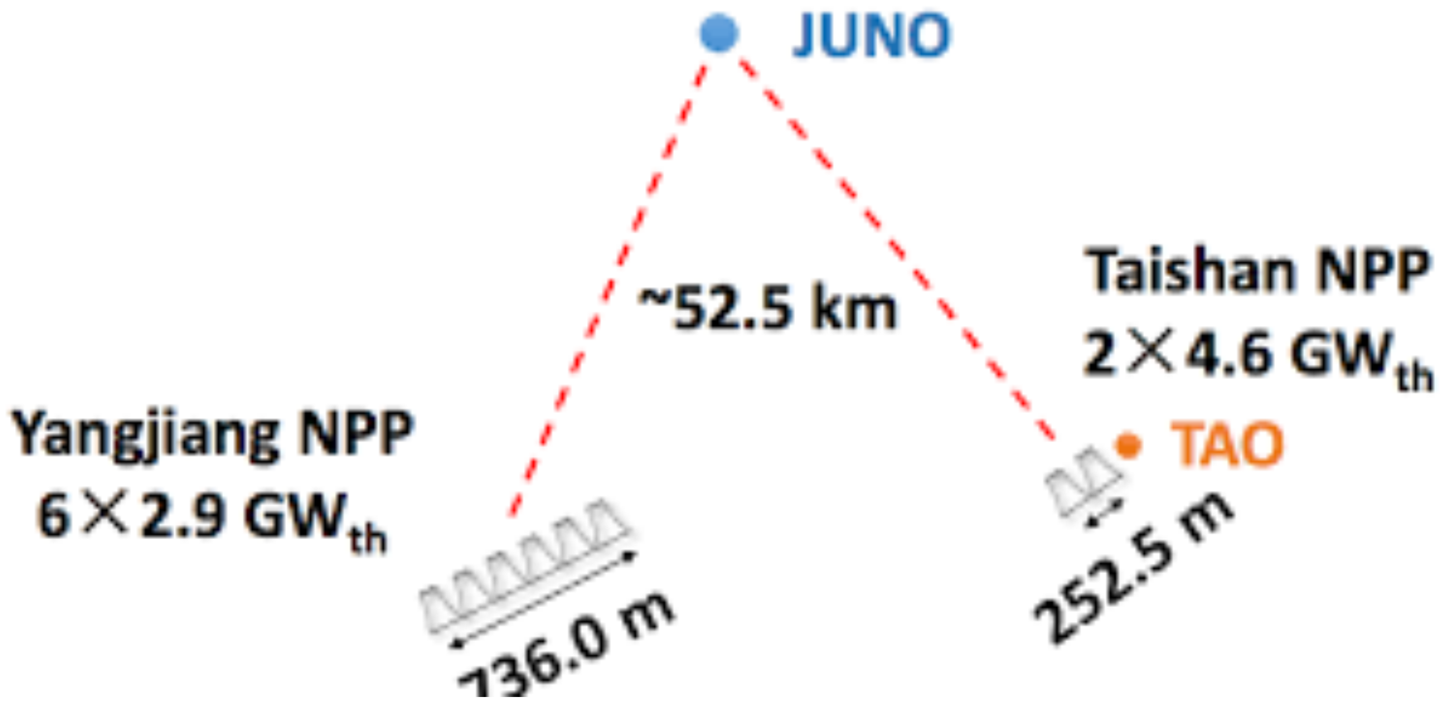


http://www.nufit.org

$|\Delta m^2_{32}|^{IO}_{e} > \Delta m^2_{31}|^{NO}_{e}$

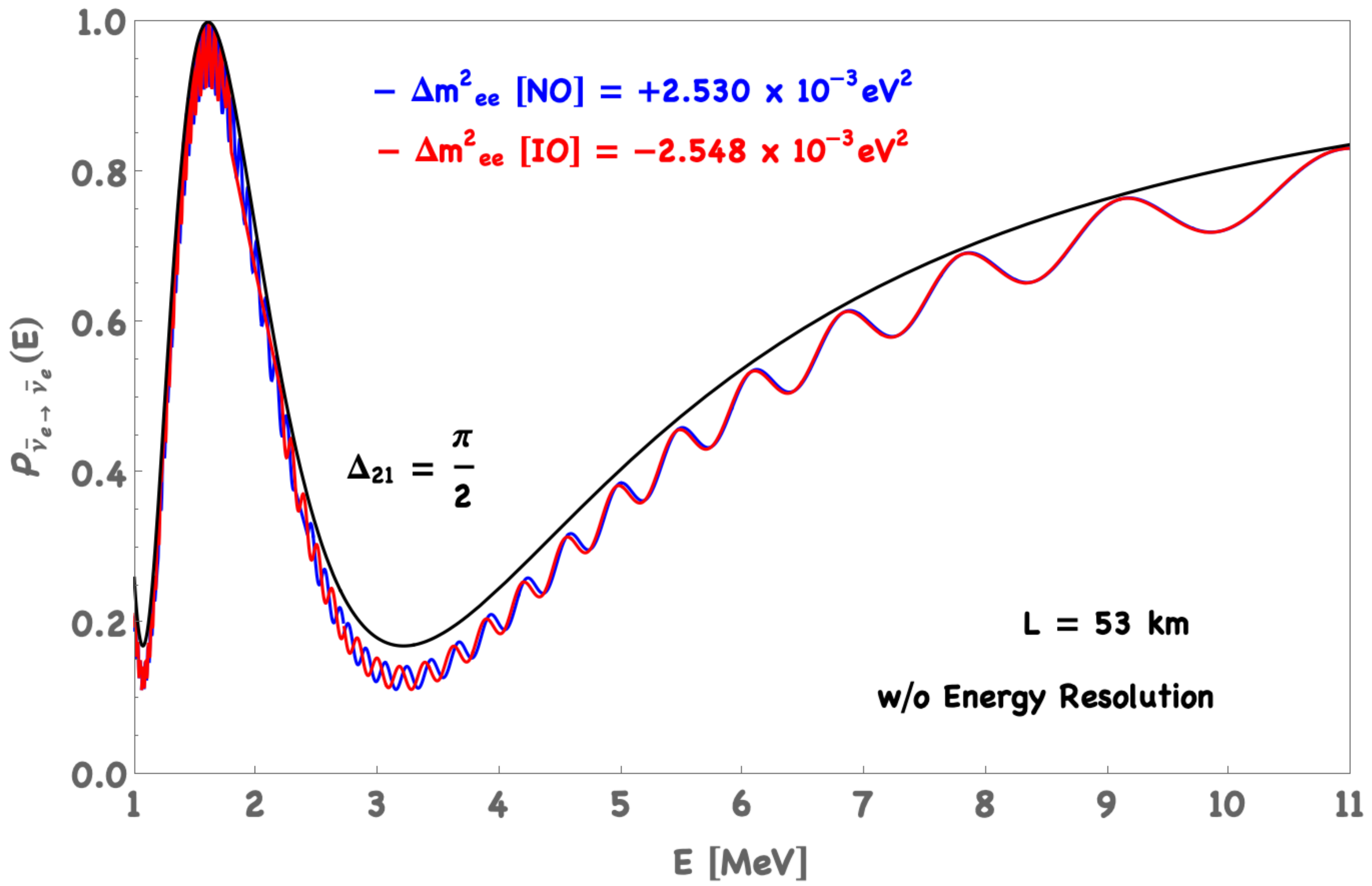
JUNO Disappearance Measurement

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$



	100 days	6 years
Δm^2_{21}	1.0 %	0.3 %
$\sin^2\theta_{12}$	1.9 %	0.5 %
$\lvert\Delta m^2_{ee}\rvert$	0.8 %	0.2 %

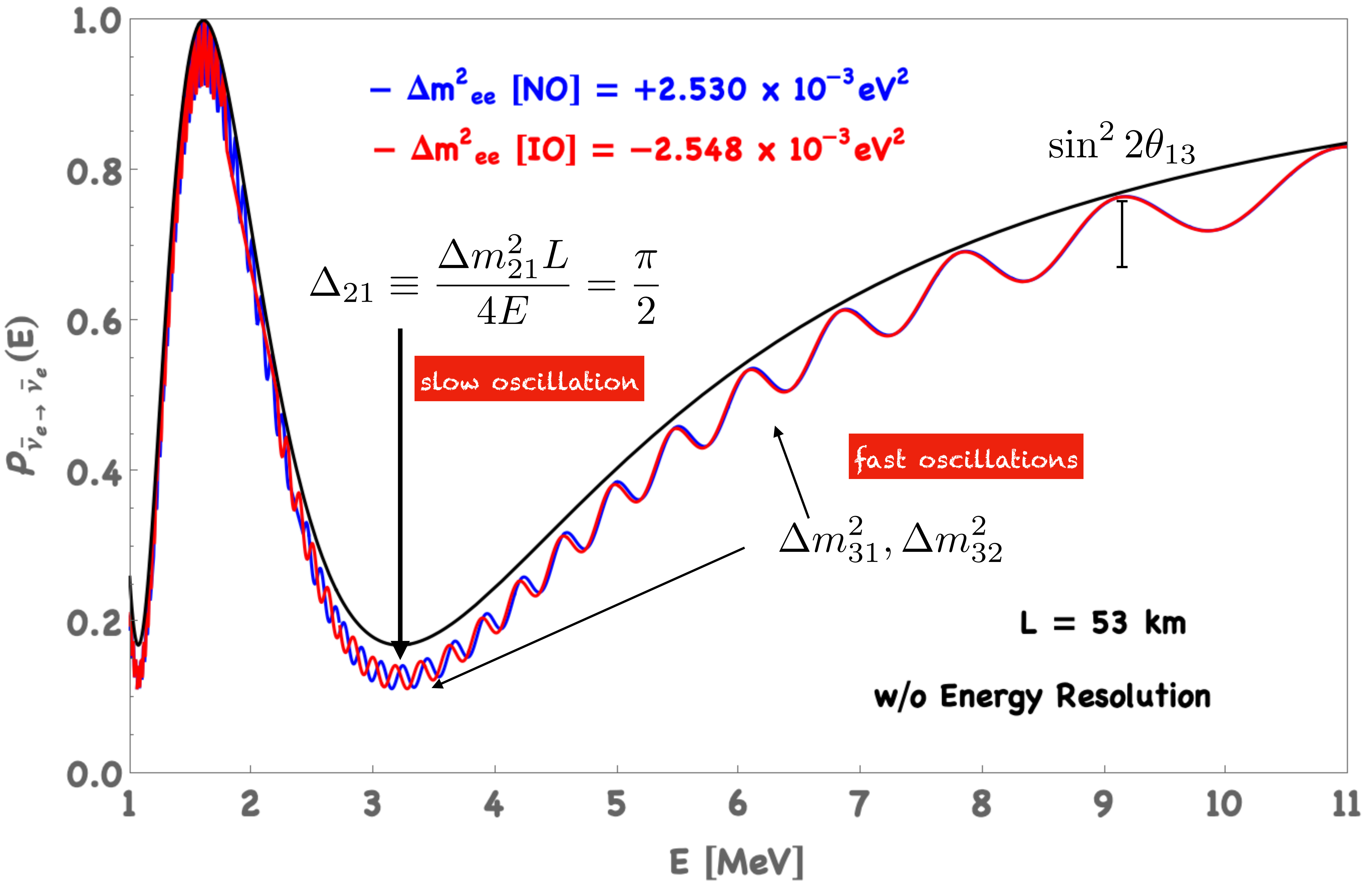
Jiangmen Underground Neutrino Observatory
- a 20 kton liquid scintillator detector
- @ 53 km from Yangjiang & Taishan Nuclear Power Plants
- 26.6 GWth
- in China - starting in 2024



REACTOR NEUTRINO EXPERIMENT

$\bar{\nu}_e \rightarrow \bar{\nu}_e$

medium baseline reactor neutrino detector



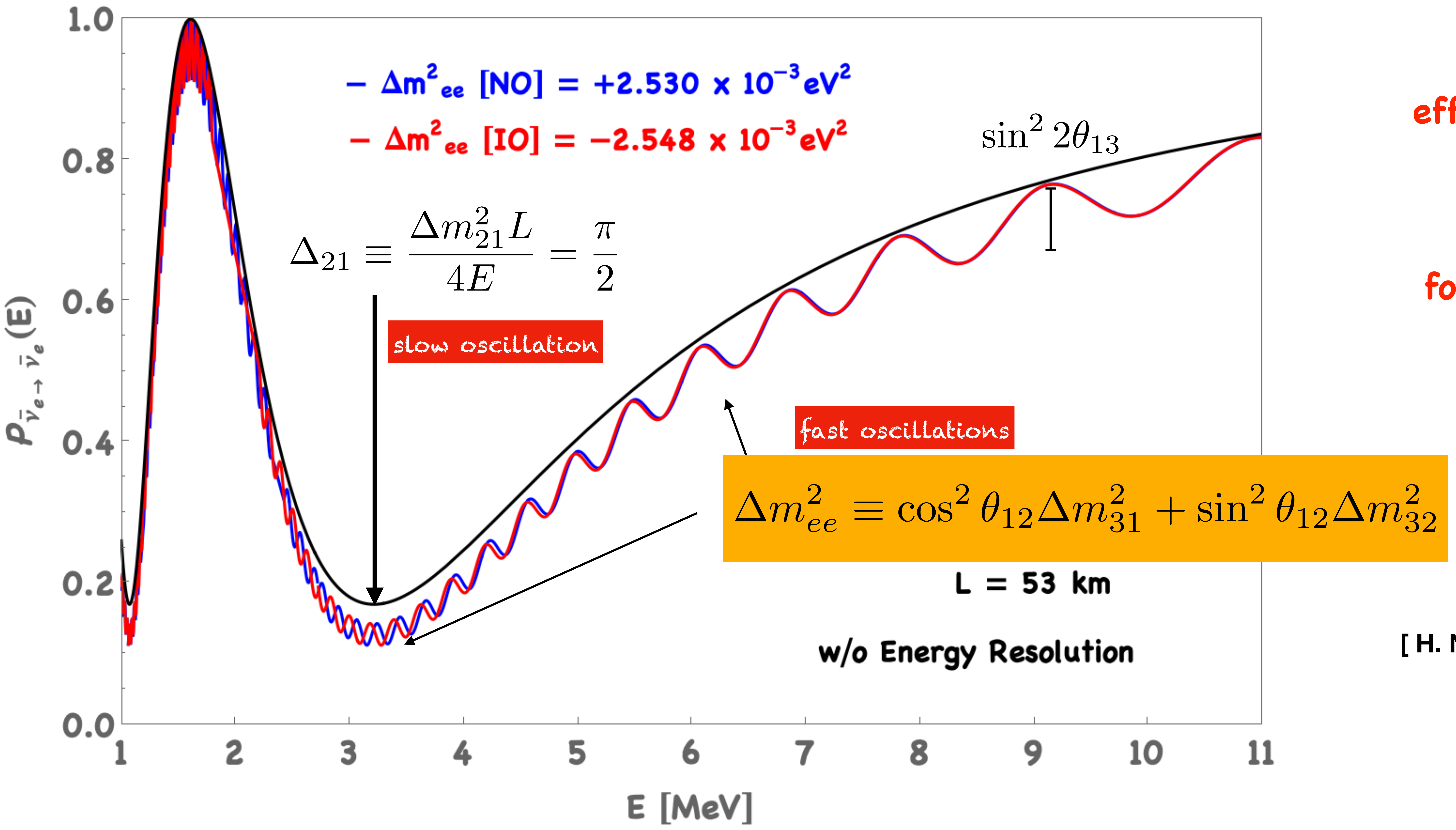
low and high frequency modes present

[S.T.Petcov & M Piai (2002)
&
S. Choubey et al. (2003)]

REACTOR NEUTRINO EXPERIMENT

$\bar{\nu}_e \rightarrow \bar{\nu}_e$

medium baseline reactor neutrino detector



effective atmospheric

$\Delta m^2_{\rm atm}$

for reactor experiments

[H. Nunokawa, S. J. Parke, RZF (2005)]

$\bar{\nu}_e$-Disappearance Measurement

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

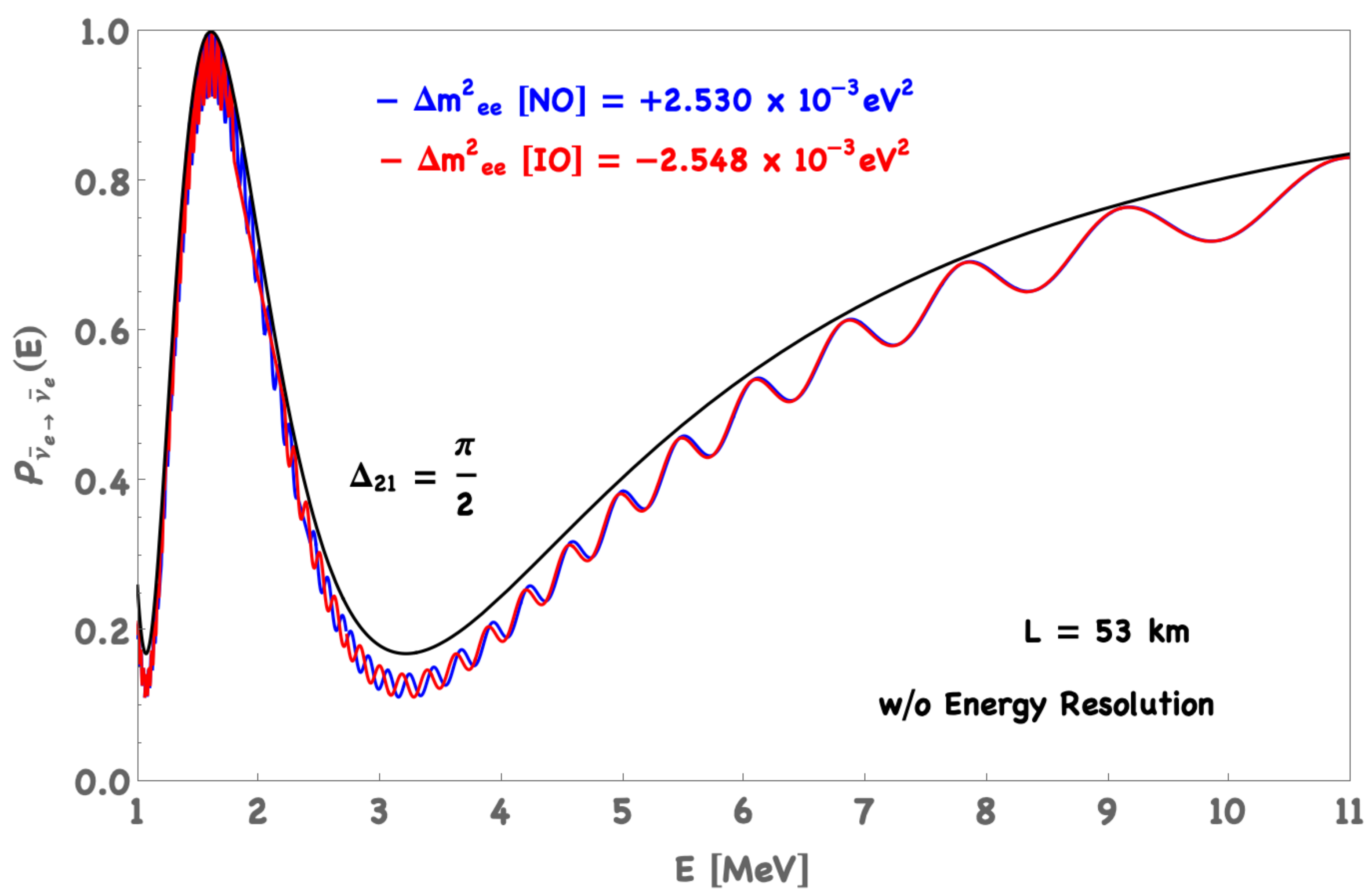
$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\ \cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$



$\bar{\nu}_e$-Disappearance Measurement

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

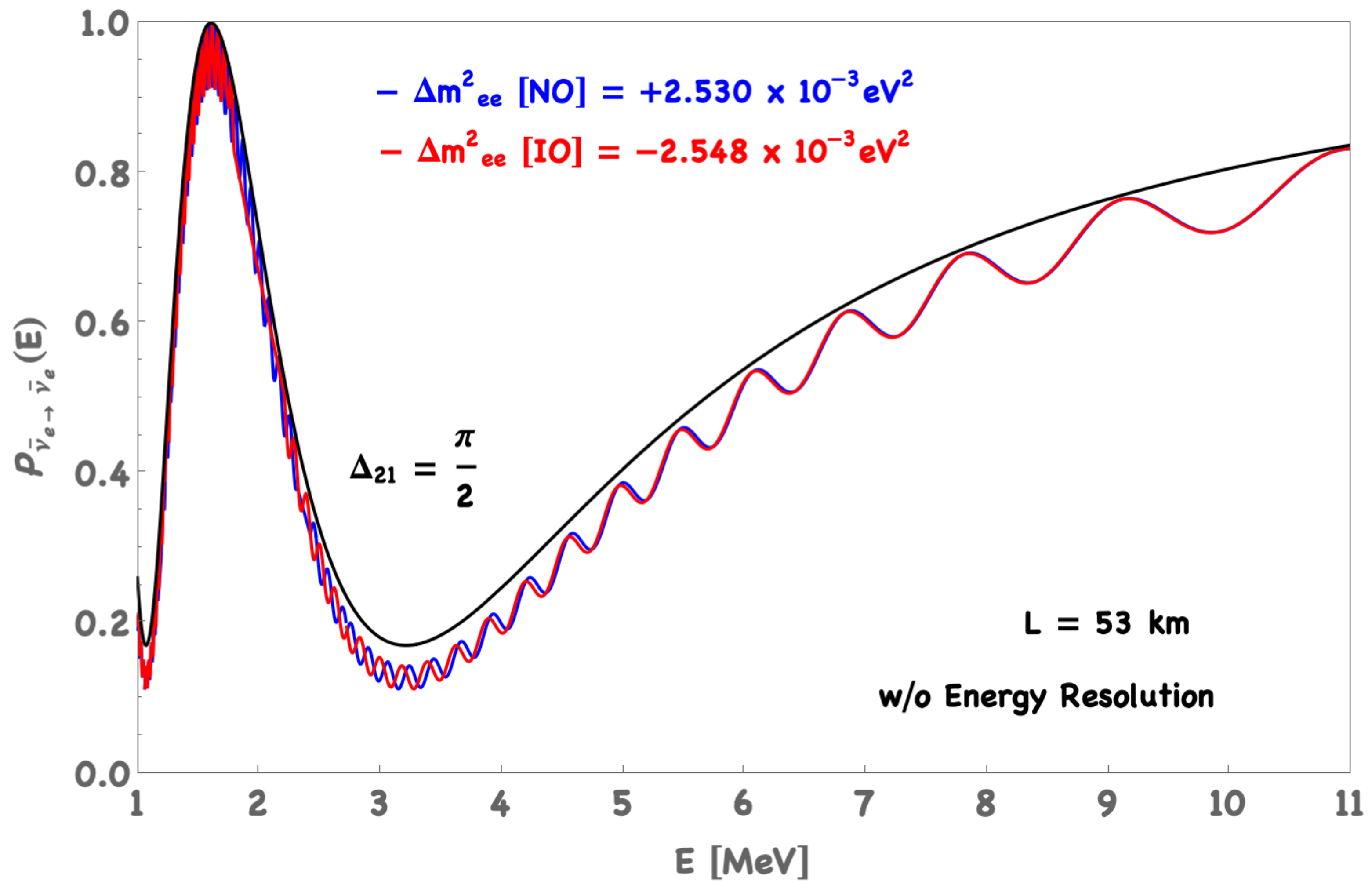
$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_\odot) \right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21}$$



$\bar{\nu}_e$-Disappearance Measurement

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

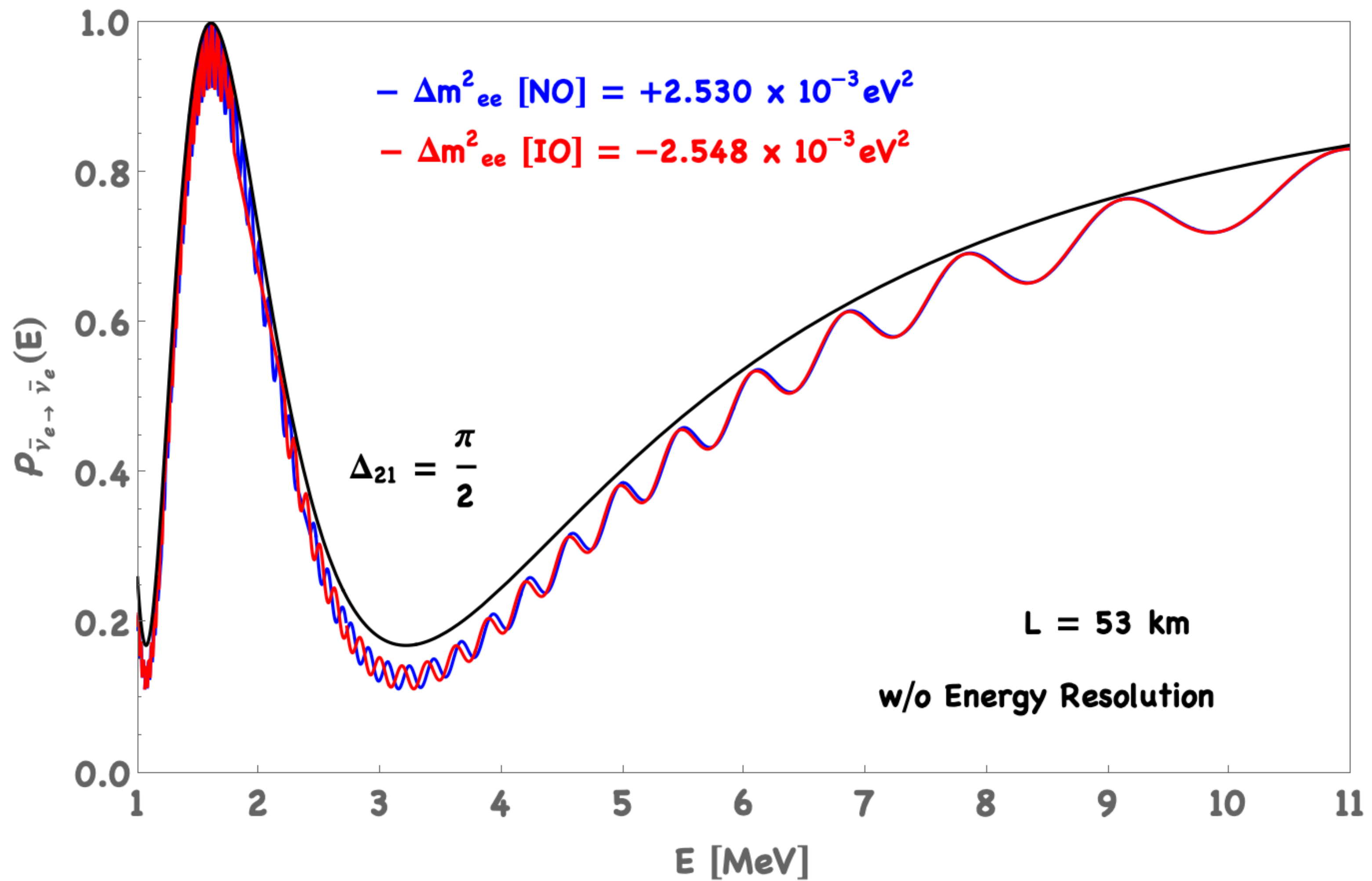
$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \, \cos(2|\Delta_{ee}| \pm \Phi_\odot) \right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21}$$



$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

$\bar{\nu}_e$-Disappearance Measurement

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

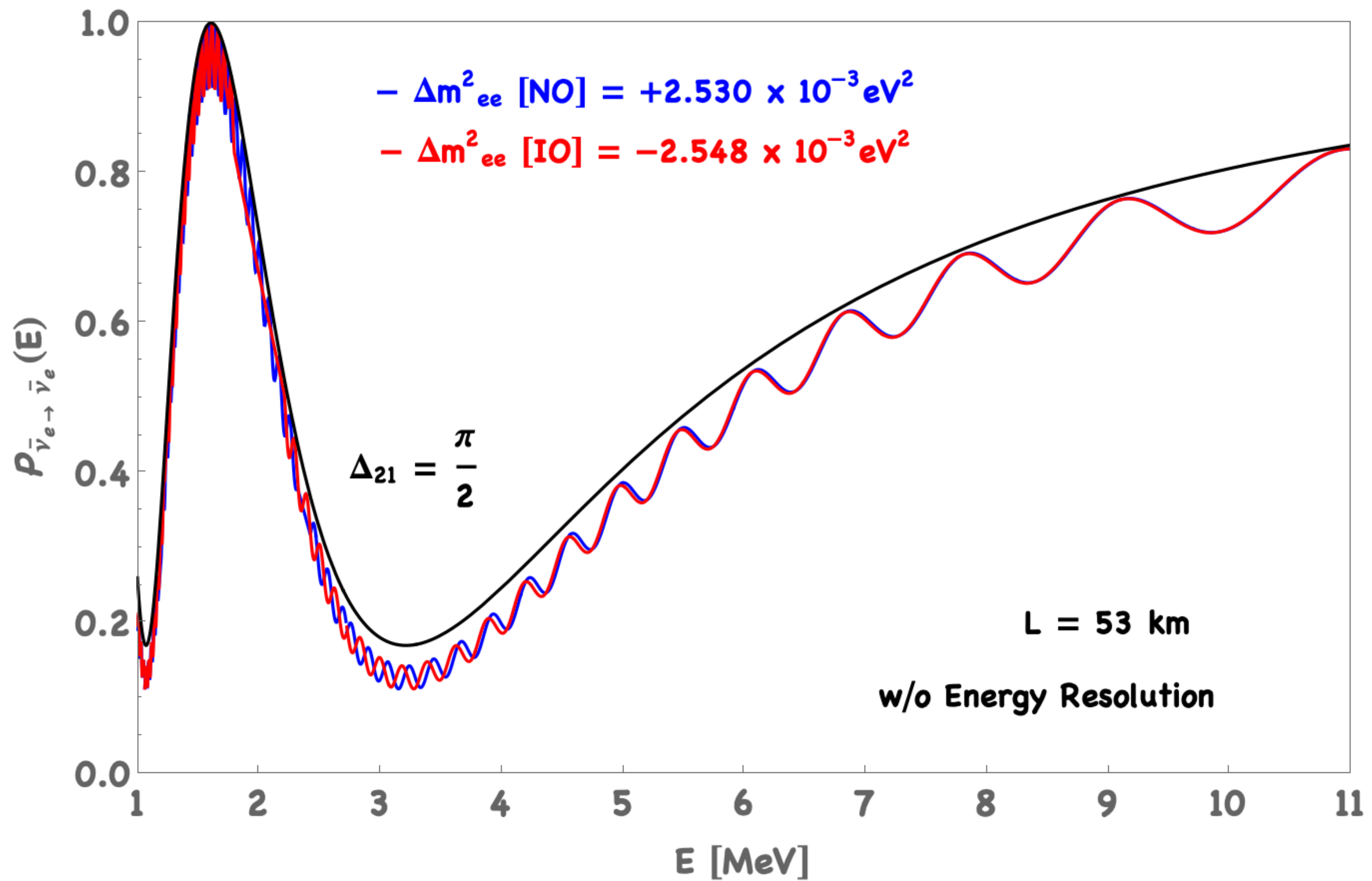
in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

phase

$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$



$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{21}$$

$\bar{\nu}_e$-Disappearance Measurement

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\;\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

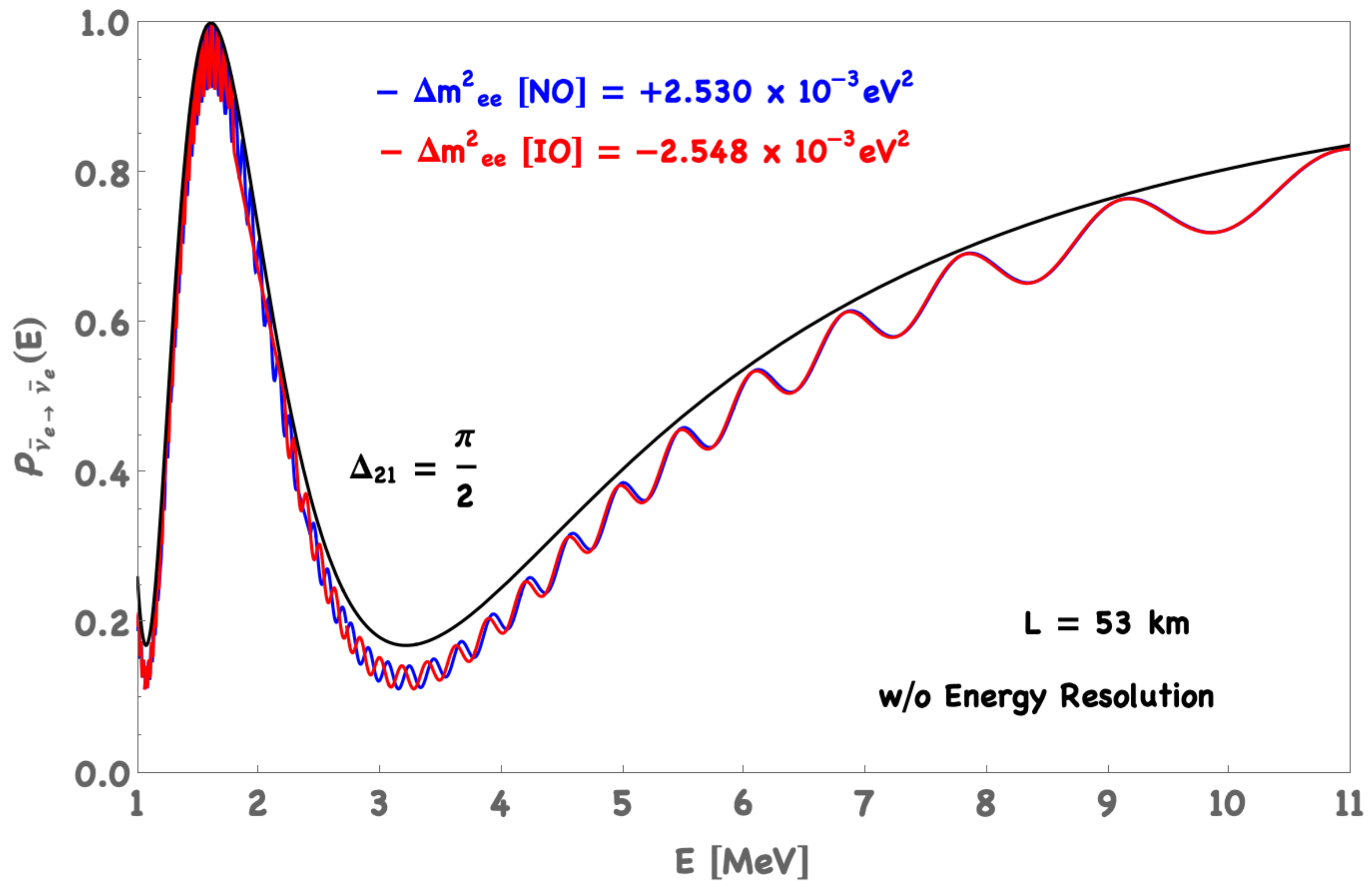
in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

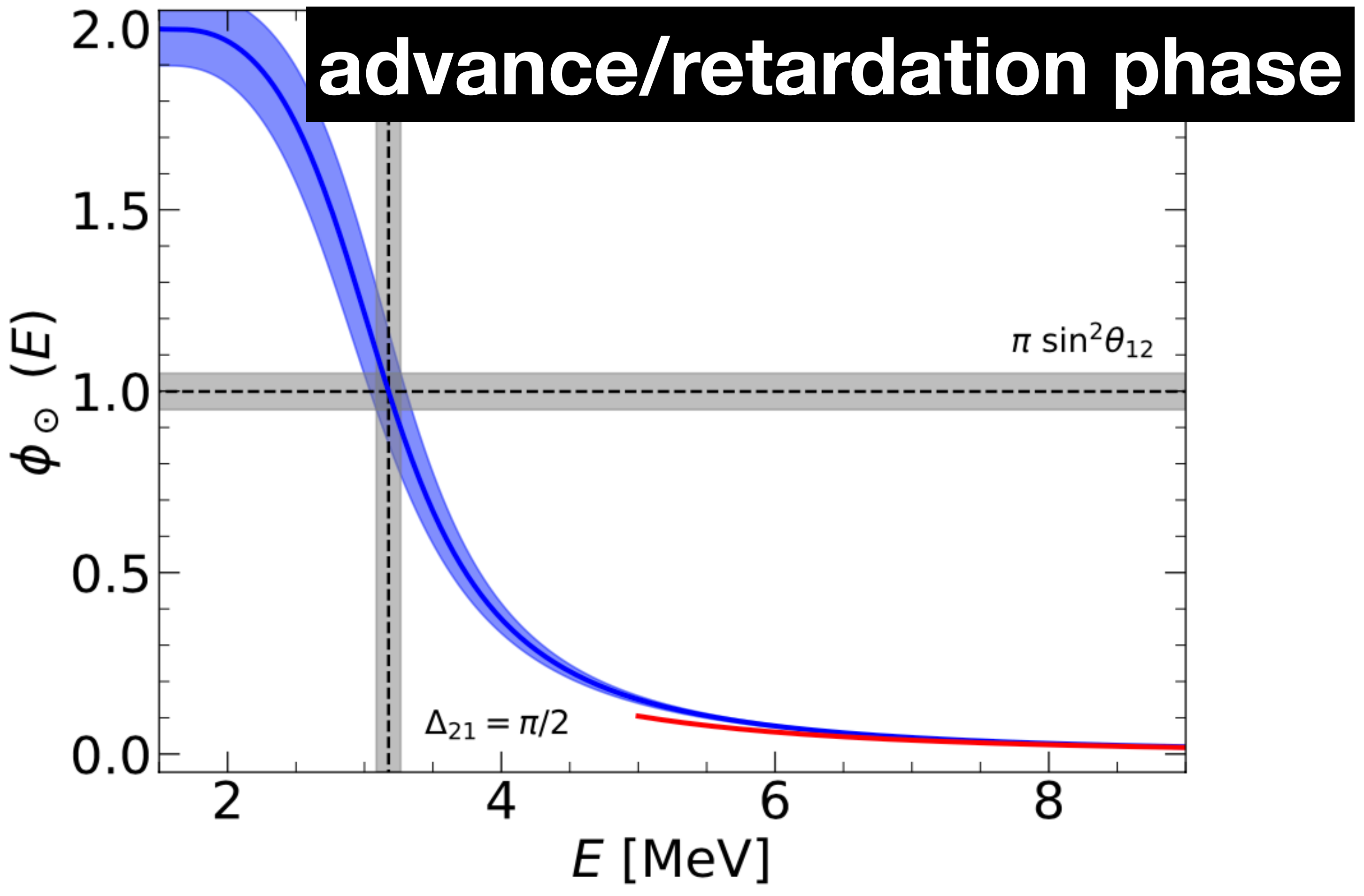
phase

$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$



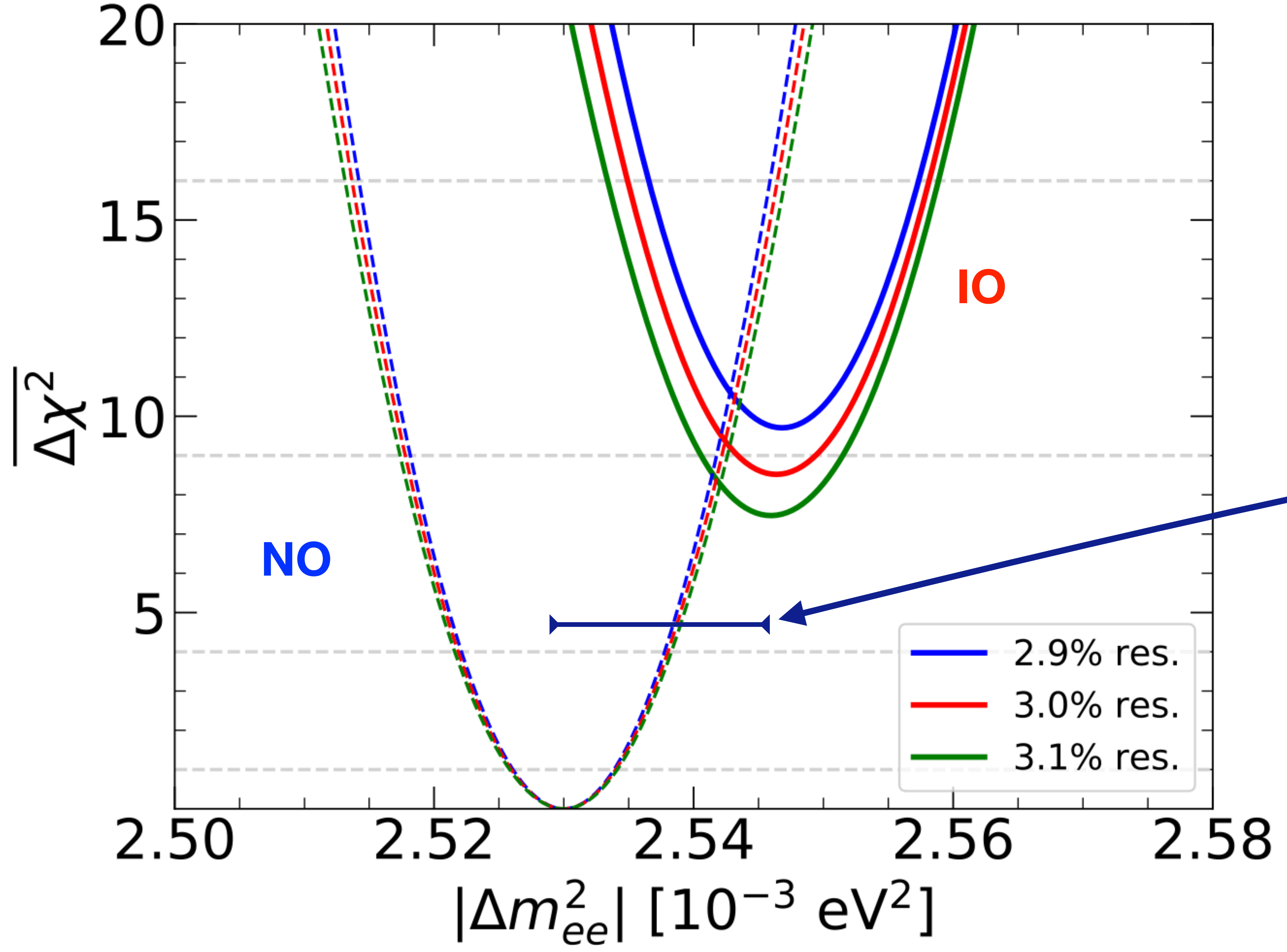
$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{21}$$

retardation/advancement of the phase result in a change of the "effective fast oscillation scale



JUNO Disappearance Measurement

$\overline{\nu}_e \rightarrow \overline{\nu}_e$



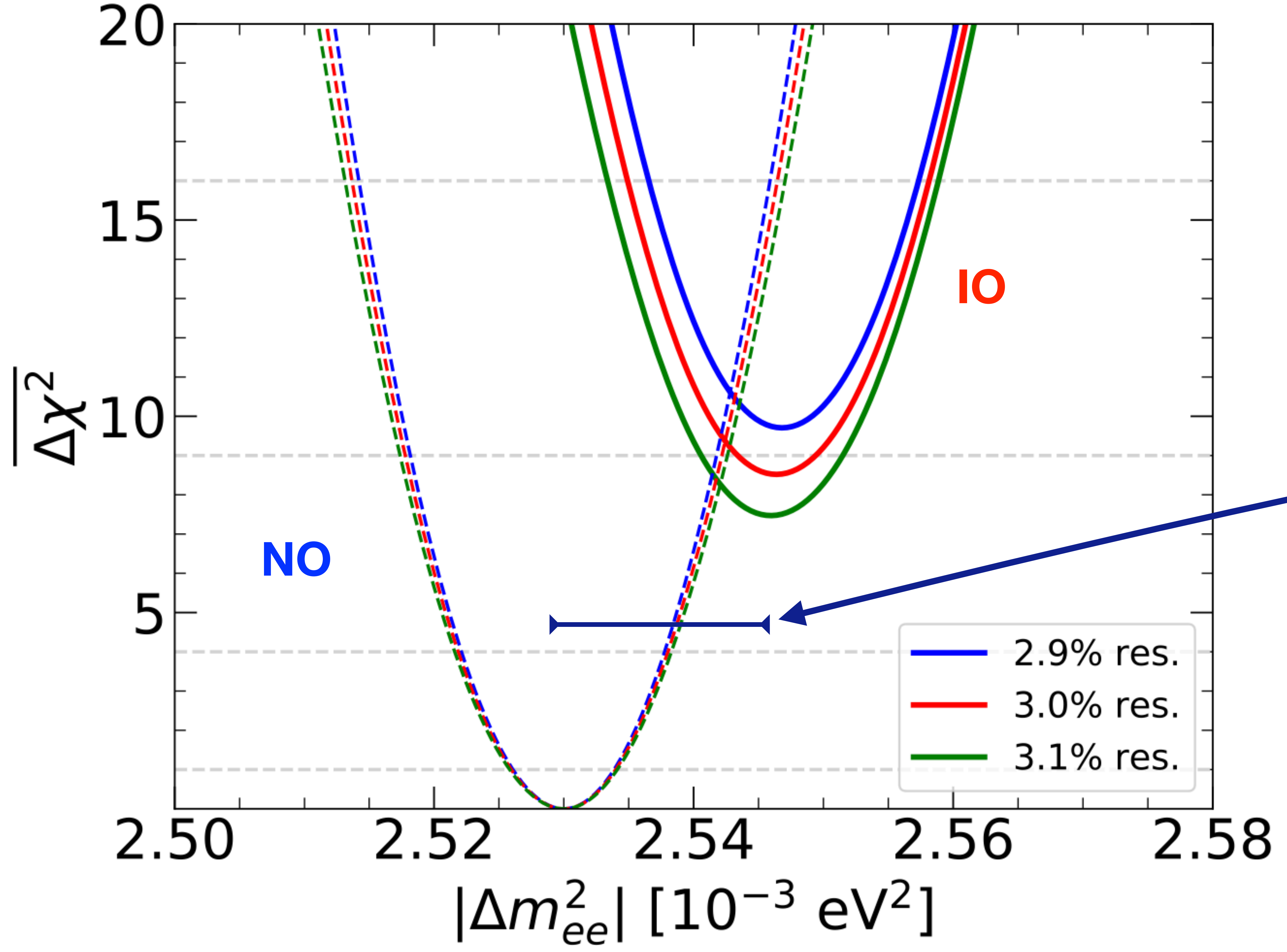
Mass Ordering
Flagship measurement

[D. V. Forero, S. J. Parke, C. A. Ternes, RZF (2021)]

$$|\Delta m^2_{ee}|^{IO}_{JUNO} = \Delta m^2_{ee}|^{NO}_{JUNO} + 1.8 \times 10^{-5}\ \text{eV}^2$$

JUNO Disappearance Measurement

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$



Mass Ordering
Flagship measurement

[D. V. Forero, S. J. Parke, C. A. Ternes, RZF (2021)]

$$|\Delta m^2_{ee}|^{IO}_{JUNO} = \Delta m^2_{ee}|^{NO}_{JUNO} + 1.8 \times 10^{-5}\ \text{eV}^2$$

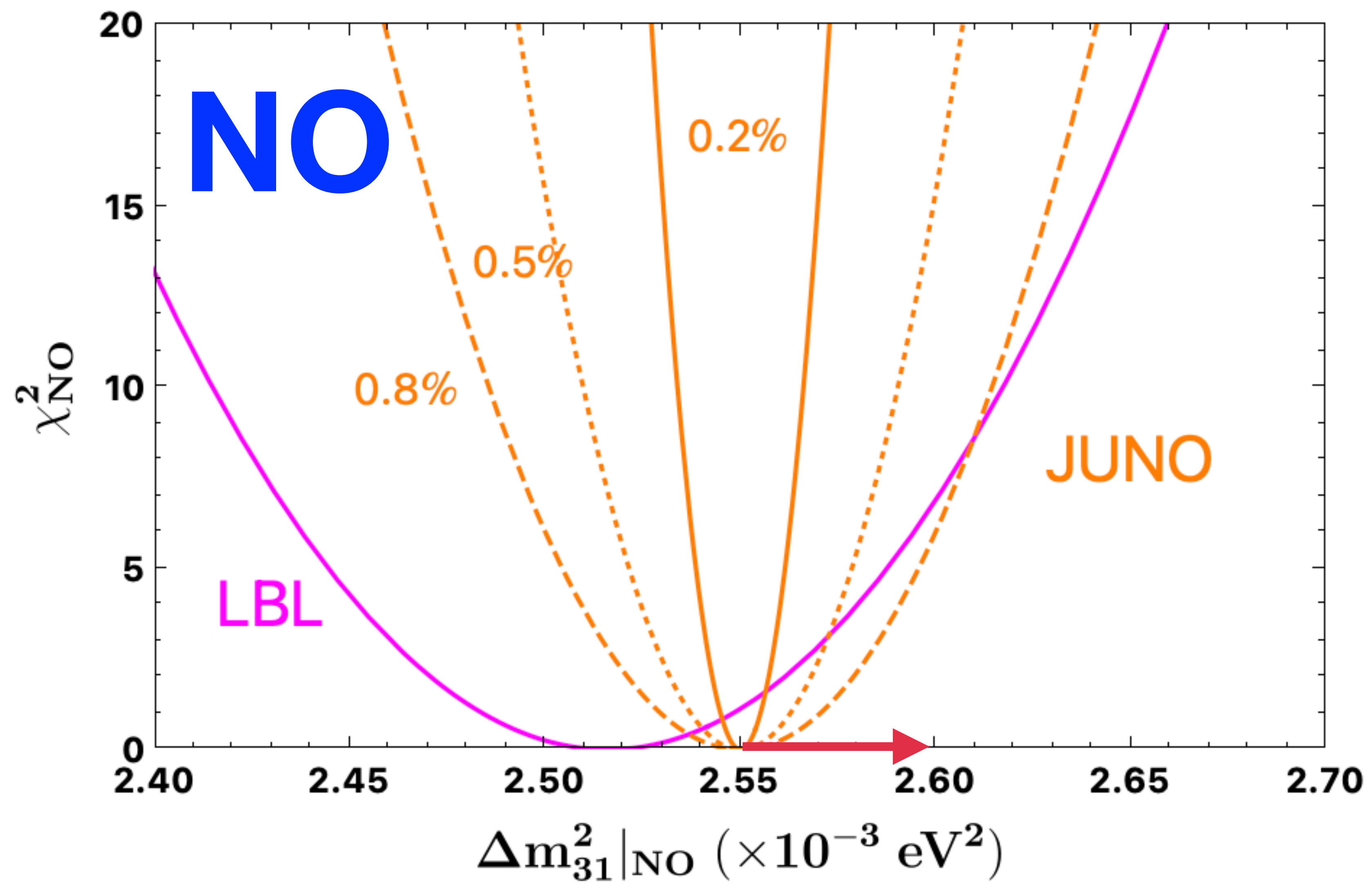
Even before they can determine the Ordering …
They can determine both values precisely
i.e. two degenerate solutions

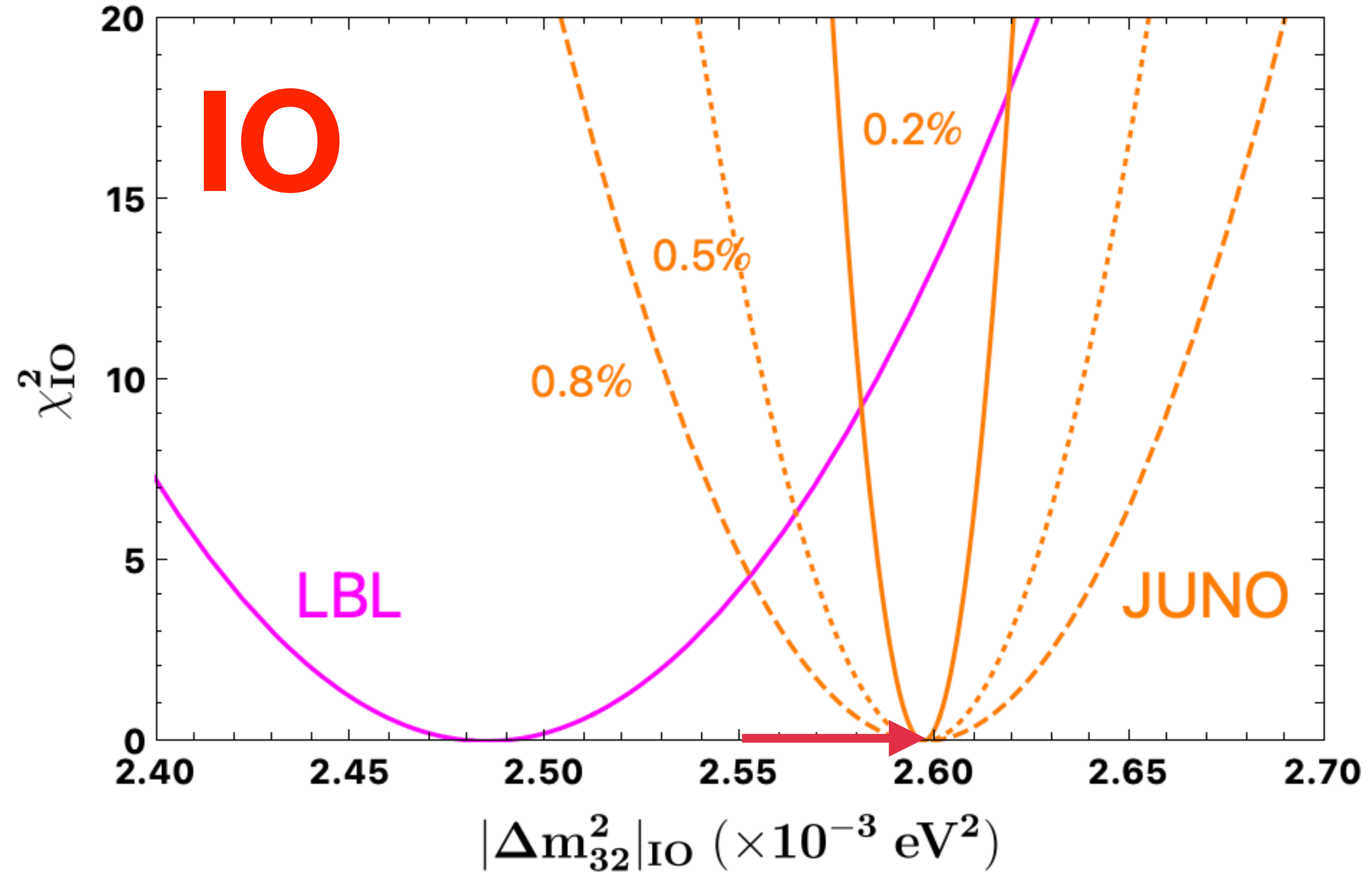
[JUNO Collab. arXiv:2204.13249]

$2.4\,\%$ (DayaBay) $\rightarrow 0.8\,\%$ **in 100 days**

FAN PLOT

[S. J. Parke, RZF arXiv:2404.0873]





$$|\Delta m^2_{32}|^{IO}_{JUNO} = \Delta m^2_{31}|^{NO}_{JUNO} + 4.7 \times 10^{-5} \text{eV}^2$$

$$\overline{\nu}_e \rightarrow \overline{\nu}_e$$

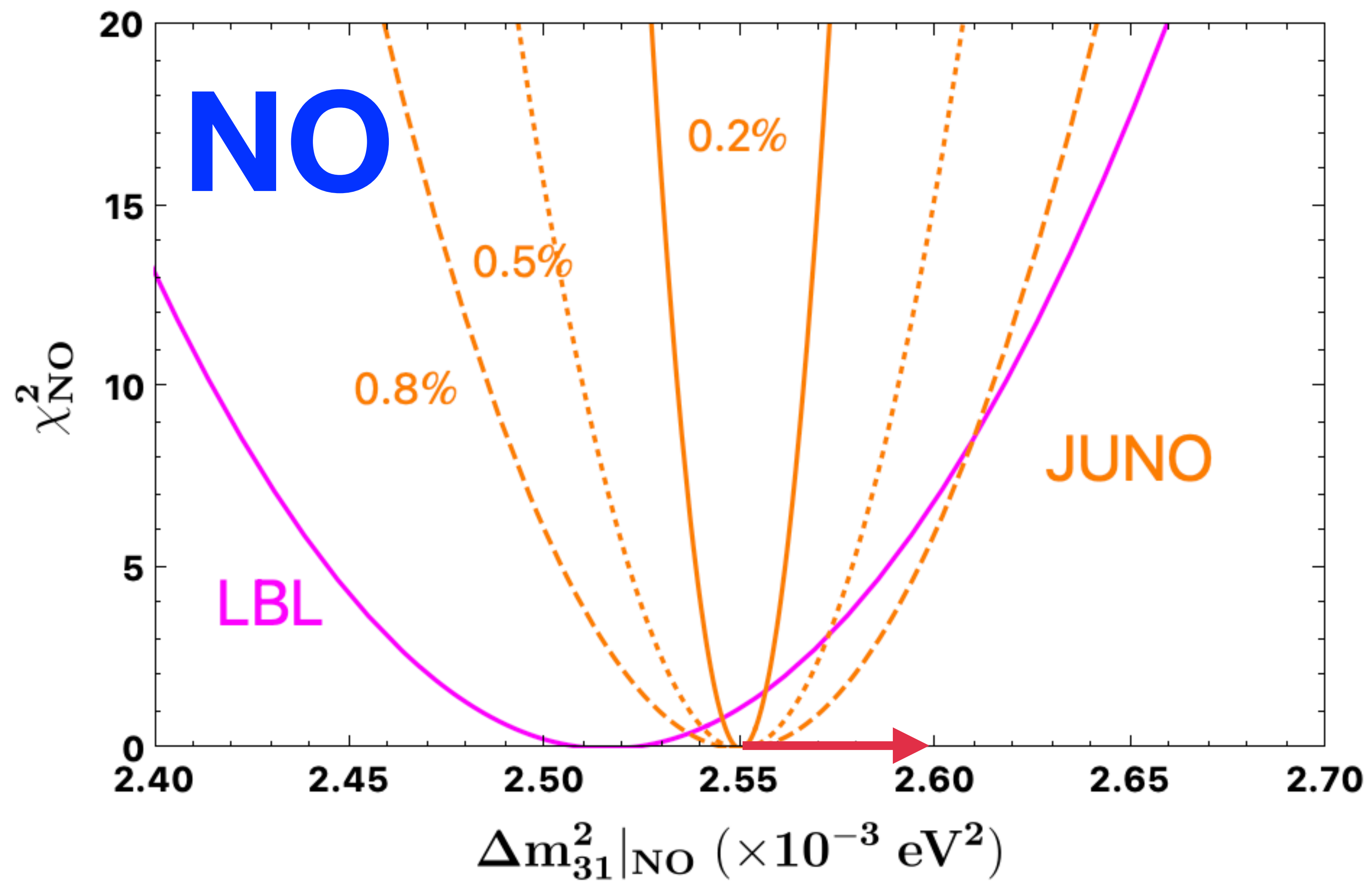
Matter Effect - A.N. Khan, H.Nunokawa, S.J. Parke (2020)

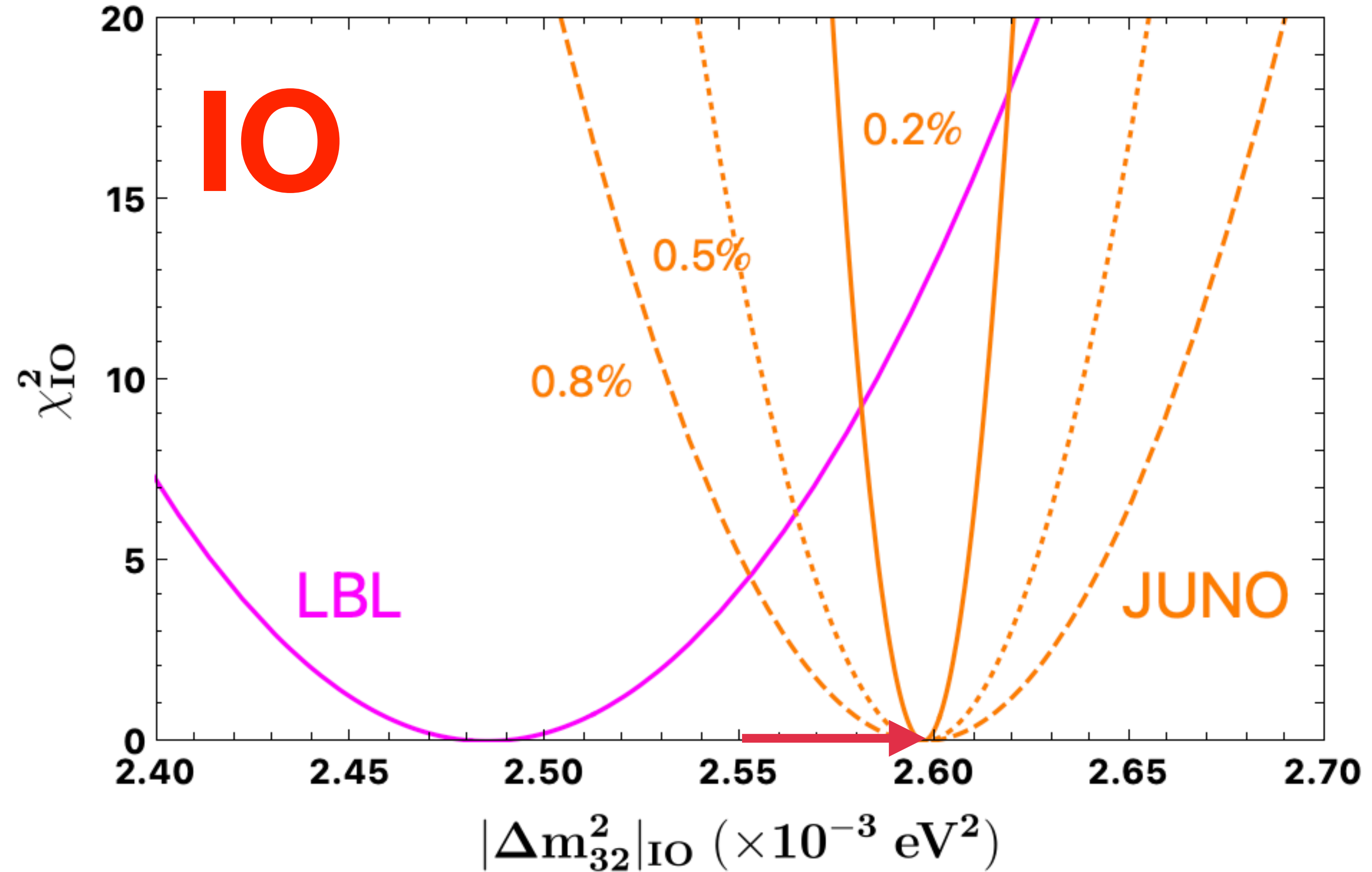
$\Delta m^2_{21} \rightarrow -1.1\,\%$

$\sin^2\theta_{12} \rightarrow 0.2\,\%$

FAN PLOT

[S. J. Parke, RZF arXiv:2404.0873]





$$|\Delta m^2_{32}|^{IO}_{JUNO} = \Delta m^2_{31}|^{NO}_{JUNO} + 4.7 \times 10^{-5} \text{eV}^2$$

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

Matter Effect - A.N. Khan, H.Nunokawa, S.J. Parke (2020)

$\Delta m^2_{21} \rightarrow -1.1\,\%$

$\sin^2\theta_{12} \rightarrow 0.2\,\%$

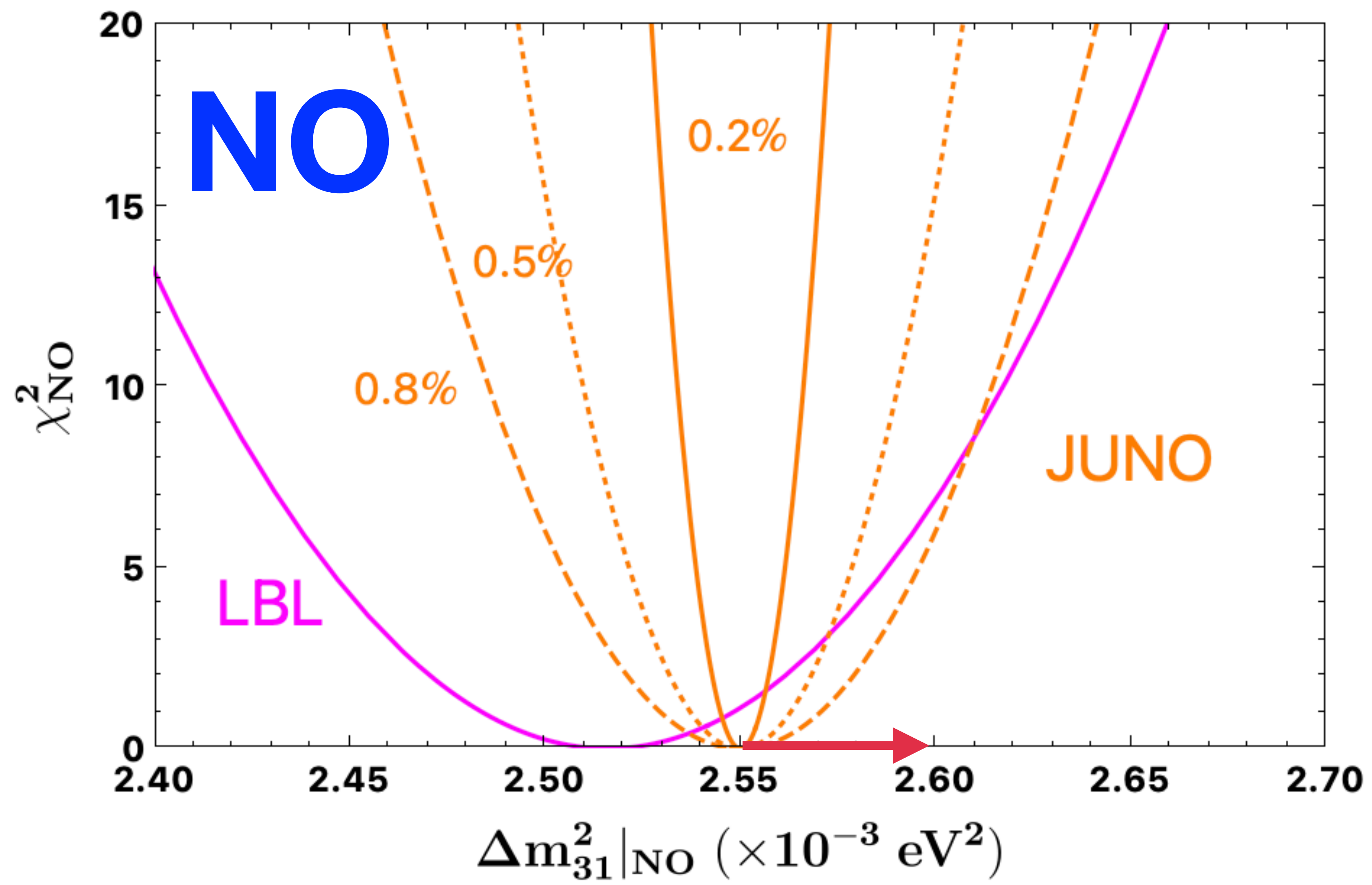
$$\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3} \text{eV}^2$$

$$\nu_\mu \rightarrow \nu_\mu / \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$$

$$|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3} \text{eV}^2$$

FAN PLOT

[S. J. Parke, RZF arXiv:2404.0873]



$|\Delta m^2_{32}|^{IO}_{JUNO} = \Delta m^2_{31}|^{NO}_{JUNO} + 4.7 \times 10^{-5} eV^2$

$\bar{\nu}_e \to \bar{\nu}_e$

Matter Effect - A.N. Khan, H.Nunokawa, S.J. Parke (2020)

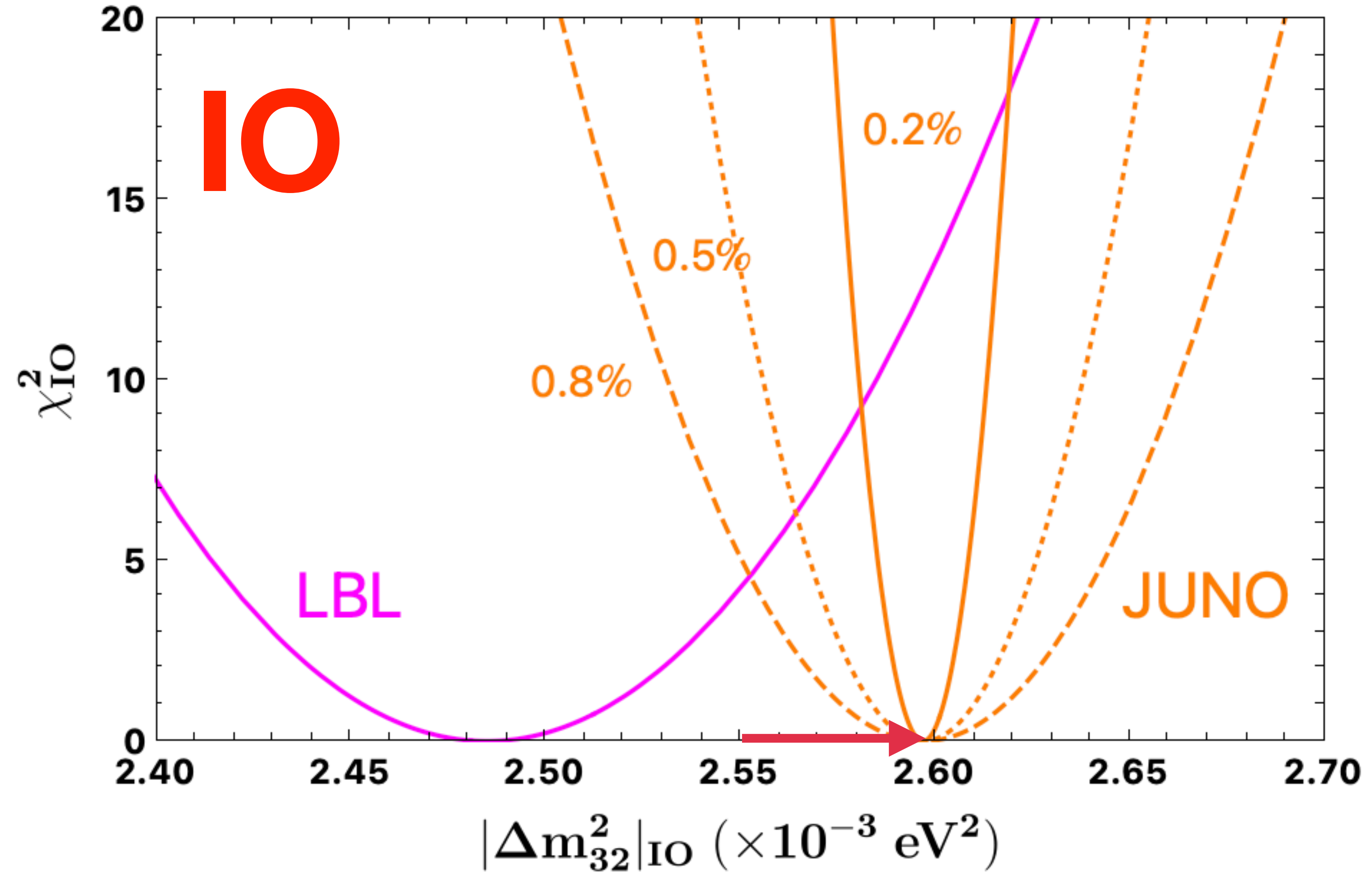
$\Delta m^2_{21} \to -1.1\,\%$

$\sin^2\theta_{12} \to 0.2\,\%$

$\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3} eV^2$

Matter Effect - P. B. Denton, S.J. Parke (2024)

matter effects in $\nu_\mu \to \nu_e$ cancel $\nu_\mu \to \nu_\tau$

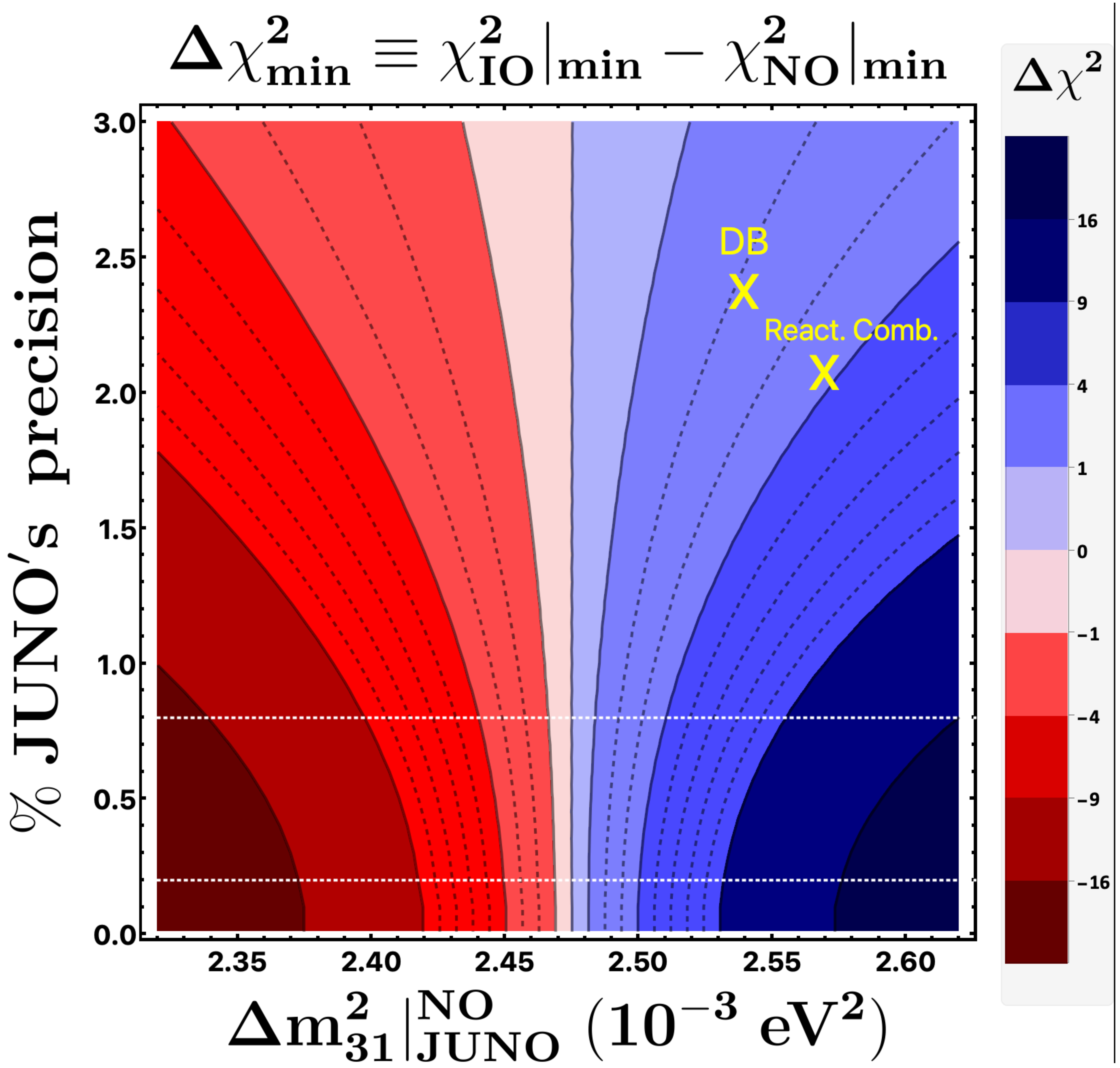


$\nu_\mu \to \nu_\mu/\bar{\nu}_\mu \to \bar{\nu}_\mu$

$|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3} eV^2$

FAN PLOT

[S. J. Parke, RZF arXiv:2404.0873]



100 days

6 years

Conclusion

- the determination of the neutrino mass ordering is relevant for:
  - model building
  - neutrinoless double beta decay experiments
  - beta decay experiments
  - cosmic neutrinos background
  - cosmology

- it is possible that we will know the ordering soon before DUNE/HYPER-K (matter effect)

  by combining two types of disappearance measurements in vacuum

Another possible way to determine the Neutrino Mass Hierarchy

Hiroshi Nunokawa[1],[*] Stephen Parke[2],[†] and Renata Zukanovich Funchal[3‡]