

Lepton Flavor Violation by Two Units

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Based on: PLB '24 [[2401.09580](#)] with Mikheil Sokhashvili



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Standard Model of Particle Physics

BARYONS/QUARKS mass → charge → spin →	$\approx 2.3 \text{ MeV}/c^2$ $2/3$ $1/2$ u up	$\approx 1.275 \text{ GeV}/c^2$ $2/3$ $1/2$ c charm	$\approx 173.07 \text{ GeV}/c^2$ $2/3$ $1/2$ t top	0 0 1 g gluon
	$\approx 4.8 \text{ MeV}/c^2$ $-1/3$ $1/2$ d down	$\approx 95 \text{ MeV}/c^2$ $-1/3$ $1/2$ s strange	$\approx 4.18 \text{ GeV}/c^2$ $-1/3$ $1/2$ b bottom	0 0 1 γ photon
	$0.511 \text{ MeV}/c^2$ -1 $1/2$ e electron	$105.7 \text{ MeV}/c^2$ -1 $1/2$ μ muon	$1.777 \text{ GeV}/c^2$ -1 $1/2$ τ tau	$91.2 \text{ GeV}/c^2$ 0 1 Z Z boson
	$< 2.2 \text{ eV}/c^2$ 0 $1/2$ ν_e electron neutrino	$< 0.17 \text{ MeV}/c^2$ 0 $1/2$ ν_μ muon neutrino	$< 15.5 \text{ MeV}/c^2$ 0 $1/2$ ν_τ tau neutrino	$80.4 \text{ GeV}/c^2$ ± 1 1 W W boson

$\approx 126 \text{ GeV}/c^2$
 0
 0
H
 Higgs boson

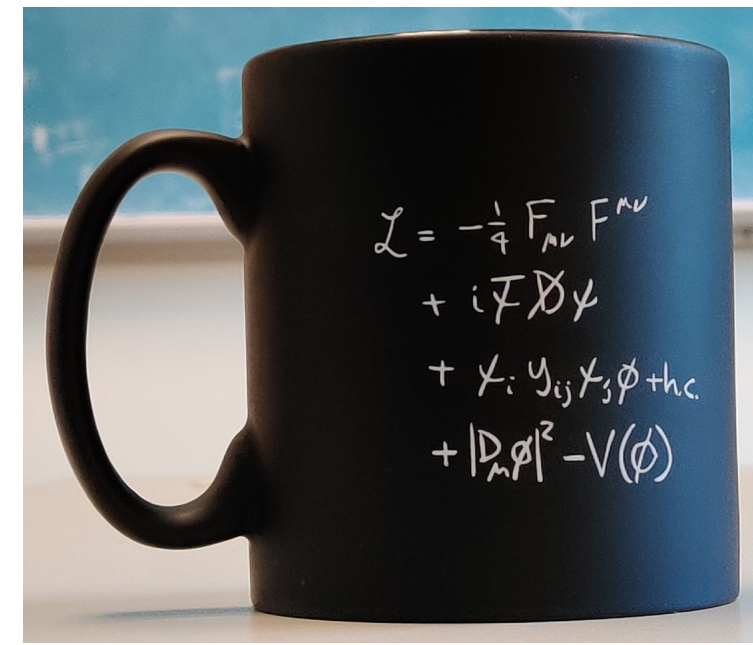
SCALARS



Englert & Higgs '13



SU(3)_c x SU(2)_L x U(1)_Y GAUGE BOSONS



[wikipedia]

Symmetries of the Standard Model

- Rephasing quark and lepton fields:

$$U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

=

$$U(1)_B \times U(1)_L \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} \cdot$$

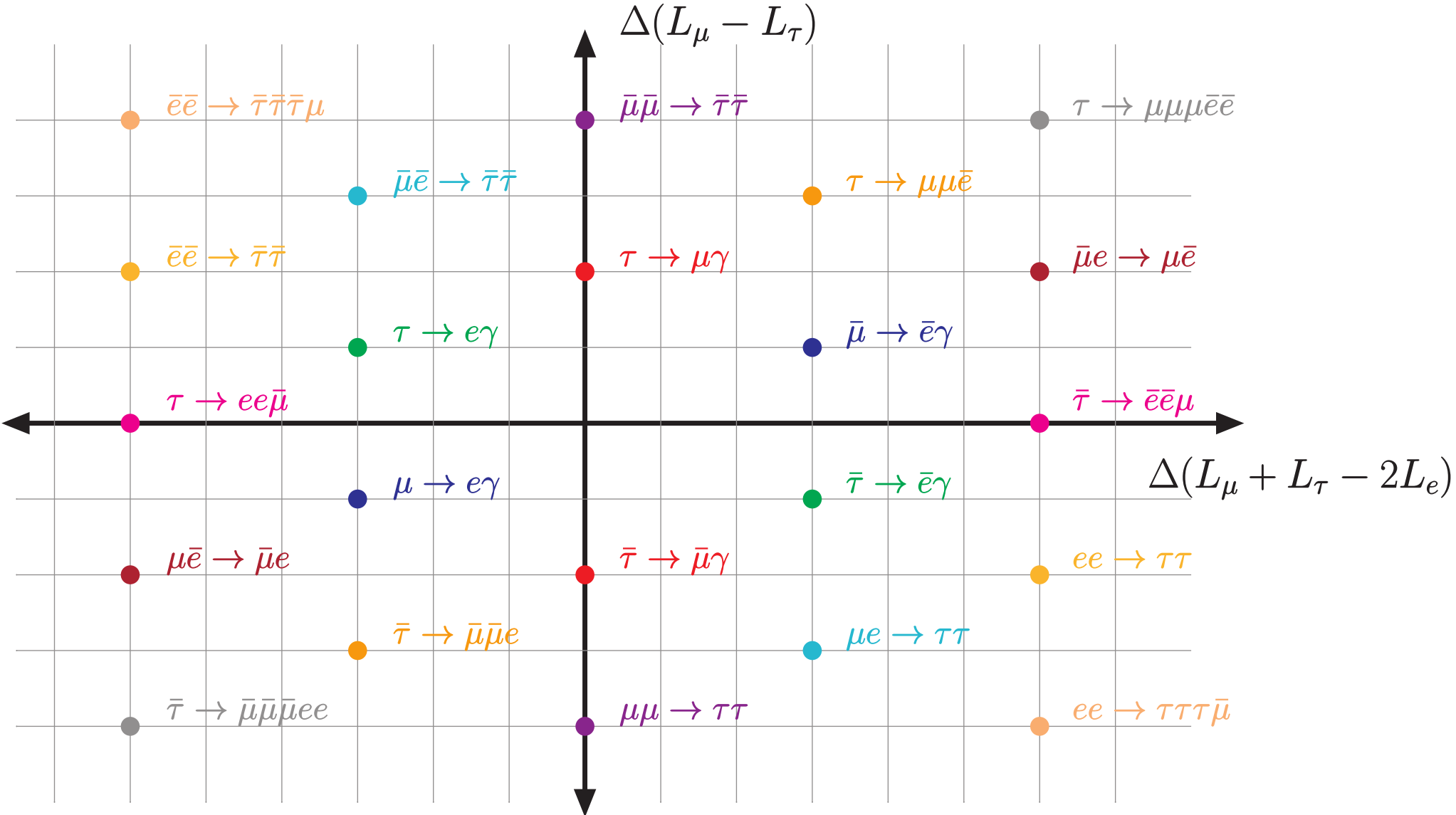

⇒ Lepton flavor conservation

- $(U(1))_{B+L}$ **broken** to Z_3 non-perturbatively, but unobservable.)
[’t Hooft, PRL ‘76]

Four conservation laws **predicted** by SM

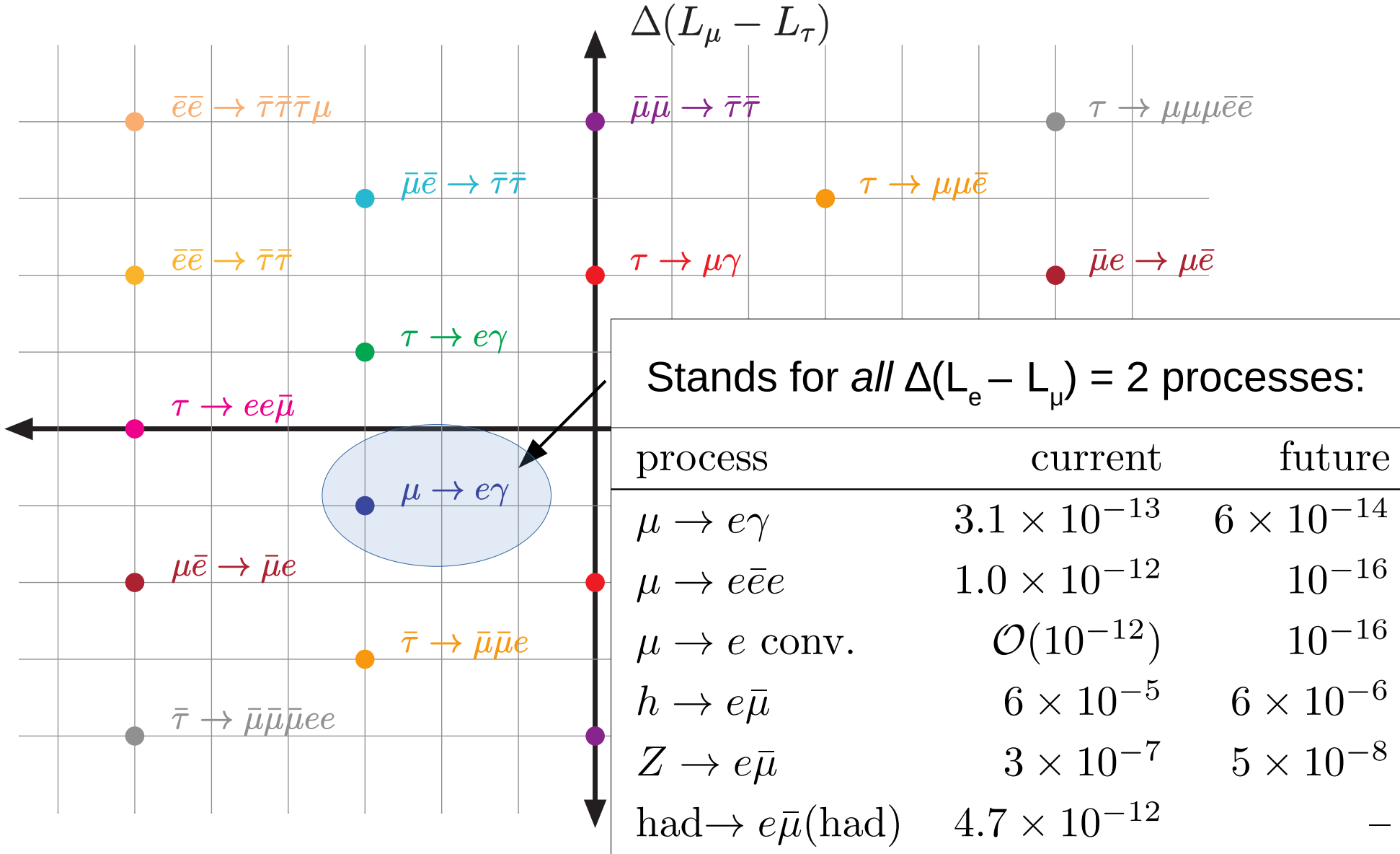
CLFV = breaking of $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$

[JH, 1610.07623]



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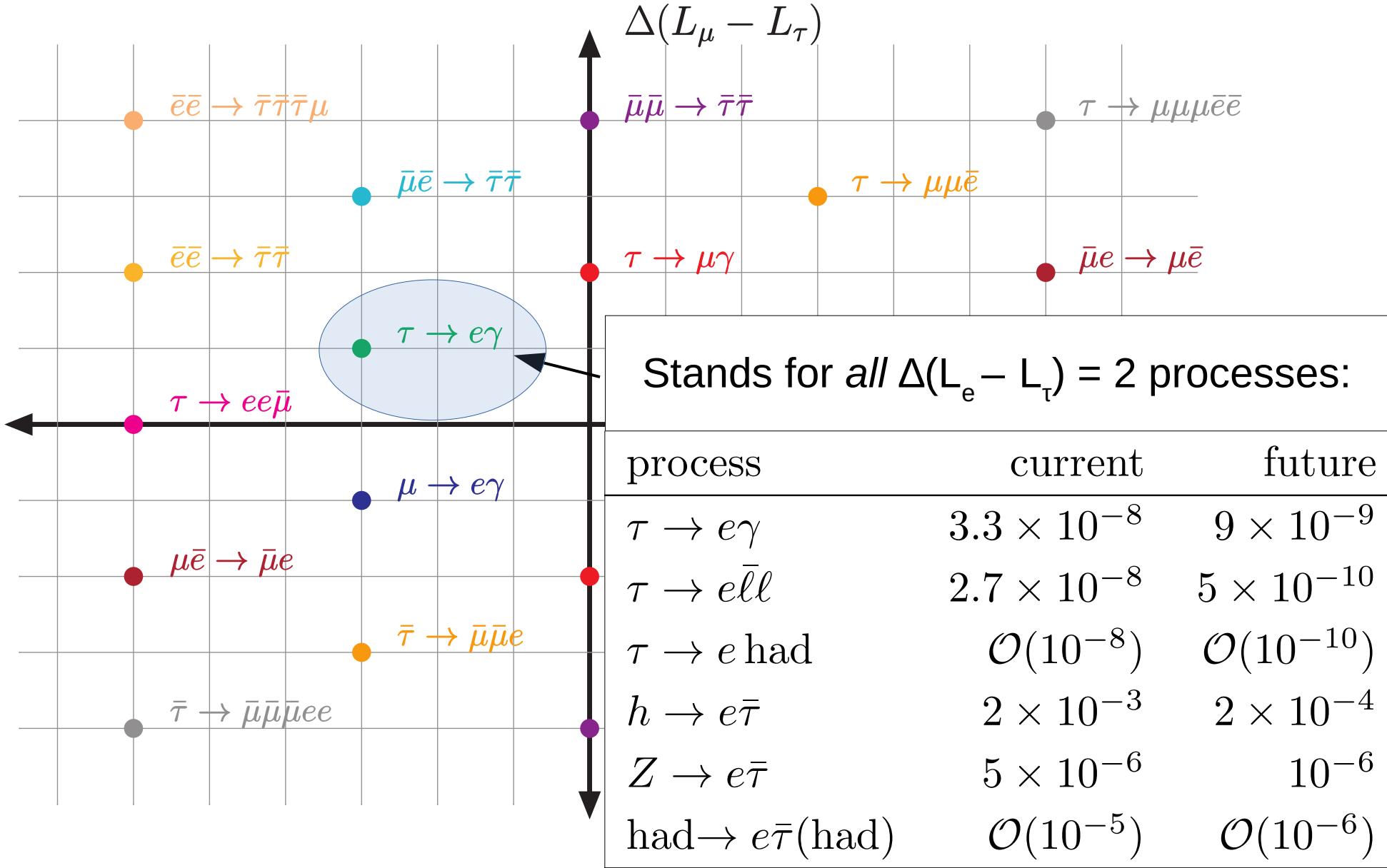
[JH, 1610.07623]



(See talk by [Ana Teixeira.](#))

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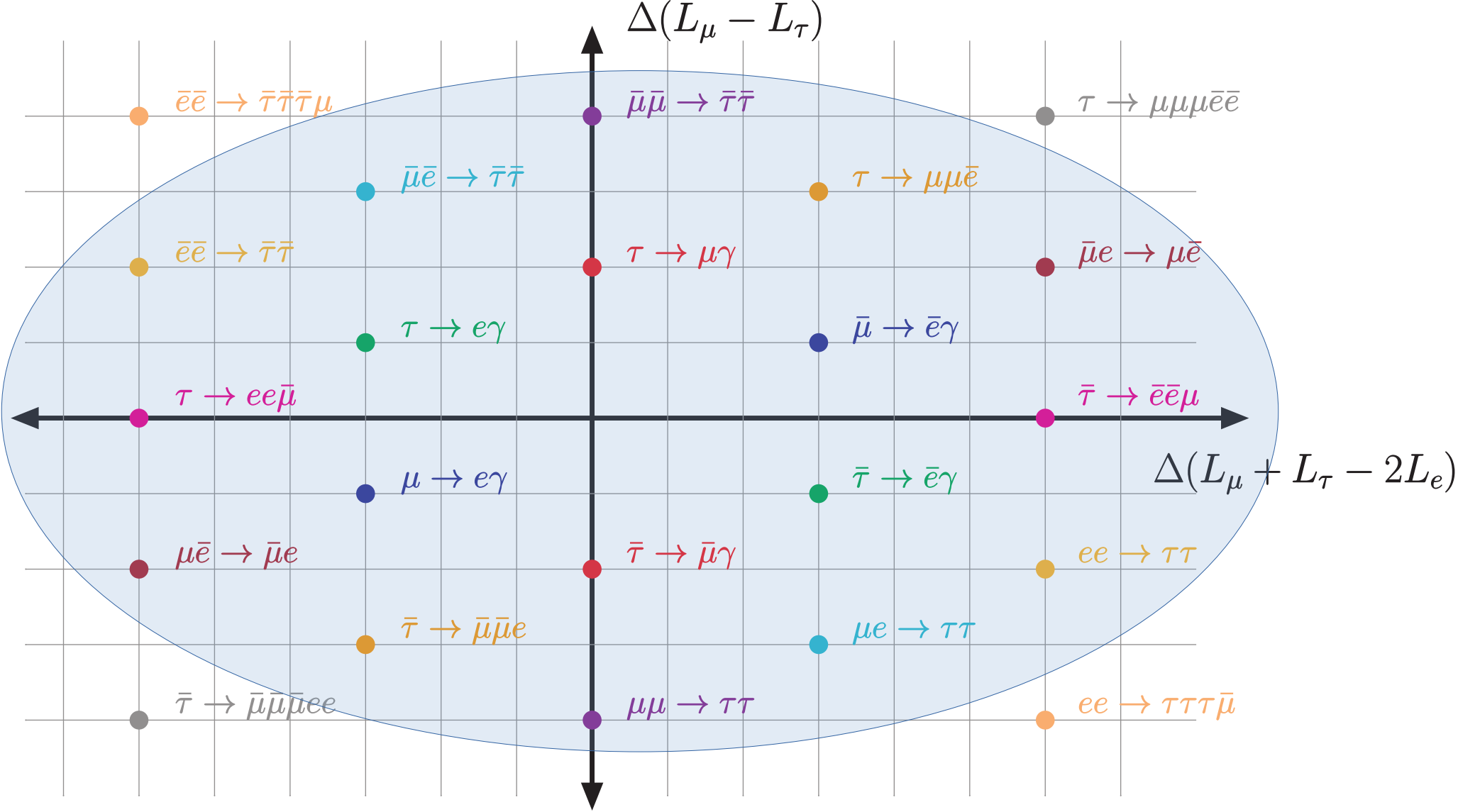
[JH, 1610.07623]



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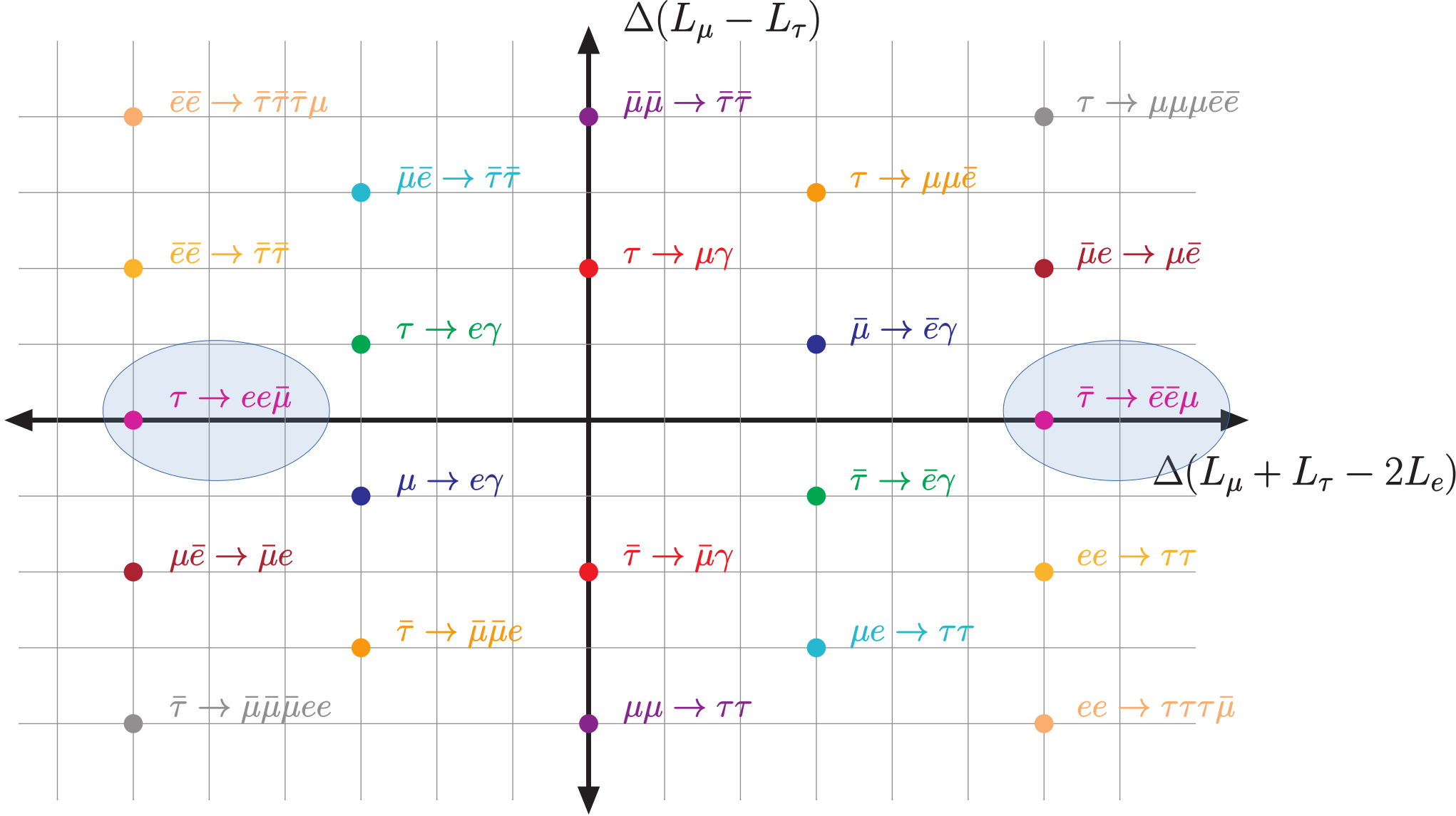
Dimension 6 SMEFT operators

[JH, 1610.07623]



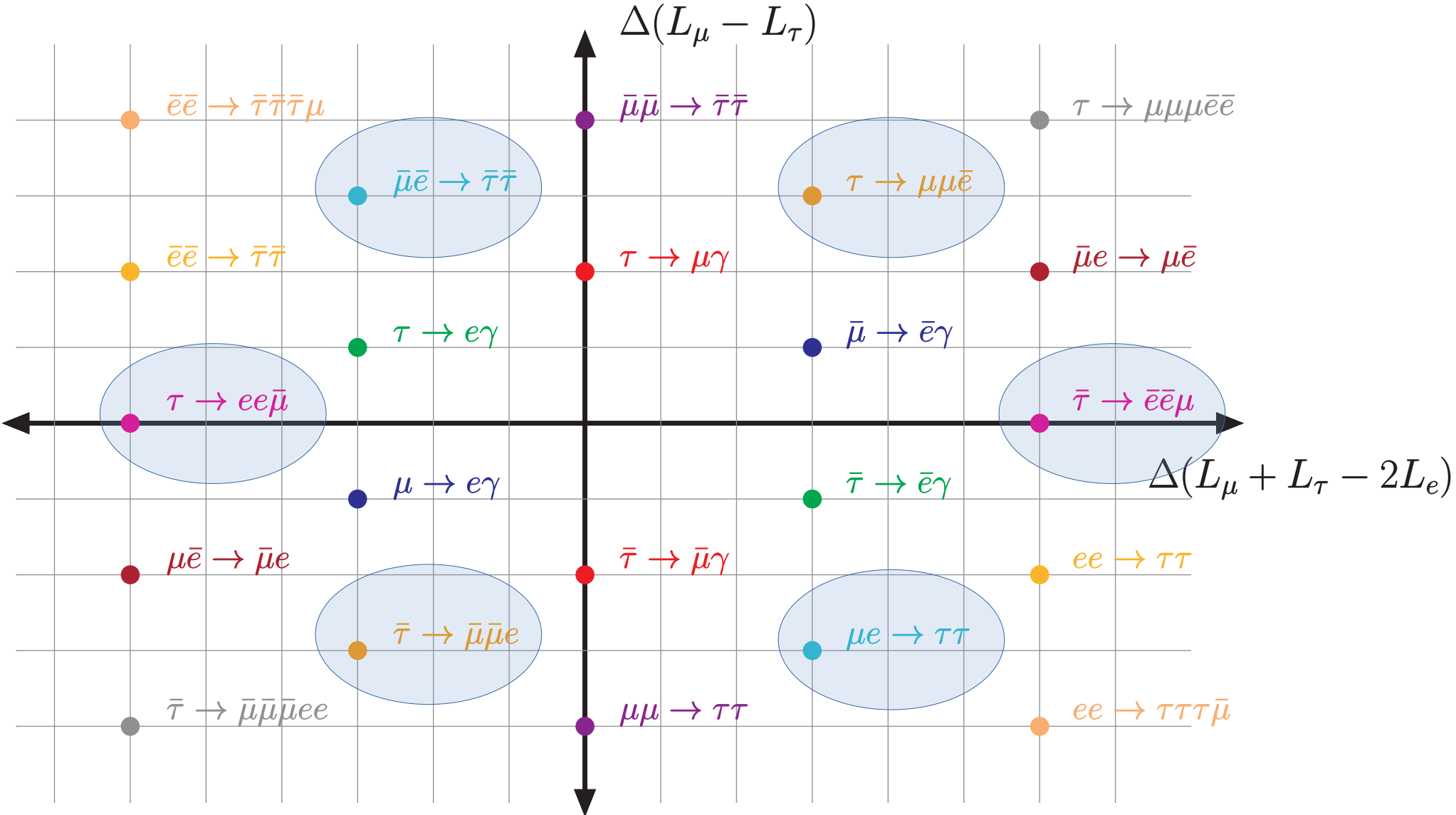
Impose $U(1)_{L_\mu - L_\tau}$ to make $\tau \rightarrow ee\bar{\mu}$ dominant.

[JH, 1610.07623]



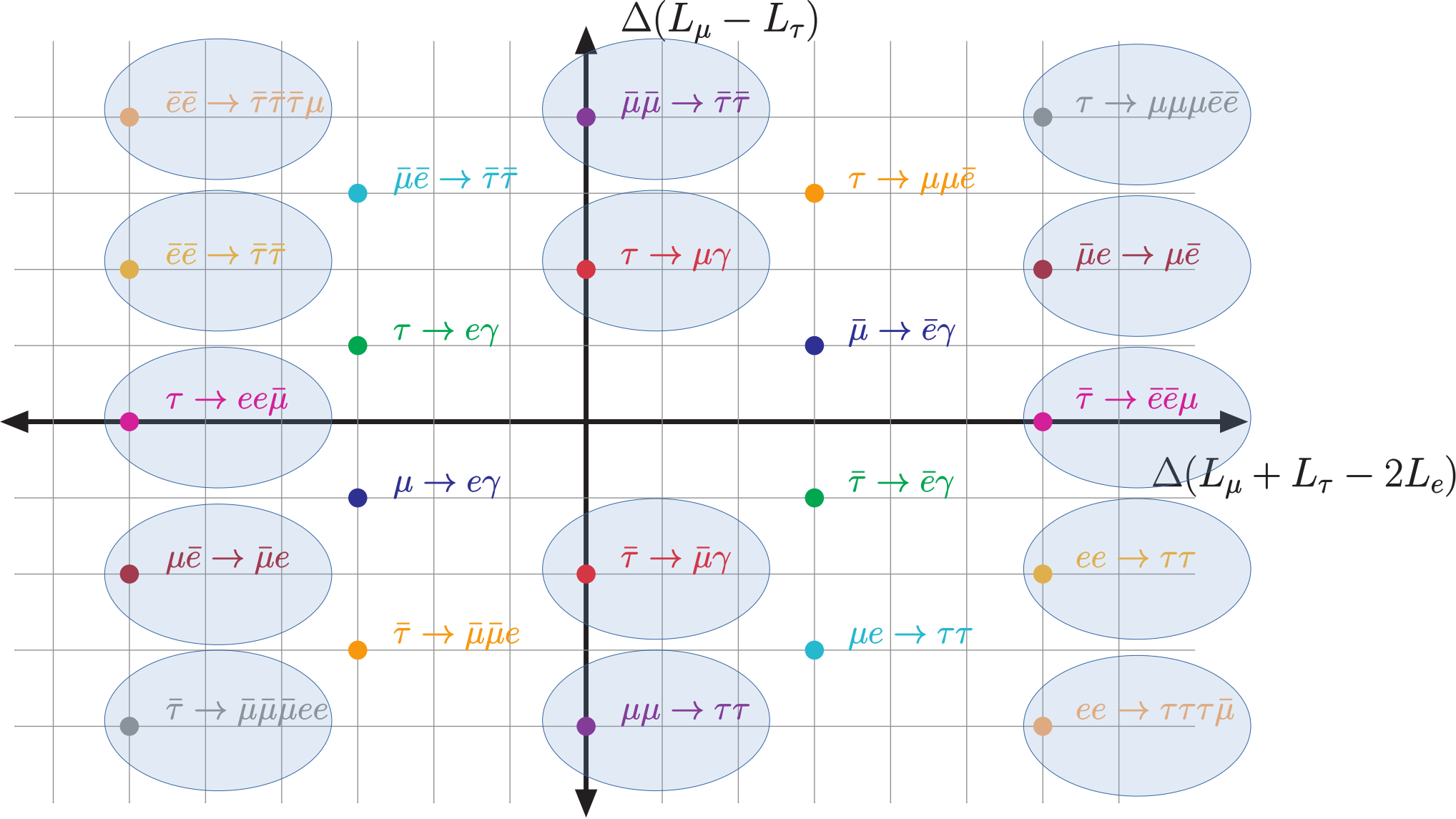
Impose lepton triality \mathbb{Z}_3 . [Altarelli & Feruglio, NPB '06]

[JH, 1610.07623]



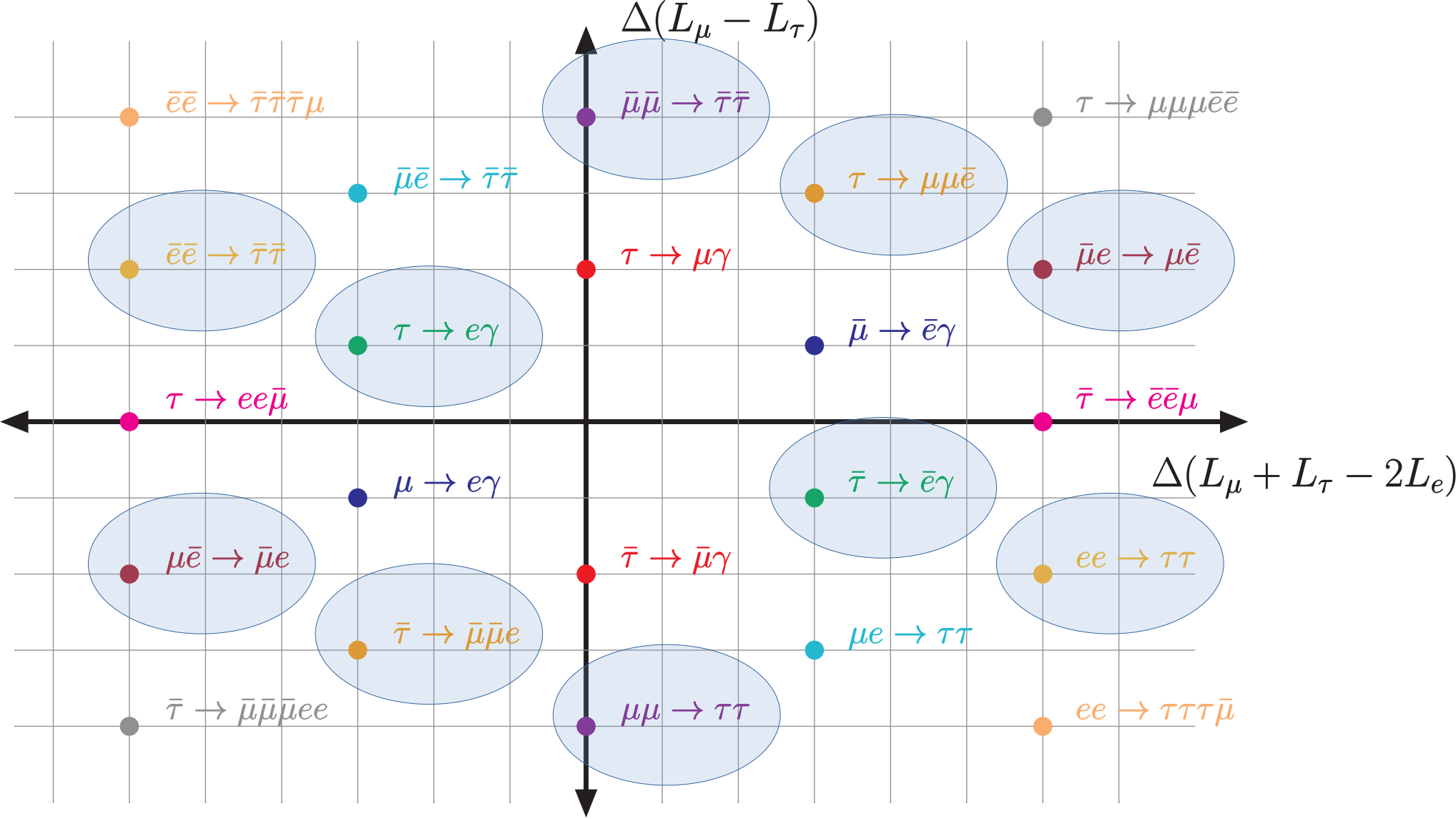
Impose \mathbb{Z}_2 under which e is odd.

[JH, 1610.07623]



Impose \mathbb{Z}_2 under which μ is odd.

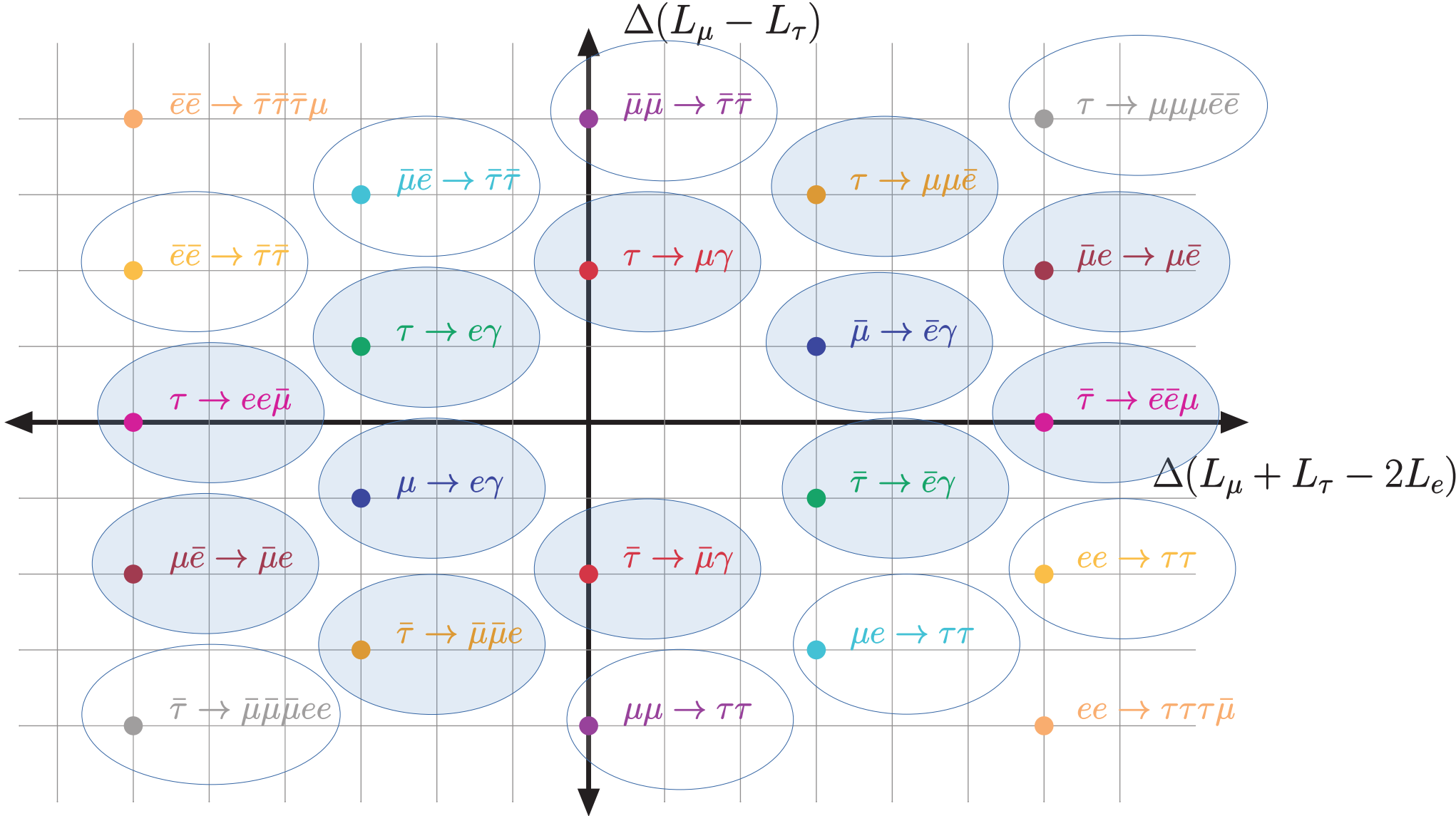
[JH, 1610.07623]

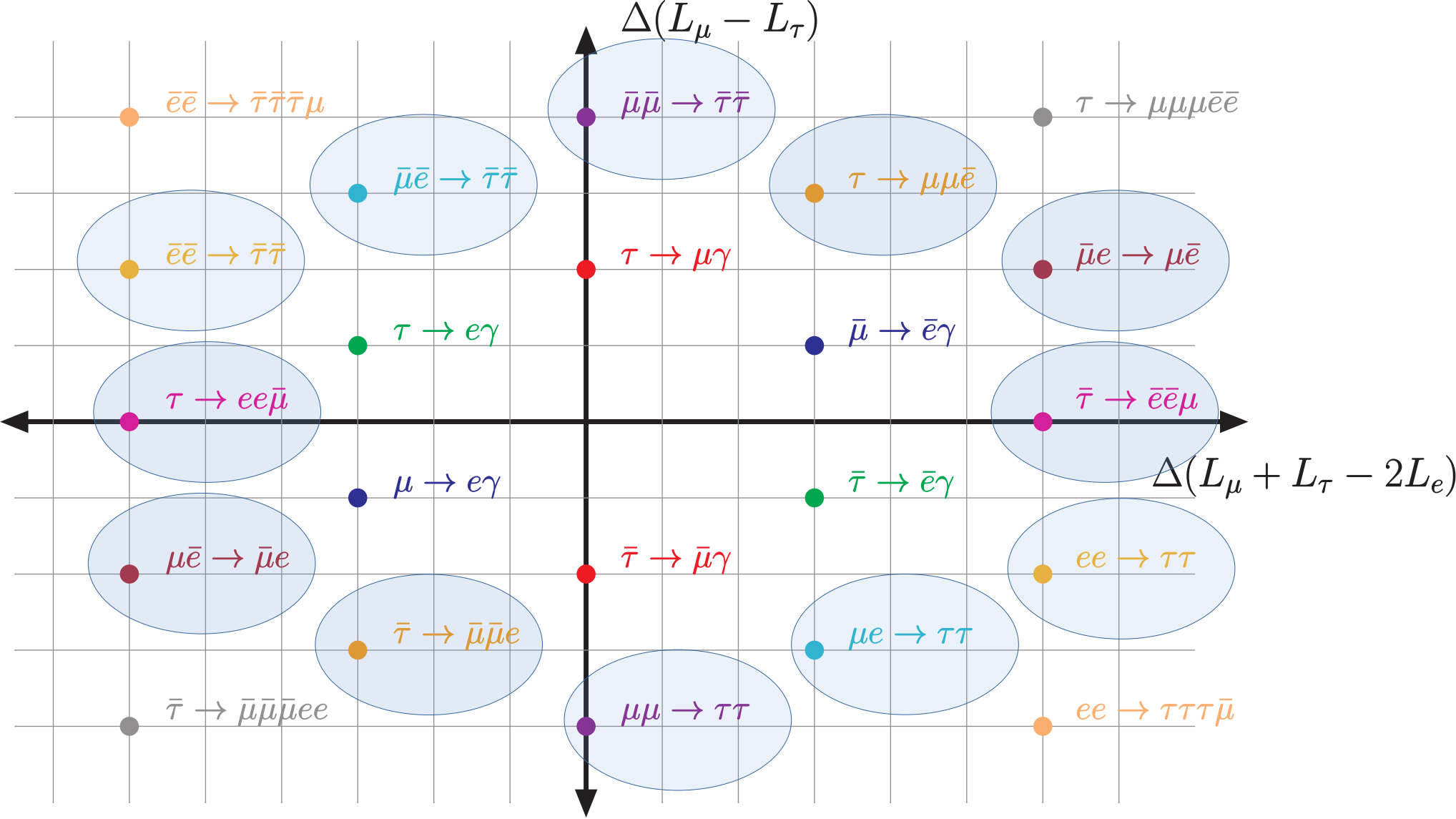


Etc. etc. etc.

Currently being probed: Future:

[JH, 1610.07623]





Standard Model Effective Field Theory

- 888 CLFV operators at $d=6$:

$$\frac{C_{ij}}{\Lambda^2} l_i^c \sigma_{\alpha\beta} l_j H F^{\alpha\beta}, \quad \frac{C_{ij}}{\Lambda^2} l_i^c \gamma^\alpha l_j H^\dagger D_\alpha H, \quad \frac{C_{ijnm}}{\Lambda^2} l_i^c l_j q_n^c q_m, \quad \frac{C_{ijnm}}{\Lambda^2} l_i^c l_j l_n^c l_m$$

[Weinberg '79; Buchmüller & Wyler, '86; Grzadkowski++, '10; Fonseca, '17]

- Model-dependent coefficients, easy to UV complete.

Standard Model Effective Field Theory

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[Weinberg '79; Buchmüller & Wyler, '86; Grzadkowski++, '10; Fonseca, '17]

- Model-dependent coefficients, easy to UV complete.
- Four-lepton operators can break lepton flavor by **2 units**:

$$\mathcal{L} \supset \sum_{a,b,c,d=e,\mu,\tau} \left[y_{abcd}^{LL} \bar{L}_a \gamma^\alpha L_b \bar{L}_c \gamma_\alpha L_d \right. \\ \left. + y_{abcd}^{LR} \bar{L}_a \gamma^\alpha L_b \bar{\ell}_c \gamma_\alpha \ell_d + y_{abcd}^{RR} \bar{\ell}_a \gamma^\alpha \ell_b \bar{\ell}_c \gamma_\alpha \ell_d \right] + \text{h.c.}$$

- UV completions by neutral or doubly-charged bosons.
- **21 different operators**; how do we look for them?

$$\Delta L_\mu = -\Delta L_e = 2, \Delta L_\tau = 0$$

- Well-known scenario, 3 operators:

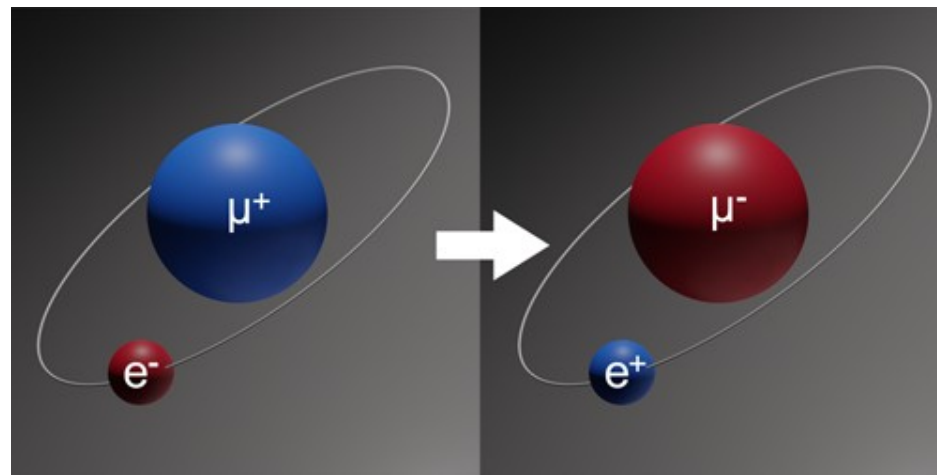
$$\mathcal{L} \supset y_{\mu e \mu e}^{LL} \bar{L}_\mu \gamma^\alpha L_e \bar{L}_\mu \gamma_\alpha L_e + y_{\mu e \mu e}^{LR} \bar{L}_\mu \gamma^\alpha L_e \bar{\ell}_\mu \gamma_\alpha \ell_e + y_{\mu e \mu e}^{RR} \bar{\ell}_\mu \gamma^\alpha \ell_e \bar{\ell}_\mu \gamma_\alpha \ell_e$$

- Contribute to **muonium-antimuonium conversion**:

$$P \simeq \frac{7.58 \times 10^{-7}}{G_F^2} |y_{\mu e \mu e}^{LL} + y_{\mu e \mu e}^{RR} - 1.68 y_{\mu e \mu e}^{LR}|^2 + \frac{4.27 \times 10^{-7}}{G_F^2} |y_{\mu e \mu e}^{LL} + y_{\mu e \mu e}^{RR} + 0.68 y_{\mu e \mu e}^{LR}|^2$$

[Conlin & Petrov, 2005.10276; Fukuyama, Mimura, Uesaka, 2108.10736; ...]

- MACS@PSI '99: $P < 8 \times 10^{-11}$, improve e.g. at MACE. [2203.11406]



[APS, Physics 15, s9]

$$\Delta L_\mu = -\Delta L_e = 2, \quad \Delta L_\tau = 0$$

- Well-known scenario, 3 operators:

$$\mathcal{L} \supset y_{\mu e \mu e}^{LL} \bar{L}_\mu \gamma^\alpha L_e \bar{L}_\mu \gamma_\alpha L_e + y_{\mu e \mu e}^{LR} \bar{L}_\mu \gamma^\alpha L_e \bar{\ell}_\mu \gamma_\alpha \ell_e + y_{\mu e \mu e}^{RR} \bar{\ell}_\mu \gamma^\alpha \ell_e \bar{\ell}_\mu \gamma_\alpha \ell_e$$

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- MACS@PSI '99: $P < 8 \times 10^{-11}$, improve e.g. at MACE. [2203.11406]
- Probes only 2 *linear combinations* of the 3 couplings:

$$|y_{\mu e \mu e}^{LL} + y_{\mu e \mu e}^{RR}| < (2.9 \text{ TeV})^{-2} \quad \& \quad |y_{\mu e \mu e}^{LR}| < (3.4 \text{ TeV})^{-2}$$

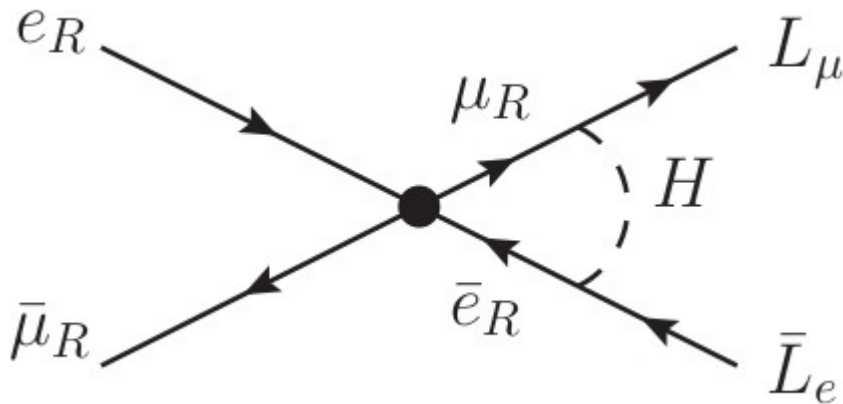
- $y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR}$ is **unconstrained**, comes from $\bar{\mu} \gamma_\alpha e \bar{\mu} \gamma^\alpha \gamma_5 e$.

$$\Delta L_\mu = -\Delta L_e = 2, \Delta L_\tau = 0$$

- Well-known scenario, 3 operators:

$$\mathcal{L} \supset y_{\mu e \mu e}^{LL} \bar{L}_\mu \gamma^\alpha L_e \bar{L}_\mu \gamma_\alpha L_e + y_{\mu e \mu e}^{LR} \bar{L}_\mu \gamma^\alpha L_e \bar{\ell}_\mu \gamma_\alpha \ell_e + y_{\mu e \mu e}^{RR} \bar{\ell}_\mu \gamma^\alpha \ell_e \bar{\ell}_\mu \gamma_\alpha \ell_e$$

- $y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR}$ is **unconstrained**, comes from $\bar{\mu} \gamma_\alpha e \bar{\mu} \gamma^\alpha \gamma_5 e$.
- Also contributes to μ conversion, but G_F suppressed.
[Conlin & Petrov, 2005.10276]
- RGE mixing?



$$y_{\mu e \mu e}^{LR} \simeq \underbrace{\frac{y_e y_\mu}{16\pi^2}}_{10^{-11}} (y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR})$$

Too small...

$$\Delta L_\mu = -\Delta L_e = 2, \quad \Delta L_\tau = 0$$

- Well-known scenario, 3 operators:

$$\mathcal{L} \supset y_{\mu e \mu e}^{LL} \bar{L}_\mu \gamma^\alpha L_e \bar{L}_\mu \gamma_\alpha L_e + y_{\mu e \mu e}^{LR} \bar{L}_\mu \gamma^\alpha L_e \bar{\ell}_\mu \gamma_\alpha \ell_e + y_{\mu e \mu e}^{RR} \bar{\ell}_\mu \gamma^\alpha \ell_e \bar{\ell}_\mu \gamma_\alpha \ell_e$$

- $y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR}$ is **unconstrained**, comes from $\bar{\mu} \gamma_\alpha e \bar{\mu} \gamma^\alpha \gamma_5 e$.
- Also contributes to μ conversion, but G_F suppressed.
- Operator contains **neutrinos**: contributes to μ decay!
 - No interference, but same electron spectrum:

$$\Gamma(\mu \rightarrow e \nu \bar{\nu}) = \Gamma_\mu^{\text{SM}} \left(1 + \frac{|2y_{\mu e \mu e}^{LL}|^2 + |y_{\mu e \mu e}^{LR}|^2}{(2\sqrt{2}G_F)^2} \right)$$

- Lepton universality $\Gamma(\mu \rightarrow e \nu \bar{\nu}) / \Gamma(\tau \rightarrow \mu \nu \bar{\nu})$ gives limit

$$|y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR}| < (0.74 \text{ TeV})^{-2}$$

[HFLAV, PDG]
[Belle II, 2405.14625]

$$\Delta L_e = 2, \Delta L_\tau = \Delta L_\mu = -1$$

- Easiest scenario, 4 operators:

$$\begin{aligned} \mathcal{L} \supset & y_{\mu e \tau e}^{LL} \bar{L}_\mu \gamma^\alpha L_e \bar{L}_\tau \gamma_\alpha L_e + y_{\mu e \tau e}^{LR} \bar{L}_\mu \gamma^\alpha L_e \bar{l}_\tau \gamma_\alpha l_e \\ & + y_{\tau e \mu e}^{LR} \bar{L}_\tau \gamma^\alpha L_e \bar{l}_\mu \gamma_\alpha l_e + y_{\mu e \tau e}^{RR} \bar{l}_\mu \gamma^\alpha l_e \bar{l}_\tau \gamma_\alpha l_e \end{aligned}$$

- Leads to $\tau^+ \rightarrow e^+ e^+ \mu^-$: $\Gamma \simeq \frac{m_\tau^5 (|y_{\mu e \tau e}^{LL}|^2 + |y_{\mu e \tau e}^{LR}|^2 + |y_{\tau e \mu e}^{LR}|^2 + |y_{\tau e \mu e}^{RR}|^2)}{1536\pi^3}$
- All coefficients $< (10 \text{ TeV})^{-2}$ from Belle, [Belle, 1001.3221]
to be improved with Belle II to $(29.0 \text{ TeV})^{-2}$. [Banerjee, 2209.11639]
- Analogous for the four $\Delta L_\mu = 2, \Delta L_\tau = \Delta L_e = -1$ operators:
 $\tau^+ \rightarrow \mu^+ \mu^+ e^-$ gives coefficient limits $(8.8 \text{ TeV})^{-2}$.

$\Delta L_\tau = 1$ is easy due to clean [tau decays](#)

$$\Delta L_e = -\Delta L_\tau = 2$$

- Difficult scenario, 3 operators:

$$\mathcal{L} \supset y_{\tau e \tau e}^{LL} \bar{L}_\tau \gamma^\alpha L_e \bar{L}_\tau \gamma_\alpha L_e + y_{\tau e \tau e}^{LR} \bar{L}_\tau \gamma^\alpha L_e \bar{l}_\tau \gamma_\alpha l_e + y_{\tau e \tau e}^{RR} \bar{l}_\tau \gamma^\alpha l_e \bar{l}_\tau \gamma_\alpha l_e$$

- No tauonium conversion to look for.
- Left-handed operators: contribution to **tau decay**.

$$\Gamma_{\tau \rightarrow e \nu \nu} = \Gamma_{\tau \rightarrow e \nu \nu}^{\text{SM}} \left(1 + \frac{|2y_{\tau e \tau e}^{LL}|^2 + |y_{\tau e \tau e}^{LR}|^2}{(2\sqrt{2}G_F)^2} \right)$$

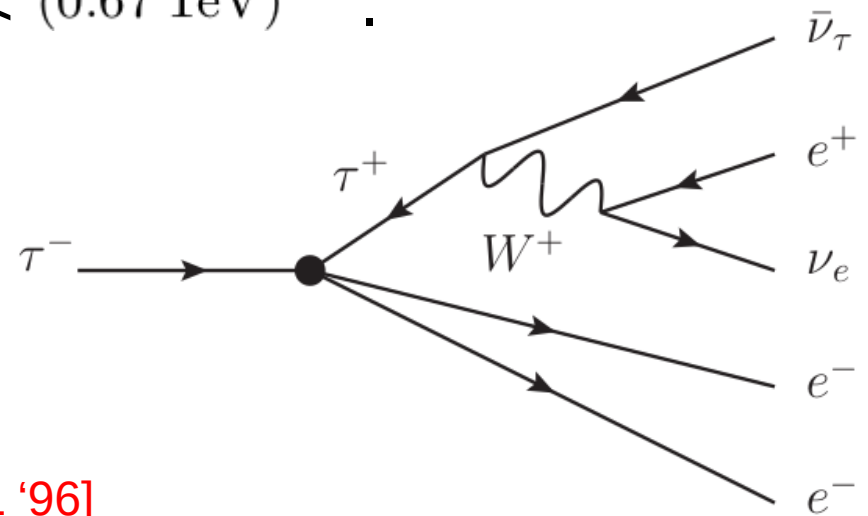
- Flavor universality: $|2y_{\tau e \tau e}^{LL}|, |y_{\tau e \tau e}^{LR}| < (0.67 \text{ TeV})^{-2}$.

- Right-handed operator?

$$- \text{BR}(\tau^- \rightarrow e^- e^- \ell^+ \nu_\ell \bar{\nu}_\tau)$$

$$\simeq 8.2 \times 10^{-15} \left| \frac{y_{\tau e \tau e}^{RR}}{(0.1 \text{ TeV})^{-2}} \right|^2$$

$$- \text{CLEO: } |y_{\tau e \tau e}^{RR}| < (0.5 \text{ GeV})^{-2} \text{ [PRL '96]}$$



$$\Delta L_e = -\Delta L_\tau = 2$$

- Difficult scenario, 3 operators:

$$\mathcal{L} \supset y_{\tau e \tau e}^{LL} \bar{L}_\tau \gamma^\alpha L_e \bar{L}_\tau \gamma_\alpha L_e + y_{\tau e \tau e}^{LR} \bar{L}_\tau \gamma^\alpha L_e \bar{l}_\tau \gamma_\alpha l_e + y_{\tau e \tau e}^{RR} \bar{l}_\tau \gamma^\alpha l_e \bar{l}_\tau \gamma_\alpha l_e$$

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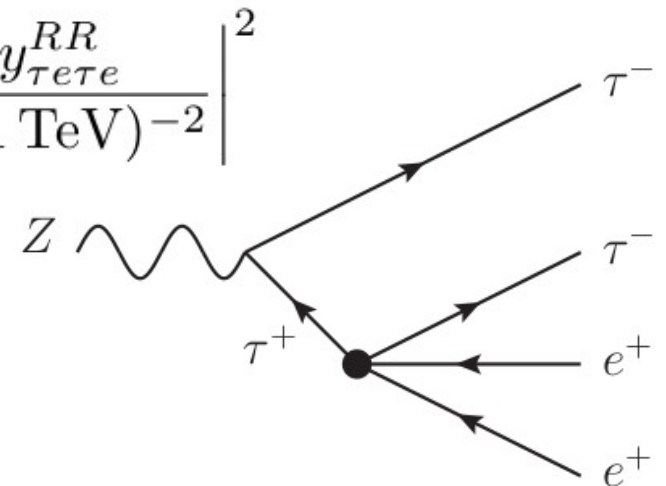
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- Flavor universality: $|2y_{\tau e \tau e}^{LL}|, |y_{\tau e \tau e}^{LR}| < (0.67 \text{ TeV})^{-2}$.
- Right-handed operator?

$$- \text{BR}(Z \rightarrow \tau^\pm \tau^\pm e^\mp e^\mp) \simeq 4.18 \times 10^{-11} \left| \frac{y_{\tau e \tau e}^{RR}}{(0.1 \text{ TeV})^{-2}} \right|^2$$

$$- \text{Z width: } |y_{\tau e \tau e}^{RR}| < (1.2 \text{ GeV})^{-2}$$

$$- \text{Z factory reach: } |y_{\tau e \tau e}^{RR}| < (250 \text{ GeV})^{-2}$$



Similar for other $\Delta L_\tau = 2$ operators.

Summary

[JH & M. Sokhashvili, 2401.09580]

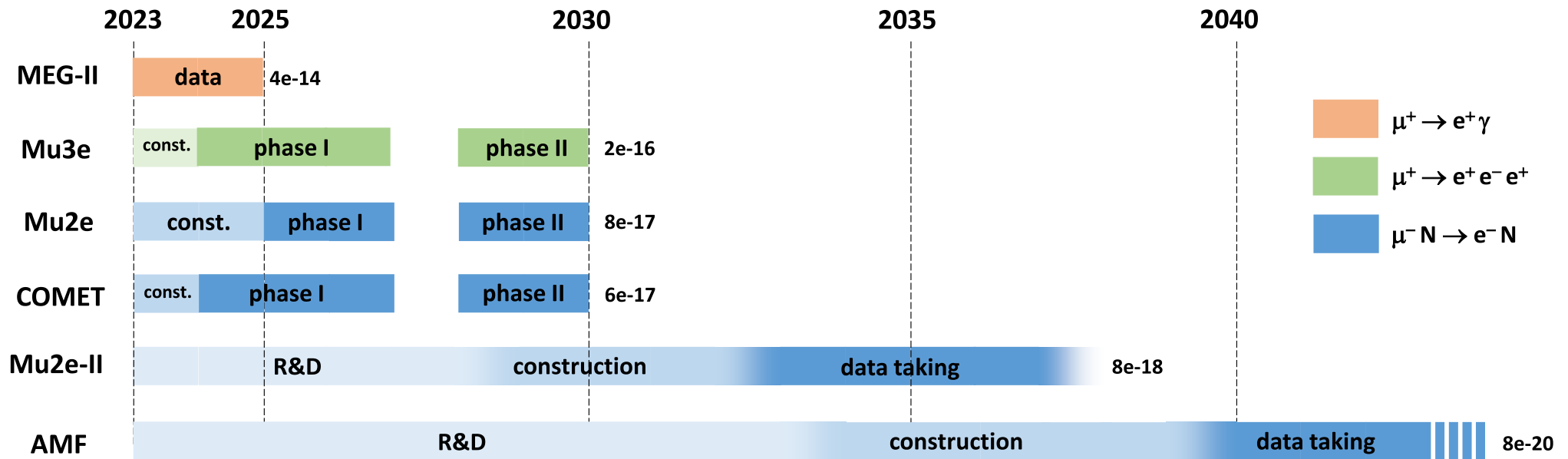
Wilson coefficient	Upper limit	Process	Violated quantum numbers
$ y_{\mu e \mu e}^{LL} + y_{\mu e \mu e}^{RR} $	$(3.2 \text{ TeV})^{-2}$ [90% C.L.]	Mu-to- $\bar{\text{M}}\mu$	$\Delta L_\mu = -\Delta L_e = 2$
$ y_{\mu e \mu e}^{LR} $	$(3.8 \text{ TeV})^{-2}$ [90% C.L.]	Mu-to- $\bar{\text{M}}\mu$	$\Delta L_\mu = -\Delta L_e = 2$
$ y_{\mu e \mu e}^{LL} - y_{\mu e \mu e}^{RR} $	$(0.74 \text{ TeV})^{-2}$ [95% C.L.]	$\Gamma(\mu \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$	$\Delta L_\mu = -\Delta L_e = 2$
$ 2y_{\tau e \tau e}^{LL} , y_{\tau e \tau e}^{LR} $	$(0.67 \text{ TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$	$\Delta L_\tau = -\Delta L_e = 2$
$ y_{\tau e \tau e}^{RR} $	$(1.2 \text{ GeV})^{-2}$ [95% C.L.]	$Z \rightarrow \tau^\pm \tau^\pm e^\mp e^\mp$	$\Delta L_\tau = -\Delta L_e = 2$
$ 2y_{\tau \mu \tau \mu}^{LL} , y_{\tau \mu \tau \mu}^{LR} $	$(0.63 \text{ TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau \rightarrow \mu\nu\bar{\nu})/\Gamma(\tau \rightarrow e\nu\bar{\nu})$	$\Delta L_\tau = -\Delta L_\mu = 2$
$ y_{\tau \mu \tau \mu}^{RR} $	$(1.2 \text{ GeV})^{-2}$ [95% C.L.]	$Z \rightarrow \tau^\pm \tau^\pm \mu^\mp \mu^\mp$	$\Delta L_\tau = -\Delta L_\mu = 2$
$ y_{e \tau \mu \tau}^{LL} , y_{\mu \tau e \tau}^{LR} $	$(0.60 \text{ TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\mu \rightarrow e\nu\bar{\nu})$	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$ y_{e \tau \mu \tau}^{LR} $	$(0.55 \text{ TeV})^{-2}$ [95% C.L.]	$\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$ y_{e \tau \mu \tau}^{RR} $	$(1 \text{ GeV})^{-2}$ [95% C.L.]	$Z \rightarrow \tau^\pm \tau^\pm e^\mp \mu^\mp$	$\Delta L_\tau = -2\Delta L_\mu = -2\Delta L_e = 2$
$ y_{\mu e \tau e}^{LL} , y_{\mu e \tau e}^{LR} , y_{\tau e \mu e}^{LR} , y_{\mu e \tau e}^{RR} $	$(10 \text{ TeV})^{-2}$ [90% C.L.]	$\tau \rightarrow \bar{\mu} e e$	$\Delta L_e = -2\Delta L_\tau = -2\Delta L_\mu = 2$
$ y_{e \mu \tau \mu}^{LL} , y_{e \mu \tau \mu}^{LR} , y_{\tau \mu e \mu}^{LR} , y_{e \mu \tau \mu}^{RR} $	$(8.8 \text{ TeV})^{-2}$ [90% C.L.]	$\tau \rightarrow \bar{e} \mu \mu$	$\Delta L_\mu = -2\Delta L_\tau = -2\Delta L_e = 2$

- The 21 d=6 SMEFT CLFV ops with $\Delta L_\alpha = 2$ often overlooked.
- Overall less clean than $\Delta L_\alpha = 1$, but good/relevant limits for 18.
- The 3 right-right $\Delta L_\tau = 2$ ops need Z factory or μ collider.

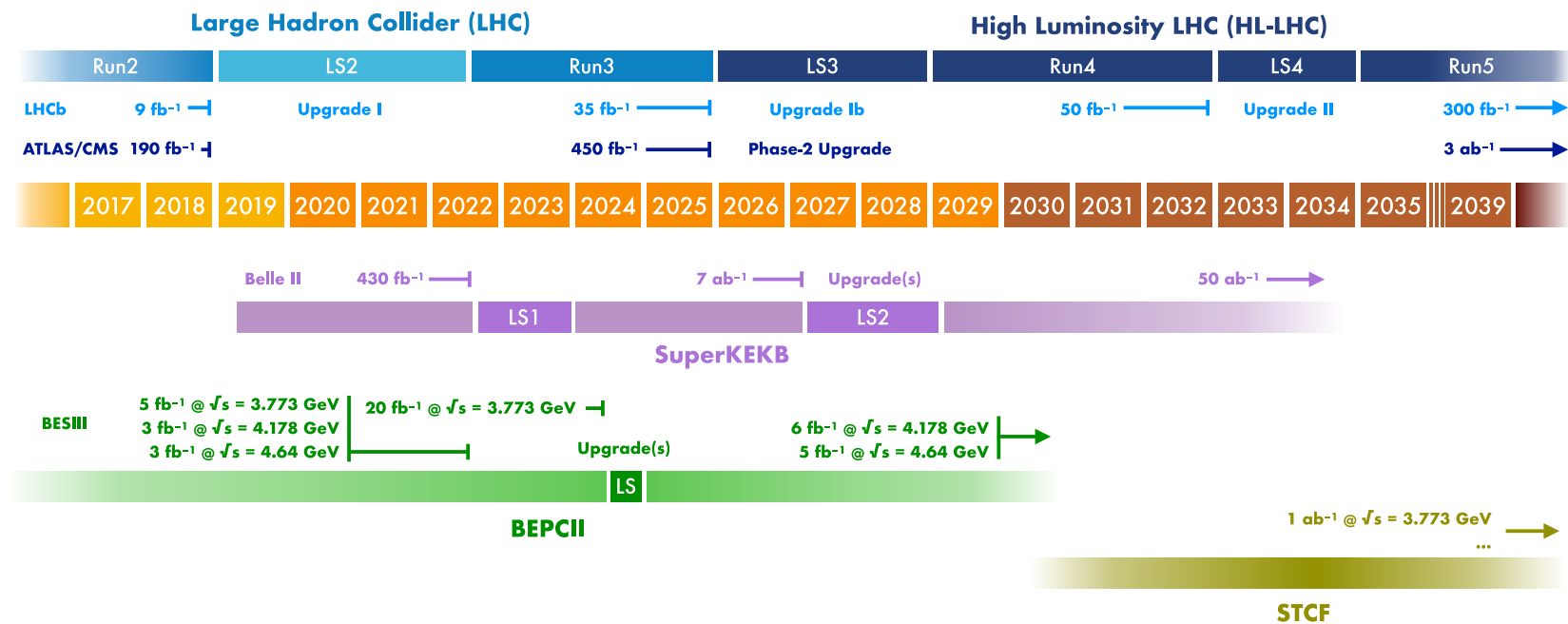
CLFV not always as clean as we think!

Backup

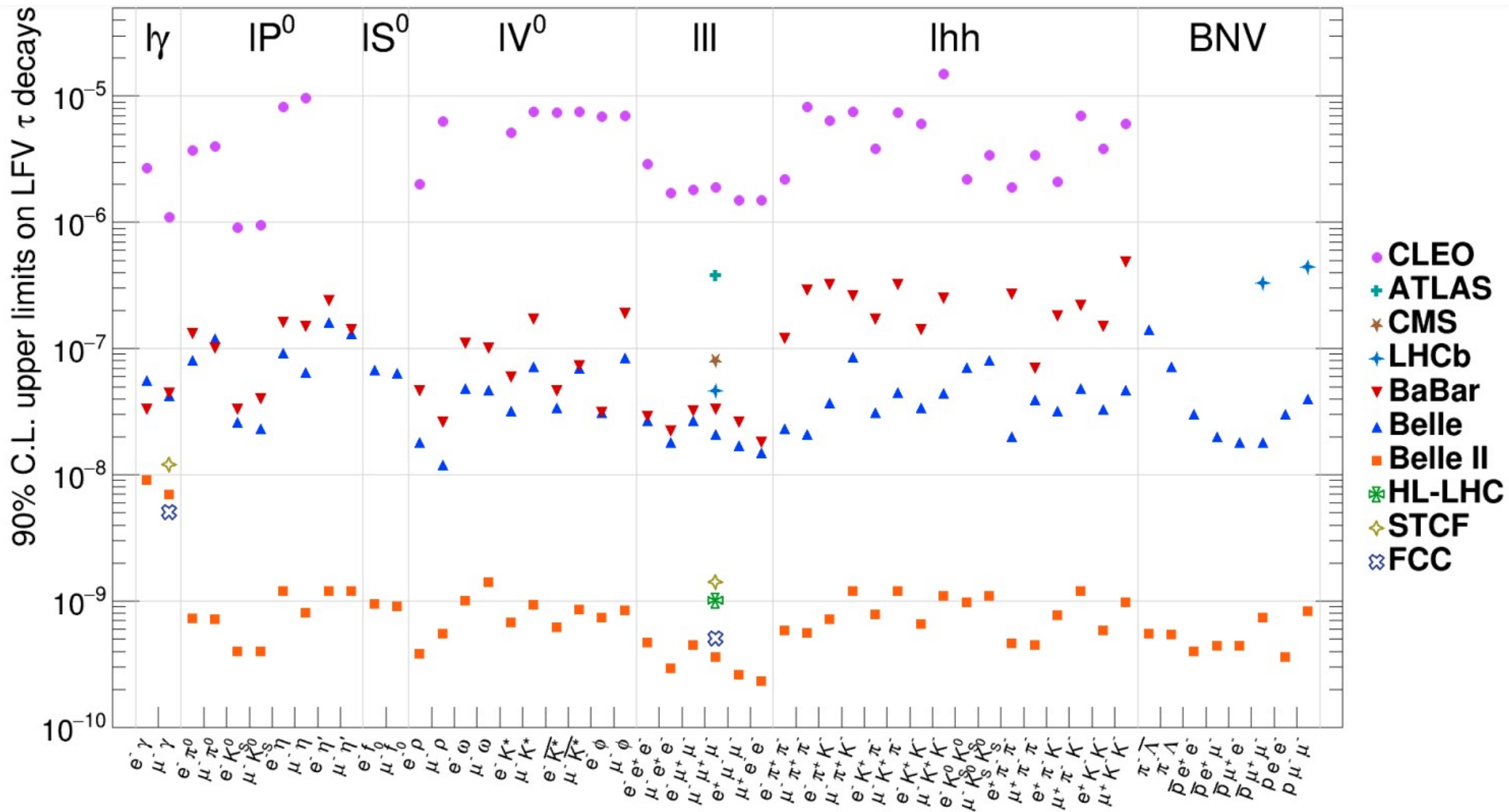
LFV Timelines



[Artuso, Bernstein, Petrov, *Snowmass 2021 Rare and Precision Frontier Report*, 2210.04765]



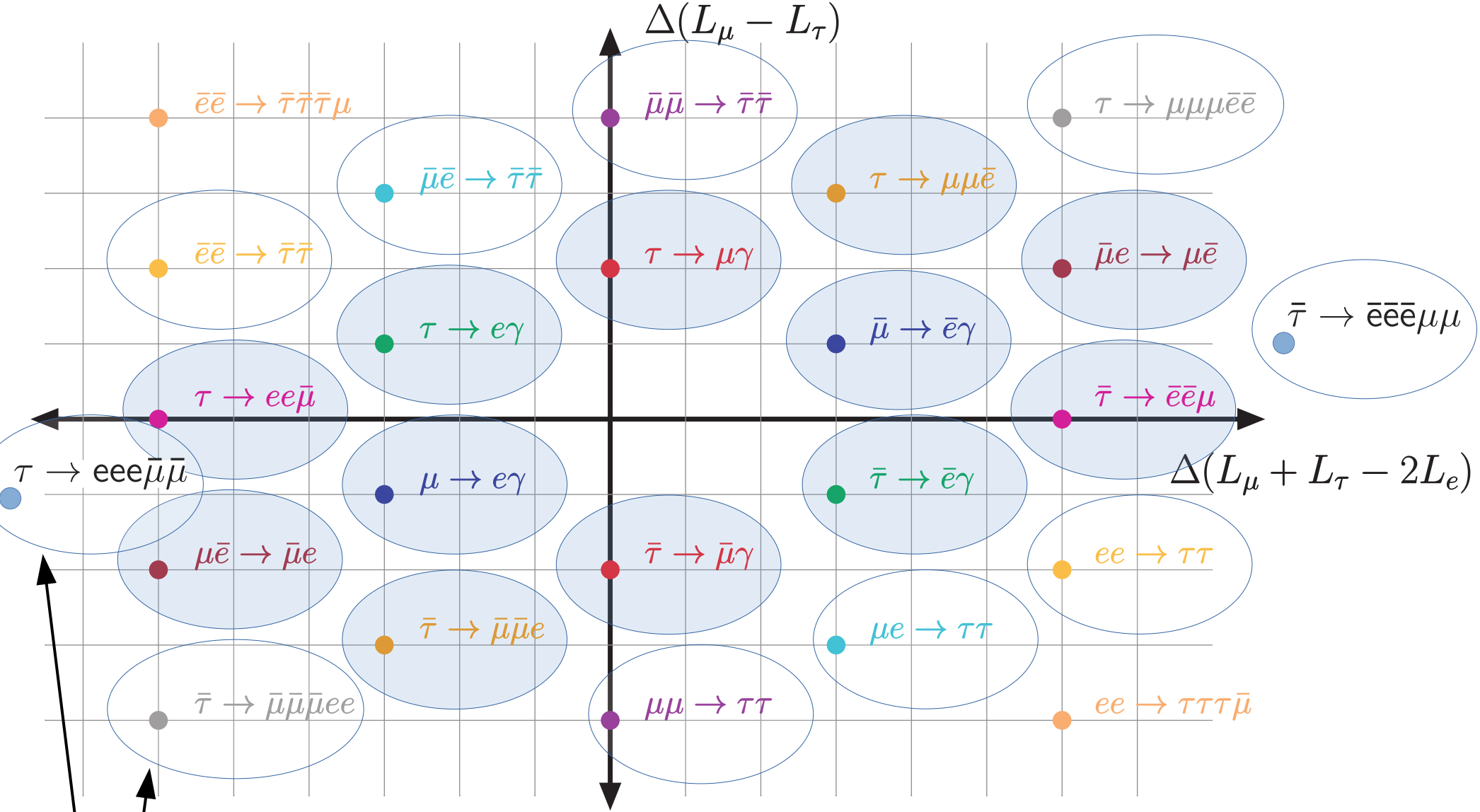
LFV in tau sector



[Banerjee++, 2203.14919]

Currently being probed: Future:

[JH, 1610.07623]

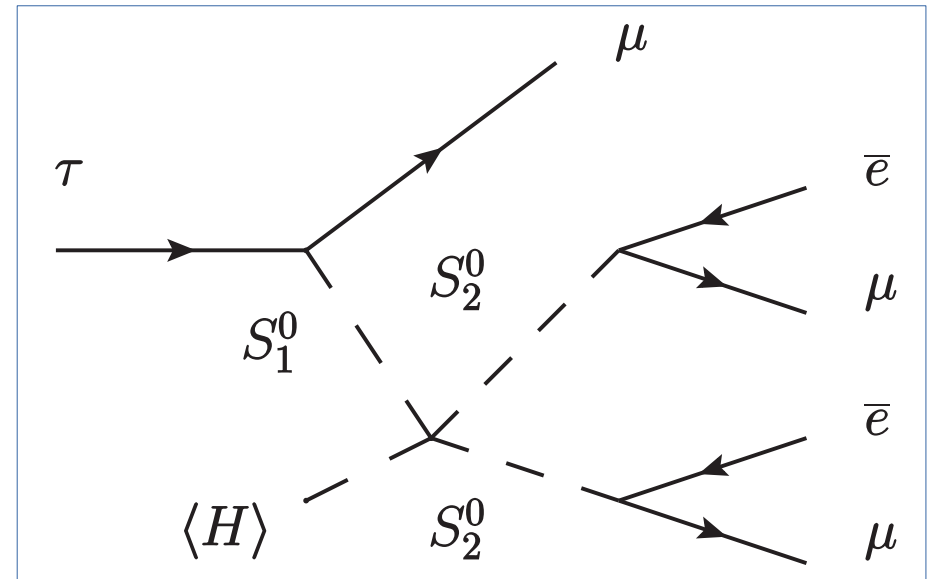


Belle II, but d=10 operator!

$\tau^- \rightarrow \mu^- \mu^- \mu^- e^+ e^+$?

- Impose $L_\mu + 4L_e - 5L_\tau$ to forbid all other LFV.
- Experimentally clean, but rate is suppressed:

$$\text{BR} \sim 10^{-10} \left(\frac{30 \text{ GeV}}{m_S} \right)^{12} .$$



- Secretly **dimension 10** operator.
- Better constraints on S from $Z \rightarrow SS$ etc.? [JH, M. Sokhashvili, Thapa, in progress]

Requires full models