CP-Violating Sources for Electroweak Baryogenesis

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Europe/Berlin timezone

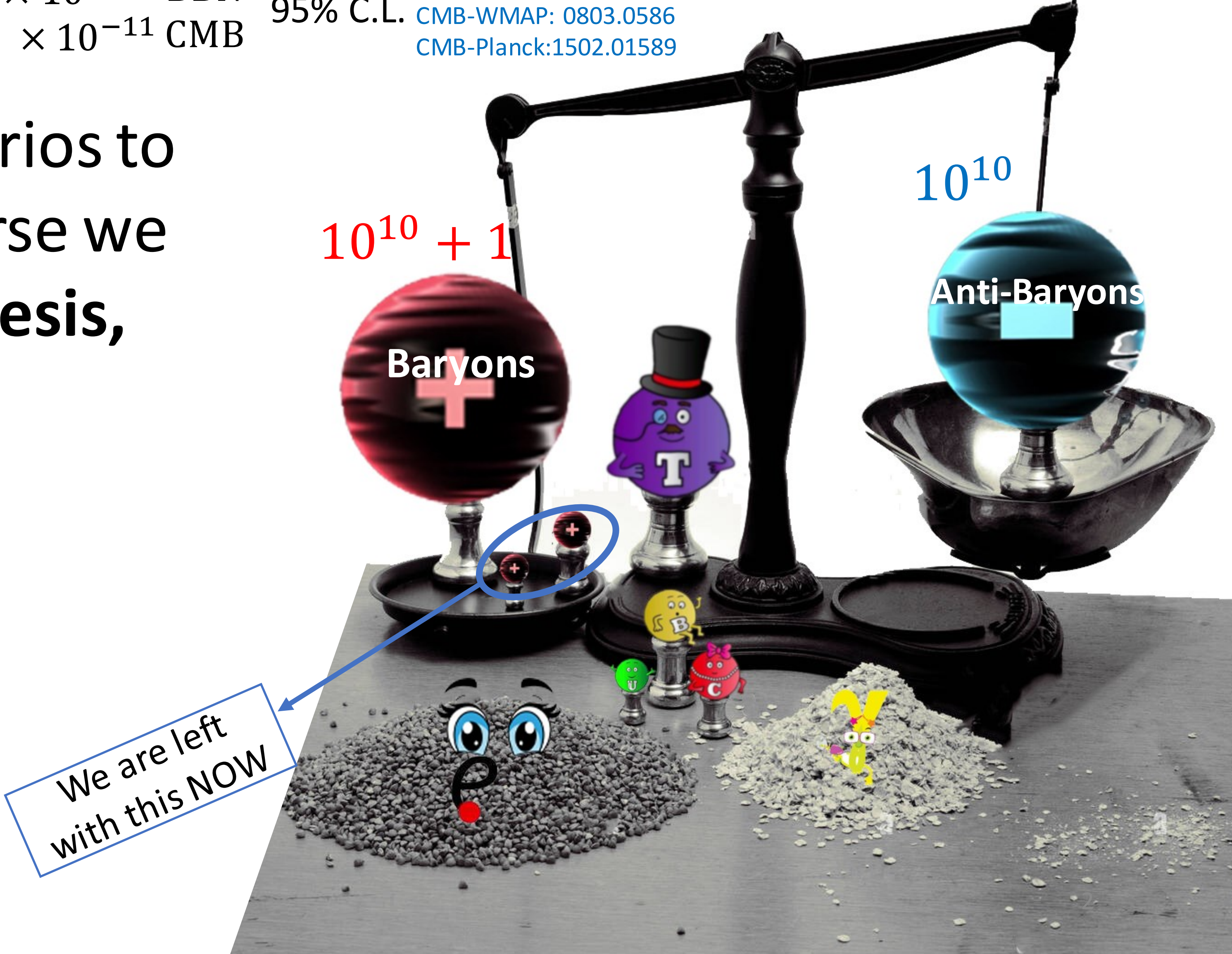
One More Open Question:

Matter-Anti-Matter Asymmetry

$$\begin{matrix} 6.7\times10^{-11} \\ 8.36\times10^{-11} \end{matrix} \leq Y_B = \frac{n_b - n_{\bar{b}}}{s} \leq \begin{matrix} 9.2\times10^{-11} & \text{BBN} \\ 9.32\ \times10^{-11} & \text{CMB} \end{matrix} \quad \text{95\% C.L.}$$

BBN: J. Phys. G 33 (2006) 1. (PDG)
CMB-WMAP: 0803.0586
CMB-Planck:1502.01589

Among many different scenarios to explain the BAU in the universe we study **Electroweak Baryogenesis,** but why we choose EWBG?



Today's Goals

- Compare three methods for calculating CP-violating sources for Electroweak Baryogenesis.
- Identify whether the sources induced by fermions mixing can be computed with different methods (spinor decomposition vs VEV-insertion approximation).
- Provide **a ready to plug** equation of the CP-violating source to be used by Phenomenologists in their favourite model.

To do that we:

- Review Methods of CP-violating sources for EWBG.
- Present Modified dispersion relation are necessary for self-consistent calculations.
- Apply these onto a the case of mixing of 2 fermionic flavours.

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Sakharov Conditions for EWBG:

- **Baryon Number violation:** Sphaleron process.
- **C and CP violation:** CP-odd phase in the mass matrix.
- **Departure from Thermodynamic equilibrium:** first-order electroweak phase transition.

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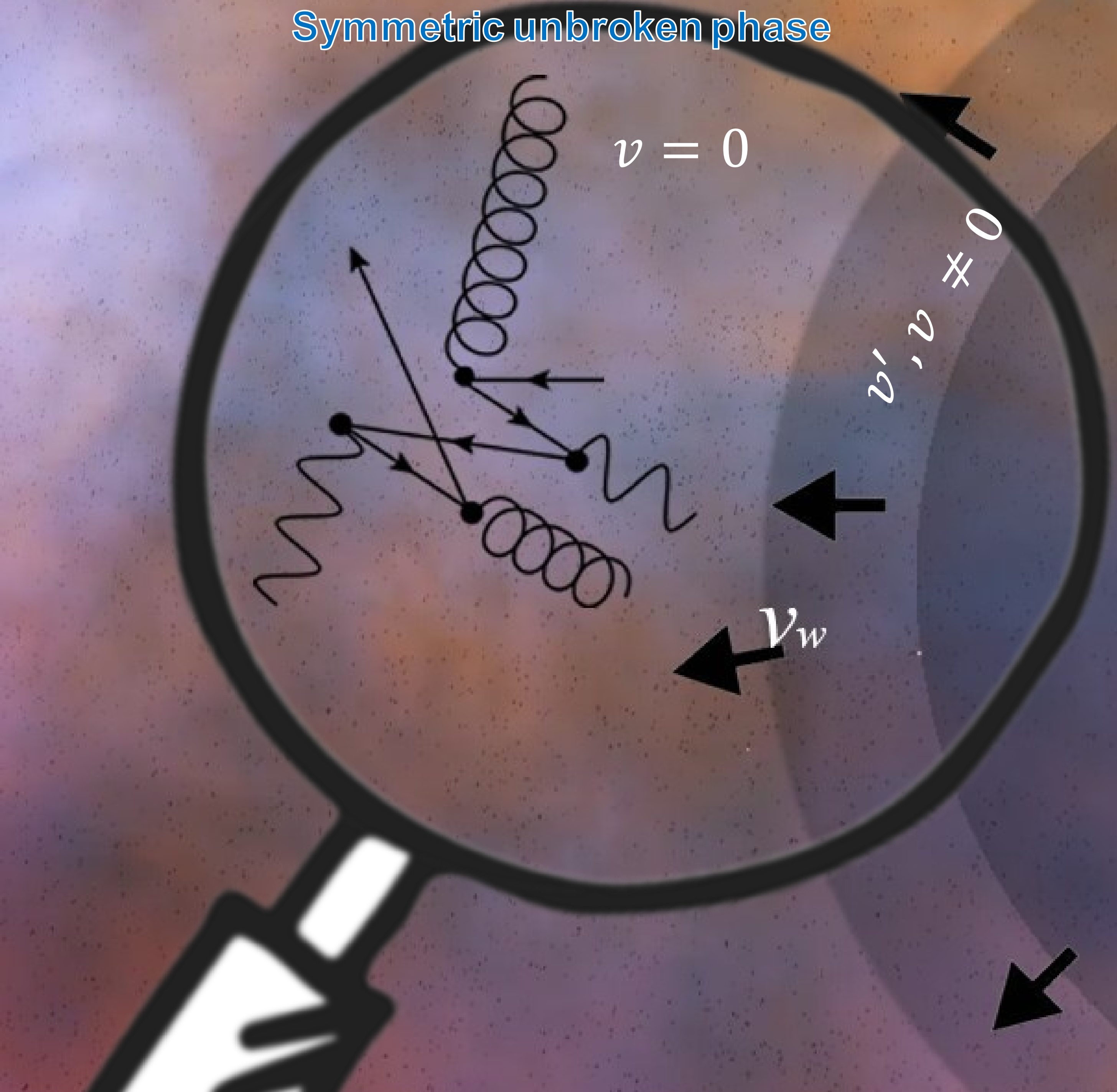
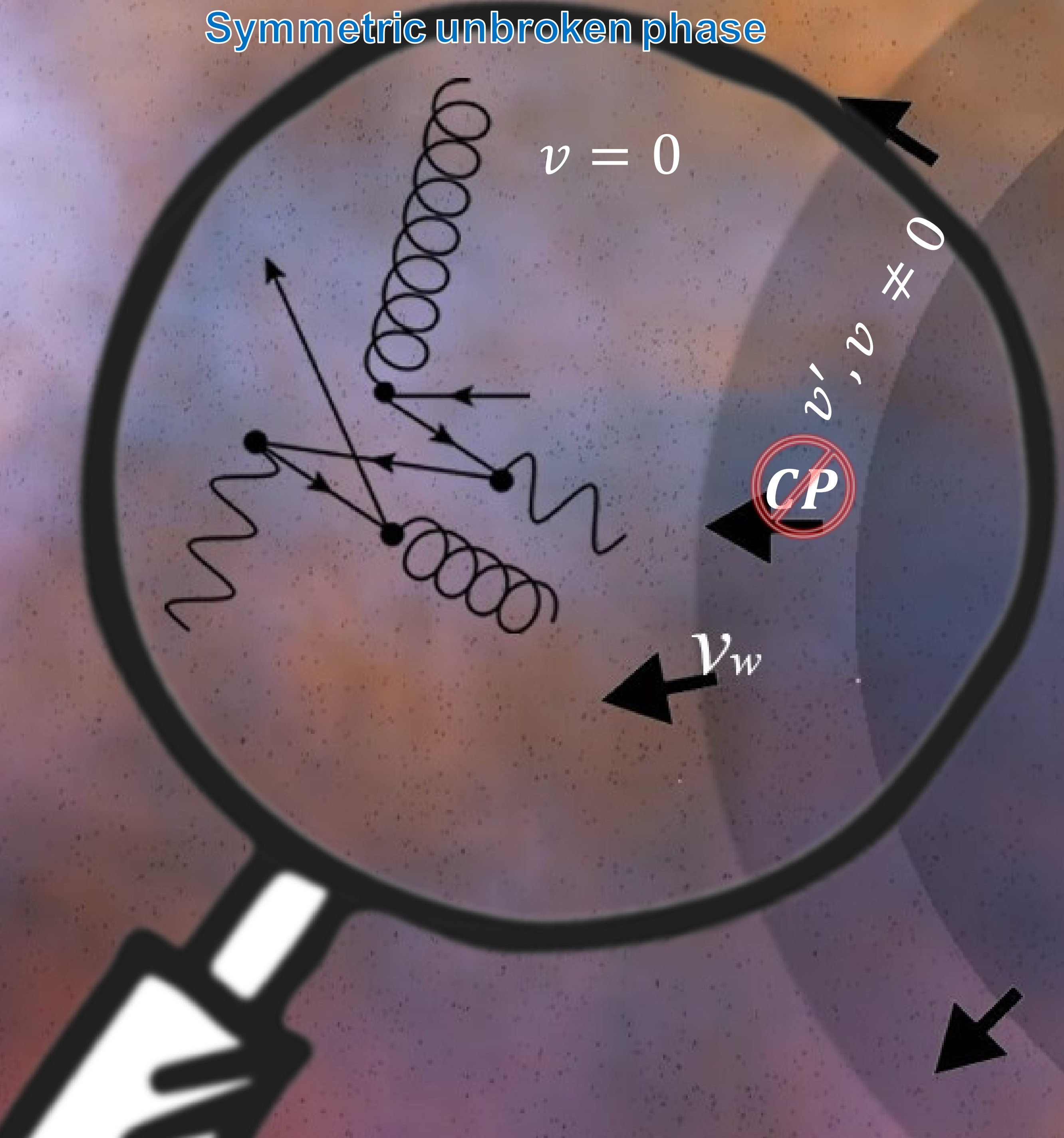


Scenario:

- Electroweak symmetry breaking.
- Injection of CP violation into the symmetric phase.
- Conversion of left handed fermion number to Baryon number through the sphaleron process

Broken phase



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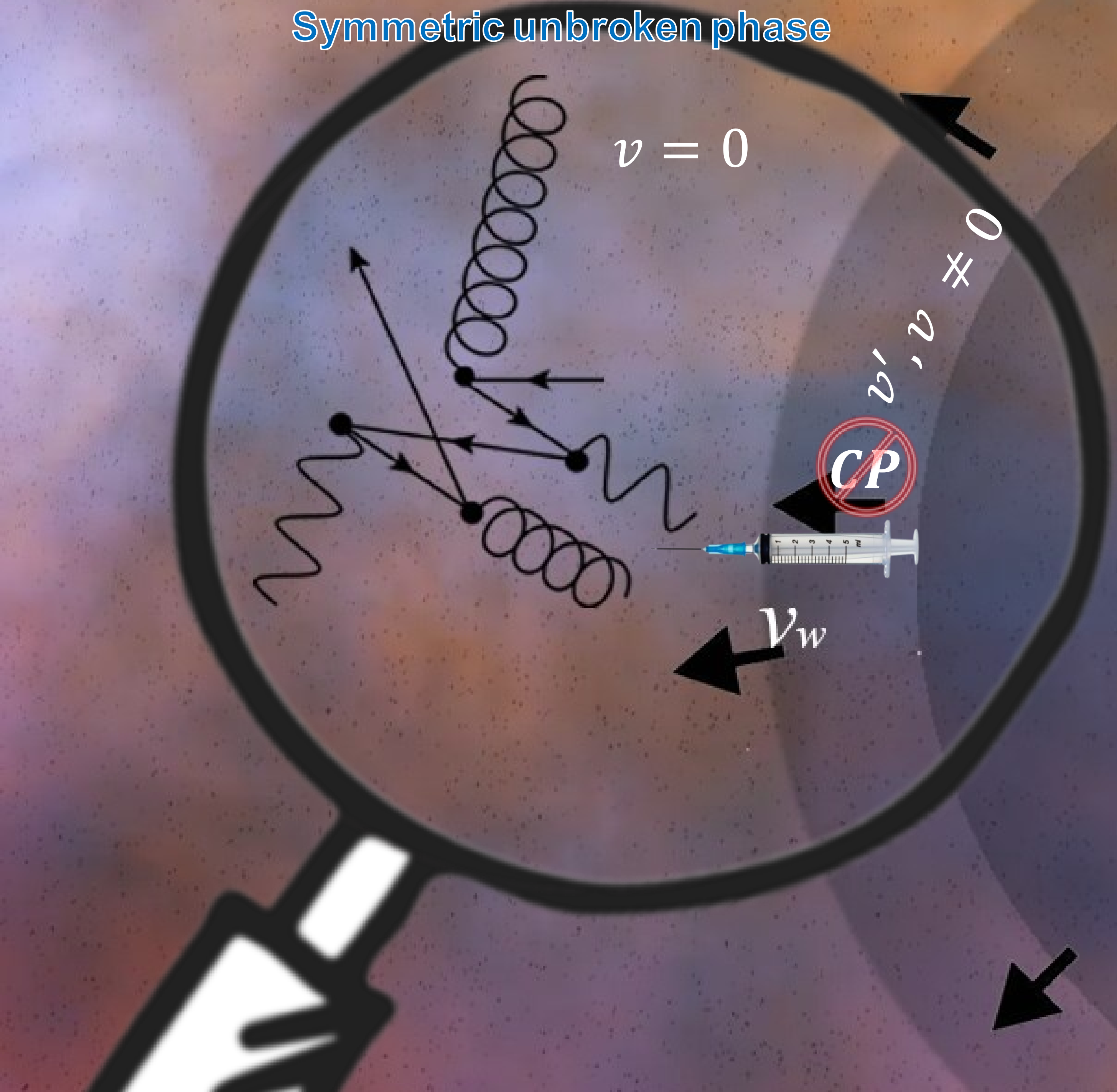

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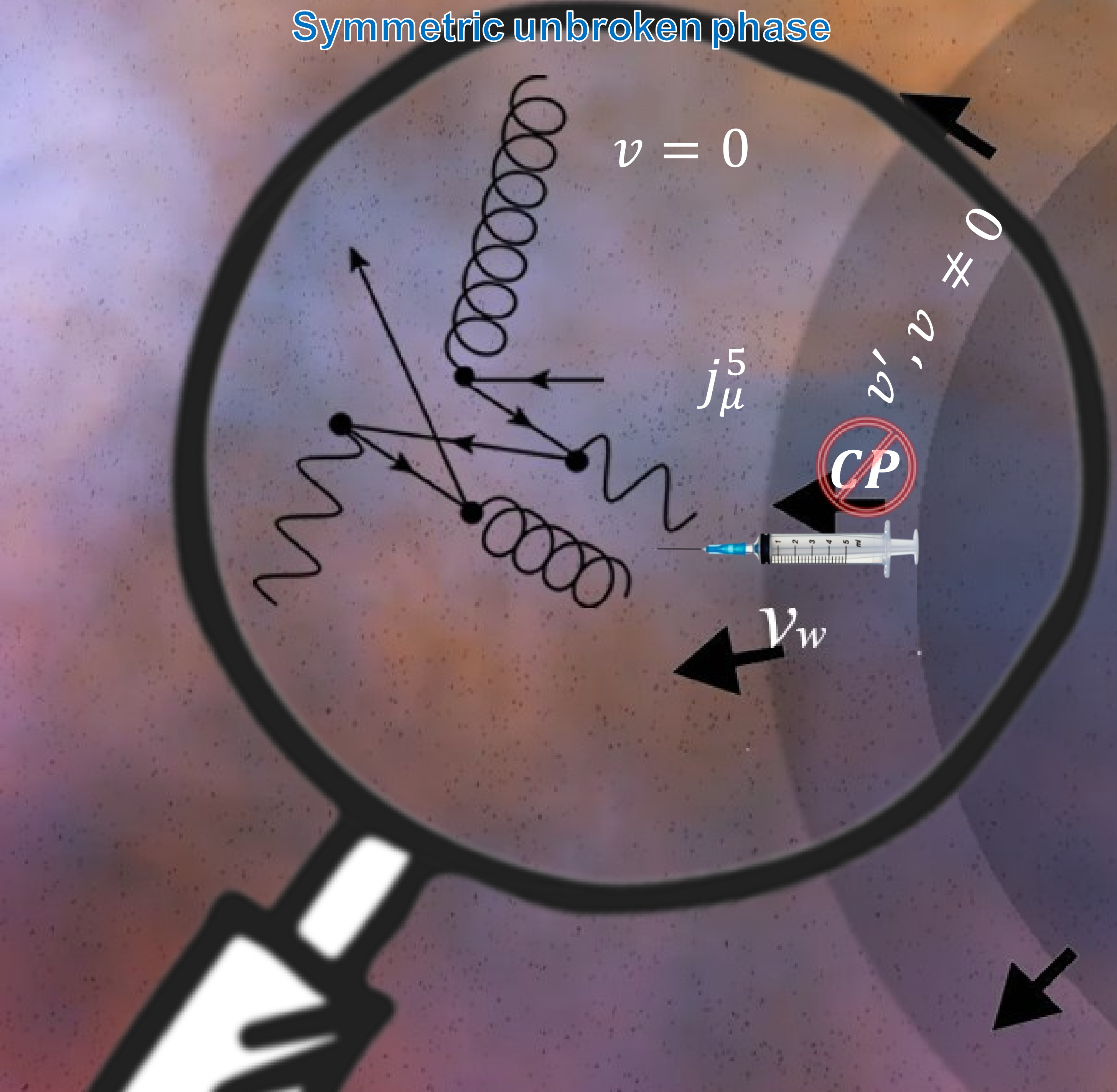

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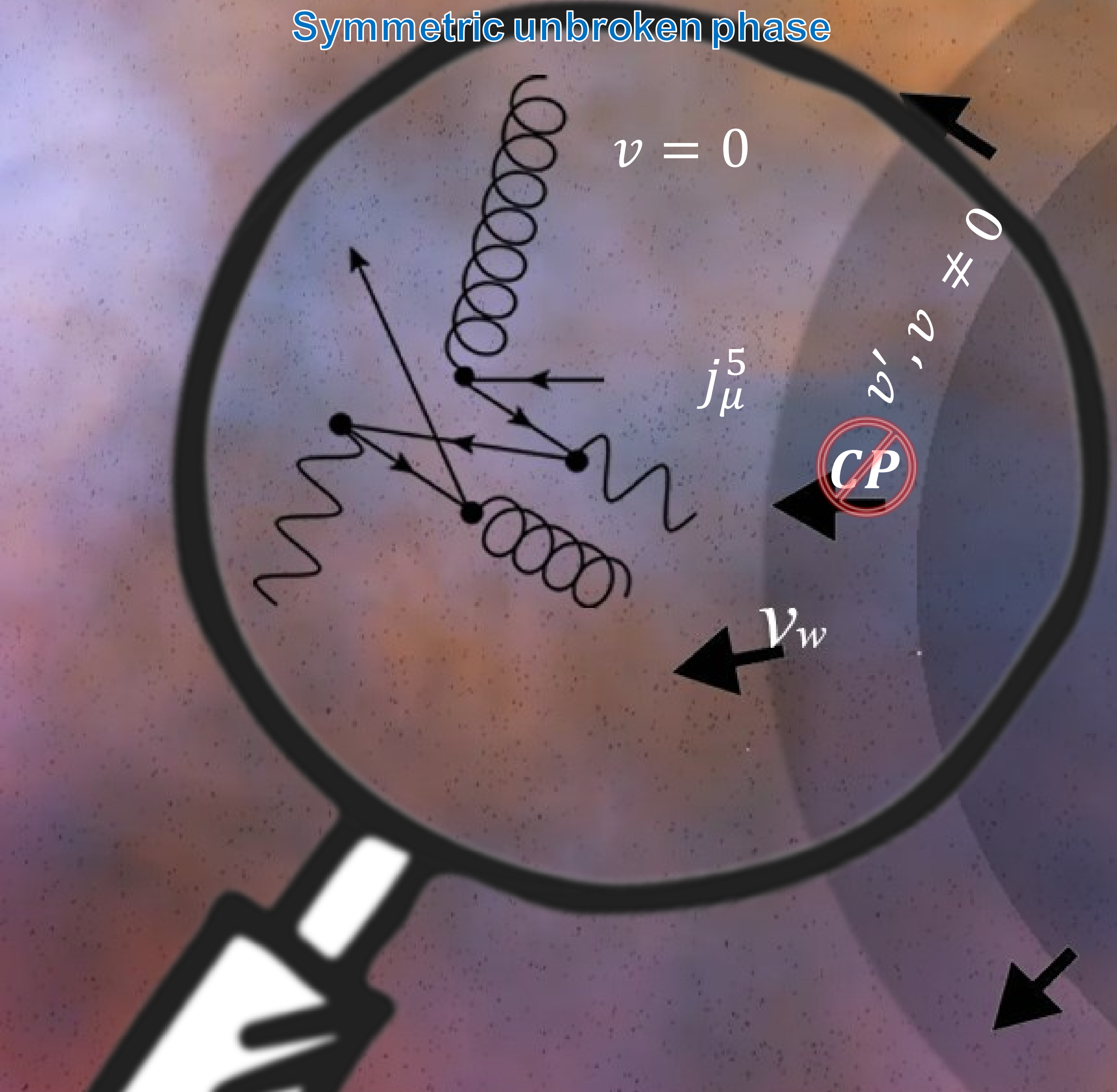

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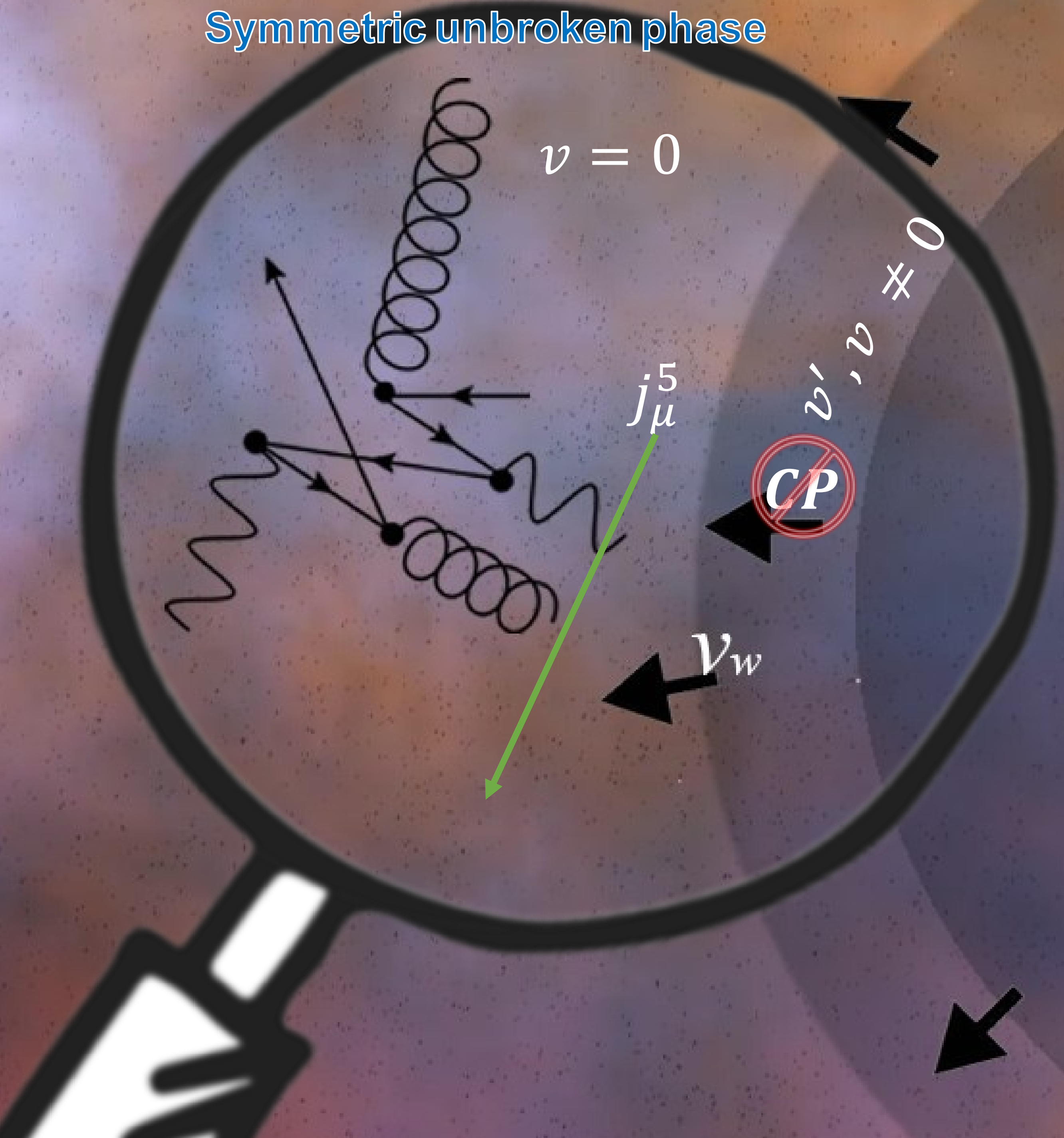

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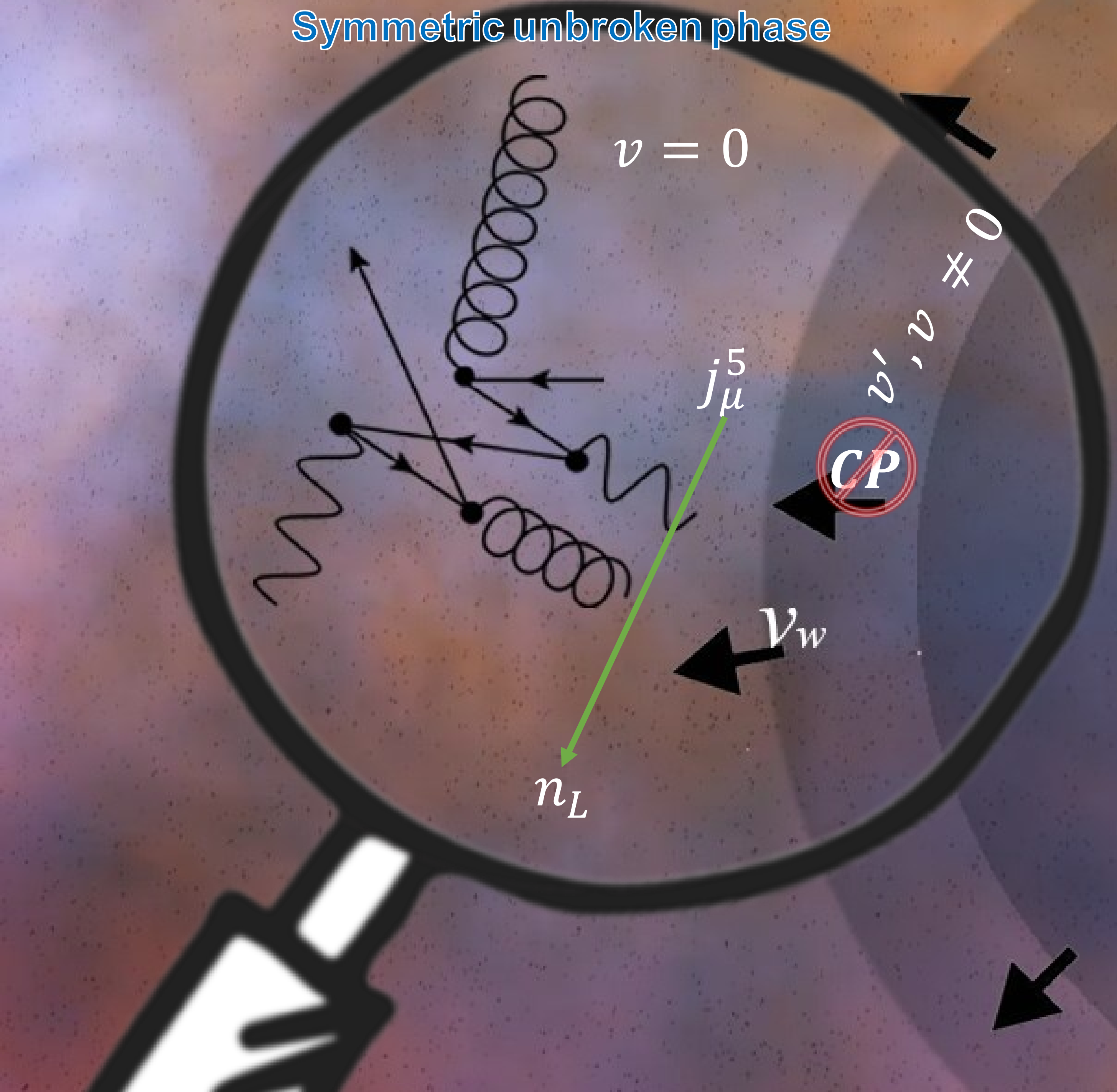

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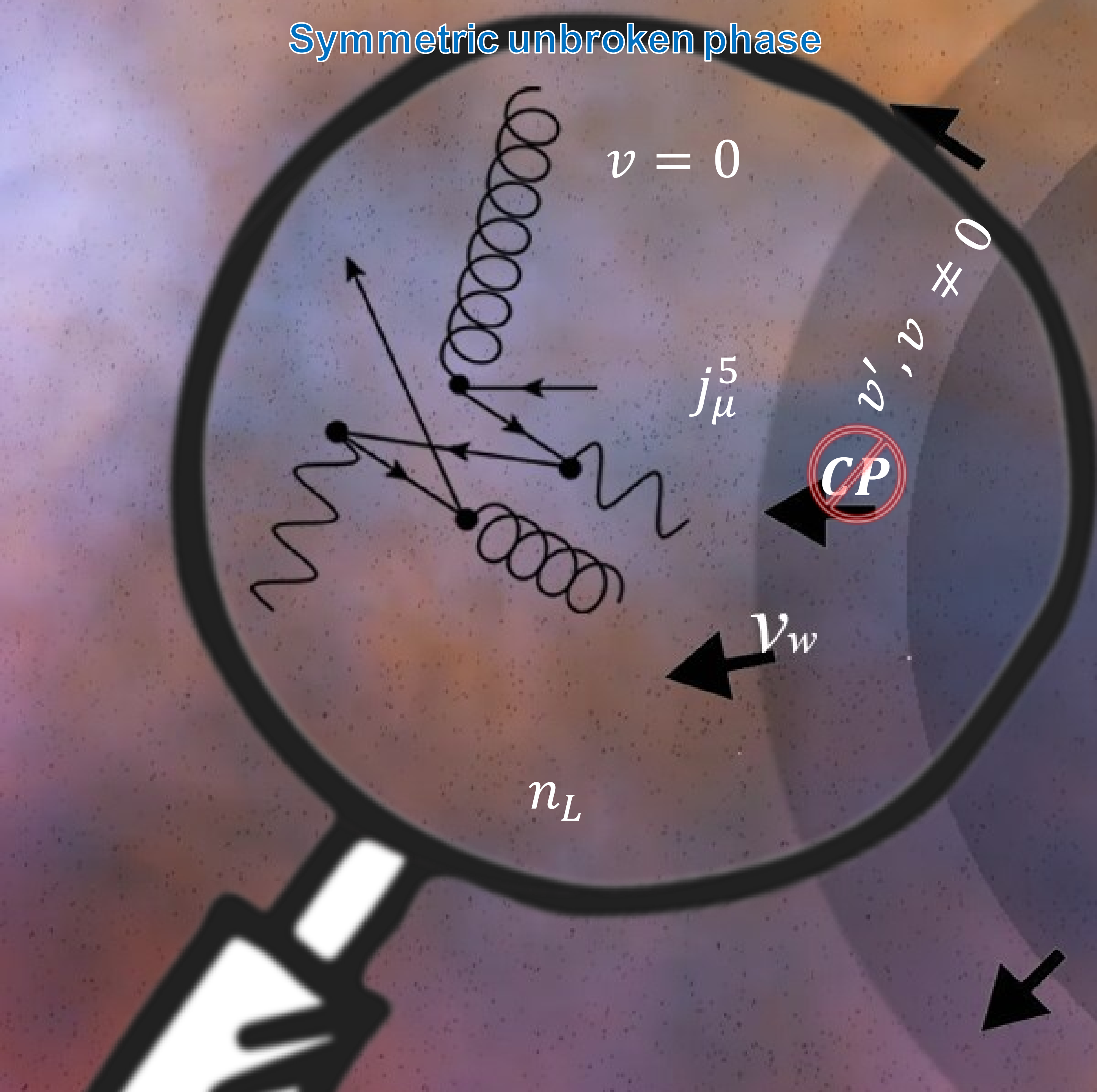

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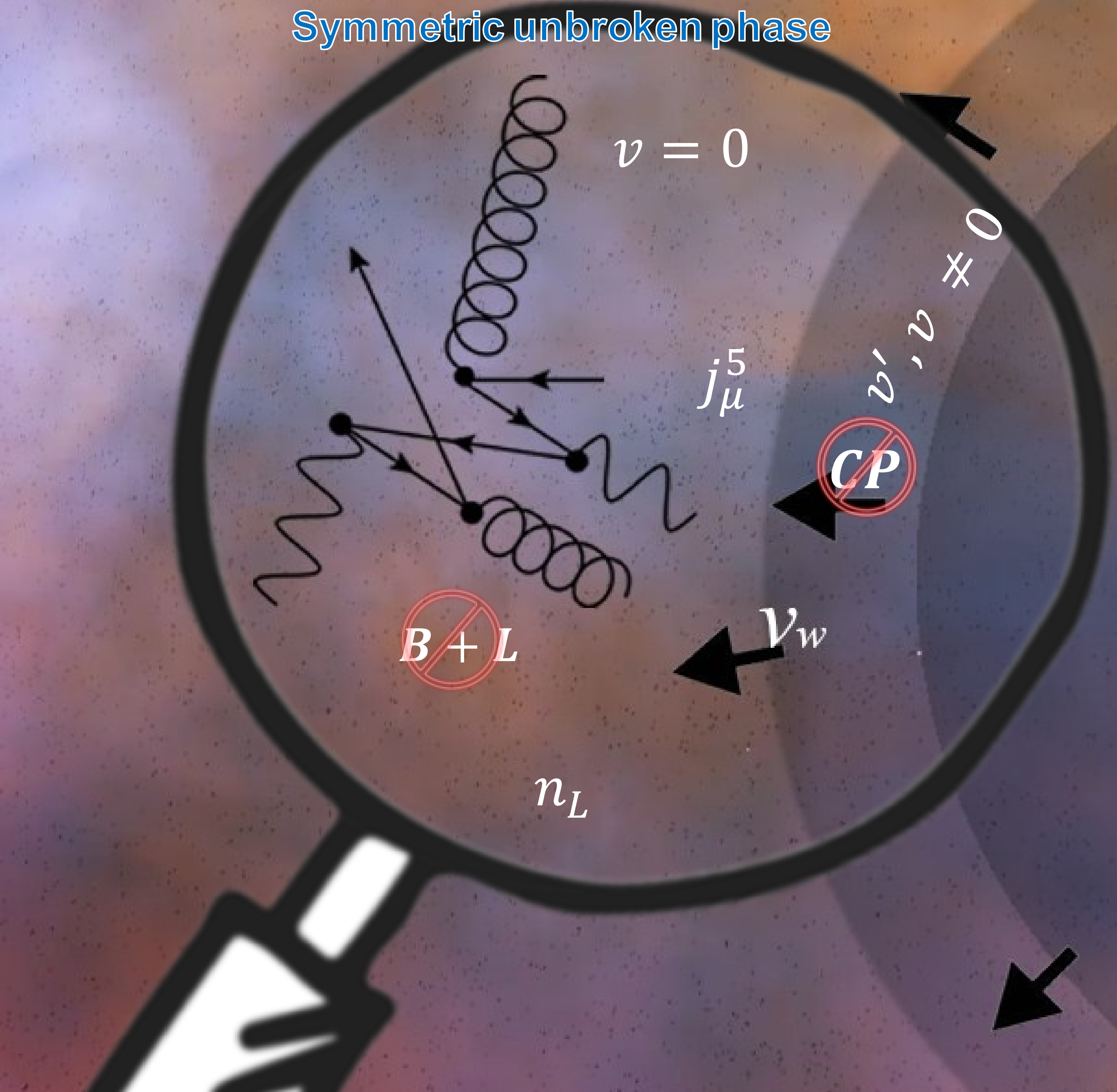

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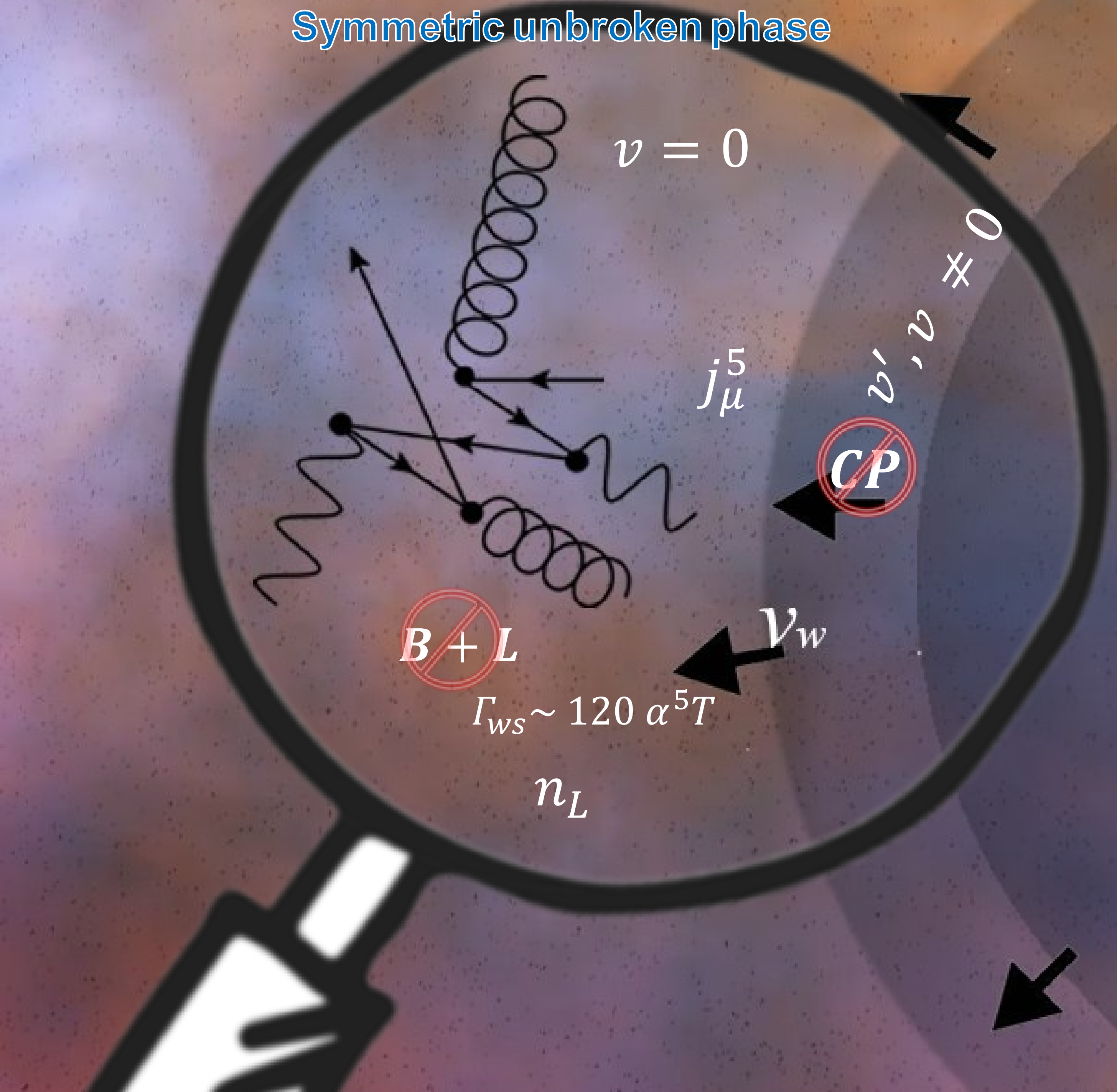

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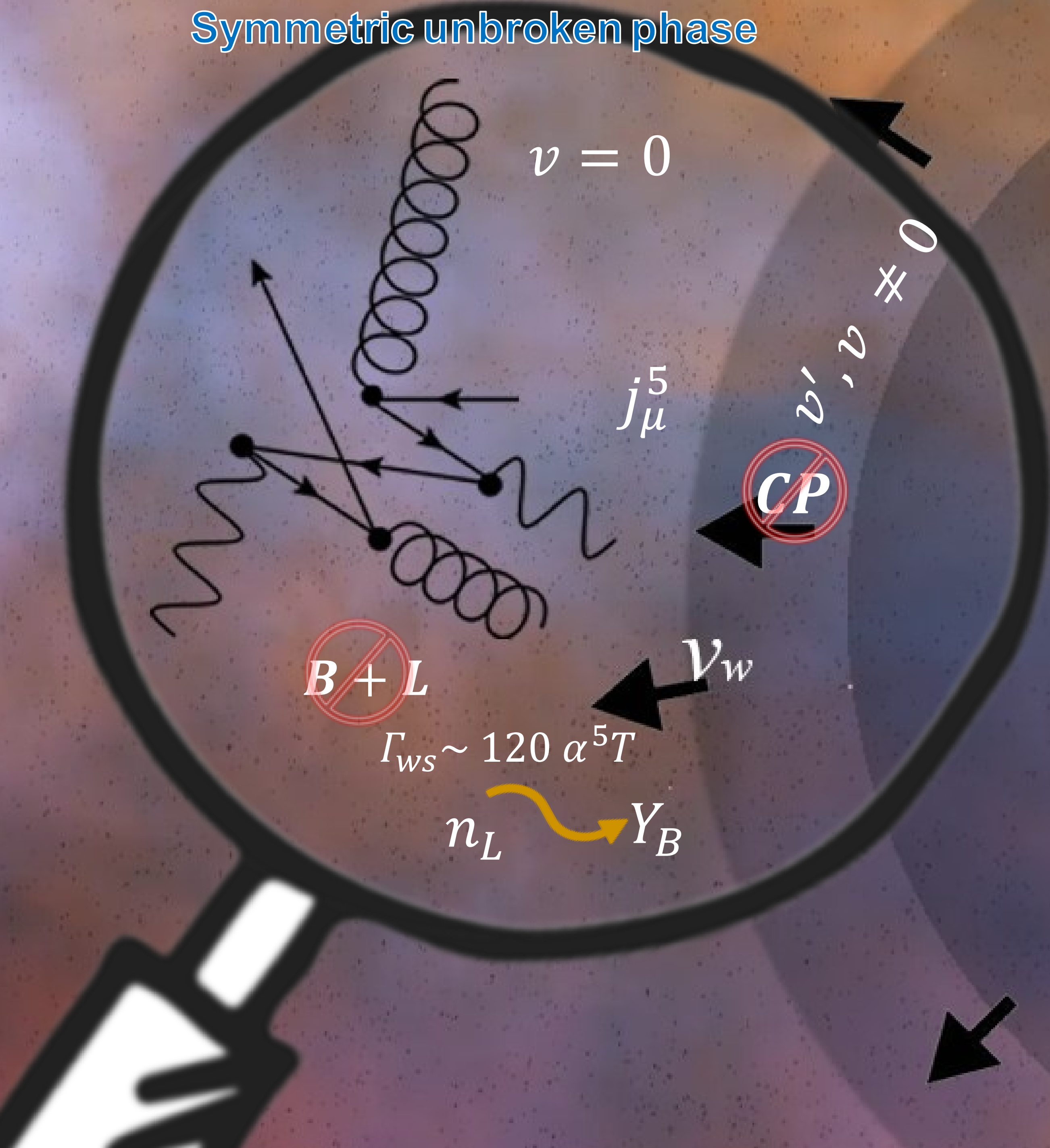

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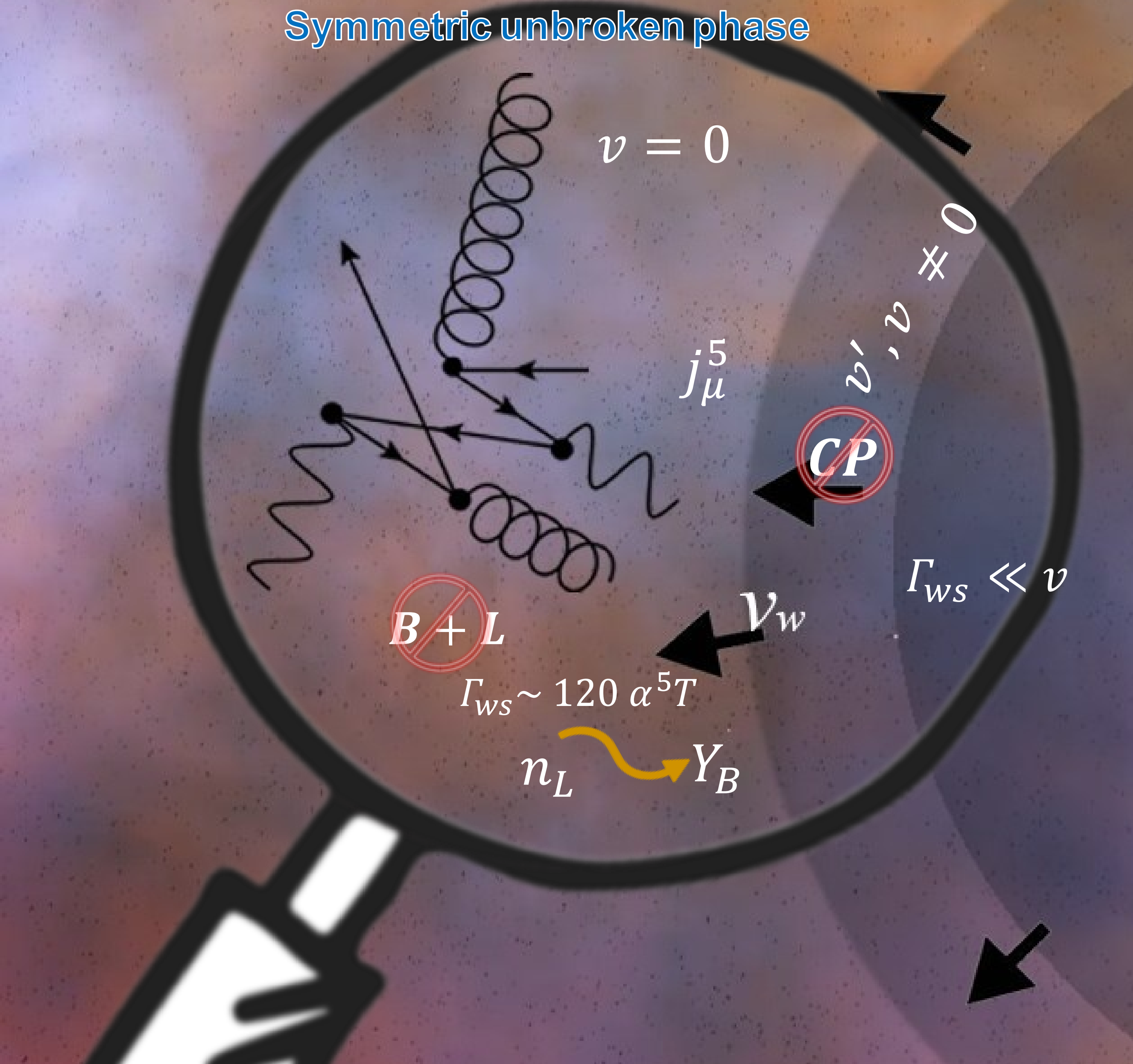

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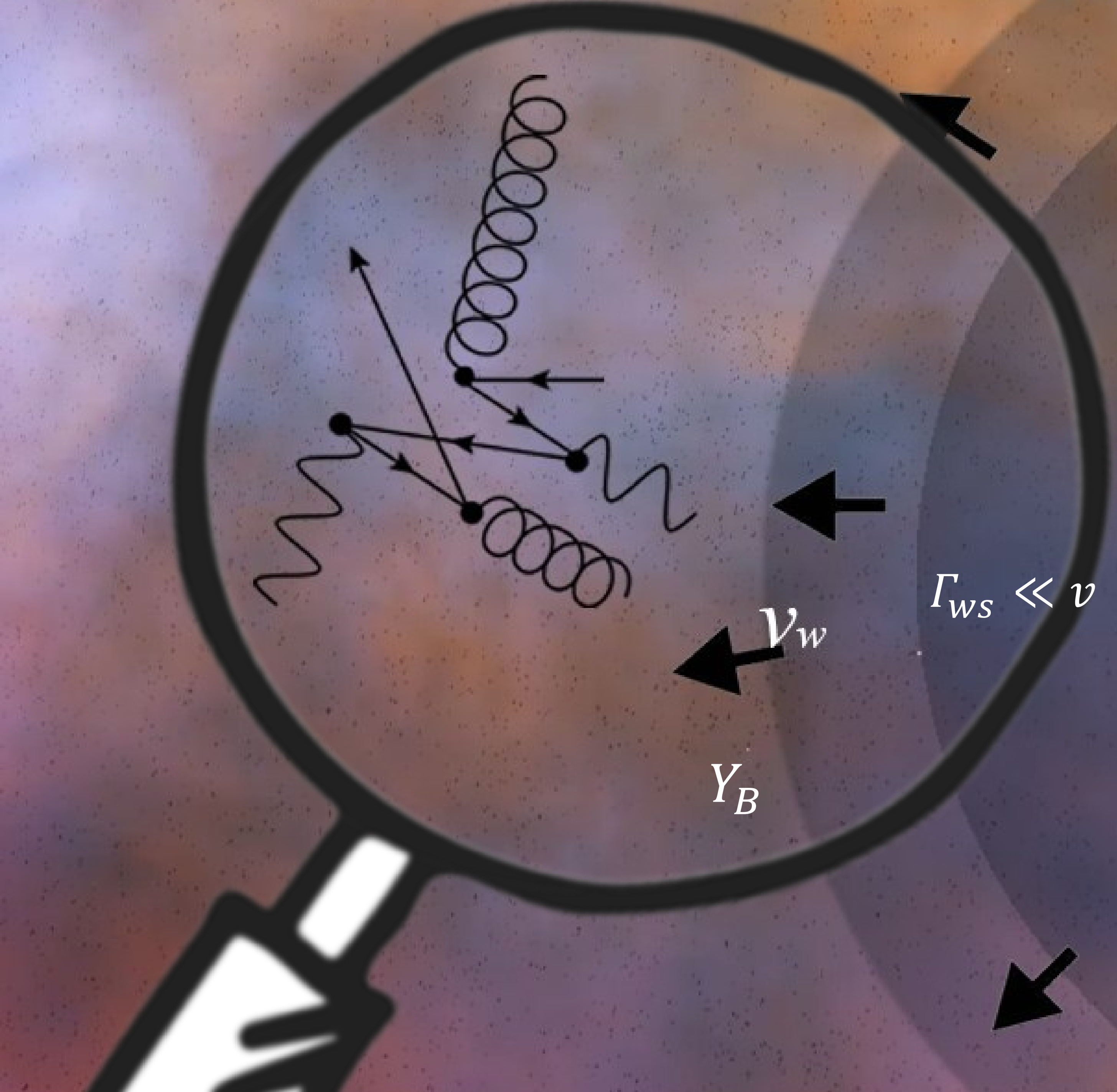

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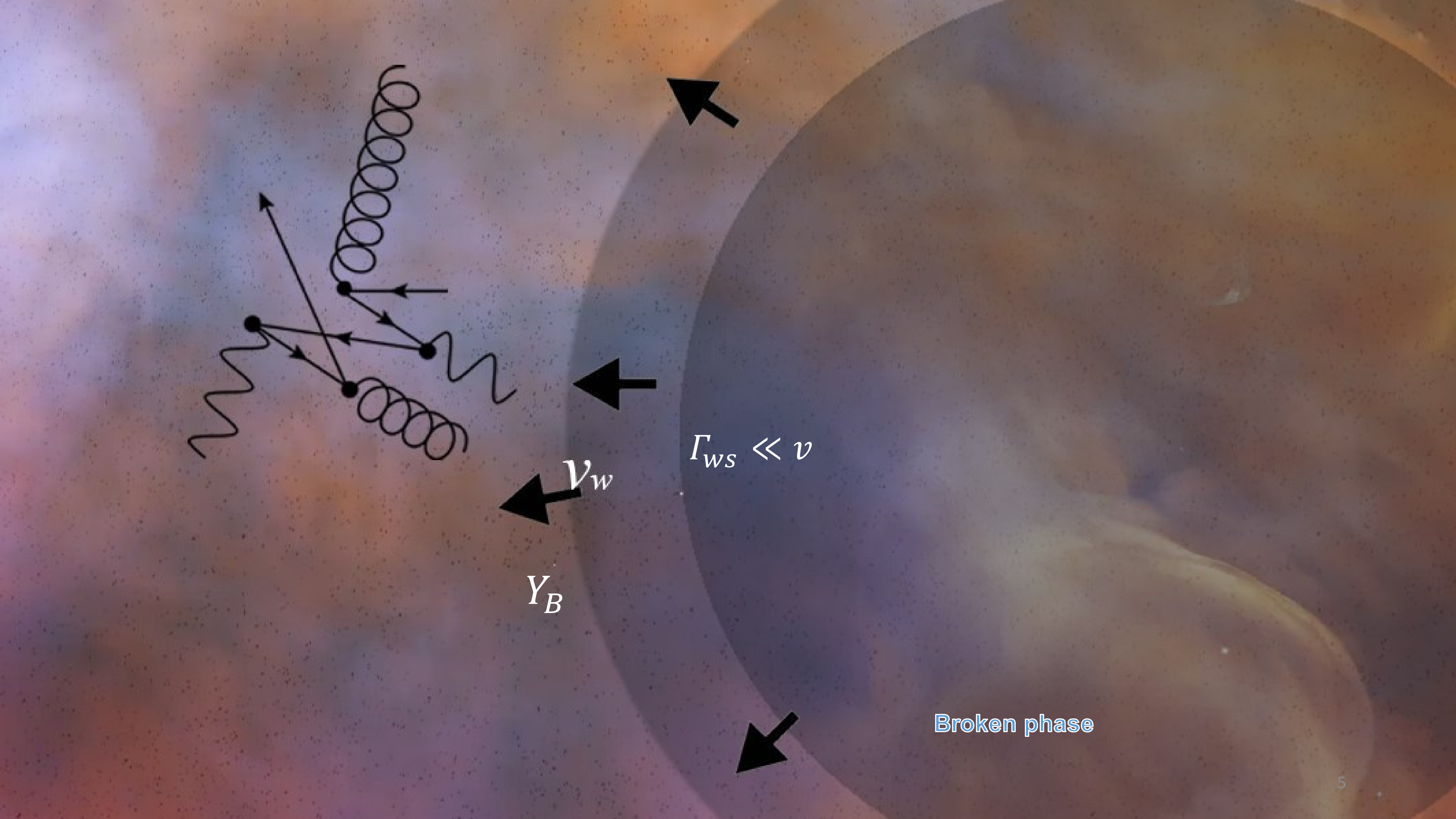

$\Gamma_{ws} \ll v$
v_w
Y_B
Broken phase


$\Gamma_{ws} \ll v$
Y_B
v
Broken phase

v

$\Gamma_{ws} \ll v$

Y_B 9.2×10^{-11}

Broken phase

Part 1 Methods

WKB Method

[Joyce, Prokopec, Turok]
[Cline, Joyce, Kainulainen, Prokopec]

- Solve Dirac eq. with space-time mass with the WKB approximation

$$\left(\mathrm{i}\partial\!\!\!/ - M^{\mathrm{H}}(z) - \mathrm{i}\gamma^5 M^{\mathrm{A}}(z)\right)\Psi = 0 \quad \Psi_s \equiv e^{-\mathrm{i}\omega t}\begin{pmatrix} L_s \\ R_s \end{pmatrix} \otimes \xi_s, \quad \sigma_3\,\xi_s \equiv s\,\xi_s, \quad , s = \pm.$$

dispersion relations

$$L_s \equiv \omega \exp\left[\mathrm{i}\int_z k_s(z')dz'\right].$$

- Dirac equation fixes $\boldsymbol{k_s}$ in terms of $\boldsymbol{\omega}$

$$\omega_s = \omega_0 \mp s\frac{\theta'}{4\,\omega_0}$$

- Using Canonical EOM:

$$v_g = \frac{\partial\omega_s}{\partial k_s} \qquad \dot{k}_s = -\frac{\partial\omega_s}{\partial x}$$

- We get

$$F_s = \dot{k}_s = \omega_s\,\dot{v}_g = -\frac{m_i\,m_i'}{\omega_s} \pm s\frac{(m_i^2\theta')'}{2\omega_0^2},$$

Fluid equations in the WKB approximation

- Boltzmann equations assumed to be of the form

$$(\partial_t + \mathbf{v}_g \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}}) f_i = C[f_i, f_j, \ldots].$$

- **Particles** and **antiparticles** are treated separately, resulting in different forces that depend on spin: CP violation.
- Taking moments of Boltzmann equations one derives fluid / diffusion equations for the particle asymmetries of different species

Getting the modified dispersion relations is **Simple**, but writing the Boltzmann is not from 1st principles.

The closed time path formalism

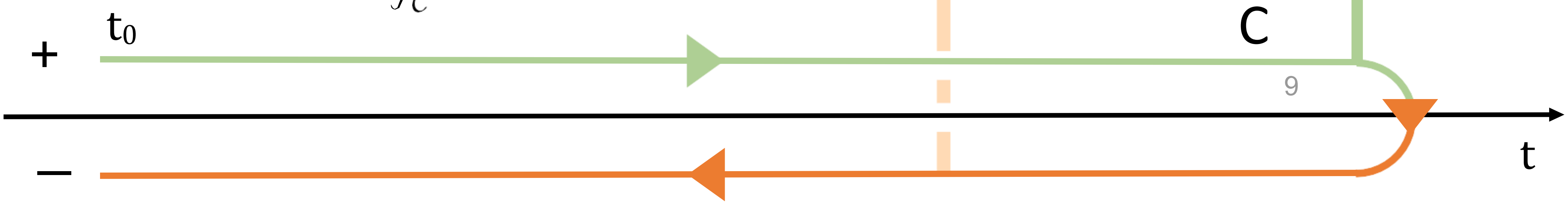
- **Time-dependent observables** in QFT can be related to a **path integration** over a **closed time path**

$$\langle S(t)|\hat{O}(t)|S(t)\rangle = \int \mathcal{D}\phi(t) \langle S(t_0)|e^{-i\hat{H}(t-t_0)}\hat{O}(t)|\phi\rangle_t \, {}_t\langle\phi|e^{i\hat{H}(t-t_0)}|S(t_0)\rangle .$$

Path integral from t to t_0

Path integral from t_0 to t

$$= \int_{\mathcal{C}} \mathcal{D}\phi e^{\mathrm{i}S[\phi]}$$



CTP Propagators

- **Propagators** carry indices a,b= ± from the time branches of the field insertions

$$iS_{ab}(x,y) = \langle T_{\mathcal{C}}\psi_a(x)\bar{\psi}_b(y)\rangle \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)} iS_{ab}\left(k, \frac{x+y}{2}\right)$$

Wigner transf.

- Contain **info** about the **shell** and **number densities** of propagating d.o.f.s

$$iS^{+-}_{\text{free}}(k) \equiv iS^{<}_{\text{free}}(k) = -2\pi\delta(k^2-m^2)(\not{k}+m)\left[\theta(k^0)f(\mathbf{k}) - \theta(-k^0)(1-\bar{f}(-\mathbf{k})\right]$$

- They satisfy quantum equations of motion: **Schwinger-Dyson** eqs. in contour *C*

$$\left[i\not{\partial} - M^{\mathrm{H}} - i\gamma^5 M^{\mathrm{A}}\right] iS^{ab}(x,y) = a\delta_{ab} i\delta^4(x-y) + \sum_c c\int d^4z\, \Sigma^{ac}(x,z) iS^{cb}(z,y)$$

Self-energy (1PI)

This leads to **Boltzmann / fluid equations from first principles**!

and for in the collision-less limit:

$$\left(\not{k} + \frac{i}{2}\not{\partial} - M^{\mathrm{H}}(z)e^{-i\diamond} - i\gamma^5 M^{\mathrm{A}}(z)e^{-i\diamond}\right) S^{<}(k;z) = 0,$$

After Wigner transf.

$$\diamond = \frac{1}{2}\partial_k^{S_2}\partial_X^{S_1} - \frac{1}{2}\partial_k^{S_1}\partial_X^{S_2} + \frac{1}{2}\partial_X^{M_1}(\partial_k^{S_1}+\partial_k^{S_2}) - \frac{1}{2}\partial_X^{M_2}(\partial_k^{S_1}-\partial_k^{S_2}).$$

CTP: VEV insertion approximation

- Consider **Schwinger-Dyson equation without a 1PI self-energy** and $m = m_0 + \delta m(x)$

$$\left(i\partial\!\!\!/ - m_0 - \delta m(x)\right) iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

- Start with **solutions** iS_0^{ab} to the **homogeneous** case $m = m_0$

$$\left(i\partial\!\!\!/ - m_0\right) iS_0^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

- The first equation is solved then by a **geometric series** [Postma]

$$S^{ab}(x,y) = S_0^{ab}(x,y) + \int d^4z\, c\, S_0^{ac}(x,z)\, \delta M(z)\, S_0^{cb}(z,y)$$
$$+ \int d^4z\, d^4w\, cd\; S_0^{ac}(x,z)\, \delta M(z)\, S_0^{cd}(z,w)\, \delta M(w)\, S_0^{db}(w,y) + \cdots$$

CTP: VEV insertion approximation

- **$\delta m(x)$ effects** can be **absorbed** into a "**self-energy-like**" contribution in eq. for S^{ab}

$$\left(i\partial\!\!\!/ - m_0 + \delta m(x)\right) iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

$$\updownarrow$$

$$(i\partial\!\!\!/ - m)S^{ab}(x,y) = a\delta^{ab}\delta^4(x-y) + \int d^4z\, c\, \delta\Sigma_0^{ac}(x,z) S^{cb}(z,y),$$

$$\delta\Sigma_0^{ab}(x,y) = a\delta^{ab}\,\delta M\delta(x-y) + \delta M(x) S_0^{ab}(x,y)\,\delta M(y) + \int d^4z\, d\,\delta M(x) S_0^{ad}(x,z)\,\delta M(z)\, S_0^{db}(z,y)\,\delta M(y) + \ldots$$

- Schwinger Dyson equations can be solved in terms of **powers of S_0, δM**
- $S_0 \propto \delta(k^0 - m_0^2)$ so the **full mass shell cannot be directly recovered**

CTP: Spin decomposition

[Kainulainen, Prokopec, Schmidt, Weinstock, Konstandin]

- For a **planar wall** in x,y directions, S_z **is conserved**
- Can **expand** propagators in **structures that commute with S_z**

$$iS_s^< = -\frac{1}{2}(\mathbf{1} + sS^z)\left[s\gamma^3\gamma^5 g_0^{s<} - s\gamma^3 g_3^{s<} + \mathbf{1} g_1^{s<} - i\gamma^5 g_2^{s,<}\right]$$

- **Schwinger-Dyson equations** solved in terms of functions g_i
- Combining eqs. → **algebraic constraints** that determine **modified mass shells**

$$(k^2 - m_{\rm eff}^2(x))g_i^<(x,k) = 0 \Rightarrow g_i^<(x,k) \propto \delta(k^2 - m_{\rm eff}^2(x))$$

$$\left(\not{k} + \frac{i}{2}\not{\partial} - M^{\rm H}(z)e^{-i\diamond} - i\gamma^5 M^{\rm A}(z)e^{-i\diamond}\right) S^<(k;z) = 0,$$

$$\begin{aligned}
2\mathrm{i}\hat{k}^0 g_0^s - 2\mathrm{i}s\hat{k}^z g_3^s - 2\mathrm{i}M^{\rm H}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_1^s - 2\mathrm{i}M^{\rm A}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_2^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_1^s - 2s\hat{k}^z g_2^s - 2\mathrm{i}M^{\rm H}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_0^s + 2M^{\rm A}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_3^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_2^s + 2s\hat{k}^z g_1^s - 2M^{H}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_3^s - 2\mathrm{i}M^{\rm A}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_0^s &= 0\,,\\
2\mathrm{i}\hat{k}^0 g_3^s - 2\mathrm{i}s\hat{k}^z g_0^s + 2M^{\rm H}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_2^s - 2M^{\rm A}\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}\cdot\overrightarrow{\partial}_k} g_1^s &= 0\,.
\end{aligned}$$

After expanding to second order in mass gradients, finally can write the master kinetic equation:

[Kainulainen, Prokopec, Schmidt, Weinstock]

$$
\begin{aligned}
& k^z \frac{\partial}{\partial z} g^s_{\mathrm{L}} + \underbrace{\frac{\mathrm{i}}{2}\left[MM^\dagger, g^s_{\mathrm{L}}\right]}_{\text{mixing term}} \\
& \underbrace{-\frac{1}{4}\left\{\left(MM^\dagger\right)', \partial_{k^z} g^s_{\mathrm{L}}\right\}}_{\text{classical force}} \underbrace{-\frac{1}{4k^z}\left(M' g^s_{\mathrm{R}} M^\dagger + M g^s_{\mathrm{R}} M'^\dagger\right) + \frac{1}{4k^z}\left(M' M^\dagger g^s_{\mathrm{L}} + g^s_{\mathrm{L}} M M'^\dagger\right)}_{\text{gradient-mixing terms}} \\
& \underbrace{+\frac{\mathrm{i}}{8}\left(M'' M^\dagger \partial_{k^3} \frac{g^s_{\mathrm{L}}}{k^z} - \partial_{k^z} \frac{g^s_{\mathrm{L}}}{k^z} M M''^\dagger\right) - \frac{\mathrm{i}}{8}\left(M'' \partial_{k^z} \frac{g^s_{\mathrm{R}}}{k^z} M^\dagger - M \partial_{k^3} \frac{g^s_{\mathrm{R}}}{k^z} M''^\dagger\right)}_{\text{semiclassical force}} \\
& -\frac{\mathrm{i}}{16}\left[\left(MM^\dagger\right)'', \partial^2_{k^z} g^s_{\mathrm{L}}\right] + \frac{\mathrm{i}}{8k^z}\left[M' M'^\dagger, \partial_{k^z} g^s_{\mathrm{L}}\right] = 0\,.
\end{aligned}
$$

$$
\partial_z j_5^z(k,z) = \left(M^H M^{H\prime} + M^A M^{A\prime}\right) \frac{1}{k^z} \partial_{k^z} j_5^z(k,z) + \left(M^H(\partial_z^2 M^A) - M^A(\partial_z^2 M_H)\right) \frac{1}{2k^z} \partial_{k^z} \left(\frac{j_N^3(k,z)}{k^z}\right).
$$

$$
m_s^2(z) = m^2(z) - \frac{s}{\tilde{k}^0}\left(M^H(z)\partial_z M^A(z) - M^A(z)\partial_z M^H(z)\right), \quad \tilde{k}^0 \equiv \mathrm{sign}(k^0)\sqrt{(k^0)^2 - (k^1)^2 - (k^2)^2}
$$

Our input: **Modified shell** precisely leads to **cancellation of δ' terms** !

$$
\mathcal{S}_{\mathrm{sc}}(\mathbf{k}) = -\frac{\beta}{2} v_w f_0(\omega_0)(1-f_0)\left[\delta F + \frac{F_0}{\omega_0}\delta\omega + F_0\,\beta(1-f_0)\delta\omega + \frac{F_0\,\omega_0}{\omega_0^2 - \mathbf{k}_{||}^2}\delta\omega\right]
$$

Part 2 Different Sources in Literature

Motivation: CP-violation in the fluid equations

Considering fermions

- The vector Current:

$$\partial_z j^z = v_w\, \partial_z \bar{n} - D\, \partial_z^2 \bar{n} = \mathcal{S}^{\text{collision}}$$

[Musolf, Chung, Tulin,…., Konstandin, Prokopec, …etc]

- The axial current:

$$\partial_z j_5^z = v_w\, \partial_z n - D\, \partial_z^2 n = -\mathcal{S}^{\text{flow}} + \mathcal{S}^{\text{collision}}$$

[Carena, Seco…, Kainulainen, Konstandin, Prokopec, Schmidt, Weinstock]

Our focus today

Liouville/Schwinger Dyson equations (microscopic kinetic equations) → Boltzmann-like kinetic equation for distribution functions → Fluid equations for number densities/ bulk flows containing Particle densities Generated through the CP violating source. → **The Asymmetry**

CPV sources in the literature

Source type	Methods	Resonant enhancement		Gradient order	
Single flavour	WKB	No	F_s	2	[Cline, Joyce, Kainulainen, Prokopec]
	Spin decom.	No	$F_s, \partial_z j^z, \partial_z j_5^z$	2	[Kainulainen, Prokopec, Schmidt, Weinstock]
Multi-flavour	WKB	No	F_s	2	[Cline, Joyce, Kainulainen]
	Spin decom.	Diag. sources are resonant but effect compensated by flavour oscillations	j^z, j_5^z	1+2	[Konstandin, Prokopec, Schmidt, Seco]
	VIA	Yes [Postma, van de Vis, White]	~~$\partial_z j^z$~~	1	[Riotto][Carena, Moreno, Quiros, Seco, Wagner] [Lee,Cirigliano, Ramsey-Musolf]

AGREE! (single flavour)

Modified dispersion relations are key

No modified shell is needed

Part 3 What we found!

We do the same thing but now for two fermion flavours:

- We consider a **2 fermion system** with **CP-odd phases** present in **mixing terms**

$$M = \begin{bmatrix} m_1 & e^{i\varphi} v_b(z) \\ v_a(z) e^{i\gamma} & m_2 \end{bmatrix}$$

$$\mathcal{L} \supset -\bar{\psi}(M^{\mathrm{H}}(x) + \mathrm{i}\gamma^5 M^{\mathrm{A}}(x))\psi \qquad \begin{cases} M^H = \frac{1}{2}(M + M^\dagger) \\ M^A = \frac{1}{2}(M - M^\dagger) \end{cases}$$

- Have computed **CPV source** $\partial_\mu j_5^\mu$ with **two different methods**:

  - **spin decomposition**
  - **VIA expansion**

Agreement up to $\mathcal{O}$ (v^3, ∂_z^3)

2 fermion mixing: CPV source

Mixing effect. Add up to zero

Involve CP odd phases. Contribute to total source.
Resonance compensated in sum by

$$j^z_{1,1} - j^z_{2,2} = \mathcal{O}(m_1 - m_2)$$

$$(\partial_z j^z_5)_{1,1} = -\frac{(v_a v'_a - v_b v'_b)}{m_1^2 - m_2^2}(j^z_{1,1} - j^z_{2,2}) + \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v'_a v'_b + v_b v''_a + v_a v''_b)\left[\frac{1}{2k_z^2}(j^z_{1,1} - k_z \partial_{k_z} j^z_{1,1}) - \frac{1}{m_1^2 - m_2^2}(j^z_{2,2} - j^z_{1,1})\right] + \mathcal{O}(v^3, vv''', v'v''),$$

$$(\partial_z j^z_5)_{2,2} = \frac{(v_a v'_a - v_b v'_b)}{m_1^2 - m_2^2}(j^z_{1,1} - j^z_{2,2}) - \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v'_a v'_b + v_b v''_a + v_a v''_b)\left[\frac{1}{2k_z^2}(j^z_{2,2} - k_z \partial_{k_z} j^z_{2,2}) + \frac{1}{m_1^2 - m_2^2}(j^z_{1,1} - j^z_{2,2})\right] + \mathcal{O}(v^3, vv''', v'v'')$$

Mixing effect.
Add up to zero

Include δ' contributions

$$j^5_\mu = -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu \gamma^5 \, iS^<(k,x)\right]$$ **VIA**

$$j^z_5 = -2(g_0^+ - g_0^-),$$ **CTP spin decomp**

Ready to plug equations for model builders and phenomenologists

The source

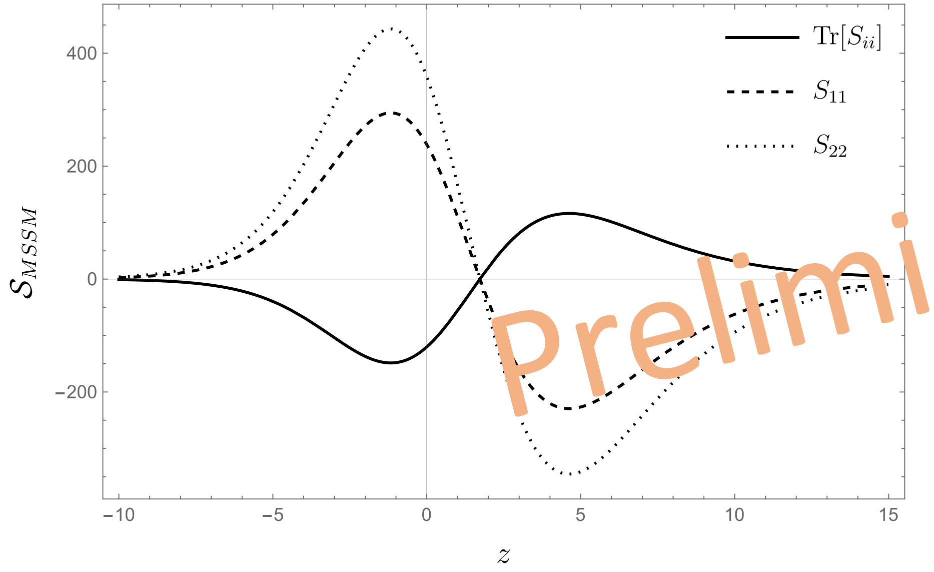
For a simple Bino-Higgsino system

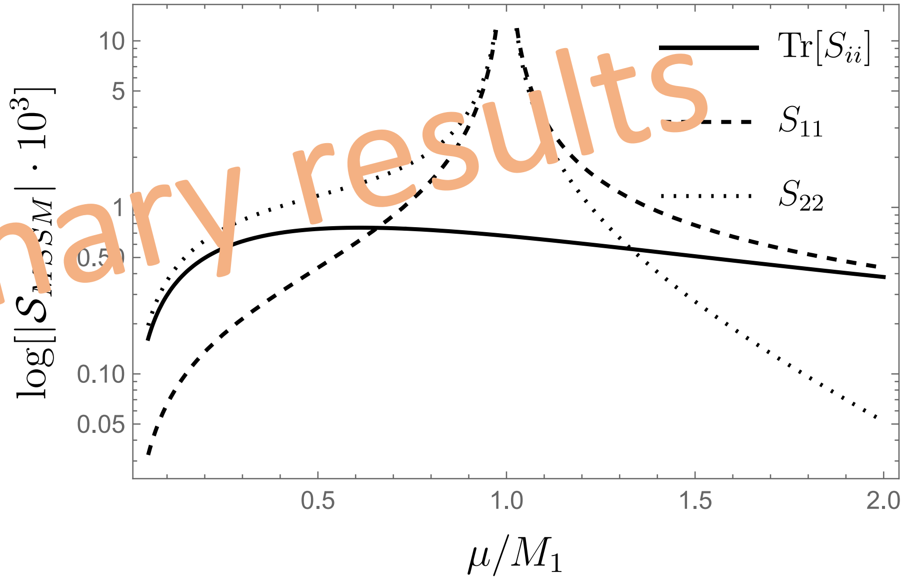
The source is:

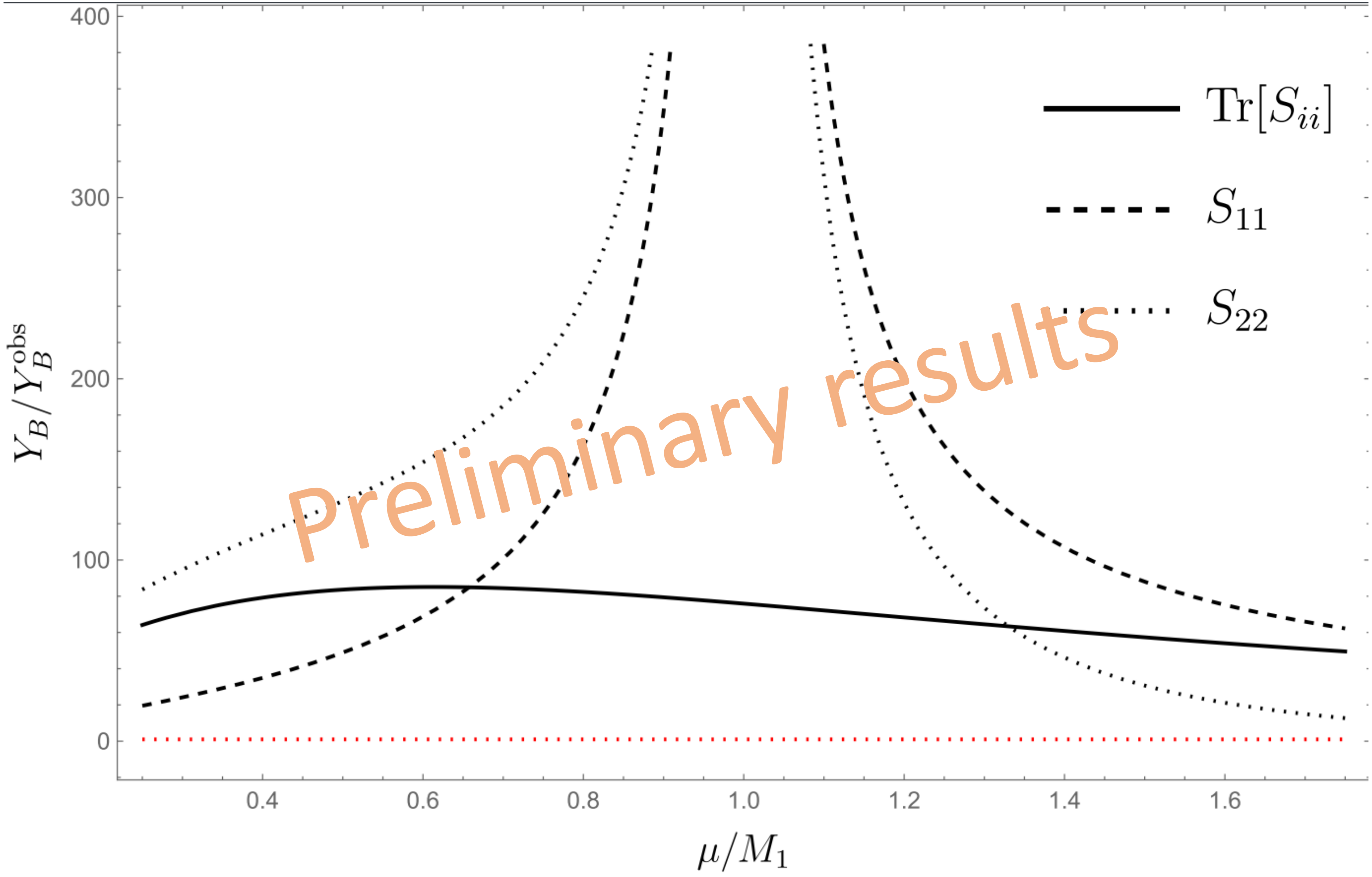
$$M_{ij} = \begin{pmatrix} M_1 & e^{i\varphi} v_b(z) \\ e^{i\gamma} v_a(z) & |\mu| \end{pmatrix}$$

$$\mathcal{S}_{11}^{\rm RM}(\mathbf{k}, x) = -\beta v_w f_{11}^0 \left(1 - f_{11}^0\right) \frac{M_1 |\mu| \left(2 v_a' v_b' + v_b v_a'' + v_a v_b''\right) \sin(\phi)}{\left(M_1^2 - |\mu|^2\right) \tilde{\omega}_{0,1}\, \omega_{0,1}}$$

$$\mathcal{S}_{22}^{\rm RM}(\mathbf{k}, x) = +\beta v_w f_{22}^0 \left(1 - f_{22}^0\right) \frac{M_1 |\mu| \left(2 v_a' v_b' + v_b v_a'' + v_a v_b''\right) \sin(\phi)}{\left(M_1^2 - |\mu|^2\right) \tilde{\omega}_{0,2}\, \omega_{0,2}}$$







$Y_B/Y_B^{\rm obs}$
μ/M_1
0
100
200
300
400
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$\mathrm{Tr}[S_{ii}]$
S_{11}
S_{22}
Preliminary results

Conclusions

- We have obtained a new consistency condition for the derivation of Boltzmann equations: all terms proportional to δ' must cancel
  - For quantum fermions, this requires **modified dispersion relations**
  - **Mass shell** can be inferred from **constrain equations** → **nontrivial check**
  - Computing the modified **dispersion relation essential** in CTP as in WKB, and **cannot be done with the VIA approach**
- We have derived **CPV sources** in a system of 2 **mixing fermions** in the CTP formalism
  - **VIA** and **spin decomposition** calculations agree, consistency condition satisfied
  - **Resonance not present** when **summing** over **flavours**
  - Sources contributing to **flavour sum** have the structure of **WKB semiclassical force**

Thank you

A consistency check

- In the quantum case, the **mass** of **on-shell excitations** can be **computed** with **constrain eqs.**

- **Cancellation of** δ' in the **kinetic equations** is then a **nontrivial consistency check**

- **Unambiguous Boltzmann equations** including CPV sources **in the CTP formalism require** to compute the **modified dispersion relation**, similar to the **WKB** case

Trivial shell: classical particle

- Consider a **classical point particle** with **space-time dependent mass**. E.o.m.s are

$$k^\mu = m(x)\frac{dx^\nu}{d\tau} \Rightarrow \frac{dk^\nu}{d\tau} = \frac{\partial m(x)}{\partial x_\nu}$$

- With $g(x,k) = \delta(k^2 - m^2(x))f(x,\mathbf{k})$ the classical **phase-space density**, **Liouville's theorem** gives

$$\frac{d}{d\tau}g(x,k) = \delta(k^2 - m^2(x))\left(\frac{k^\mu}{m}\frac{\partial f(\mathbf{k},x)}{\partial x^\mu} + \frac{dk^\mu}{d\tau}\frac{\partial f(\mathbf{k},x)}{\partial k^\mu}\right)$$

$$+2k_\mu f(\mathbf{k},x)\delta'(k^2 - m^2(x))\left(\frac{dk^\mu}{d\tau} - \frac{\partial m}{\partial x_\mu}\right)$$

Cancels! Consistent Boltzmann eq.

Nontrivial shell: single fermion

- Consider a **quantum fermion** with

$$\mathcal{L} \supset -\bar{\psi}(M^{\mathrm{H}}(x) + \mathrm{i}\gamma^5 M^{\mathrm{A}}(x))\psi$$

- The **spin decomposition method** gives a **CPV source** in static bubble frame:

$$\partial_z j_5^z(k,z) = \left(M^H M^{H\prime} + M^A M^{A\prime}\right)\frac{1}{k^z}\partial_{k^z} j_5^z(k,z) + \left(M^H(\partial_z^2 M^A) - M^A(\partial_z^2 M_H)\right)\frac{1}{2k^z}\partial_{k^z}\left(\frac{j_N^3(k,z)}{k^z}\right)$$

as well as a **modified shell**

$$m_s^2(z) = m^2(z) - \frac{s}{\tilde{k}^0}\left(M^H(z)\partial_z M^A(z) - M^A(z)\partial_z M^H(z)\right), \quad \tilde{k}^0 \equiv \mathrm{sign}(k^0)\sqrt{(k^0)^2 - (k^1)^2 - (k^2)^2}$$

[Kainulainen, Prokopec, Schmidt, Weinstock]

- **Our input**: **Modified shell** precisely leads to **cancellation of δ' terms** !

2 fermion mixing: Consistency check

The **consistency check** requires to compute the **modified dispersion relation**

$$k^2 = (m_i + \delta m_i)^2$$

$$\delta m_1^s = -\frac{s m_2 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\,(m_1^2 - m_2^2)\,\sqrt{{k_z}^2 + m_1^2}} + (s\text{-independent}) + \mathcal{O}(v^3, vv'', v'v'),$$

$$\delta m_2^s = -\frac{s m_1 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\,(m_2^2 - m_1^2)\,\sqrt{{k_z}^2 + m_2^2}} + (s\text{-independent}) + \mathcal{O}(v^3, vv'', v'v'),$$

This results lead to the cancellation of δ' terms!! ▶ Consistent Boltzmann eq.

▶ Terms that give nonzero total source lead to force terms in Boltzmann eqs.
$\propto m_i\, \partial_z(\delta m_i)$ exactly as in the WKB formalism

Using off-diagonal components:

Exchanging L ↔ R, M ↔ M†, we can write the sought-for divergence of the chiral current

$$\partial_z j_5^z = -\sum_{s=\pm 1} s(\partial_z g_R^s + \partial_z g_L^s).$$

But the main objective is to write the divergence of the chiral current along the diagonal $\partial_z j^z_{5,ii}$, in terms of the diagonal number currents.

So to do so, we need to solve for (${g^s_{R/L,ij}}^2, i \neq j$) up to the same order, so we make the following expansion:

$$g^s_{R/L,ij} = g^{s,(0)}_{R/L,ij} + g^{s,(1)}_{R/L,ij} + g^{s,(2)}_{R/L,ij} + \mathcal{O}(\delta m'^3, \delta m''\delta m', \delta m'''),$$

$$g^{s,(0)}_{R,12} = \frac{\left(m_2 \delta m_a^\dagger + \delta m_b m_1\right)\left(g^s_{R,11} - g^s_{R,22}\right)}{m_1^2 - m_2^2} + \mathcal{O}(\delta m^2),$$

$$g^{s,(0)}_{R,21} = \frac{\left(m_1 \delta m_b^\dagger + \delta m_a m_2\right)\left(g^s_{R,11} - g^s_{R,22}\right)}{m_1^2 - m_2^2} + \mathcal{O}(\delta m^2),$$

$$g^{s,(0)}_{L,12} = \frac{\left(m_1 \delta m_a^\dagger + \delta m_b m_2\right)\left(g^s_{L,11} - g^s_{L,22}\right)}{m_1^2 - m_2^2} + \mathcal{O}(\delta m^2),$$

$$g^{s,(0)}_{L,21} = \frac{\left(m_2 \delta m_b^\dagger + \delta m_a m_1\right)\left(g^s_{L,11} - g^s_{L,22}\right)}{m_1^2 - m_2^2} + \mathcal{O}(\delta m^2).$$

The resonant mixing source:

$$(\partial_z j_5^z)_{11} = \sum_{s=\pm} \Bigg[-\frac{2s\,(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\,(g_{3,11}^s - g_{3,22}^s) \\ - \frac{2s\sin(\gamma+\varphi)m_1 m_2}{k^z(m_1^2-m_2^2)}(2v_a'v_b' + v_b v_a'' + v_a v_b'')\Bigg[\frac{1}{2(k^z)^2}(g_{3,11}^s - k^z\partial_{k^z} g_{3,11}^s) \\ - \frac{1}{m_1^2-m_2^2}(g_{3,22}^s - g_{3,11}^s)\Bigg] + \mathcal{O}(\delta m^3, \delta m\delta m''', \delta m''\delta m'),$$

$$(\partial_z j_5^z)_{22} = \sum_{s=\pm} \Bigg[\frac{2s\,(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\,(g_{3,11}^s - g_{3,22}^s) \\ + \frac{2s\sin(\gamma+\varphi)m_1 m_2}{k^z(m_1^2-m_2^2)}(2v_a'v_b' + v_b v_a'' + v_a v_b'')\Bigg[\frac{1}{2(k^z)^2}(g_{3,22}^s - k^z\partial_{k^z} g_{3,22}^s) \\ - \frac{1}{m_1^2-m_2^2}(g_{3,22}^s - g_{3,11}^s)\Bigg] + \mathcal{O}(\delta m^3, \delta m\delta m''', \delta m''\delta m')$$

$$(\partial_z j_5^z)_{1,1} = \frac{\mathrm{i}(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\,\left(\mathrm{Tr}[\gamma^3 S_0^<]_{1,1} - \mathrm{Tr}[\gamma^3 S_0^<]_{2,2}\right) \\ - \frac{\mathrm{i}\sin\varphi\, m_1 m_2}{k_z(m_1^2-m_2^2)}(2v_a'v_b' + v_b v_a'' + v_a v_b'')\Bigg[\frac{1}{2k_z^2}\left(\mathrm{Tr}[\gamma^3 S_0^<]_{1,1} - k_z\partial_{k_z}\mathrm{Tr}[\gamma^3 S_0^<]_{1,1}\right) \\ - \frac{1}{m_1^2-m_2^2}\left(\mathrm{Tr}[\gamma^3 S_0^<]_{2,2} - \mathrm{Tr}[\gamma^3 S_0^<]_{1,1}\right)\Bigg],$$

$$(\partial_z j_5^z)_{2,2} = -\frac{\mathrm{i}(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\,\left(\mathrm{Tr}[\gamma^3 S_0^<]_{1,1} - \mathrm{Tr}[\gamma^3 S_0^<]_{2,2}\right) \\ + \frac{\mathrm{i}\sin\varphi\, m_1 m_2}{k_z(m_1^2-m_2^2)}(2v_a'v_b' + v_b v_a'' + v_a v_b'')\Bigg[\frac{1}{2k_z^2}\left(\mathrm{Tr}[\gamma^3 S_0^<]_{2,2} - k_z\partial_{k_z}\mathrm{Tr}[\gamma^3 S_0^<]_{2,2}\right) \\ + \frac{1}{m_1^2-m_2^2}\left(\mathrm{Tr}[\gamma^3 S_0^<]_{1,1} - \mathrm{Tr}[\gamma^3 S_0^<]_{2,2}\right)\Bigg].$$

Writing an appropriate ansatz from the constraint equations:

From the constraints equations to the second order in mass gradients, we can write

$$g^s_{0,ii} = \sum_j c^s_{ij}(k,x)\,\delta(k^2 - m_j^2 - 2m_j\delta m_j^s).$$

$$g^{s,(0)}_{3,ii} = \frac{k^z s\tilde{k}_0}{k^{z2} + m_i^2}\, g_{0,ii} = 2\pi \frac{k^z s}{\tilde{k}_0}\,\delta(k^2 - m_j^2 - 2m_j\delta m_j^s)\, c^s_{ii}(k,x)$$

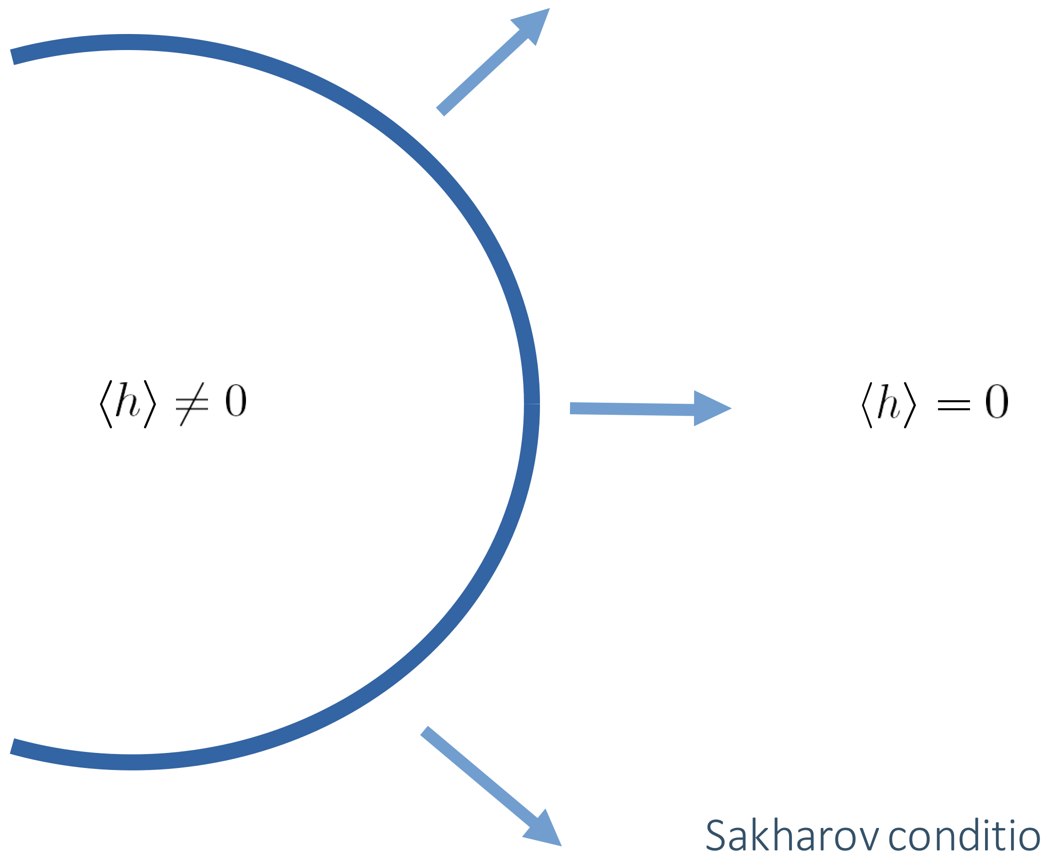
where, $c^s_{12} = O(v^2)c^s_{22}$, $\quad c^s_{21} = O(v^2)c^s_{11}$, and

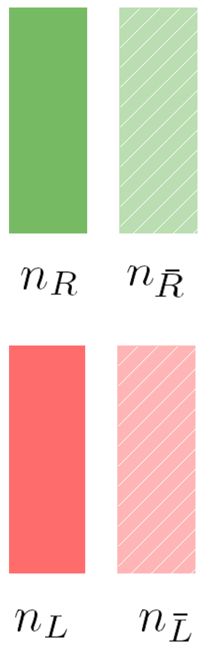
$$\delta m_1^s = -\frac{s m_2 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\left(m_1^2 - m_2^2\right)\sqrt{{k_z}^2 + m_1^2}} + (s\text{-independent}),$$

$$\delta m_2^s = -\frac{s m_1 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\left(m_2^2 - m_1^2\right)\sqrt{{k_z}^2 + m_2^2}} + (s\text{-independent}).$$

The coefficients $c^s_{ij}\,(k,x)$ contain appropriate number distribution functions.

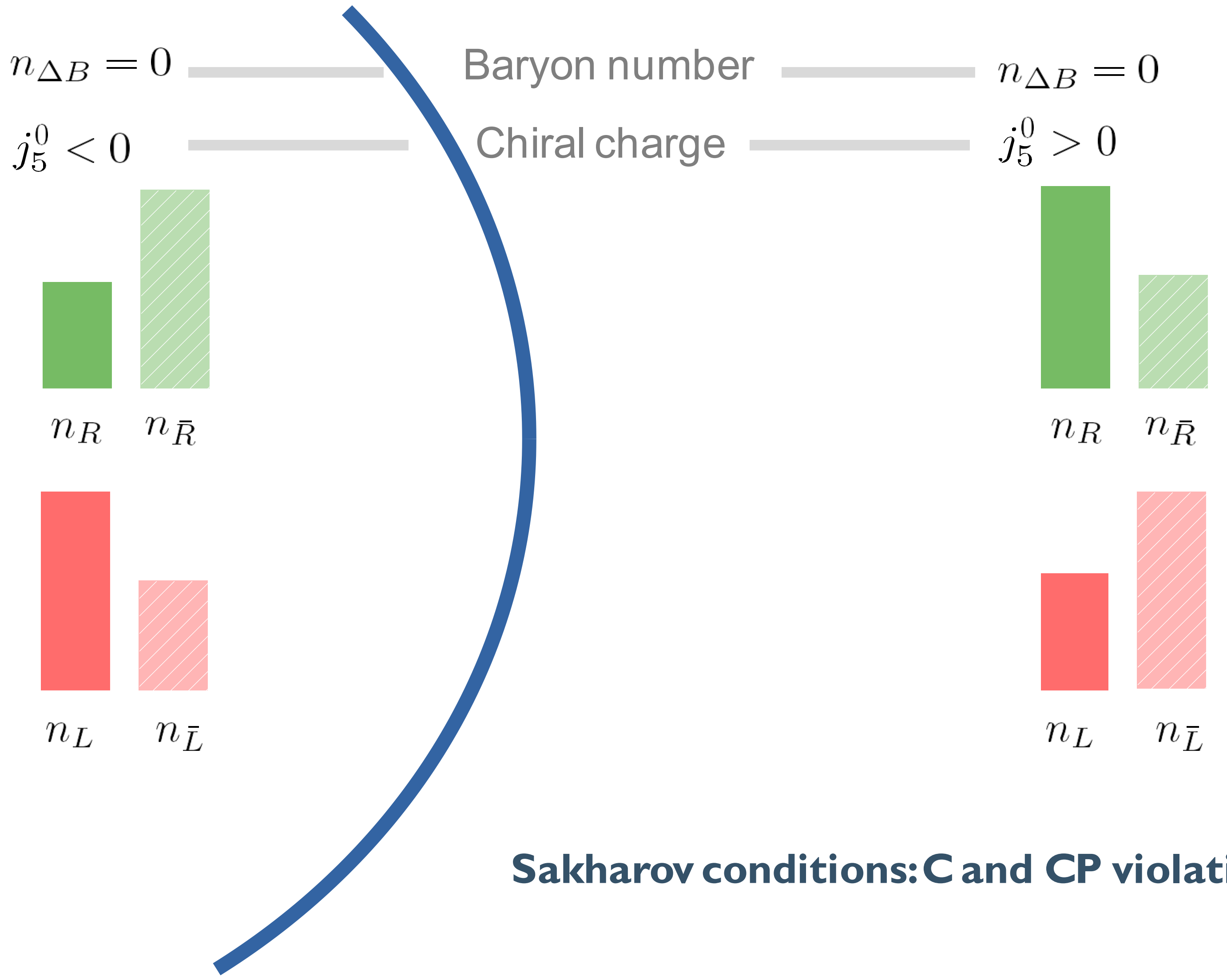
Back-up SM Sakharov





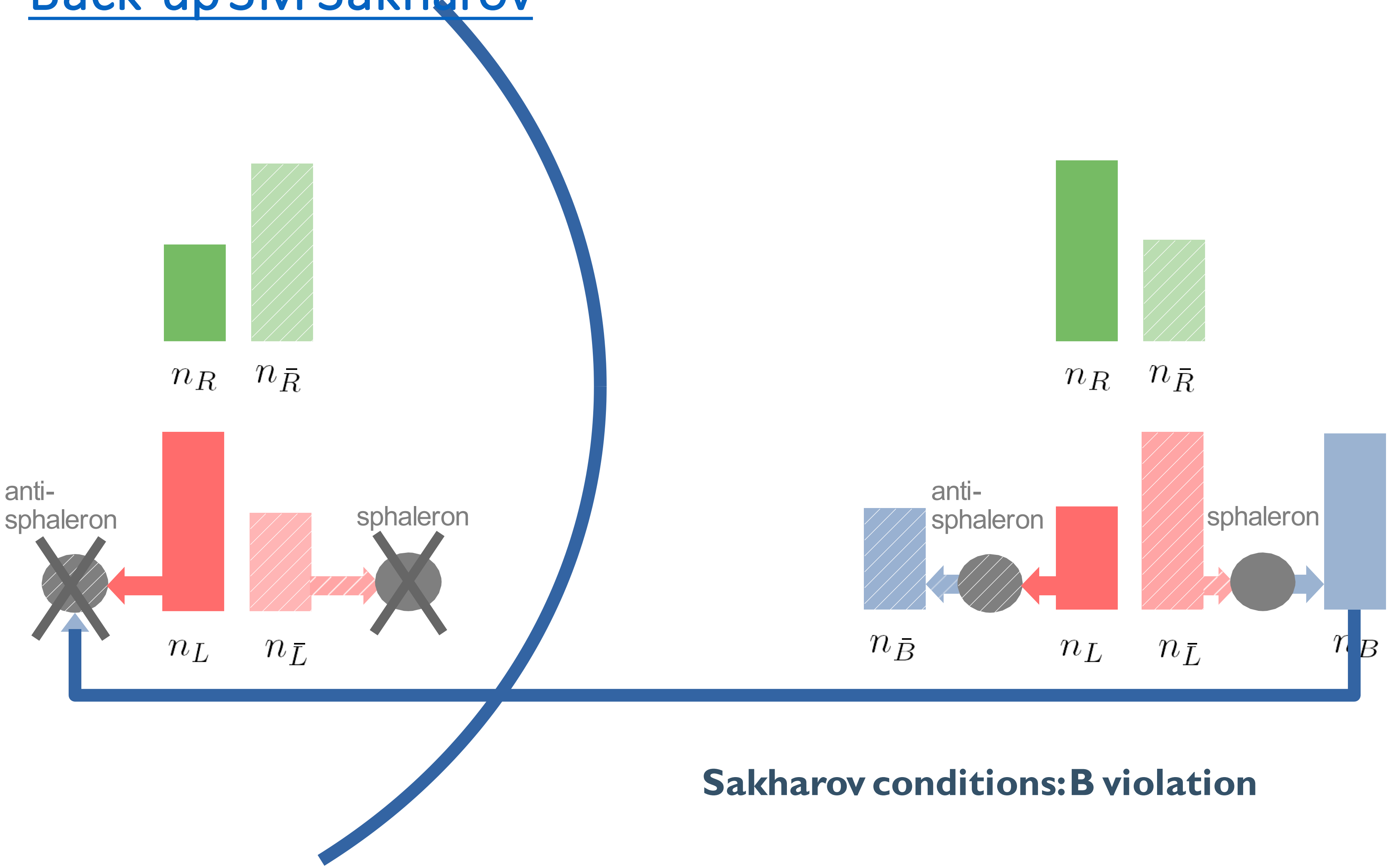
Sakharov conditions: Departure from equilibrium

Back-up SM Sakharov



Sakharov conditions: C and CP violation

Back-up SM Sakharov



Sakharov conditions: B violation

Back up – Effective action

The 2PI effective action can be expressed as

$$\Gamma[\Delta, S] = B + \mathrm{i}\,\mathrm{tr}[\Delta^{(0)}{}^{-1}\Delta] - \mathrm{i}\,\mathrm{tr}[S^{(0)}{}^{-1}S] + \mathrm{i}\,\mathrm{tr}\log\Delta^{-1} - \mathrm{i}\,\mathrm{tr}\log S^{-1} + \Gamma_2[\Delta, S]\,,$$

where B is the classical action, $\Delta^{(0)}{}^{-1}$ the Klein-Gordon, $S^{(0)}{}^{-1}$ the Dirac operator,

$$\Gamma_2[\Delta, S] \equiv -\mathrm{i} \times \text{the sum of 2PI vacuum graphs}\,,$$

Backup :Toward kinetic theory: Wigner transformation

Wanted: Equations for classical distributions (eventually fluids) that encompass relevant quantum effects.

relative coordinate $r \equiv x - y$
average coordinate $X \equiv 1/2(x + y)$

Wigner transformation of the two-point functions:

$$G(k,X) = \int d^4r\, e^{ik\cdot r} G\left(X + \frac{r}{2}, X - \frac{r}{2}\right)$$

Convolutions lead to the gradient expansion:

$$\int d^4r\, e^{ik\cdot r} \int d^4z G\left(X + \frac{r}{2}, z\right) F\left(z, X - \frac{r}{2}\right) = e^{-i\diamond} \{G(k,X)\} \{F(k,X)\}$$

$$A \diamond B = \frac{1}{2}(\partial_X^A \partial_k^B - \partial_X^B \partial_k^A)(AB).$$

The collisionless Schwinger-Dyson equations in Wigner space adopt the form:

$$\left(\not{k} + \frac{i}{2}\not{\partial} - M^{\mathrm{H}}(z)e^{-i\diamond} - i\gamma^5 M^{\mathrm{A}}(z)e^{-i\diamond}\right) S^{<}(k; z) = 0,$$

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Backup - Wigner Transform

- For an operator associated with a generalized "matrix" representation $O(x, y)$, where x and y are spacetime coordinates, the Wigner transform is defined as the Fourier transform with respect to the relative coordinate $r \equiv x - y$. Defining the average coordinate $X \equiv 1/2(x + y)$, the Wigner transform $O(k; X)$ of $O(x, y)$ is obtained as

$$O(k;X) = \int d^4r\, e^{ikr} O\left(X + \frac{r}{2}, X - \frac{r}{2}\right)$$

The corresponding inverse transform is

$$O\left(X + \frac{r}{2}, X - \frac{r}{2}\right) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ikr} O(k;X).$$

The Wigner transform of a product of operators

$$C(x,y) = \int d^4z\, A(x,z)B(z,y)$$

is known to be of the form arXiv:hep-ph/9802312.

$$C(k;X) \equiv A(k;X)\, e^{-i\diamond}\, B(k;X),$$

$$A \diamond B = \frac{1}{2}(\partial_X^A \partial_k^B - \partial_X^B \partial_k^A)(AB).$$

Back up spin diagonal

$$g_0^+ + g_0^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[s\gamma^3\gamma^5 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{charge density,}$$

$$g_3^+ + g_3^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[s\gamma^3 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{axial charge density,}$$

$$g_1^+ + g_1^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[\mathrm{i}S_s^{<,>}\right]\,, \quad \text{scalar density,}$$

$$g_2^+ + g_2^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[\mathrm{i}\gamma^5 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{pseudoscalar density,}$$

$$g_0^+ - g_0^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[\gamma^3\gamma^5 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{axial current density,}$$

$$g_3^+ - g_3^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[\gamma^3 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{current density,}$$

$$g_1^+ - g_1^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[s\mathrm{i}S_s^{<,>}\right]\,, \quad \text{spin density,}$$

$$g_2^+ - g_2^- = -\frac{1}{2}\sum_{s=\pm}\mathrm{tr}\left[s\mathrm{i}\gamma^5 \mathrm{i}S_s^{<,>}\right]\,, \quad \text{axial spin density.}$$