AsyInt for massive two-loop four-point integrals at high energies

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Based on [JHEP 09 (2024) 069]

https://gitlab.com/asyint/asyint-public

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Karlsruher Institut für Technologie

Institute for Theoretical Particle Physics

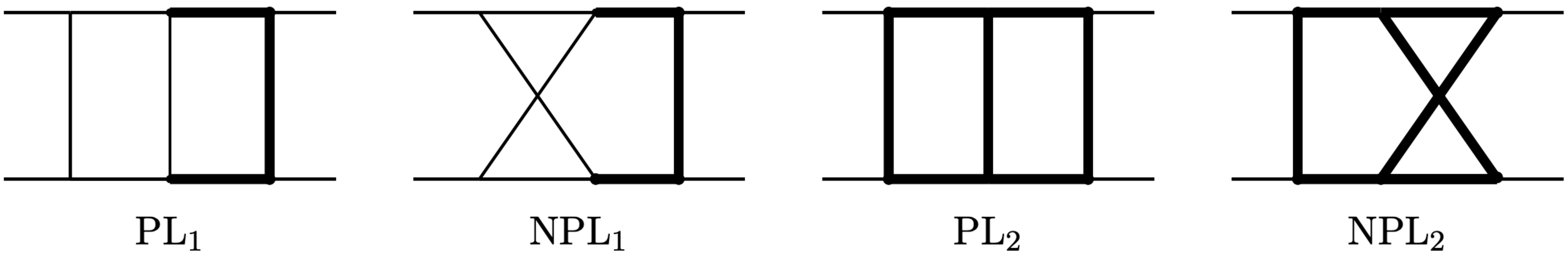
AsyInt

by Hantian Zhang — *JHEP 09 (2024) 069*

For analytic calculations of massive two-loop four-point integrals at high energies

Download at: https://gitlab.com/asyint/asyint-public

Sample Feynman diagrams calculated by AsyInt

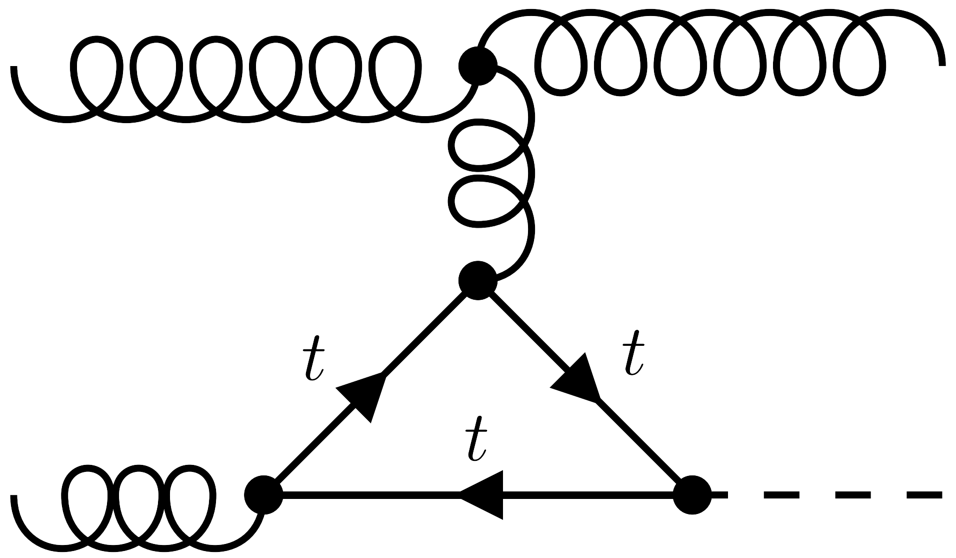


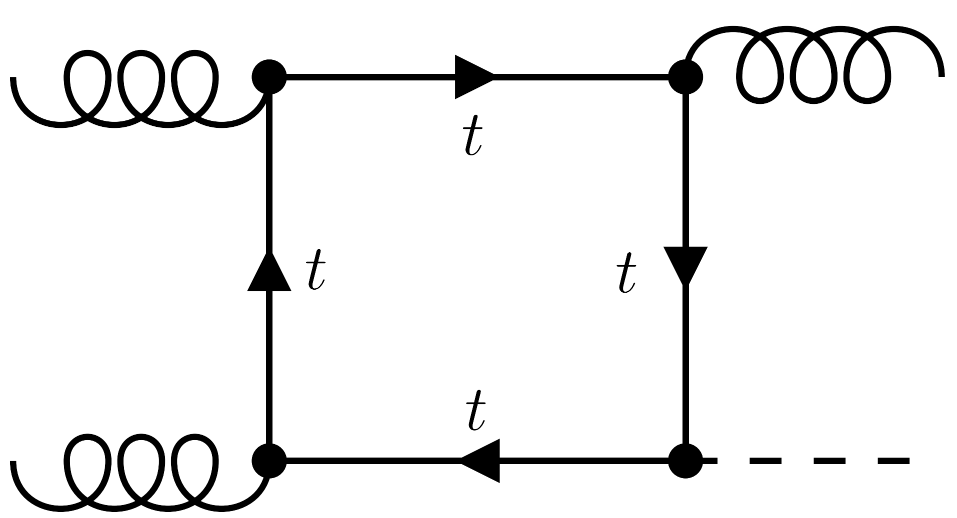
Sample planar and non-planar diagrams. Thick lines denote massive propagators

leading H.E. terms to ϵ-finite part
@ $\mathcal{O}(1/m)$ and $\mathcal{O}(m^0)$

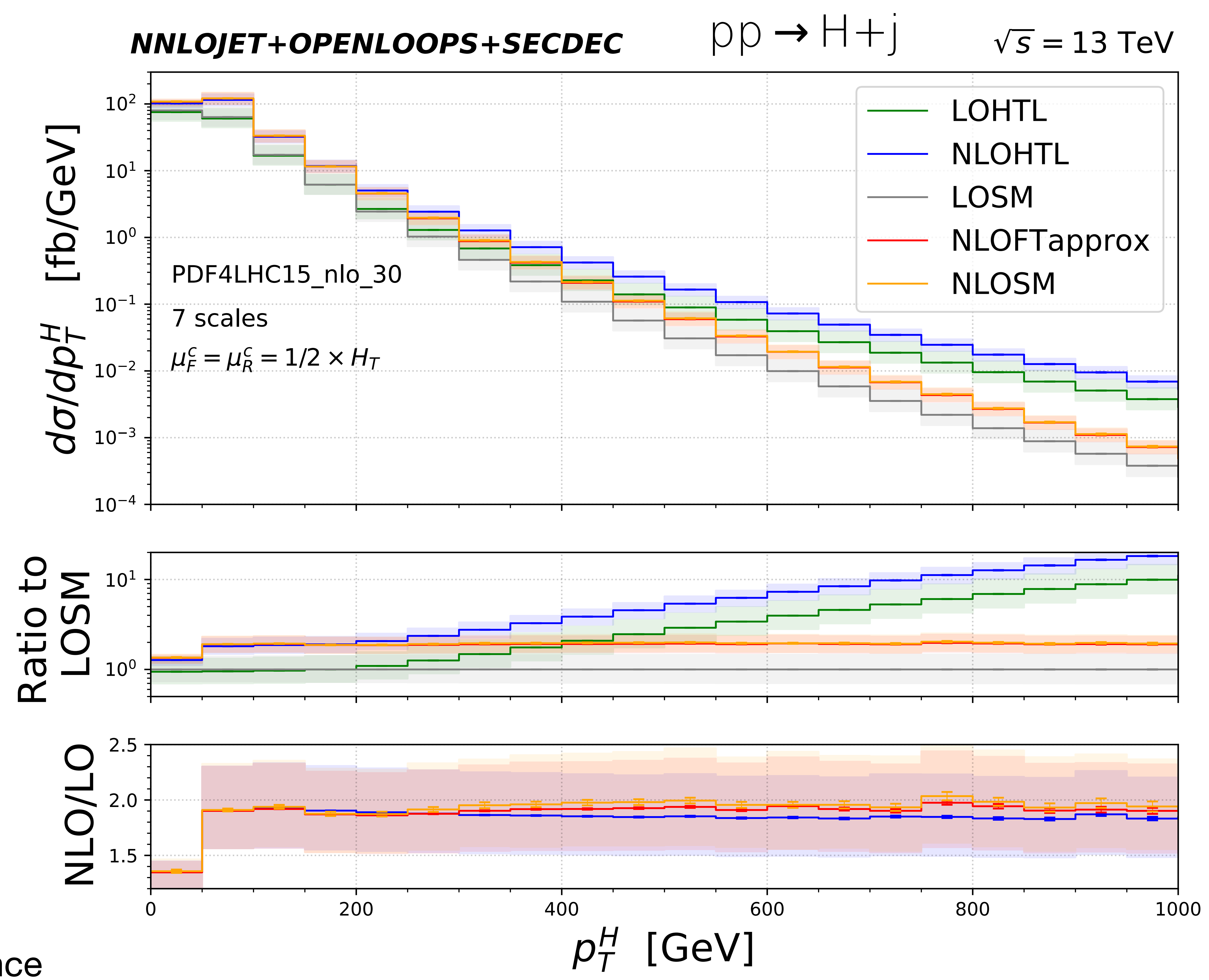
Motivation: probe boosted Higgs boson with large p_T^H

- Higgs boson plus jet production with large transversal momentum p_T^H at LHC





NLOSM ⇒ NLO QCD with top-mass dependence



High-energy region

Sensitive to new physics

Large QCD & EW corrections

Massive Feynman integrals

Precise Higgs + 2 jets @ NLO QCD also available in [Chen, Huss, Jones, Kerner, Lang, Lindert, **Zhang**, ***JHEP 03 (2022) 096***]

Motivation: probe boosted Higgs boson with large p_T^H

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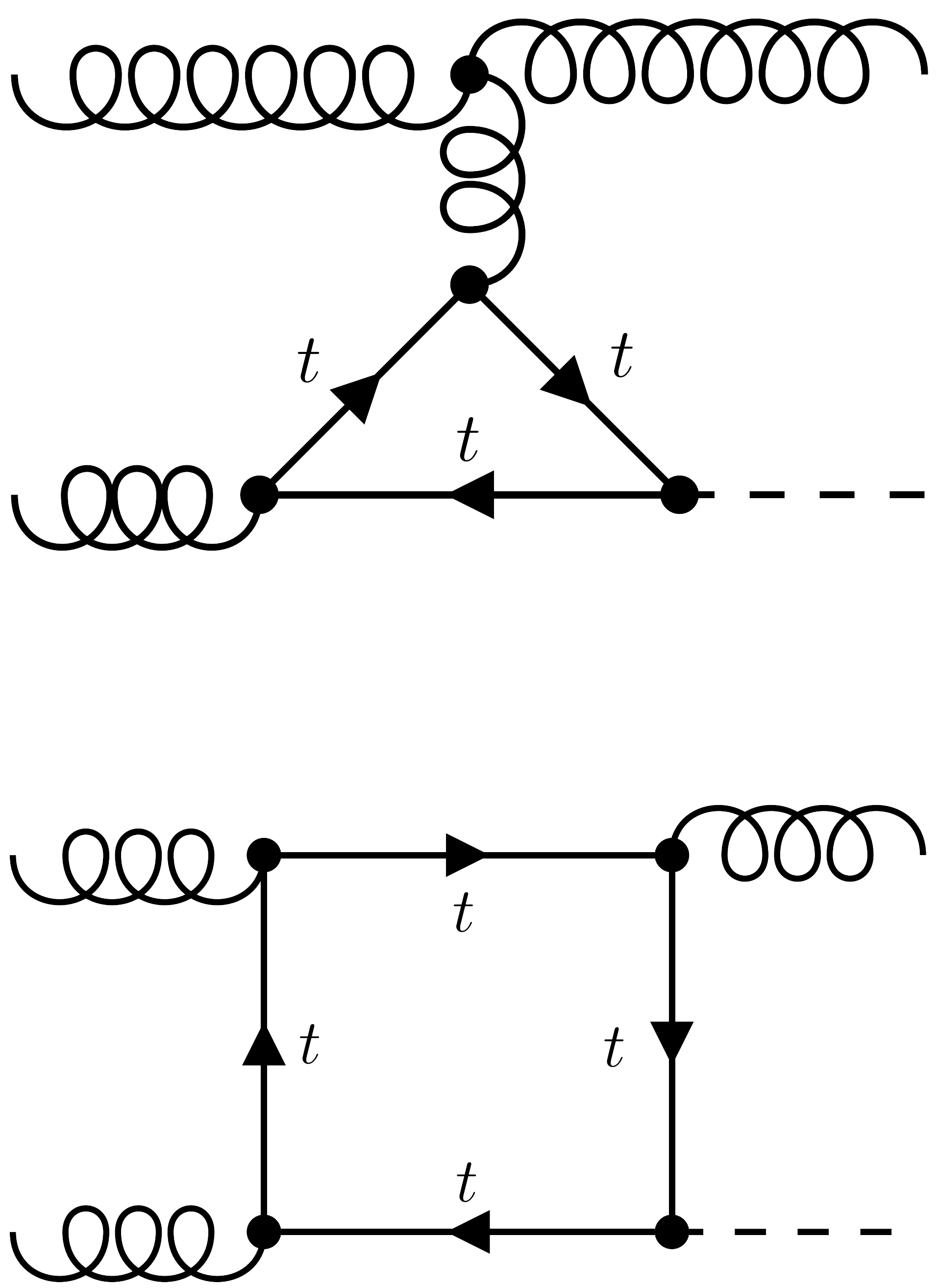

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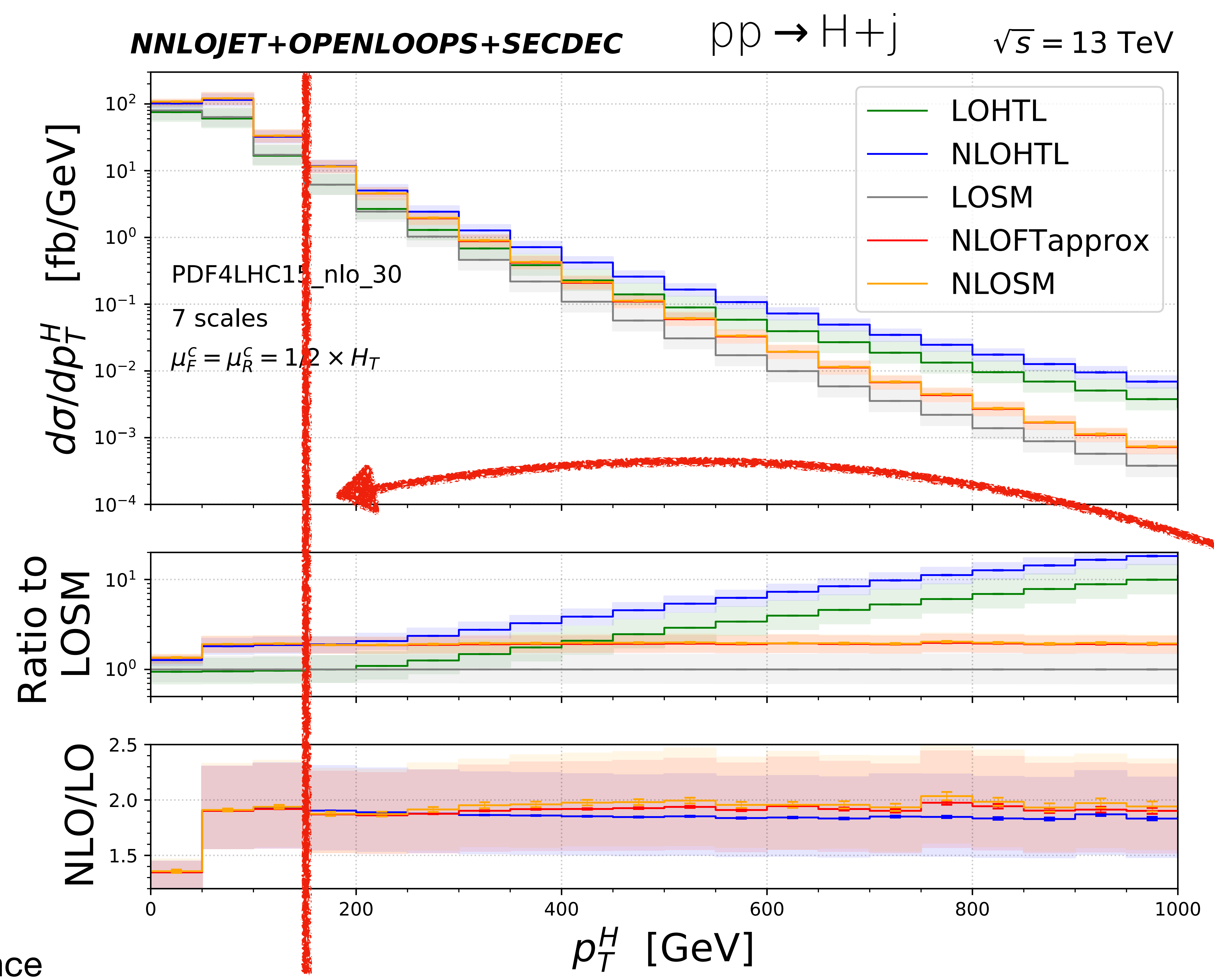

High-energy region

Sensitive to new physics

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High-energy expansion aims to cover $p_T^H > 150$ GeV region

Precise Higgs + 2 jets @ NLO QCD also available in [Chen, Huss, Jones, Kerner, Lang, Lindert, **Zhang**, ***JHEP 03 (2022) 096***]

Overview of analytic $2 \to 2$ high-energy calculations

High-energy expansion

- QCD corrections for $gg \to HH$ [Davies, Mishima, Steinhauser, Wellmann, 18']
- QCD corrections for $gg \to ZH$ [Davies, Mishima, Steinhauser, 21']
- Yukawa-top corrections for $gg \to HH$ [Davies, Mishima, Schönwald, Steinhauser, **Zhang**, 22']
- QCD master integrals in high-energy expansion [Mishima, 18']
- **AsyInt** and EW master integrals (partial results) in high-energy expansion [**Zhang**, 24']

High-energy limit & QCD factorisation

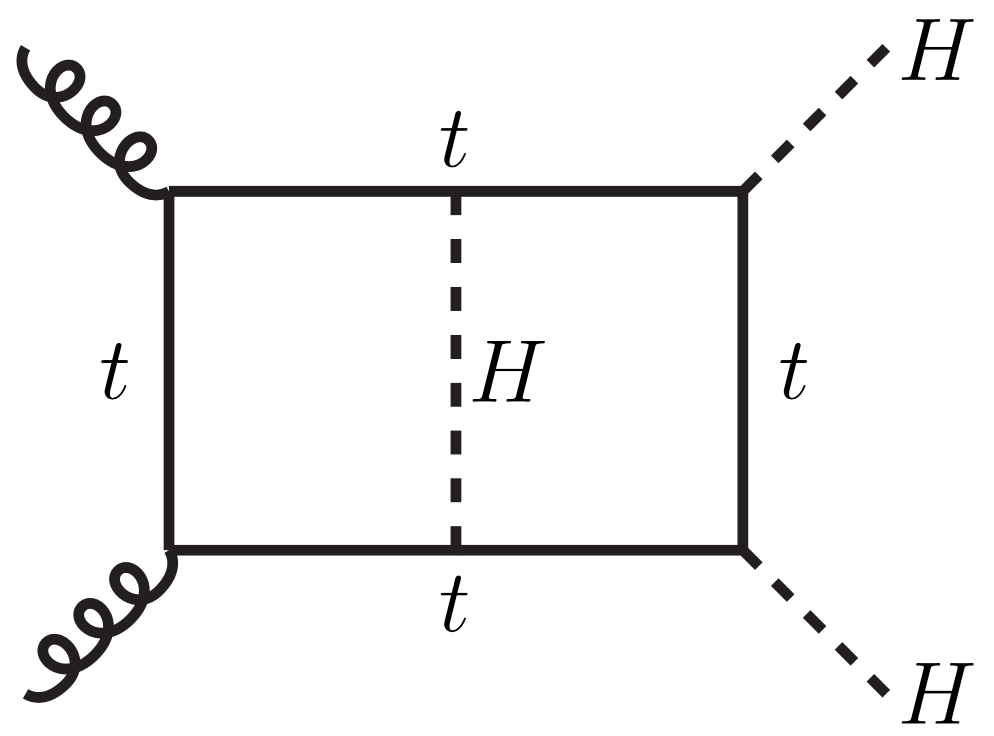
- QCD corrections for $gg \to Hg$ with small bottom mass (no top quark) [Melnikov, Tancredi, Wever, 16']
- QED corrections for massive Bhabha scattering [Penin, 06', Becher, Melnikov, 07']
- Massive QCD factorisation [Mitov, Moch, 06', Wang, Xia, Yang, Ye, 24']

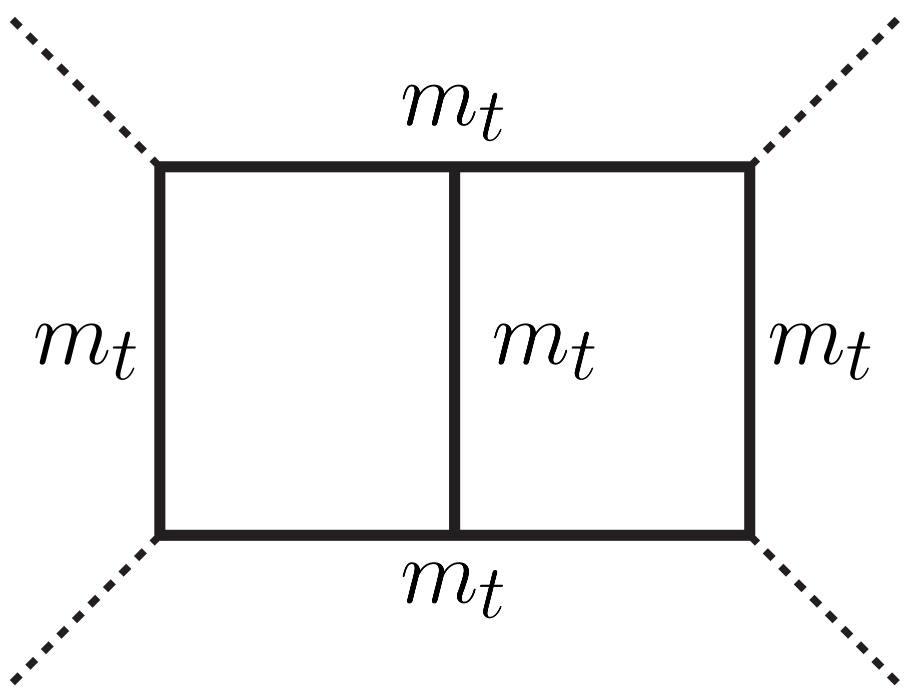
This talk: AsyInt for analytic master integrals at high energies

Expansion strategies at high energies

- At high energies, SM masses are of a similar order: $m_t \approx m_W, m_Z, m_H \ll \sqrt{s}$
- Two **fast convergent Taylor expansions**: equal-internal-mass and external-mass expansions

  e.g. convergent rates controlled by $m_H^2/s < 0.06$ and $(m_t^2 - m_H^2)/s < 0.01$ for $\sqrt{s} > 500$ GeV





$s, t \gg m_t^2$

High-energy expansion

High energy expansion of master integrals

1. **Asymptotic expansion:** $s, t \gg m_t^2$

2. **System of differential equations for Master Integrals** from IBP reduction [Kira]

$$\frac{\partial}{\partial\left(m_t^2\right)}\mathbf{I} = M(s, t, m_t^2, \epsilon)\,\mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{I}_1, \ldots, \mathcal{I}_N\right)^T$$

3. Plug in **power-log ansatz** for each master integral

$$\mathcal{I}_n = \sum C_{(n)}^{ijk}(s, t)\,\epsilon^i \left[m_t^2\right]^j \left[\log(m_t^2)\right]^k$$

4. Solve **boundary master integrals in $m_t^2/s \to 0$ to higher orders in m_t^2 and ϵ** **using AsyInt**

AsyInt toolkit I: generate MB-integral representations

- Two-loop Feynman integral with n propagators and k numerators

$$\mathscr{I}_{n,k} = \int \prod_{j=1}^{2} \mathrm{d}l_j \, \frac{N_1^{\lambda_1} \cdots N_k^{\lambda_k}}{D_1^{1+\delta_1} \cdots D_n^{1+\delta_n}}$$

δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)

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δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)

↓ Apply method-of-region ($s, t \gg m_t^2$) with **asy2.1.m** [Smirnov]

`GenerateInput / GenerateInputNum`

↙ **Hard region** ↓ **Asymptotic regions**

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↓

`GenerateInput / GenerateInputNum`

Hard region | **Asymptotic regions** → `AlphaRepForTempInt`

Hard region ↓ **Massless integrals**

Known master integrals

Independent calculation to weight-6 provided in **AsyInt** repository

`AlphaRepForTempInt` ↓ **Template integrals**

Perform parametric integration and Mellin transformations in complex plane

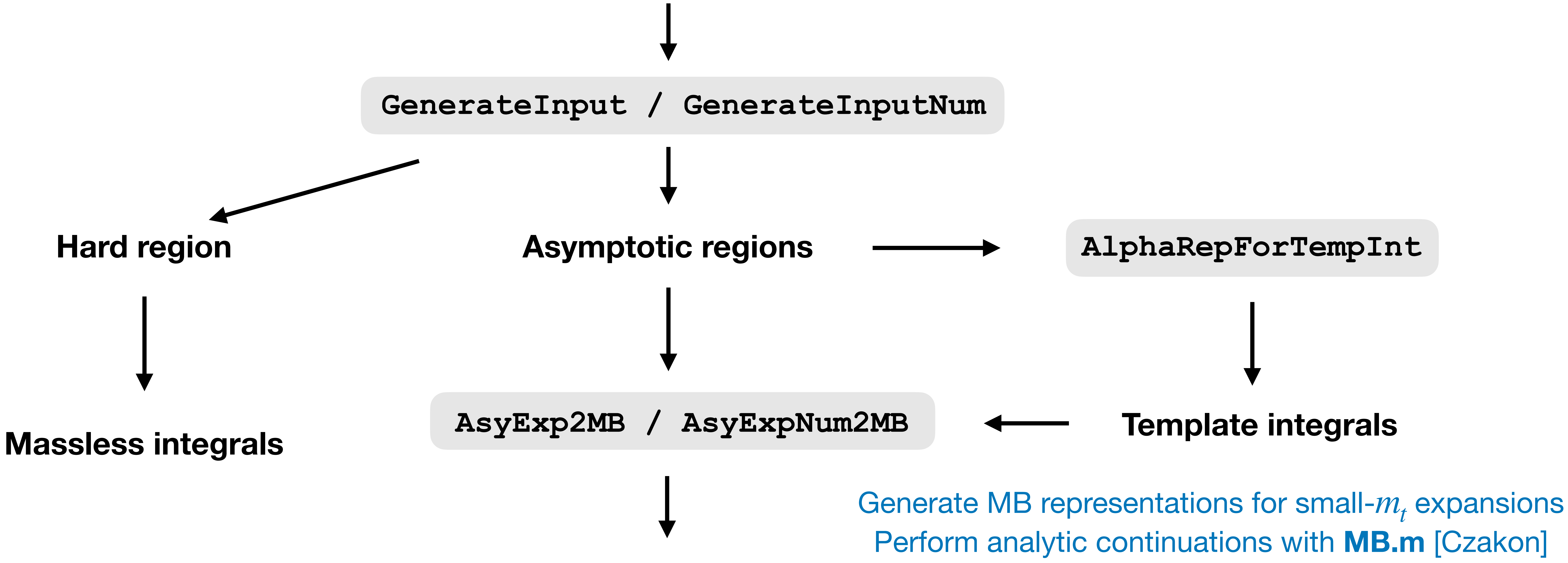
$$(x+y)^\lambda = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{\Gamma(-\lambda+z)\Gamma(-z)}{\Gamma(-\lambda)} x^z y^{\lambda-z} \quad \text{with} \quad z \in \mathbb{C}$$

AsyInt toolkit I: generate MB-integral representations

- Two-loop Feynman integral with n propagators and k numerators

$$\mathcal{I}_{n,k} = \int \prod_{j=1}^{2} \mathrm{d}l_j \, \frac{N_1^{\lambda_1} \dots N_k^{\lambda_k}}{D_1^{1+\delta_1} \dots D_n^{1+\delta_n}}$$

δ_i: additional regulators and shifts for dotted propagators (e.g. $\delta_i \to \delta_i + 1$)



Asymptotic Mellin-Barnes integrals to higher orders in m_t and ϵ

δ_i singularities cancel in sum of all asymptotic regions

AsyInt toolkit II: solve MB integrals

↓ MB dimensions reduction

Irreducible MB integrals

Numerical Reconstruction

↓

`AIRecNum1DMB`

Apply PSLQ algorithm given a basis of constants

New constant found in the fully-massive non-planar integral NPL_2

Analytic Summation

`AISum1DMB`
&
`AISum2DMB`

Apply Cauchy theorem and extract residues

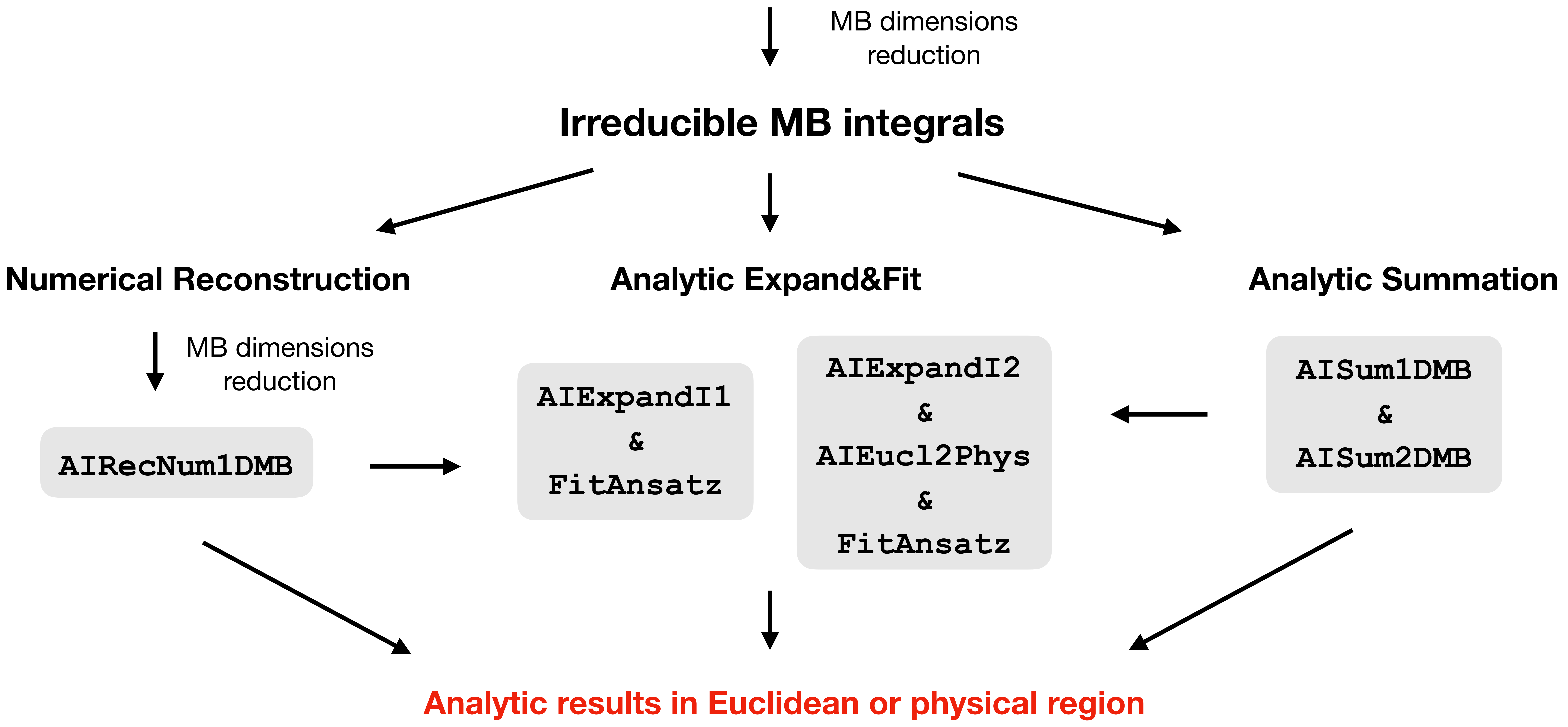
If residue series converge and no arc contributions, sum with **HarmonicSums.m** and **Sigma.m** [Ablinger, Schneider]

$$c_Z = \int_0^\infty \frac{\mathrm{d}\alpha_1\,\mathrm{d}\alpha_2}{\sqrt{\alpha_1\,\alpha_2\,\left(\alpha_1+\alpha_2+1\right)\,\left(\alpha_2\alpha_1+\alpha_1+\alpha_2\right)}}$$

Probably a weight-2 constant

$$= \sum_{k=0}^{\infty} \frac{2\,\Gamma\left(k+\frac{1}{2}\right)^4 \left[\psi^{(0)}(k+1) - 2\psi^{(0)}\left(k+\frac{1}{2}\right) + \psi^{(0)}(2k+1)\right]}{\pi(k!)^2\,\Gamma(2k+1)} = 17.695031908454309764234228747255\ldots$$

AsyInt toolkit II: solve MB integrals



Expand&Fit for complicated irreducible MB integrals

(type-1): 2-dim 1-scale MB integrals with non-vanishing arc contributions

(type-2): 2-dim 2-scale MB integrals

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]

- Close the contour to left or right semi-circle, and sum up residues, e.g.

$$\int_{-i\infty}^{+i\infty} \frac{dz_1}{2\pi i} f(z_1) = -\sum_{k=0}^{\infty} \mathrm{Res}_{z_1=k}\big[f(z_1)\big] - \int_{\mathrm{arc}} \frac{dz_1}{2\pi i} f(z_1) \quad \text{with} \quad f(z_1) = \frac{z_1^8 \Gamma(-z_1)^2 \Gamma(z_1)^2}{(z_1+1)^3 (z_1+2)^3}$$

$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, *JHEP 08 (2022) 259*]

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$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

arc integral non-zero

solve arc contribution by adding auxiliary scale:

$$\int_{\mathrm{arc}} \frac{dz_1}{2\pi i} \xi^{z_1} f(z_1) = -\sum_{k=0}^{\infty} \frac{k^6}{(1+k)^3 (2+k)^3} \xi^k \log(\xi) \overset{\xi \to 1}{=} -1$$

Analytic Expand&Fit method

Mellin-Barnes (MB) integrals with non-vanishing arc

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]

- Close the contour to left or right semi-circle, and sum up residues, e.g.

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Expand&Fit method [**Zhang**, ***2407.12107***]

- for 2-dim 1-scale MB integral with nested non-vanishing arc contributions

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

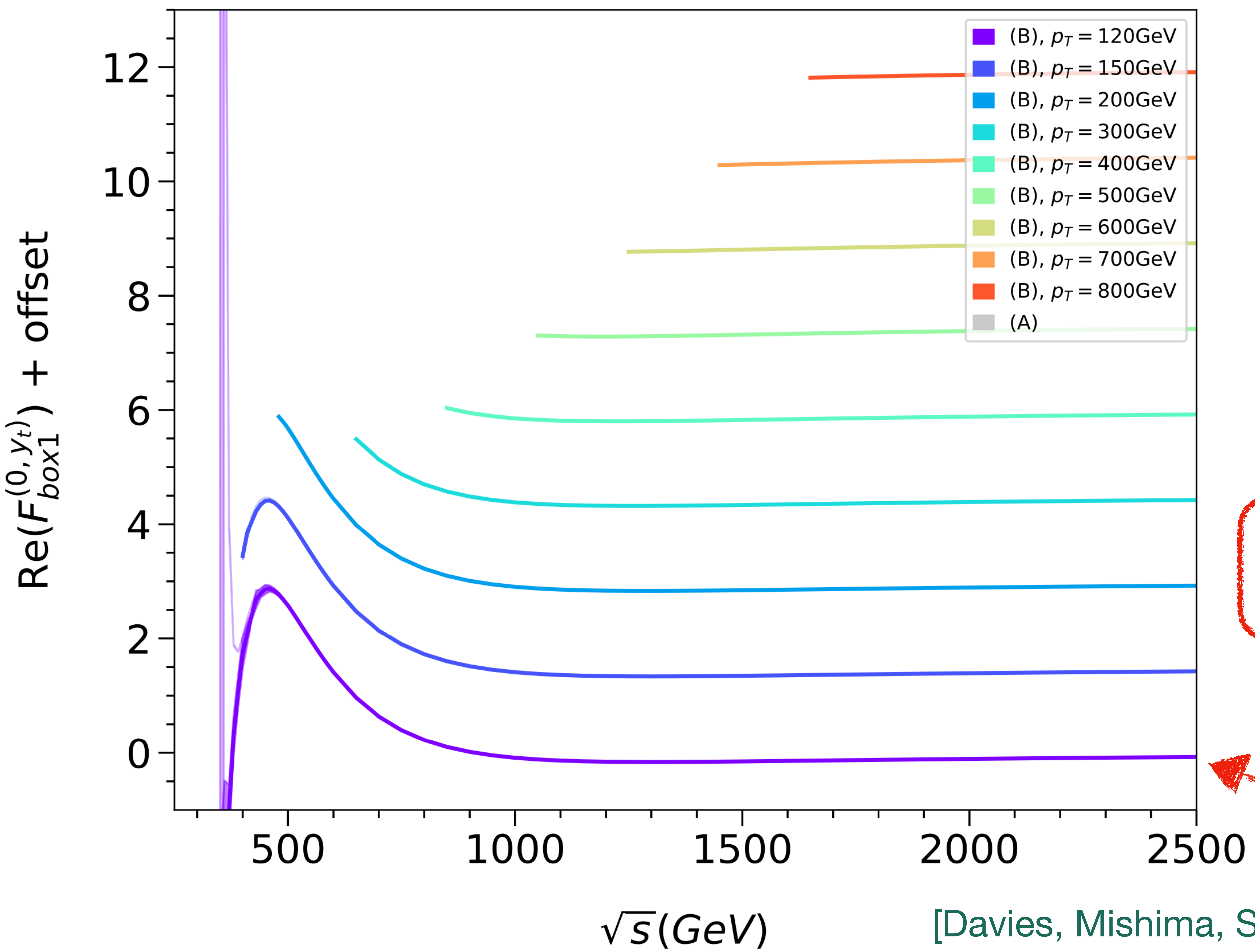
- for 2-dim 2-scale MB integral in non-planar diagrams

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{-t}{-s}\right)^{z_1} \left(\frac{-u}{-s}\right)^{z_2} f\left(\Gamma, \psi^{(i)}; z_1, z_2\right) \Rightarrow \text{HPLs}$$

(1). Expand in $(-t) \to 0$ limit to more than a hundred terms
(2). Solve expanded MB integrals exactly
(3). Reconstruct analytic results with ansat in Euclidean region (for planar integrals) or in physical region (with analytic continuation for non-planar integrals)

A demanding scenario for high energy expansion

Two-loop Yukawa correction to $gg \to HH$: heavy three-particle cuts $2m_t + m_H \approx 470$ GeV



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_1 + T_2^{\mu\nu}\,\mathscr{F}_2^{\mu\nu}$$

$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

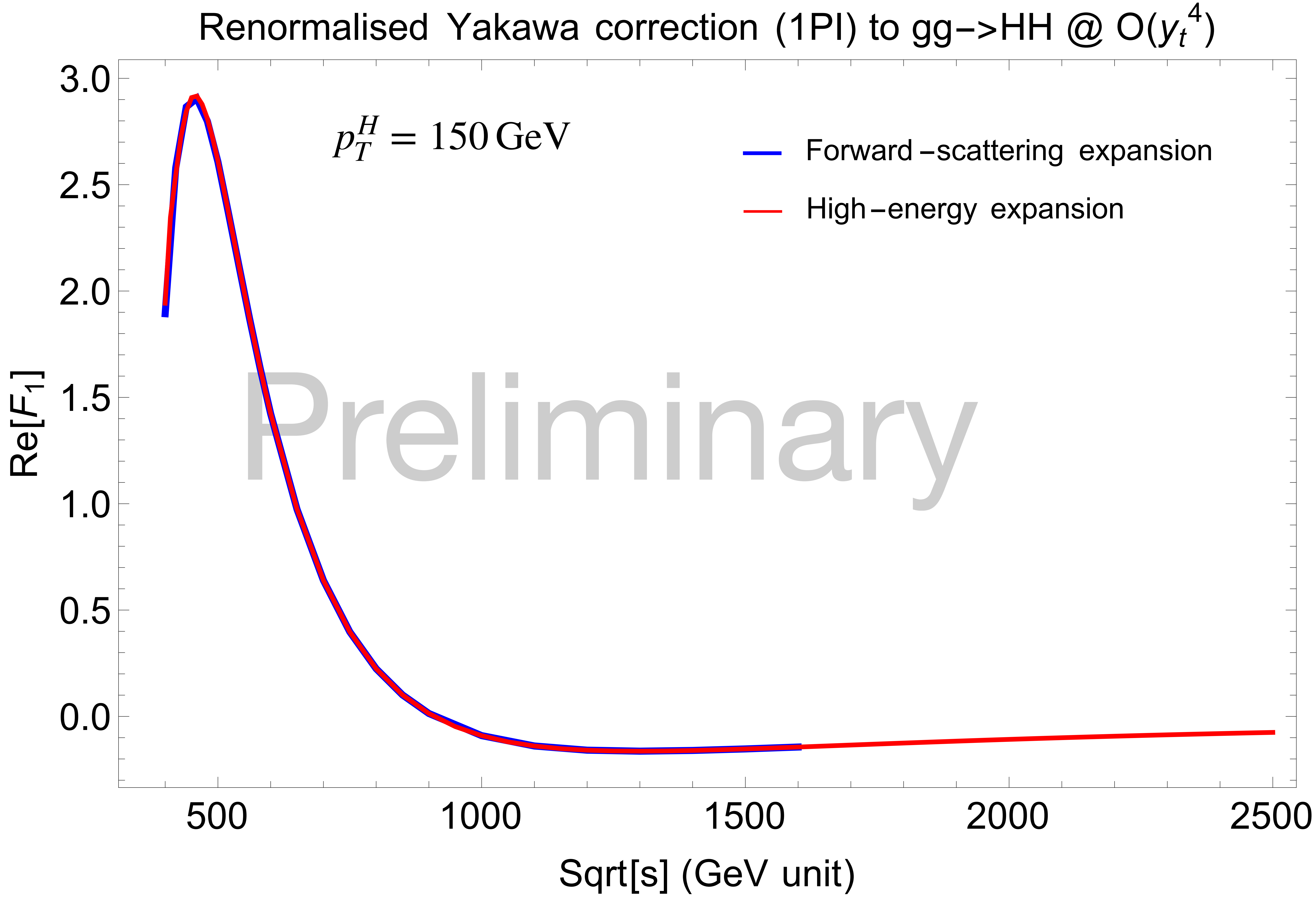
Padé improved high energy expansions converge even at $p_T^H = 120$ GeV

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]

Padé improved equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\mathrm{int}}\right)^2$ using high-energy MIs expanded to $\mathscr{O}(m_t^{116})$

Combination of forward-scattering and H.E. expansions

Two-loop Yukawa correction to $gg \to HH$



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_1 + T_2^{\mu\nu}\,\mathscr{F}_2^{\mu\nu}$$

$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

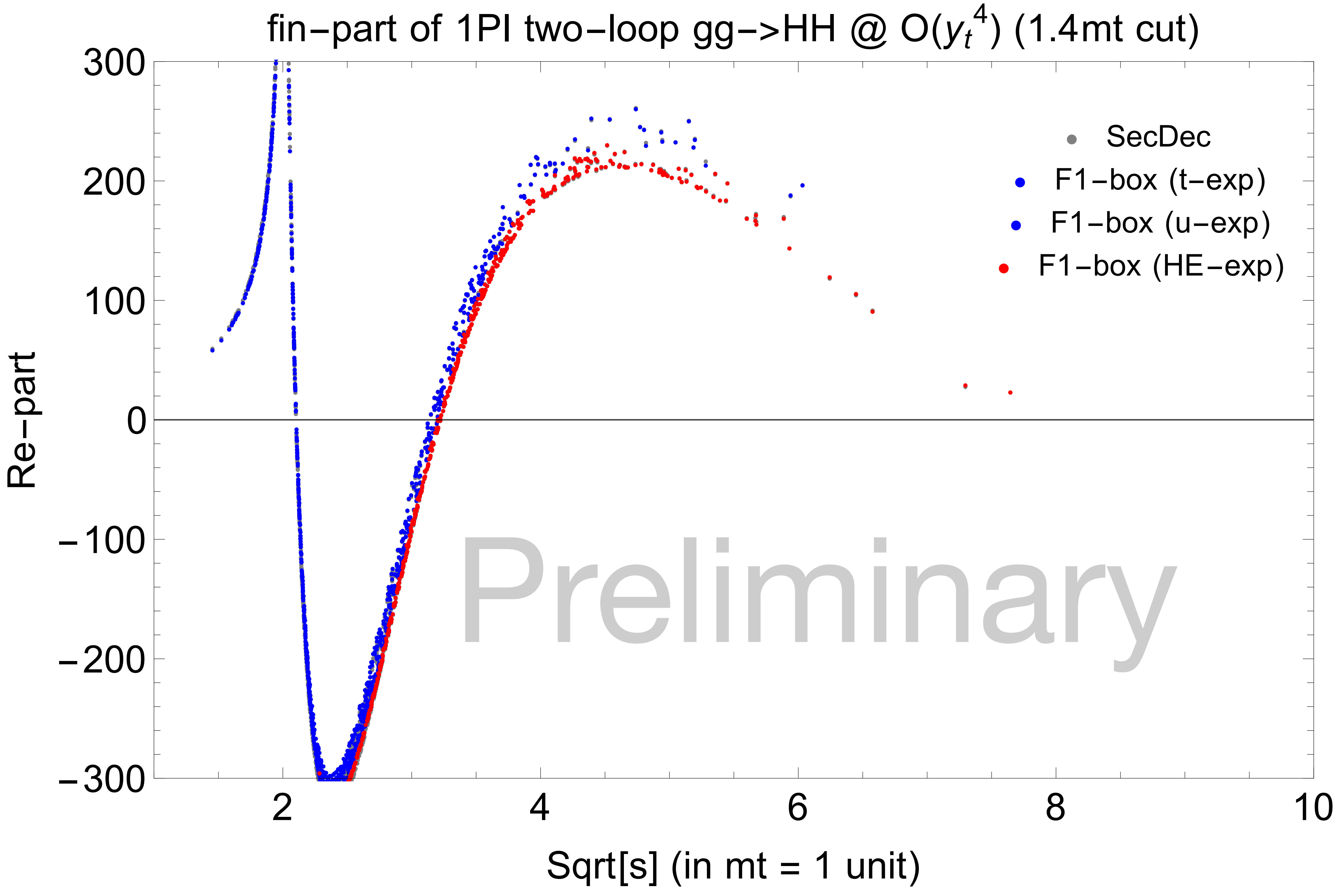
Highest available expansion terms are used

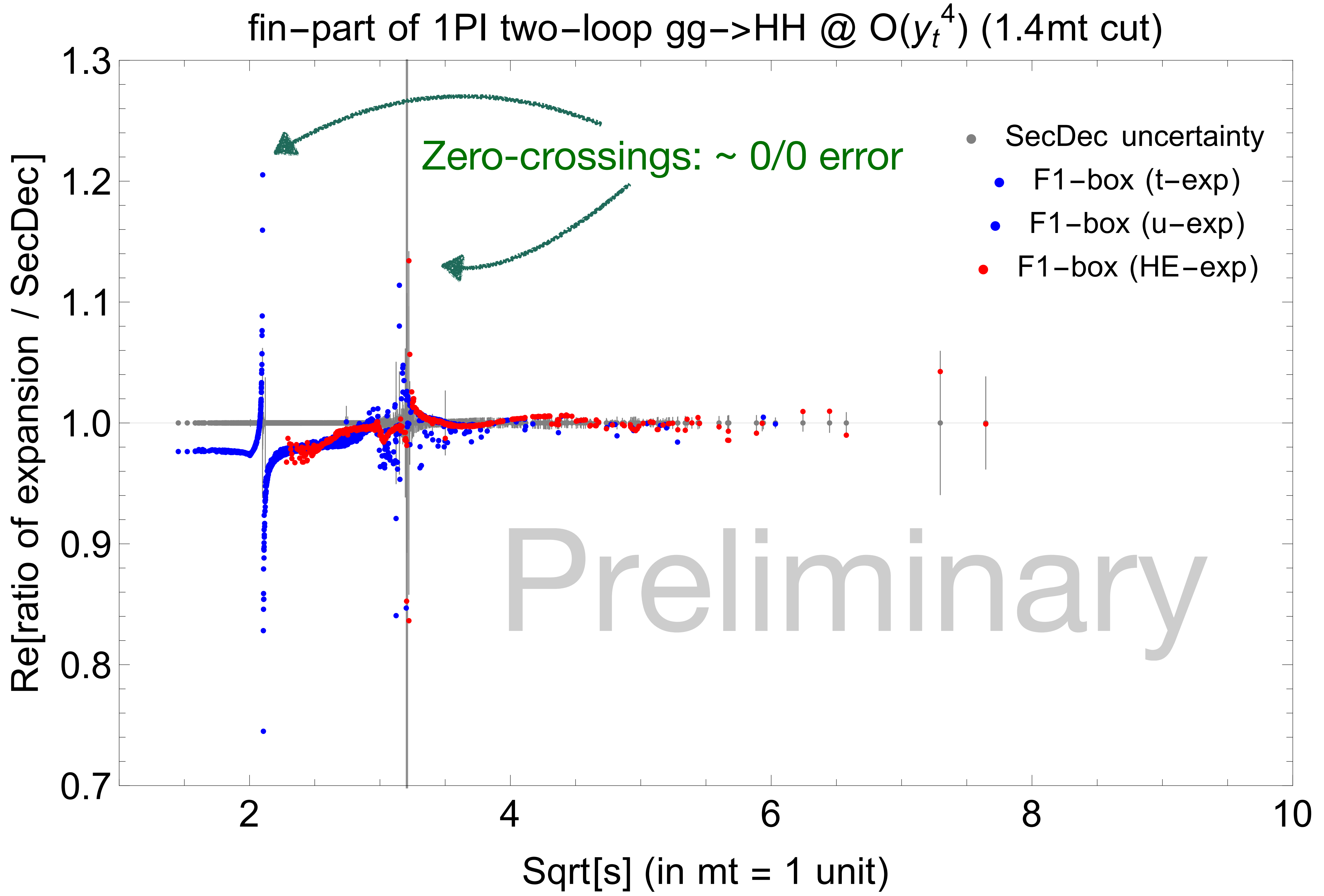
Comparison to SecDec numerical results

Two-loop Yukawa correction to $gg \to HH$

Finite part of bare two-loop form factor comparison to SecDec group

[Heinrich, Jones, Kerne, Stone, Vestner, 2407.04653]





High-energy expansion agree perfectly with SecDec results

Forward-scattering expansion under improvement

Conclusions

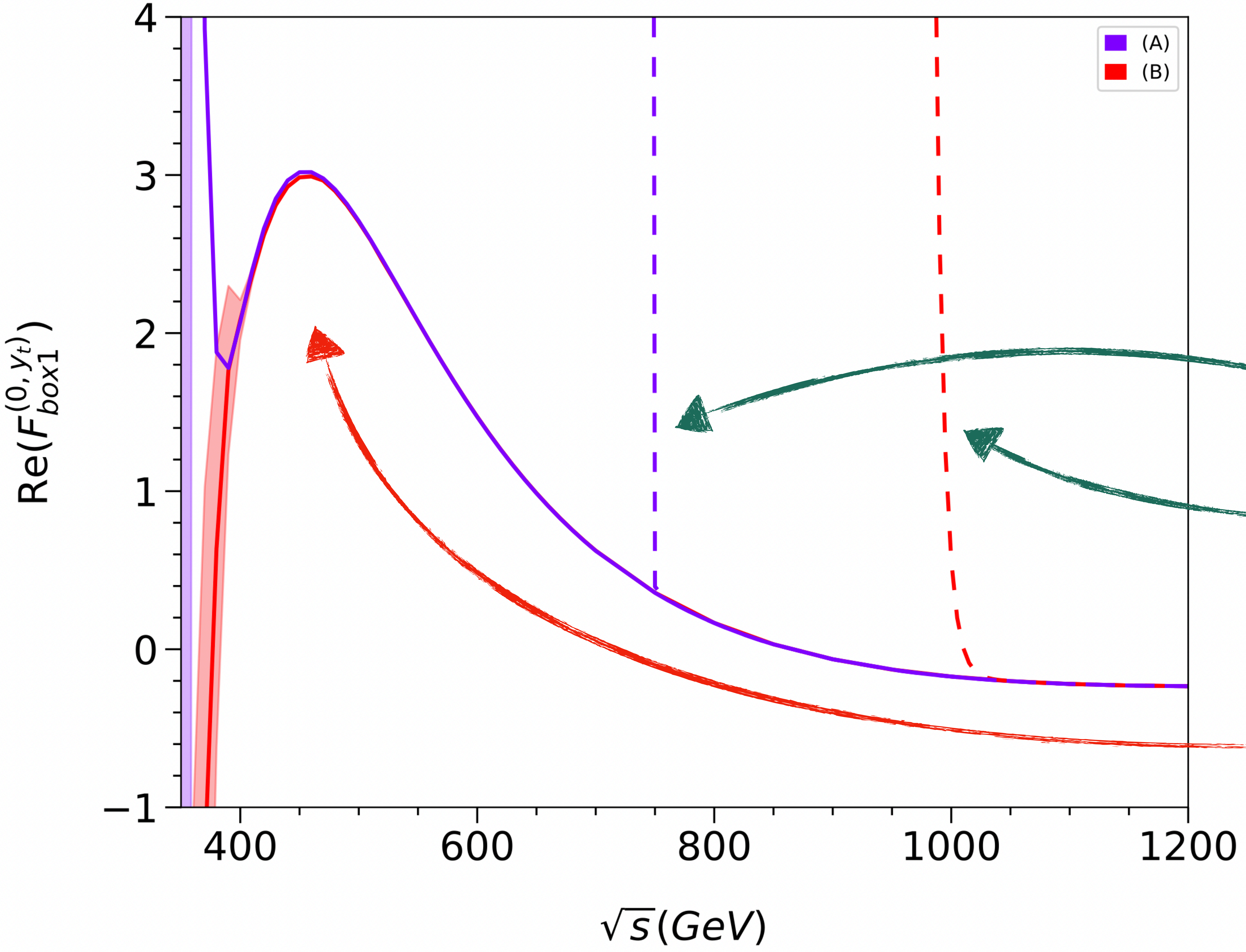
- **AsyInt** released in *[JHEP 09 (2024) 069]*
  - Toolbox for analytic massive two-loop four-point Feynman integrals at high energies
  - Download at: **https://gitlab.com/asyint/asyint-public**

- High-energy expansion works perfectly for a demanding scenario for two-loop leading Yukawa corrections to $gg \to HH$ *[JHEP 08 (2022) 259]*
  - Matches forward-scattering expansion down to $p_T = 150$ GeV
  - Matches SecDec group's numerical results

Backup Slides

High energy expansion @ NLO Yukawa

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\mathscr{F}_1 + T_2^{\mu\nu}\mathscr{F}_2^{\mu\nu}$$

Fixed scattering angle: $\theta = \frac{\pi}{2}$

Two internal-mass expansion approaches (A,B) (see backups)

Naive high-energy expansions starts to diverge at $\sqrt{s} \approx 700\ \&\ 1000$ GeV

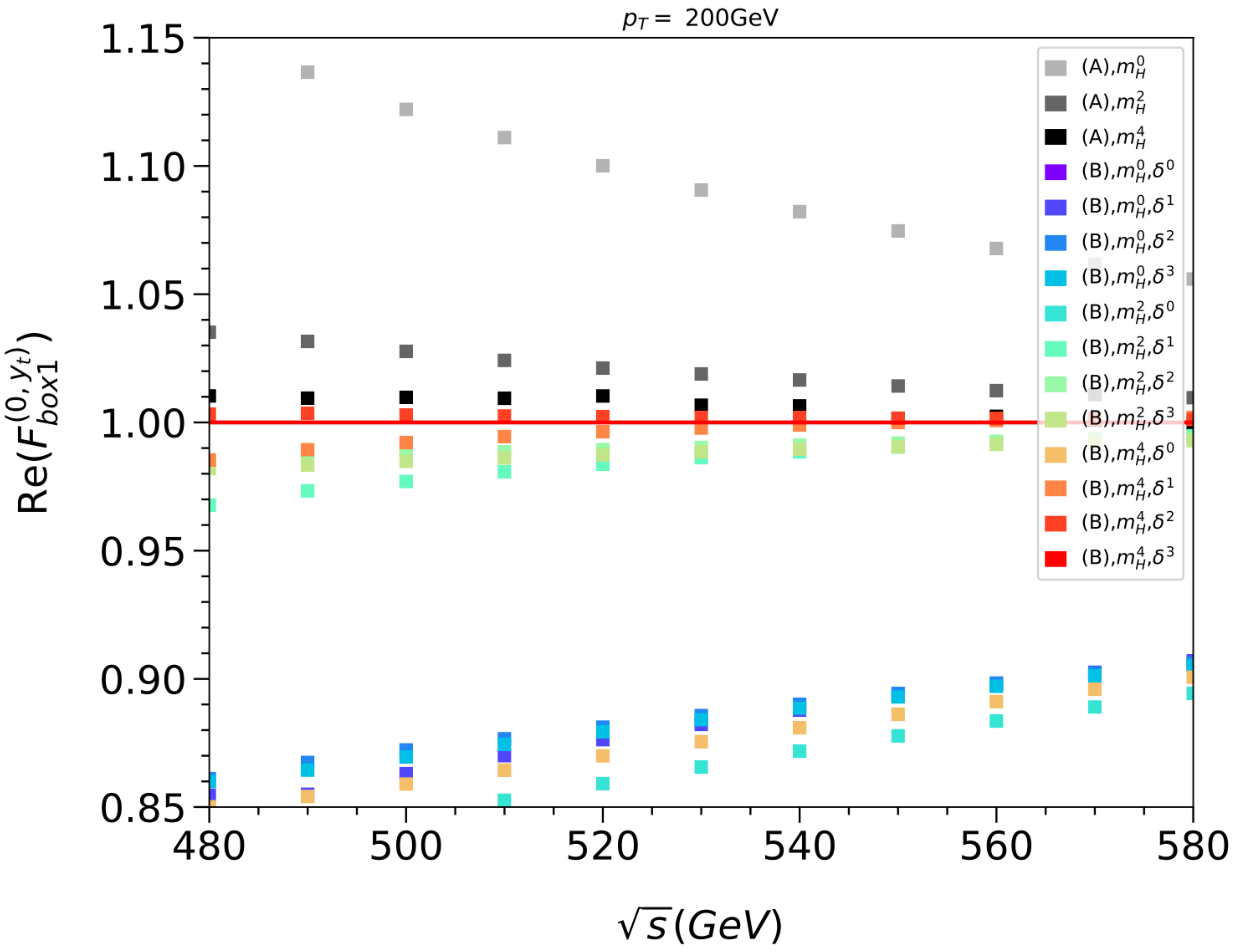
Padé improved high-energy expansions converge even at $\sqrt{s} \approx 500$ GeV

Solid color lines: Padé improved results using MIs from $\mathcal{O}(m_t^{116})$ in two expansion approaches

Dashed color lines: Naive expansions at high energies to $\mathcal{O}(m_t^{116})$

Convergence of H.E. expansions for $gg \to HH$ form factors

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_{\mathrm{box1}} + T_2^{\mu\nu}\,\mathscr{F}^{\mu\nu}_{\mathrm{box2}}$$

The benchmark is expansion at $\mathcal{O}\left(m^4_{H(\mathrm{ext})}, \delta^3, m_t^{116}\right)$.

$$\delta = 1 - \frac{m_H^{(\mathrm{int})}}{m_t}$$

Color points: Convergence plot of different expansion orders by ratios to the benchmark at fixed $p_T^H = 200$ GeV.