Nonleptonic *b*-decays with full charm mass dependence at NNLO

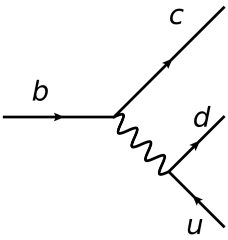
Young Scientists Meeting of the CRC TRR 257

Manuel Egner | Karlsruhe, September 26, 2024

INSTITUTE FOR THEORETICAL PARTICLE PHYSICS

based on 2406.19456 [Egner, Fael, Schönwald, Steinhauser (2024)]

Motivation



- Heavy Quark Expansion (HQE): Decay width of $B \to X_c q_2 q_3$ and $B \to X_u q_2 q_3$ as sum of decay width of $b \to q_1 q_2 q_3$ and corrections suppressed by the mass m_b:

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \cdots + 16\pi^2 \left(\Gamma_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \Gamma_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right).$$

- $\Gamma_3 \leftrightarrow$ decay of free b-quark.

- Our goal: Hadronic decay channels $b \to q_1 q_2 q_3$ at $\mathcal{O}\left(\alpha_s^2\right)$ with all charm mass effects.
- Input for the calculation of B-meson lifetimes in HQE.
- Knowing the $\mathcal{O}\left(\alpha_s^2\right)$ contributions will reduce the uncertainty induced by the renormalization scale variation.

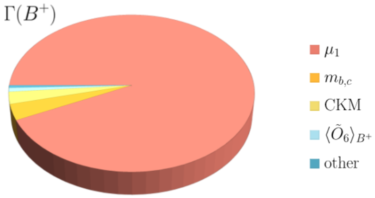
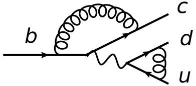


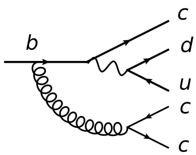
Figure: Uncertainty contributions on B-meson lifetimes [Albrecht, Bernlochner, Lenz, Rusov (2024)]

Decay channels at NNLO

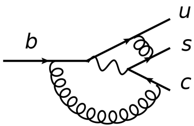
Four decay channels with charm quarks in the final state at $\mathcal{O}(\alpha_s^2)$:

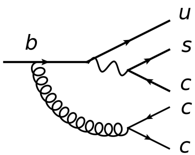
- $b \to c\bar{u}d$ ($\propto V_{cb}$)





- $b \to u\bar{c}s$ ($\propto V_{ub}$)





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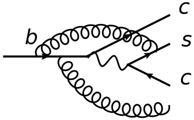

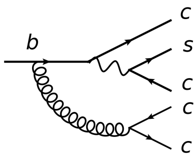

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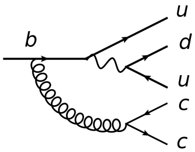


What is known?

Previous calculations including finite charm quark mass for Γ_3: Hadronic decay channel:

- $b \to c\bar{u}d$: $\mathcal{O}\left(\alpha_s^1\right)$ [Bagan, Ball, Braun, Gosdzinsky (1994)]
- $b \to c\bar{c}s$: $\mathcal{O}\left(\alpha_s^1\right)$ [Bagan, Ball, Fiol, Gosdzinsky (1995)],[Krinner, Lenz, Rauh (2013)]
- $\mathcal{O}\left(\alpha_s^2\right)$: first steps in [Czarnecki, Slusarczyk, Tkachov (2006)]

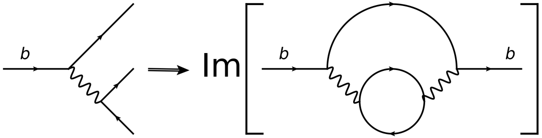
What we did:

- NNLO calculation of semileptonic decay channel $b \to cl\bar{\nu}$ to set up and check calculation procedure [Egner, Fael, Schönwald,Steinhauser (2023)]
- NNLO calculation of nonleptonic decay channels $b \to q_1\overline{q_2}q_3$ [Egner, Fael, Schönwald,Steinhauser (2024)]

Calculation setup

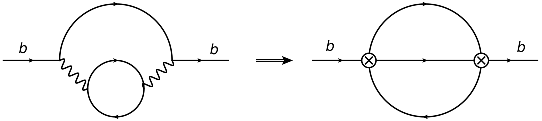
- Optical theorem:

$$\Gamma = \frac{1}{m_b}\mathrm{Im}\left[\mathcal{M}(b \to b)\right]$$

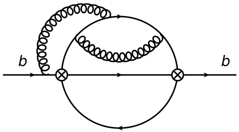


- Integrate out W-boson

$$\frac{1}{(m_W^2 - p^2)} \to \frac{1}{m_W^2}$$



- At $\mathcal{O}\left(\alpha_s^2\right)$ calculate imaginary part of 4-loop diagrams



Effective operators

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} V_{q_1 b} V^*_{q_2 q_3} \left(C_1(\mu_b) O_1^{q_1 q_2 q_3} + C_2(\mu_b) O_2^{q_1 q_2 q_3} \right) + \text{h.c.}$$

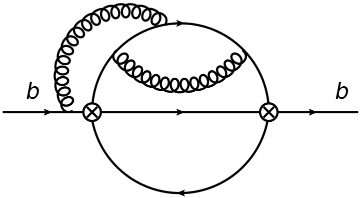
with physical operators

$$O_1^{q_1 q_2 q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha),$$
$$O_2^{q_1 q_2 q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\alpha)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\beta).$$

- Consider all possible combinations of operators:

$$\{O_1 \times O_1, O_1 \times O_2, O_2 \times O_1, \dots\}$$

- What to do with γ_5 in P_L?

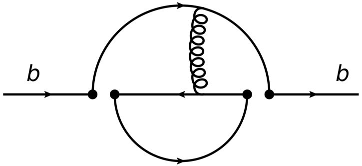


Treatment of γ_5

- At NLO and NNLO traces with one γ_5 matrix appears

$$\mathrm{Tr}\left[\gamma_5\gamma_\nu\gamma_\mu\gamma_\rho\gamma_\sigma\right]$$

  in $d = 4 - 2\epsilon$ dimensions



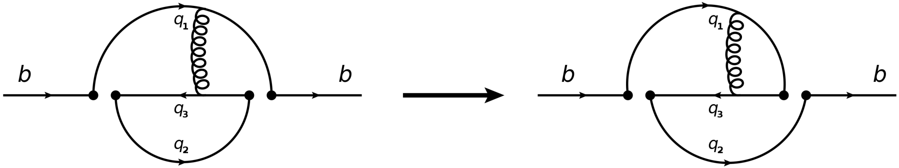
- Use anticommuting γ_5 to be consistent with calculation of matching coefficients and anomalous dimensions [Buras, Weisz (1990)], [Gorbahn, Haisch (2005)]
- Solution: Fierz identities [Bagan, Ball, Braun, Gosdzinsky (1994)]:

$$O_1^{q_1 q_2 q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha) \xrightarrow{\text{Fierz}} (\bar{q}_2^\beta \gamma^\mu P_L b^\beta)(\bar{q}_1^\alpha \gamma_\mu P_L q_3^\alpha)$$

Treatment of γ_5

- Fierzed operators correspond to unfierzed operators with switched color indices

$$O_1^{q_1 q_2 q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha) \xrightarrow{\text{Fierz}} (\bar{q}_2^\beta \gamma^\mu P_L b^\beta)(\bar{q}_1^\alpha \gamma_\mu P_L q_3^\alpha) = O_2^{q_2 q_1 q_3}$$



- Applying Fierz identities to one vertex leads to one instead of two spin lines:

$$\mathrm{Tr}\,[\ldots\gamma_5\ldots]\,\mathrm{Tr}\,[\ldots\gamma_5\ldots] \to \mathrm{Tr}\,[\ldots\gamma_5\ldots\gamma_5\ldots]$$

- We can use anticommuting γ_5!
- But: Fierz identities are valid in $d = 4$ dimensions!

Evanescent operators

- Fierz symmetry can be restored order by order in perturbation theory by choosing the correct evanescent operators [Buras, Weisz (1990)], [Herrlich, Nierste (1994)]

$$E_1^{(1),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\alpha) - (16 - 4\epsilon + A_1\epsilon^2) O_1^{q_1q_2q_3},$$
$$E_2^{(1),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\beta) - (16 - 4\epsilon + A_1\epsilon^2) O_2^{q_1q_2q_3},$$
$$E_1^{(2),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\alpha) - (256 - 224\epsilon + B_1\epsilon^2) O_1^{q_1q_2q_3},$$
$$E_2^{(2),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\beta) - (256 - 224\epsilon + B_2\epsilon^2) O_2^{q_1q_2q_3}$$

- Evanescent operators and physical operators mix under renormalization:

$$\begin{pmatrix} O_P \\ O_E \end{pmatrix} = Z \begin{pmatrix} O_{P,B} \\ O_{E,B} \end{pmatrix}$$

$\rightarrow$ Evanescent operators contribute to physical result at higher orders.

- $\mathcal{O}(\epsilon)$ piece $\leftrightarrow$ Fierz symmetries at NLO
- $\mathcal{O}(\epsilon^2)$ piece $\leftrightarrow$ Fierz symmetries at NNLO

Evanescent operators

- Fix $\{A_1, B_1, B_2\}$ by imposing a symmetric anomalous dimension matrix γ [Buras, Weisz (1990)]

$$\mu \frac{\mathrm{d}C_i}{\mathrm{d}\mu} = \gamma_{ij} C_j, \qquad \gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \quad \text{with } \gamma_{11} = \gamma_{22}, \quad \gamma_{12} = \gamma_{21}$$

- This condition yields

$$A_2 = -4, \qquad B_1 = -\frac{45936}{125}, \qquad B_2 = -\frac{115056}{115}$$

$$E_1^{(1),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\alpha) - (16 - 4\epsilon - 4\epsilon^2) O_1^{q_1q_2q_3},$$

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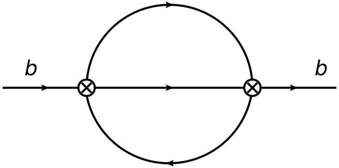
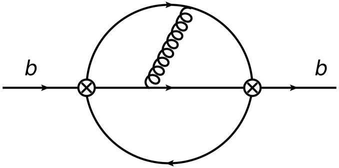
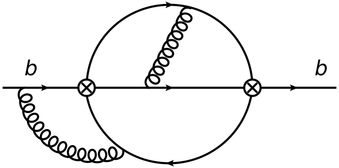
Evanescent operators



$$\{O_1, O_2, E_1^{(1)}, E_2^{(1)}, E_1^{(2)}, E_2^{(2)}\} \times \{O_1, O_2, E_1^{(1)}, E_2^{(1)}, E_1^{(2)}, E_2^{(2)}\}$$



$$\{O_1, O_2, E_1^{(1)}, E_2^{(1)}\} \times \{O_1, O_2, E_1^{(1)}, E_2^{(1)}\}$$



$$\{O_1, O_2\} \times \{O_1, O_2\}$$

Calculation setup

	$b \to c\bar{u}d$	$b \to c\bar{c}s$
■ Generate diagrams with `QGRAF` [Nogueira (1993)]	1308 diagrams	1308 diagrams
■ Mapping on integral families with `TAPIR` [Gerlach, Herren, Lang (2022)] and `EXP` [Harlander, Seidensticker, Steinhauser (1998-1999)]	42 families	49 families
■ Reduction to master integrals with `Kira` [Klappert, Lange, Maierhöfer, Usovitsch (2021)] ■ Choose good basis of master integrals, where ϵ and $\rho = m_c/m_b$ factorize, with `ImproveMasters.m` [Smirnov, Smirnov (2020)]	385 master integrals	644 master integrals

Master integrals: Expand and Match

[Fael, Lange, Schönwald, Steinhauser (2021)]

1. Set up differential equation for master integrals using IBP relations [Chetyrkin, Tkachov (1981)]

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\vec{I} = A(\epsilon, \rho) \cdot \vec{I}$$

2. Make general expansion ansatz in $\rho = m_c/m_b$ around certain point ρ_0 for integral

$$I_i(\rho, \rho_0) = \sum_{j=\epsilon_{\min}}^{\epsilon_{\max}} \sum_{m=0}^{j+|\epsilon_{\min}|} \sum_{n=n_{\min}}^{n_{\max}} c[i,j,m,n]\, \epsilon^j (\rho - \rho_0)^{n/2} \log^m(\rho - \rho_0)$$

3. Insert ansatz in DEQ $\rightarrow$ linear equations for $c[i,j,m,n]$ for every power in ρ
4. Determine remaining coefficients by matching to numerical results obtained with `AMFlow` [Liu, Ma (2022)]
5. Expansions around several expansion points, match in between
   $\rightarrow$ cover $\rho \in [0,1]$.

Master integrals: Expand and Match

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Which expansion for which expansion point?

- Depends on singular points of the $A(\epsilon,\rho)$:
  - $b \to c\bar{u}d$ & $b \to u\bar{c}s$: $\{0, 1/3, 1\}$
  - $b \to c\bar{c}s$: $\{0, 1/4, 1/2, 1\}$
  - $b \to u\bar{u}d$: $\{0, 1/2, 1\}$

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\vec{I} = A(\epsilon,\rho)\cdot\vec{I}$$

$$= \frac{A'(\epsilon,\rho)\cdot\vec{I}}{\rho - 1/3} + \frac{A''(\epsilon,\rho)\cdot\vec{I}}{\rho - 1} + \dots$$

$$I_i(\rho,\rho_0) = \sum_{j=\epsilon_{\min}}^{\epsilon_{\max}} \sum_{m=0}^{j+|\epsilon_{\min}|} \sum_{n=n_{\min}}^{n_{\max}} c[i,j,m,n]\,\epsilon^j (\rho-\rho_0)^{n/2} \log^m(\rho-\rho_0)$$

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- Expansion around $\{2, 4\}$-particle threshold

$$\to \quad I_i(\rho, \rho_0) = \sum_{j=\epsilon_{\min}}^{\epsilon_{\max}} \sum_{m=0}^{j+|\epsilon_{\min}|} \sum_{n=n_{\min}}^{n_{\max}} c[i,j,m,n]\, \epsilon^j (\rho - \rho_0)^{n/2} \log^m(\rho - \rho_0)$$

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- Expansion around $\rho_0 \in \{0, 1\}$ or 3-particle threshold

$$\to \quad I_i(\rho, \rho_0) = \sum_{j=\epsilon_{\min}}^{\epsilon_{\max}} \sum_{m=0}^{j+|\epsilon_{\min}|} \sum_{n=0}^{n_{\max}} c[i, j, m, n]\, \epsilon^j (\rho - \rho_0)^{n} \log^m(\rho - \rho_0)$$

Master integrals: Expand and Match

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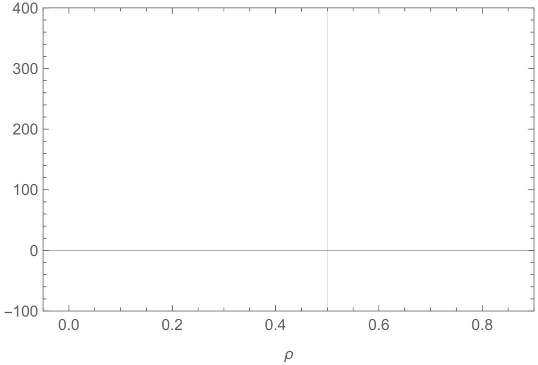
- Expansion around regular point

$$\rightarrow \quad I_i(\rho,\rho_0) = \sum_{j=\epsilon_{\min}}^{\epsilon_{\max}} \sum_{n=0}^{n_{\max}} c[i,j,0,n]\,\epsilon^j (\rho-\rho_0)^n$$

Master integrals: Example

1. Taylor expansion around $\rho_0 = 0.5$
2. Obtain numerical values of integrals at $\rho = 0.5$ with `AMFlow`
3. Determine expansion coefficients by matching expansion of step 1 to numerical results of step 2
4. Threshold expansion around $\rho_0 = 1/3$
5. Evaluate expansions of step 1 at $\rho = 0.4$
6. Determine expansion coefficients of expansion of step 4 by matching to numerical results of step 5
7. Repeat procedure for next expansion point



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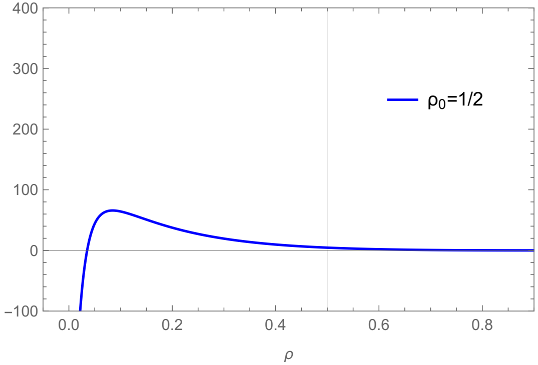

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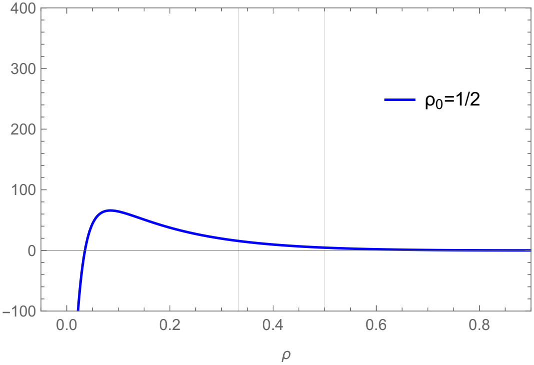

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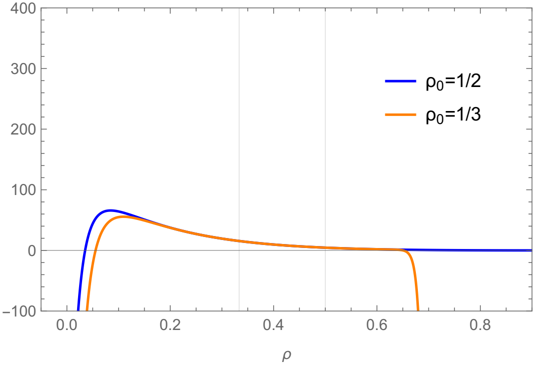

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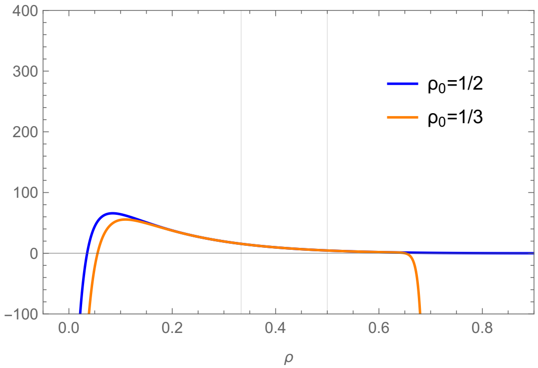
6. Determine expansion coefficients of expansion of step 4 by matching to numerical results of step 5
7. Repeat procedure for next expansion point



Master integrals: Example

1. Taylor expansion around $\rho_0 = 0.5$
2. Obtain numerical values of integrals at $\rho = 0.5$ with `AMFlow`
3. Determine expansion coefficients by matching expansion of step 1 to numerical results of step 2
4. Threshold expansion around $\rho_0 = 1/3$
5. Evaluate expansions of step 1 at $\rho = 0.4$
6. Determine expansion coefficients of expansion of step 4 by matching to numerical results of step 5
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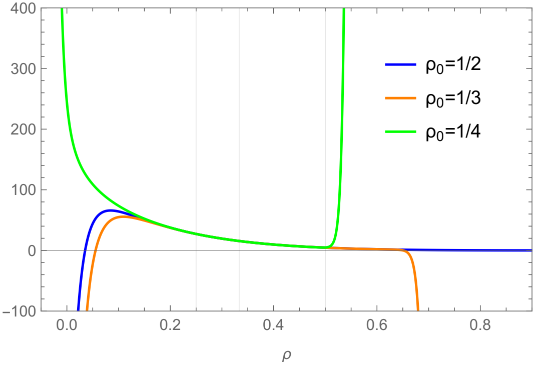

Master integrals: Example

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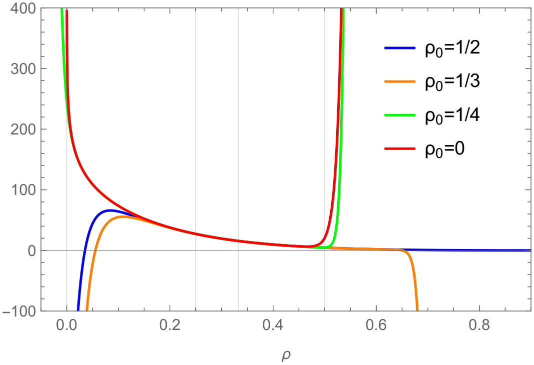


Results

$$\Gamma\left(b \to c\bar{u}d\right) = \frac{G_f^2 m_b^5 \left|V_{bc}\right|^2}{192\pi^3}\left[C_1^2(\mu)G_{11} + C_1(\mu)C_2(\mu)G_{12} + C_2^2(\mu)G_{22}\right]$$

with

$$G_{ij} = G_{ij}^{(0)} + \frac{\alpha_s}{\pi}G_{ij}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 G_{ij}^{(2)}.$$

- All $G_{ij}^{(n)}$ as functions of the mass ratio $\rho = m_c/m_b$ expanded around value ρ_0 and colour factors.
- $n_l = 3$, $n_c = 1$, $n_h = 1$ number of light, charm and bottom quarks.
- Both $G_{ij}^{(n)}$ and $C_i(\mu)$ are depending on the choice of evanescent operators and the γ_5 scheme.

decay channel	LO	NLO	NNLO
$b \to c\bar{u}d$	yes	yes	yes
$b \to u\bar{u}d$	yes	yes	yes
$b \to u\bar{c}s$	yes	yes	yes
$b \to c\bar{c}s$	yes	yes	yes

Results: $b \to c\bar{u}d$

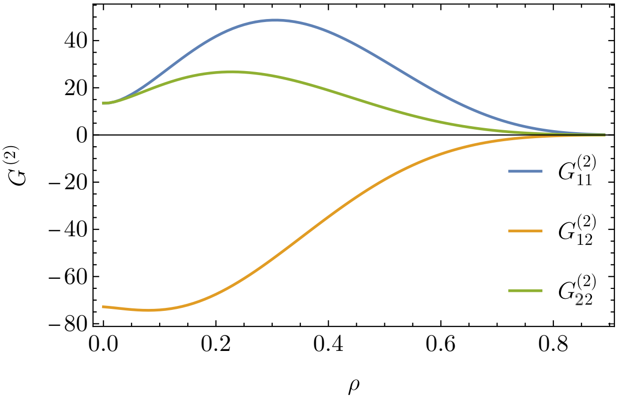
$$\Gamma(b \to c\bar{u}d) = \frac{G_f^2 m_b^5 |V_{bc}|^2}{192\pi^3}\left[C_1^2(\mu)G_{11} + C_1(\mu)C_2(\mu)G_{12} + C_2^2(\mu)G_{22}\right]$$

- Fierz identites:

$$(\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha)$$
$$\xrightarrow{\text{Fierz}} (\bar{q}_2^\beta \gamma^\mu P_L b^\beta)(\bar{q}_1^\alpha \gamma_\mu P_L q_3^\alpha)$$

- $b \to c\bar{u}d$: $q_1 = c$, $q_2 = u$
- In the limit $\rho \to 0$: $q_1 = q_2 = u$

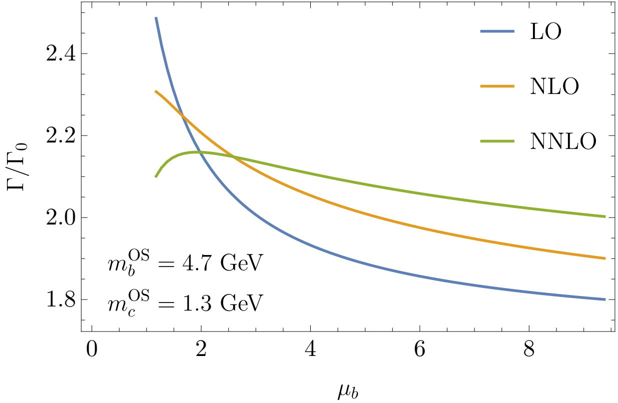
$$\to G_{11}^{(2)} = G_{22}^{(2)}$$



[Egner, Fael, Schönwald, Steinhauser (2024)]

Results $b \to c\bar{u}d$

- Scheme dependence drops out in the final result for Γ
- Renormalization scale dependece reduced by $\approx 50\%$



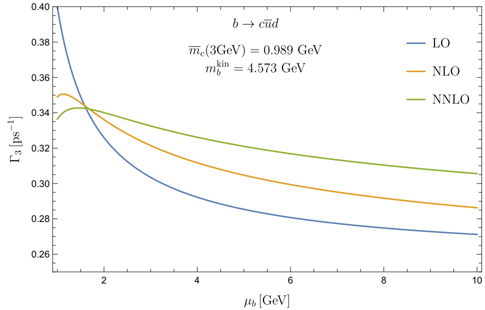
$$\Gamma\left(b \to c\bar{u}d\right) = \frac{G_f^2 m_b^5 \left|V_{bc}\right|^2}{192\pi^3}\left[C_1^2(\mu)G_{11} + C_1(\mu)C_2(\mu)G_{12} + C_2^2(\mu)G_{22}\right]$$

$$= \Gamma_0\left[1.89907 + 1.77538\left(\frac{\alpha_s}{\pi}\right) + 14.1081\left(\frac{\alpha_s}{\pi}\right)^2\right]\Bigg|_{\mu=m_b}$$

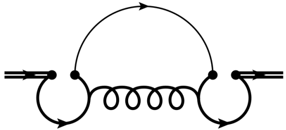
[Egner, Fael, Schönwald, Steinhauser]

Outlook

preliminary!



- Phenomenological analysis: different mass schemes
  $\to$ In collaboration with Alexander Lenz, Maria Laura Piscopo and Aleksey Rusov (University of Siegen).



- $b \to c\bar{c}s$: Penguin-like topologies with insertions of O_1 and O_2 at NLO and NNLO.

Thank you for your attention!