Towards HH at NNLO QCD: the n_h^2 Contribution

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Work in collaboration with **J. Davies, K. Schönwald, M. Steinhauser**

Higgs Physics at the LHC

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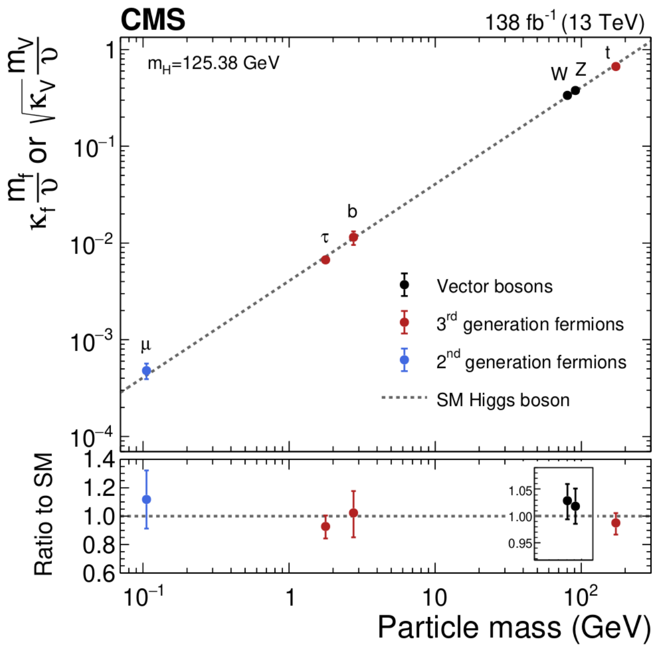
Does the discovered Higgs boson behave as the SM predicts?

What we still don't know after Run2

- Yukawa couplings of first and second generation
- Higgs total decay width
- Shape of the Higgs potential

$$V(h) = \frac{m_H^2}{2} h^2 + \lambda_3 v h^3 + \frac{\lambda_4}{4} h^4$$

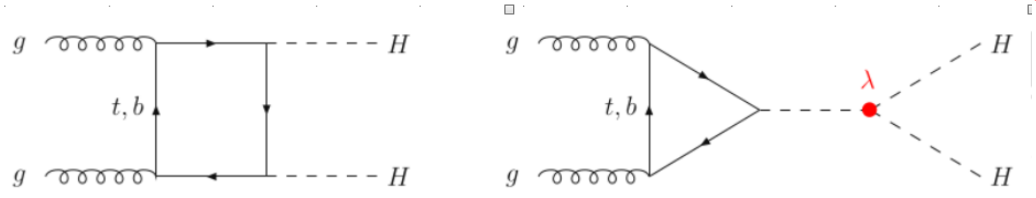
$$\lambda_4^{\mathrm{SM}} = \lambda_3^{\mathrm{SM}} = \lambda = m_H^2/(2\,v^2)$$

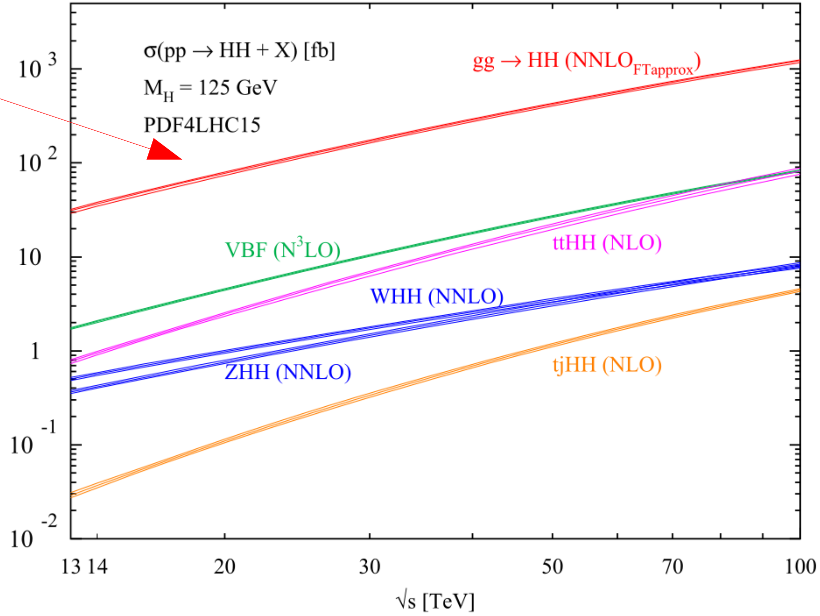


[CMS - 2207.00043]

HH Production @LHC

- Best chance to measure λ_3
- Gluon-initiated channel dominant

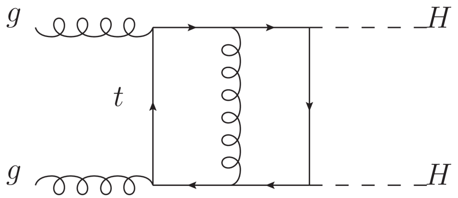




- LO computed in [Glover, van der Bij ('88); Plehn et al. (96)]
- In the SM, destructive interference between triangle (signal) and box (background)
- Accurate predictions required for both

[Di Micco et al. - 1910.00012]

NLO QCD corrections for HH



Full top-mass dependence obtained via

Multi-scale $(s, t, m_{H,} m_t)$ two-loop integrals
No exact analytic results available

- Numerical evaluation
  [Borowka et al. - 1604.06447, 1608.04798; Baglio et al. - 1811.05692]
- Analytic approximations
  Large Mass Expansion [Dawson, Dittmaier, Spira '98]
  pT expansion [Bonciani et al. - 1806.11564]
  High-Energy expansion [Davies et al. - 1811.05489]
  Small-mass expansion [Wang et al. - 2010.15649]
  Full phase space covered in [Bellafronte et al. – 2202.12157; Davies et al. - 2302.01356]

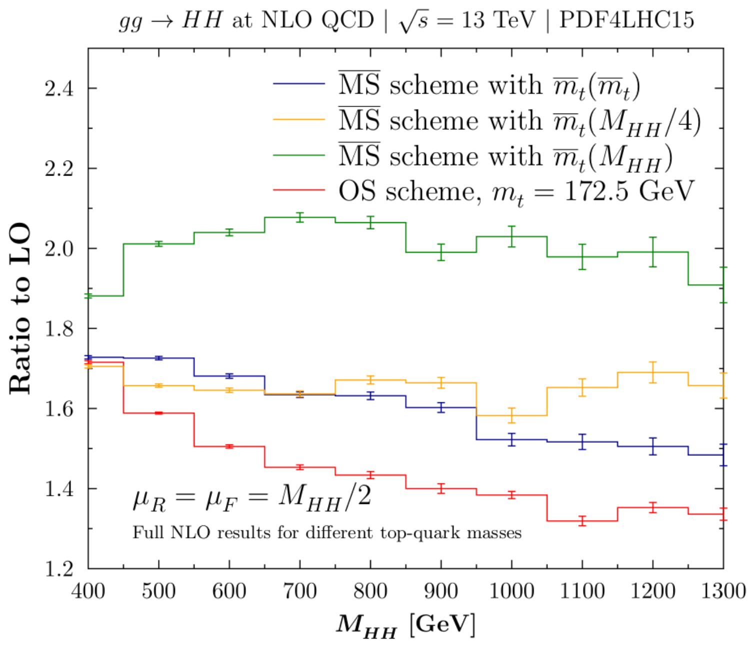
Theoretical Uncertainties at NLO QCD

- Scale uncertainties reduced to O(15%)

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$
On-Shell	$18.22^{29.5\%}_{-21.3\%}$	-	$30.93^{13.7\%}_{-12.7\%}$	-

[Bagnaschi, Degrassi, Gröber - 2309.10525]

- Large uncertainty due to choice of renormalization scheme and scale for the top mass
- NNLO would still be desirable



[Baglio et al. - 2008.11626]

Analytic approximations for NNLO QCD

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Exploit **hierarchies** of masses/kinematic invariants

Pros: simplified integral structures; can change parameters easily

Cons: proliferation of integrals; restricted to specific phase-space regions

$m_t \to \infty$ limit (N3LO)

[De Florian, Mazzitelli 1305.5206 and 1309.6594; . Grigo, Melnikov and Steinhauser – 1408.2422; Chen et al. - 1909.06808 and 1912.13001;]

Finite $1/m_t$ effects

[Grigo, Hoff, Steinhauser - 1508.00909;
Davies, Steinhauser – 1909.01361; Davies et al. 2110.03697]

Forward Expansions

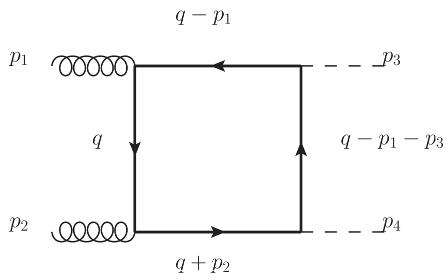
- pT expansion $m_H^2, p_T^2 \ll m_t^2, \hat{s}$

  [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

- $t \to 0$ expansion $m_H^2, \hat{t} \ll m_t^2, \hat{s}$

  [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]

pT Expansion $m_H^2, p_T^2 \ll m_t^2, \hat{s}$



1) Consider a **one-loop** box integral

$$\int d^D q \, \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2}(q\cdot p_2)^{n_3}(q\cdot p_3)^{n_4}}{(q^2-m_t^2)[(q+p_2)^2-m_t^2][(q-p_1-p_3)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; explicit transverse momentum (Sudakov)

$$p_3^\mu = -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + r_\perp^\mu \qquad \frac{t'}{s'} = -\frac{1}{2}\left\{1-\sqrt{1-2\frac{p_T^2+m_H^2}{s'}}\right\}$$

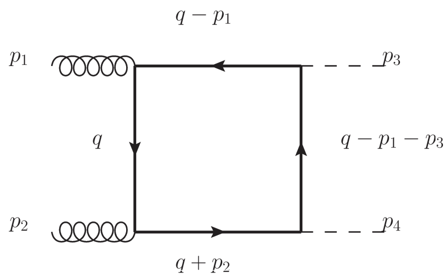
$$r_\perp^2 = -p_T^2$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

Taylor expansion →
$$\int d^D q \, \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2'}(q\cdot p_2)^{n_3'}(q\cdot r_\perp)^{n_4'}}{(q^2-m_t^2)^{l_1}[(q+p_2)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

4) Tensor + IBP Reduction $\rightarrow$ Dependence on $r_\perp$ removed

pT Expansion $m_H^2, p_T^2 \ll m_t^2, \hat{s}$



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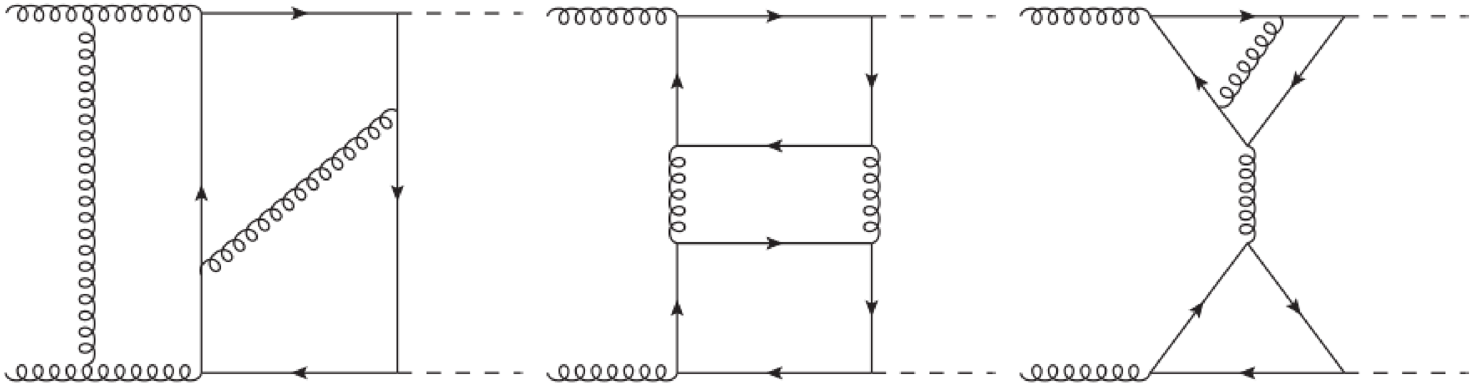
$$I(\hat{s}, p_T^2, m_H^2, m_t^2) \rightarrow \mathrm{MI}(\hat{s}/mt^2)$$ **single-scale integrals**

Going to NNLO QCD...

Can we use the forward expansion for higher orders?

- Classes of three loop diagrams



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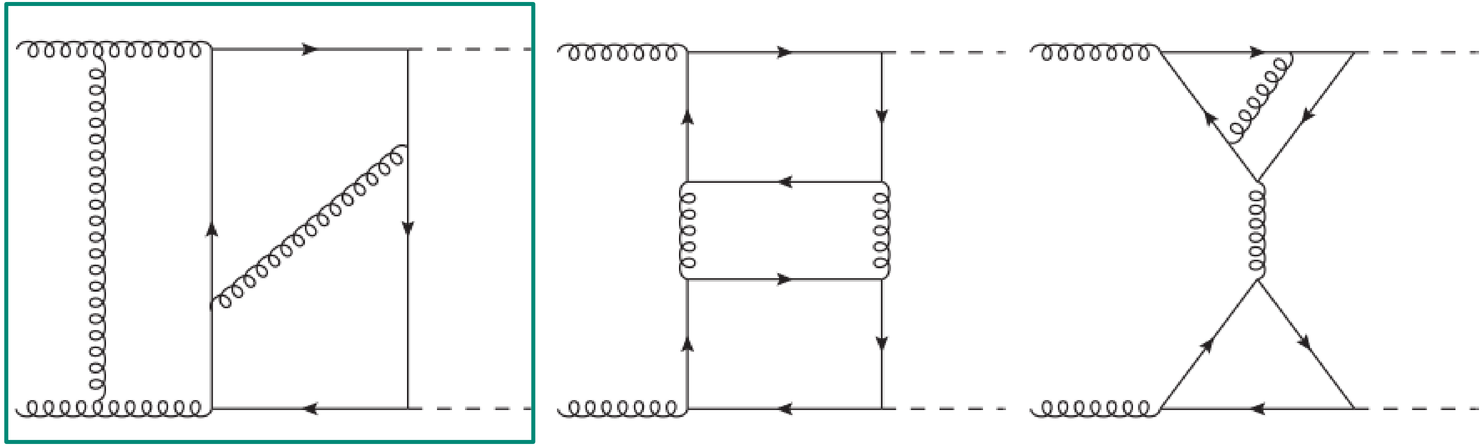
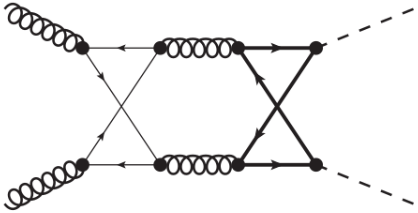


Conceptually yes

Practical implementation promising for the *t*→*0* expansion $\{t^0, m_H^0\}$

[Davies, Schönwald, Steinhauser 2307.04796]



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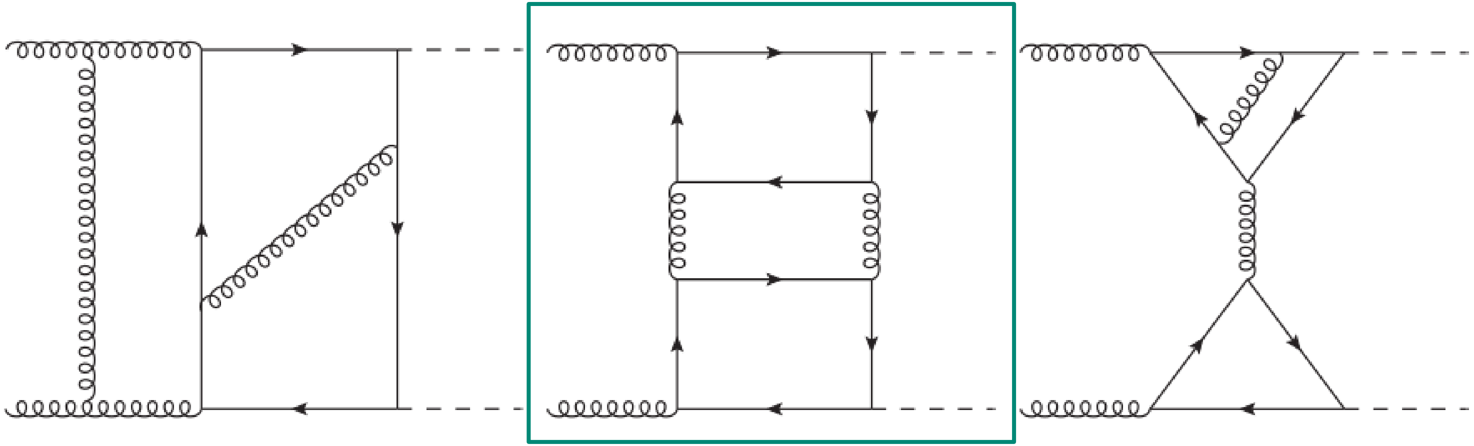

More involved due to t-channel cuts
through massless lines
$\Rightarrow$ A Taylor expansion is not sufficient

Going to NNLO QCD...

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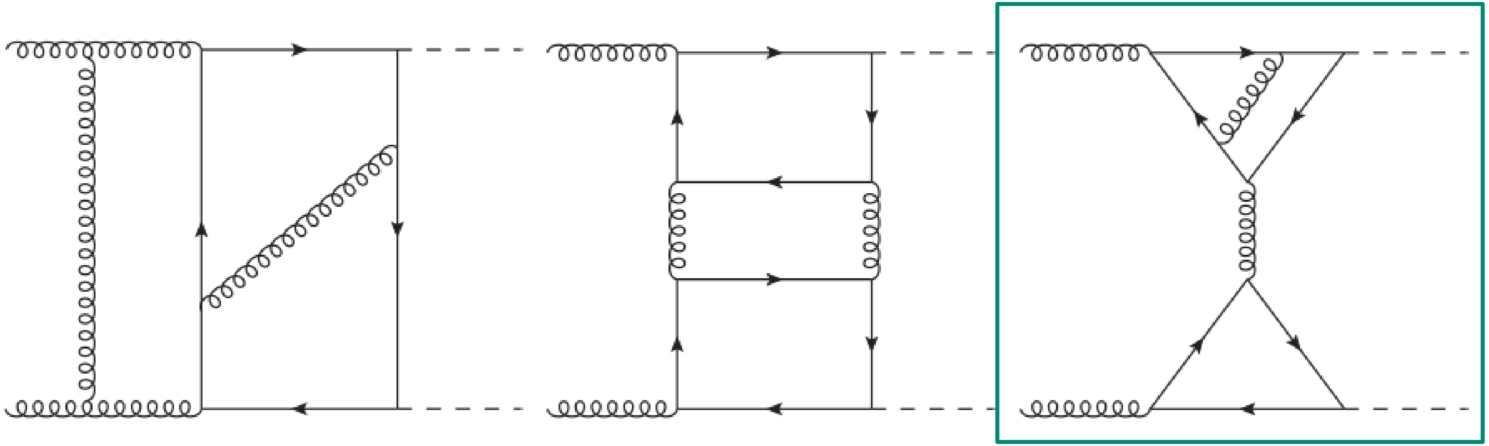
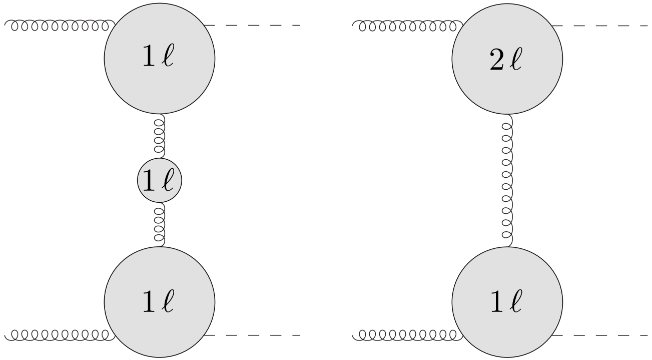


Start by studying the 1PR piece

1PR Contribution to *gg*→*HH* @ 3 Loops

[Davies, Schönwald, Steinhauser, MV - 2405.20372]

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0 s\left(F_1 A_1^{\mu\nu} + F_2 A_2^{\mu\nu}\right)$$

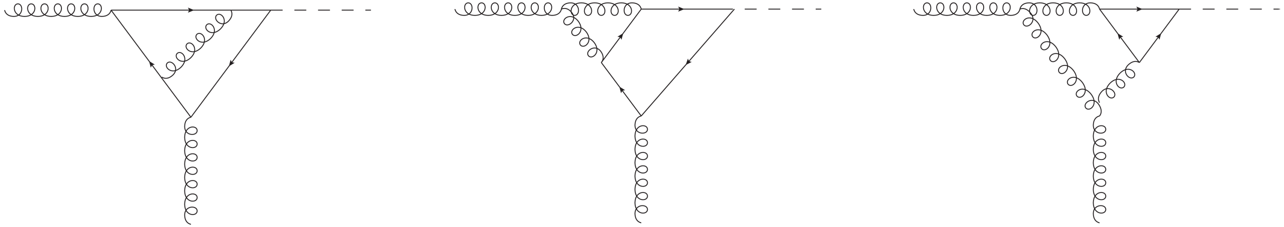


Goal: compute $F_1^{(3\ell,\ 1\mathrm{PR})}$ $F_2^{(3\ell,\ 1\mathrm{PR})}$

Approach: construct the *gg*→*HH* form factors from the 1PI gg*H subamplitudes

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = F_a\ g^{\alpha\beta}(q_s\cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$

$$q_2^2 = 0, q_s^2 \neq 0$$
$$m_H \neq 0$$



Outline of Calculation

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1. Generation of diagrams with qgraf [Nogueira, '93]

2. Manipulation with tapir [Gerlach, Herren, Lang - 2201.05618], q2e/exp [Harlander, Seidensticker Steinhauser – '97], FORM [Ruijl, Ueda, Vermaseren - 1707.06453]

3. IBP reduction (KIRA [Klappert, Lange, Maierhöfer, Usovitsch – 2008.06494])

4. Perform asymptotic expansions in two limits

$$m_H^2 \ll q_s^2, m_t^2$$

$$q_s^2 \ll m_H^2, m_t^2$$

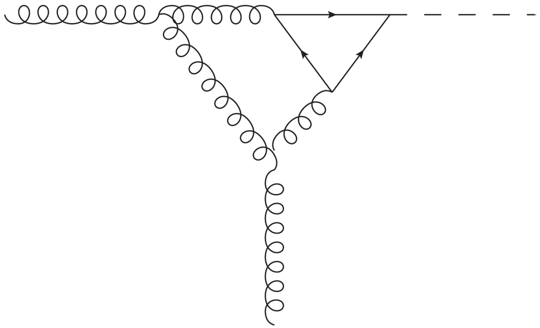
Same MIs from t → 0 expansion at NLO
Evaluated using "expand and match" approach

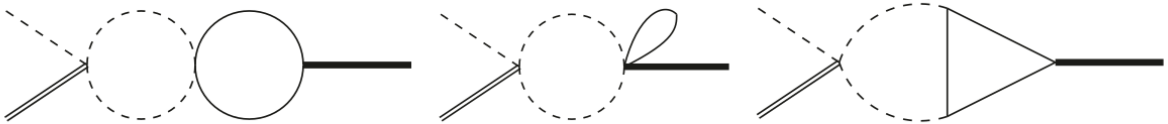
[Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]

Results expressed in terms of HPLs

*gg*H* Form Factors

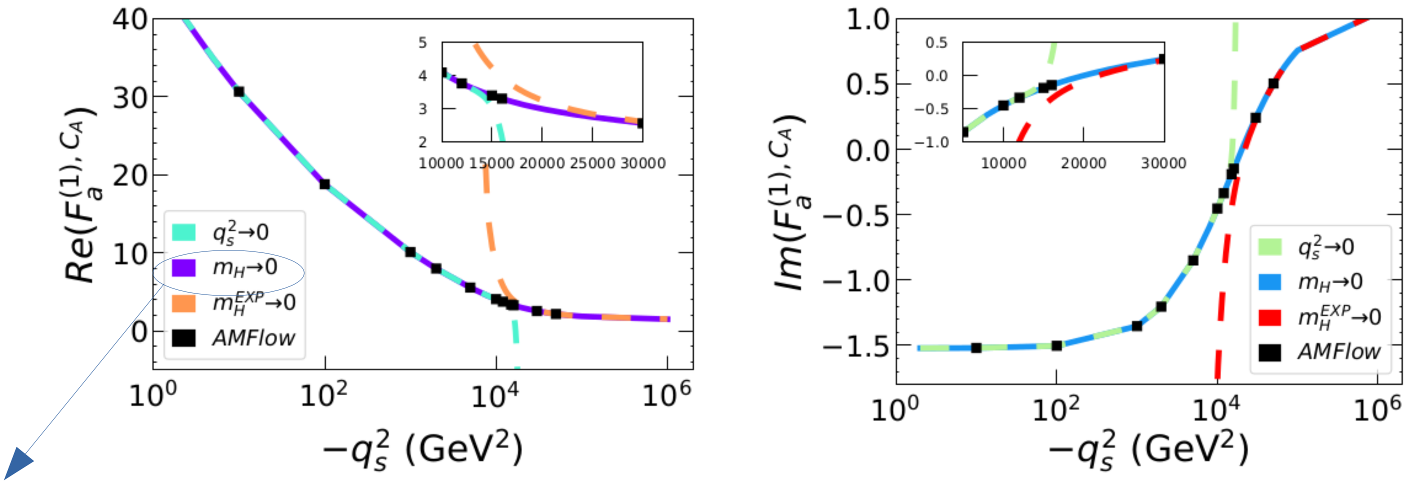
- A Taylor expansion of the two-loop integrals is not possible due to diagrams where the off-shell gluon couples to massless internal lines
- Three topologies require an asymptotic expansion





*gg*H* Form Factors

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = \boxed{F_a} g^{\alpha\beta}(q_s \cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$



- Use expanded MIs but keep coefficients exact ($m_H \to 0$)
- Results checked with AMFlow [Liu, Ma - 2201.11669]

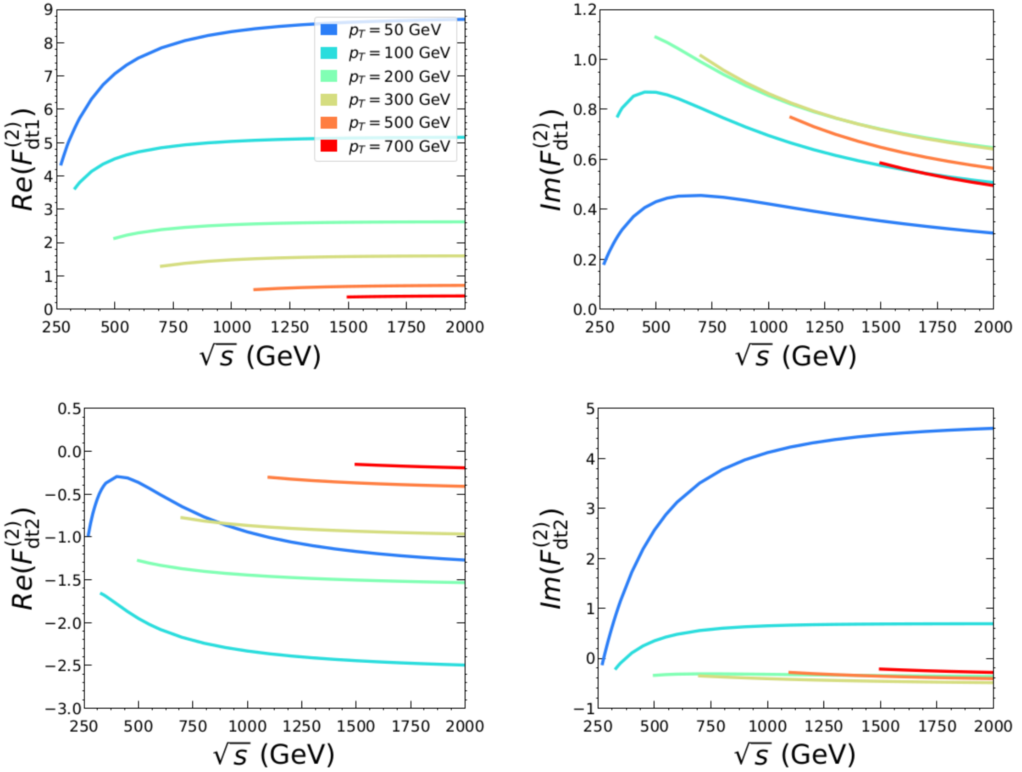
$gg \to HH$ Form Factors

$$\tilde{F}^{(2)}_{\rm dt1}(t) = F^{(0)}_a(t)\left[F^{(1)}_a(t) + \frac{1}{2}\,F^{(0)}_a(t)\,\Pi_{gg}(t) - \frac{s\,(\epsilon\,(m_H^2 - 2p_T^2 + t) + 2p_T^2)}{(1-2\epsilon)(m_H^2 - s)t}\,F^{(1)}_d(t)\right]$$

$$\tilde{F}^{(2)}_{\rm dt2}(t) = F^{(0)}_a(t)\left[\frac{p_T^2}{t}\left(F^{(1)}_a(t) + \frac{1}{2}\,F^{(0)}_a(t)\,\Pi_{gg}(t)\right) - \frac{s\,(\epsilon\,(2p_T^2 - m_H^2 - t) + m_H^2 + t)}{(1-2\epsilon)(m_H^2 - s)t}\,F^{(1)}_d(t)\right]$$

- Agreement with LME result of

  [Davies, Steinhauser - 1909.01361]



Conclusions

- Including NNLO QCD effects in $gg \to HH$ would allow full control over scale and top-mass-scheme uncertainties
- An expansion in the forward-scattering limit is a promising way to obtain fast and flexible results *and* a wide coverage of the phase space
- At three loops, asymptotic expansions are necessary to account for the n_h^2 contribution, already in the reducible piece

Outlook

- Computation of the 1PI n_h^2 contribution (work in progress)
- Combination of all virtual corrections for complete account of NNLO effects
- Extend application to other *gg*-initiated $2 \to 2$ processes

Thank you for your attention

Backup

$gg \to ZH$ @ NLO QCD

Inclusive cross section

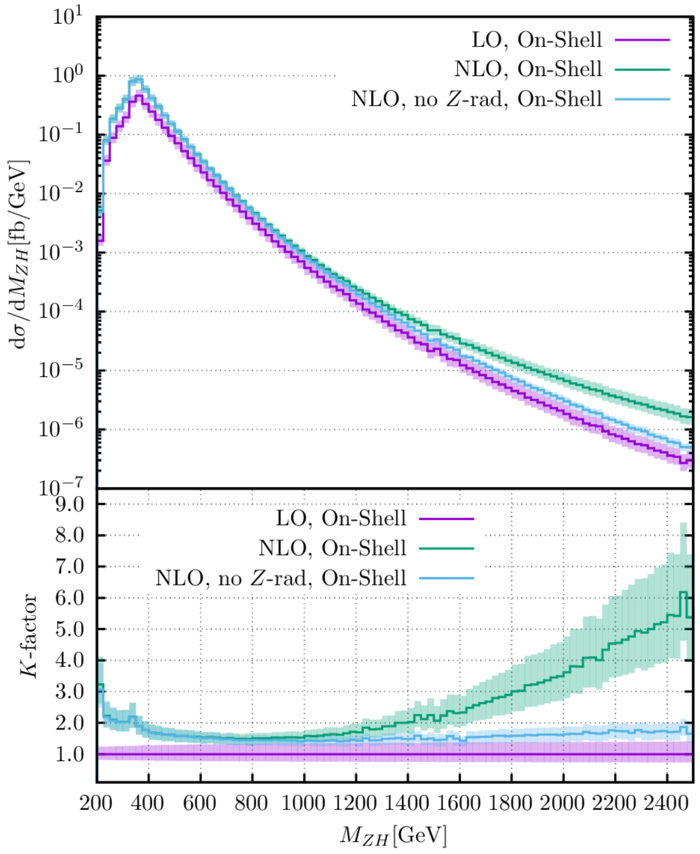
$$\sqrt{s} = 13\text{TeV}$$
$$\mu_r = \mu_f = M_{ZH}/2$$

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 30% wrt LO

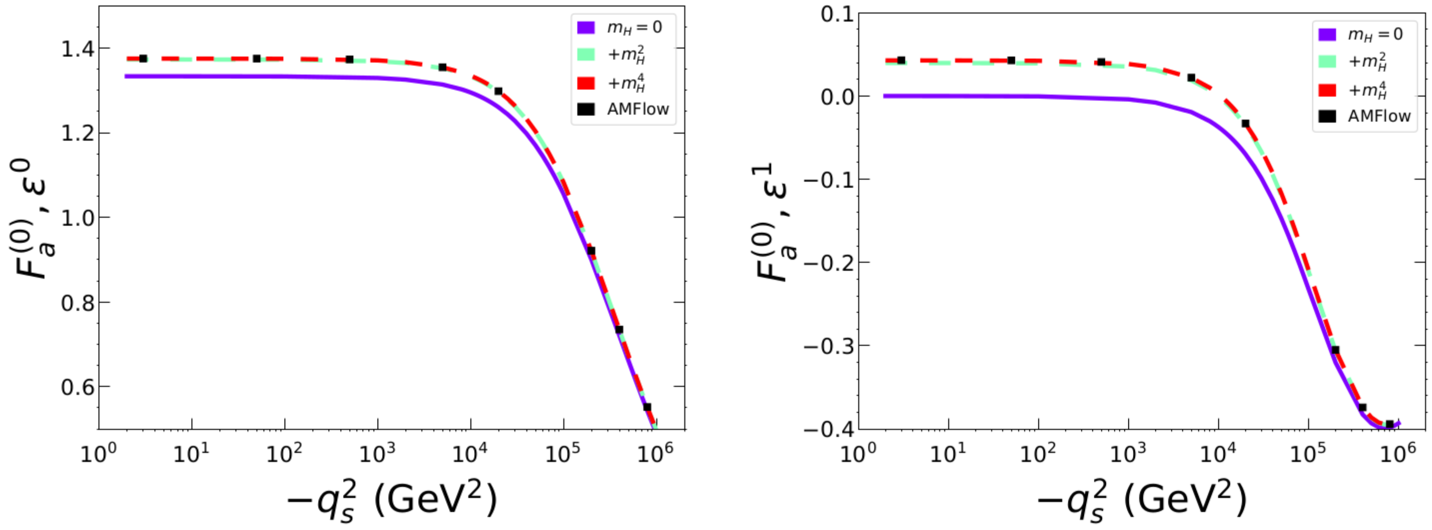
Invariant-mass distribution

- K-factor is not flat over M_{ZH} range
- Large NLO enhancement in the high-energy tail ($M_{ZH} > 1\,\text{TeV}$)



[Degrassi, Gröber, MV, Zhao - 2205.02769]

LO Validation



- Two-loop: results in agreement with [Degrassi, Giardino, Gröber – 1603.00385]
- NEW: inclusion of $O(\varepsilon^2)$ terms (renormalization and IR subtraction)