Three-loop heavy-to-light form factors in QCD

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TP1 Theoretical Particle Physics

CPPS Center for Particle Physics Siegen

Collaborative Research Center TRR 257

P3H

Particle Physics Phenomenology after the Higgs Discovery

in collaboration with Matteo Fael, Tobias Huber, Fabian Lange, Kay Schönwald and Matthias Steinhauser;

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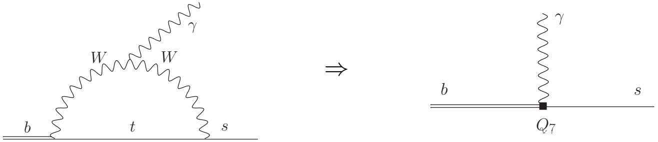
Motivation and applications

- Rare b-quark decays like $b \to s\gamma$, $b \to sll$ or $b \to ul\nu$ are golden modes to test the flavour sector of the SM.
- CKM or loop-suppression (Flavour-changing neutral currents) $\Rightarrow$ Precise measurement and theoretical predictions: Indirect search for physics beyond the SM.
- Example: $b \to s\gamma$ transition $\Rightarrow$ Electromagnetic dipole operator Q_7.

[Bertolini,Borzumati,Masiero'87] [Grinstein,Springer,Wise'88] [Misiak,Rehman,Steinhauser'17,'20 (latest update)] [...]



- Factorisation theorem for the photon energy spectrum in $B \to X_s\gamma$: $\mathrm{N^3LL'}$ analysis.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \propto \boxed{H} \int J \times S + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

- Jet and soft function J and S known at N^{3}LO.
- Hard function H only at N^{2}LO (before this paper). [Becher,Neubert'05'06] [Ali,Greub,Pecjak'07] [Bell,Beneke,Huber,Li'10] [Ligeti,Stewart,Tackmann'08] [Brüser,Liu,Stahlhofen'18'19] [SIMBA'20] [Dehnadi,Novikov,Tackmann'22]

Motivation and applications

- Determination of $|V_{ub}|$ from inclusive semi-leptonic $B \to X_u \ell \nu$ decays:

$$|V_{ub}^{\text{excl.}}|/|V_{ub}^{\text{incl.}}| = 0.84 \pm 0.04$$

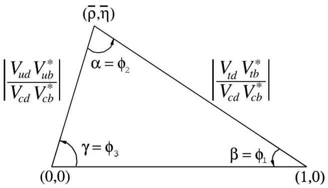
[HFLAV'22]

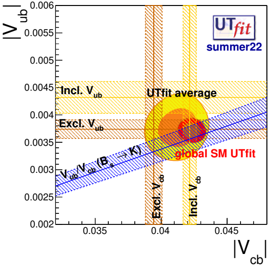
  - However, most recent extraction by Belle compatible with unity. [Belle'23]

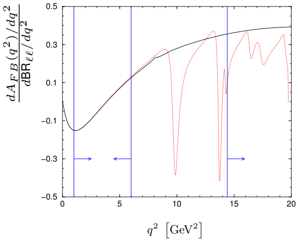
- Studying the decay $t \to Wb$: Precise differential top-quark decay width.

- $B \to X_s \ell\ell$: Extraction of zero-crossing of the forward-backward asymmetry at N^3LO.

[At NNLO: Bell,Beneke,Huber,Li'10]

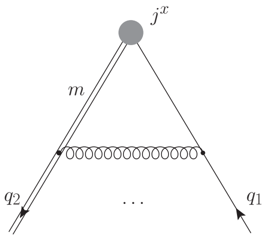






[Sanchez et al.(LHCb)'12][Bona et al.(UTfit)'22][Ghinculov,Hurth,Isidori,Yao'03]

Setup of the calculation



- Kinematics: $q_1^2 = 0,\ q_2^2 = m^2$
  $s \equiv q^2 = (q_1 - q_2)^2$
- External currents:
  $$j^x = \bar{\psi}_Q\{1, \mathrm{i}\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \mathrm{i}\sigma^{\mu\nu}\}\psi_q$$
- Depending on the phenomenological application: momentum transfer $s = 0$ or $s \neq 0$.

- General structure of the amplitude:

$$\int \frac{\mathrm{d}^4 y}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}q\cdot y} \langle \psi_Q^{\text{out}}(q_2, s_2)|j^x(y)|\psi_q^{\text{in}}(q_1, s_1)\rangle = \bar{u}(q_2, s_2)\Gamma(q_1, q_2)u(q_1, s_1)\delta^{(4)}(q - q_1 - q_2)$$

- Example: Vertex function for the tensor current:

$$\Gamma^t_{\mu\nu}(q_1, q_2) = \mathrm{i}F_1^t(q^2)\sigma_{\mu\nu} + \frac{F_2^t(q^2)}{m}(q_{1,\mu}\gamma_\nu - q_{1,\nu}\gamma_\mu) + \frac{F_3^t(q^2)}{m}(q_{2,\mu}\gamma_\nu - q_{2,\nu}\gamma_\mu) + \frac{F_4^t(q^2)}{m^2}(q_{1,\mu}q_{2,\nu} - q_{1,\nu}q_{2,\mu})$$

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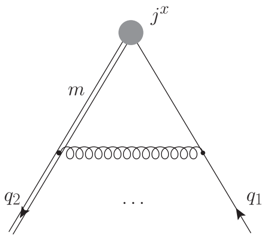

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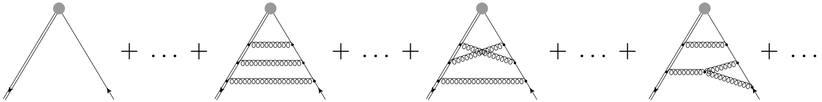
- Depending on the phenomenological application: momentum transfer $s = 0$ or $s \neq 0$.

- Two-loop corrections to heavy-to-light form factors are known. [Asatrian et al.'06] [Ali,Greub,Pecjak'07] [Asatrian,Greub,Pecjak'08] [Ligeti,Stewart,Tackmann'08] [Bell'08] [Bonicani,Ferroglia'08] [Beneke,Huber,Li'08] [Huber'09] [Bell,Beneke,Huber,Li'10]

- Analytical three-loop $\propto N_c^3$ corrections to heavy-to-light form factors appeared last year. [Chen,Wang'18] [Datta,Rana,Ravindran,Sarkar'23]

Calculation steps

1. Generate all possible Feynman diagrams.



2. Apply Feynman rules.

3. Simplify the colour, tensor and Dirac structure and obtain scalar integral topologies.

4. Reduce them to Master Integrals using Integration-by-parts techniques.

5. Calculate the Master Integrals up to the desired order in the dimensional regulator ϵ.

6. Perform UV renormalization and IR subtraction (matching onto SCET).

7. Use results for a phenomenological analysis.

Simplification of the amplitude

- Main calculation ($s \neq 0$): Carried out in a well-established setup:
  - Apply projectors to the amplitude to obtain scalar expressions for general QCD gauge parameter ξ.
  - Based on the tools `qgraf`, `tapir`, `exp`, `calc`.

[Nogueira'93] [Gerlach,Herren,Lang'22][Harlander,Seidensticker,Steinhauser'97] [Seidensticker'99]

- Cross-check: Tensor current at $s = 0$: Different setup:
  - Generate all diagrams using `qgraf`.
  - Apply the Feynman rules to setup the amplitude for Feynman gauge $\xi = 1$.
  - Apply the Dirac equation and use the Passarino-Veltman procedure to simplify the tensor structures (`FeynHelpers`, `Fermat`). [Shtabovenko'16][Lewis'86]
  - Identify momenta mappings between different integral topologies and zero-sectors (`FEYNSON`).

[Magerya'22]

- Results in a minimal number of integral families with distinct, linearly independent denominators. (We find 47 families.)

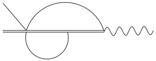
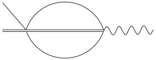
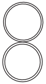
Integration-by-parts reduction

- IBP reduction to MIs: Automated implementation in `Kira`:

  [Maierhöfer,Usovitsch,Uwer'17][Klappert,Lange,Maierhöfer,Usovitsch'20]

  1. IBP reduction (familywise) of sample integrals to find all MIs.
  2. Find "better" MIs using `ImproveMasters.m` to avoid potential "bad" denominators. [Smirnov,Smirnov'20]
     - Denominators should factorize in space-time d and the kinematic variables s and m^2.
  3. IBP reduction (familywise) of the amplitude with the so obtained MIs.
  4. Final IBP reduction of all MIs for all integral families to find further symmetries.

- Two-loop IBP example:

 $= -\frac{1}{m^2}$  $+ \frac{d-2}{(2d-6)m^4}$ 

- We obtain 429 MIs for all form factors ($s \neq 0$) and 246 MIs (tensor, $s = 0$) at three-loop level. (Full two-loop: 18 MIs).

Master Integrals

- One- and two-loop MIs:
  - Re-calculated analytically to higher orders in ϵ.
- Three-loop MIs:
  - Different methods depending on the off/on-shellness condition $s \neq 0$ or $s = 0$ and on the topology.
    $\Rightarrow$ Limit $s \to 0$ of the full amplitude possible.
- Three-loop MIs for $s \neq 0$:
  - Differential equations with respect to $x = s/m^2$:
    `LiteRed` and subsequent reduction with `Kira`. [Lee'23]

$$\boxed{\frac{\partial}{\partial x} M_n = A_{nm}(\epsilon, x) M_m}$$

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- Method 1: "Expand and Match":

[Fael,Lange,Schönwald,Steinhauser'21,'22,'23]

  - Series expansions about regular and singular points of the DE.
  - Neighboring expansions are then numerically matched at a point where both expansions converge.
  - Here: Expansion points with 50 expansion terms each:

$$x = \{-\infty, -60, -40, -30, -20, -15, -10, -8, -7, -6, -5, -4, -3, -2, -1, -1/2, 0, 1/4, 1/2, 3/4, 7/8, 1\}$$

  - Except for $x = 1$ and $x = -\infty$: Taylor expansions, else power-log ansatz.
  - Boundary conditions: `AMFlow` with 100 digits in $x = 0$. [Liu,Ma'23]

Master Integrals

- Method 2: [Ablinger,Blümlein,Marquard,Rana,Schneider'18]
  - Decoupling of blocks of the DE into higher-order ones.
  - Solve these via factorization of the differential operator and variation of constants.
  - No canonical bases.
  - Iterated integrals over the alphabet:

$$\frac{1}{x}, \quad \frac{1}{1 \pm x}, \quad \frac{1}{2-x}$$

  - Boundary conditions:
    - Direct integration (at $x = 0$).
    - Mellin-Barnes techniques (at $x = 0$).
    - `PSLQ` on numerical results obtained from `AMFlow` (at $x = 0$). [Bailey,Ferguson'18]
    - Regularity conditions (in $x = 0$ and $x = 1$).

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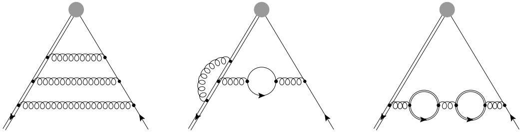


Topology	Result	Method
all	Semi-analytically	M1
N_C^3	Analytically	M2
$C_F T_F^2 n_l^2$	Analytically	M2
$C_F T_F^2 n_h^2$	Analytically	M2
$C_F T_F^2 n_l n_h$	Analytically	M2
$C_F^2 T_F n_l$	Analytically	M2
$C_F C_A T_F n_l$	Analytically	M2

UV renormalization

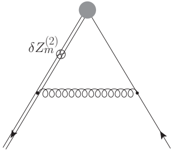
$$F^x = Z_x \left(Z_{2,Q}^{\rm OS}\right)^{1/2} \left(Z_{2,q}^{\rm OS}\right)^{1/2} F^{x,{\rm bare}}\Big|_{\alpha_s^{\rm bare}=Z_{\alpha_s}\alpha_s^{(n_f)},\, m^{\rm bare}=Z_m^{\rm OS} m^{\rm OS},\, \alpha_s^{(n_f)}=\zeta_{\alpha_s}^{-1}\alpha_s^{(n_l)}}$$

- $\overline{\rm MS}$ scheme for the strong coupling α_s.
- On-shell scheme for the heavy-quark mass m: Explicit mass counterterm insertions in one- and two-loop diagrams.
  (Switch to $\overline{\rm MS}$ scheme possible in the electronic files.)

[https://www.ttp.kit.edu/preprints/2024/ttp24-017/.]

- Decoupling relation in d dimensions:
  $\alpha_s^{(n_f)}(\mu) \to \alpha_s^{(n_l)}(\mu)$ $(n_f = n_l + n_h)$

- Example: Diagram for mass renormalization:



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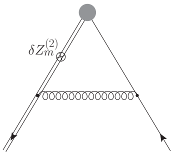
- Decoupling relation in d dimensions:
  $\alpha_s^{(n_f)}(\mu) \to \alpha_s^{(n_l)}(\mu)$ $(n_f = n_l + n_h)$
- Anomalous dimensions:
  - vector and axialvector current: $Z_v = Z_a = 1$.
  - scalar and pseudoscalar current: related to the mass renormalization: $Z_s = Z_p = Z_m$.
  - tensor current: cannot be related to other renormalization factors.

- Example: Diagram for mass renormalization:



- On-shell wave function renormalization constants:
  - heavy quark: $Z_{2,Q}^{\text{OS}}$.
  - light quark: $Z_{2,q}^{\text{OS}}$ (starting at two-loops).

IR subtraction

- Form factors F^x are still IR divergent!
- Universal renormalization constant Z stemming from the SCET approach for any of the UV renormalized form factors F^x:

$$\boxed{C = Z^{-1} F^x}$$

- Z is given by the
  - anomalous dimensions of the light and heavy quark γ^q and γ^Q ($\gamma^H = \gamma^q + \gamma^Q$)
  - light-like cusp anomalous dimension γ^{cusp} and the QCD β function

$$\ln Z = \frac{\alpha_s^{(n_l)}}{4\pi}\left[\frac{\Gamma_0'}{4\epsilon^2} + \frac{\Gamma_0}{2\epsilon}\right] + \left(\frac{\alpha_s^{(n_l)}}{4\pi}\right)^2 \left[-\frac{3\beta_0\Gamma_0'}{16\epsilon^3} + \frac{\Gamma_1' - 4\beta_0\Gamma_0}{16\epsilon^2} + \frac{\Gamma_1}{4\epsilon}\right]$$
$$+ \left(\frac{\alpha_s^{(n_l)}}{4\pi}\right)^3 \left[\frac{11\beta_0^2\Gamma_0'}{72\epsilon^4} - \frac{5\beta_0\Gamma_1' + 8\beta_1\Gamma_0' - 12\beta_0^2\Gamma_0}{72\epsilon^3} + \frac{\Gamma_2' - 6\beta_0\Gamma_1 - 6\beta_1\Gamma_0}{36\epsilon^2} + \frac{\Gamma_2}{6\epsilon}\right] + \mathcal{O}(\alpha_s^4),$$

$$\Gamma = \gamma^H(\alpha_s^{(n_l)}) - \gamma^{\text{cusp}}(\alpha_s^{(n_l)}) \ln\left(\frac{\mu}{m(1-x)}\right), \quad \Gamma' = \frac{\partial}{\partial \ln \mu}\Gamma = -\gamma^{\text{cusp}}(\alpha_s^{(n_l)})$$

- All ingredients for the renormalization procedure are known.

Renormalization Group Equations

- The two-fold structure of the RGE

$$\frac{d}{d\ln(\mu)}\,C(s,\mu)=\left[\gamma^{\rm cusp}(\alpha_s^{(n_l)})\,\ln\left(\frac{(1-x)m}{\mu}\right)+\gamma^{H}(\alpha_s^{(n_l)})+\gamma^{\rm QCD}(\alpha_s^{(n_f)})\right]\,C(s,\mu)$$

can be used to distinguish two scales μ (SCET) and ν (QCD)

$$\frac{d}{d\ln(\mu)}\,C(s,\mu,\nu)=\left[\gamma^{\rm cusp}(\alpha_s^{(n_l)}(\mu))\,\ln\left(\frac{(1-x)m}{\mu}\right)+\gamma^{H}(\alpha_s^{(n_l)}(\mu))\right]\,C(s,\mu,\nu)$$

$$\frac{d}{d\ln(\nu)}\,C(s,\mu,\nu)=\;\gamma^{\rm QCD}(\alpha_s^{(n_f)}(\nu))\,C(s,\mu,\nu)$$

- The dependence of the matching coefficients C on $L_\mu=\ln(\mu^2/m^2)$ and $L_\nu=\ln(\nu^2/m^2)$ is then predicted from lower loops using the
  - decoupling relation: $\alpha_s^{(n_f)}(\mu)\to\alpha_s^{(n_l)}(\mu)$
  - running of the strong coupling: $\alpha_s^{(n_f)}(\nu)\to\alpha_s^{(n_f)}(\mu)$

- Cross-check of the genuine three-loop calculation.

Ward identity, pole cancellations and further checks

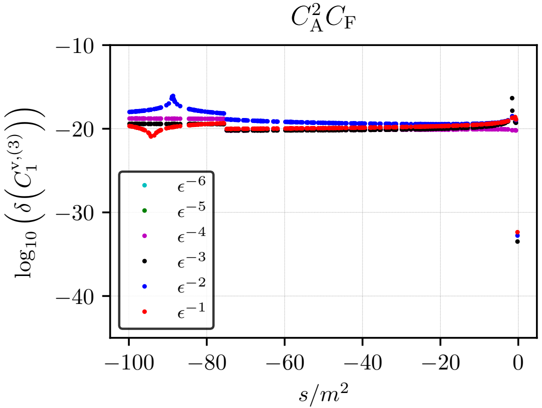
- QCD gauge parameter ξ drops out after UV renormalization.
- Equations of motion $\Rightarrow$ Ward identities:

$$-q^\mu \Gamma^v_\mu = m\,\Gamma^s \Rightarrow F^v_1 - \frac{2s}{m^2} F^v_3 = F^s$$

- Cancellation of poles in $1/\epsilon$:

$$\delta\left(C^{(3)}\Big|_{\epsilon^i}\right) = \frac{F^{(3)}\Big|_{\epsilon^i} + F^{(\text{CT}+Z)}\Big|_{\epsilon^i}}{F^{(\text{CT}+Z)}\Big|_{\epsilon^i}}$$

  - In the range $-75 < s < 15/16$: cancellation of at least 16 digits for each colour of each form factor and each $1/\epsilon$ pole.



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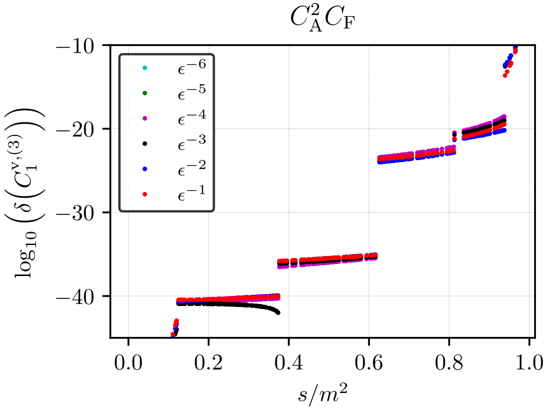

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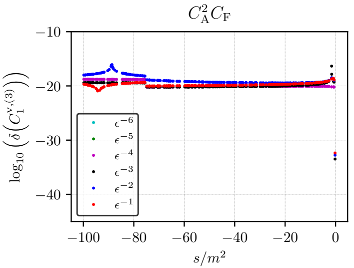
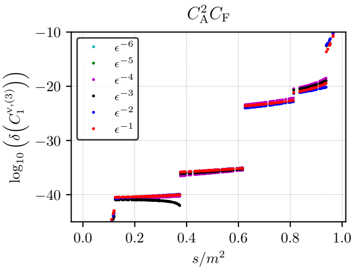
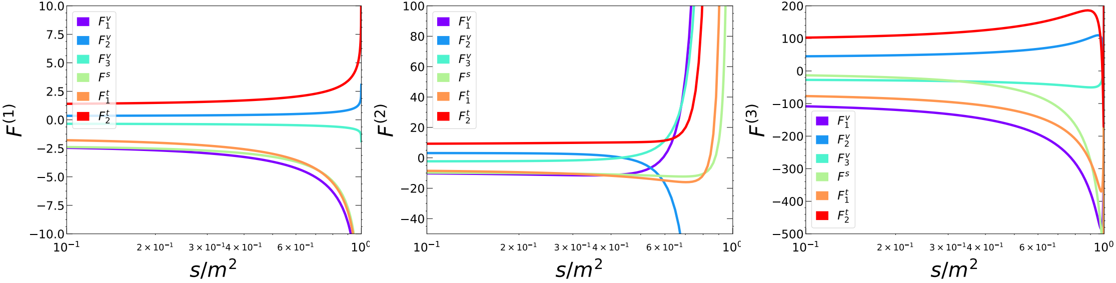
  - In the range $-75 < s < 15/16$: cancellation of at least 16 digits for each colour of each form factor and each $1/\epsilon$ pole.
- We find agreement with analytical three-loop $\propto N_c^3$ corrections to heavy-to-light form factors appeared in `2308.12169`. [Chen,Wang'18] [Datta,Rana,Ravindran,Sarkar'23]
- Subsequent analytical calculation of all fermionic pieces (except linear in n_h) in `2407.14550` confirms our results. [Datta,Rana'24]





Numerical results

- Plots of the finite pieces of the one-, two- and three loop form factors F^x at $\mu^2 = m^2$:



- Complete expressions can be obtained online and are implemented in the `Fortran` library `FFh2l` (with `Mathematica` interface).

[https://www.ttp.kit.edu/preprints/2024/ttp24-017/.] [https://gitlab.com/formfactors3l/ffh2l/.]

Analytical results

- Example: Analytical result for the n_l^2-part of the finite three-loop matching coefficient C_1^t at $\nu^2 = \mu^2 = m^2$:

$$C_1^{t,(3),n_l^2} = -\frac{370949}{419904} - \frac{221\pi^4}{38880} - \pi^2\left(\frac{829}{3888} - \frac{(3-11x)H_1}{81x} + \frac{1}{27}H_{0,1} + \frac{2}{27}H_{1,1}\right) + \frac{(657-1430x)H_1}{1458x}$$
$$+ \frac{(48-121x)H_{0,1}}{162x} + \frac{(48-121x)H_{1,1}}{81x} + \frac{(3-11x)H_{0,0,1}}{27x} + \frac{2(3-11x)H_{0,1,1}}{27x} + \frac{2(3-11x)H_{1,0,1}}{27x}$$
$$- \frac{4}{9}H_{1,1,0,1} + \frac{4(3-11x)H_{1,1,1}}{27x} - \frac{1}{9}H_{0,0,0,1} - \frac{2}{9}H_{0,0,1,1} - \frac{2}{9}H_{0,1,0,1} - \frac{4}{9}H_{0,1,1,1}$$
$$- \frac{2}{9}H_{1,0,0,1} - \frac{4}{9}H_{1,0,1,1} - \frac{8}{9}H_{1,1,1,1} - \frac{\left(323 + 126H_1\right)\zeta_3}{486}$$

- The function space is given by harmonic polylogarithms (if we allow evaluation at argument $1-x$).

- Complete expressions can be obtained online and are implemented in the `Fortran` library `FFh2l` (with `Mathematica` interface).

[https://www.ttp.kit.edu/preprints/2024/ttp24-017/.] [https://gitlab.com/formfactors3l/ffh2l/.]

Hard function in $B \to X_s\gamma$ to three-loops

- SCET-based approach for the photon energy spectrum of $B \to X_s\gamma$:
  $\mathrm{N^3LL'}$ analysis requires the hard function H to three-loops.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \propto \boxed{H} \int J \times S + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

- We have to consider the electromagnetic dipole operator Q_7:

$$Q_7 = -\frac{e\,\overline{m}_b(\mu)}{4\pi^2}\,(\bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R) \xrightarrow[\text{SCET}]{\text{matching}} J^A = (\bar{\xi} W_{hc}) \not{\epsilon}_\perp (1-\gamma_5) h_v$$

- On-shell matching yields for momentum transfer $s = 0$ (after IR-subtraction):

$$\langle s\gamma | Q_7 | b \rangle = -\frac{e\,\overline{m}_b\, 2E_\gamma}{4\pi^2} \underbrace{\left(C_1^t - \frac{1}{2}\, C_2^t\right)\Big|_{s=0}}_{\equiv C_\gamma} \times J^A$$

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- The hard function is given by:

$$H(\mu) = \left|C_\gamma\Big|_{L_\nu=0}\right|^2 \Rightarrow \boxed{H^{(3)}(m_b) = -181.16173810663548219}$$

- More than a factor two outside of the nuisance parameter range assumed in `2211.07663`.

[Dehnadi,Novikov,Tackmann'22]

Conclusion

- We calculated the three-loop corrections of $\mathcal{O}(\alpha_s^3)$ to heavy-to-light form factors for generic external currents.
- The master integrals are obtained:
  - analytically in the case of all fermionic contributions (except linear in n_h).
  - semi-analytically using the "expand and match" method for all topologies.
- We calculated the hard matching coefficients in SCET for all currents.
- The results are available in electronic form and implemented in the `Fortran` code `FFh2l` (with `Mathematica` interface) for numerical evaluations in the relavent phase space.

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- The tensor coefficients are used to extract the hard function in the factorization theorem of $B \to X_s\gamma$ to three-loops.
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Thank you for your attention!