Higher-order QED$\times$QCD Corrections to Semi-leptonic Decays

Based on JHEP01(2023)159

Francesco Moretti

Institute for Theoretical Particle Physics (TTP)
Karlsruhe Institute of Technology (KIT)

CRC YS Meeting - Karlsruhe
25-27 Sep 2024

Karlsruher Institut für Technologie

Institute for Theoretical Particle Physics

Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Outline

1 Introduction

2 Hadronic Matrix Elements
- Lattice QCD
  - Lattice Renormalisation
  - $\overline{\mathrm{MS}}$ Renormalisation
  - $\overline{\mathrm{MS}}$ Matching

3 Short-Distance Contribution
- $\overline{\mathrm{MS}}$

4 Conclusions

Why?

- (Semi-)Leptonic decays of light hadrons probe the CKM matrix
  - Precision Electroweak test of the Standard Model
- Kaon and nuclear decays $\rightarrow$ Information on V_{us} and V_{ud}
  - Unitarity test on the first row

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \quad \longrightarrow \quad \Delta_{\mathrm{CKM}}^{\mathrm{SM}} = 0$$

- Recent analyses uncovered a tension up to 3σ [Hardy & Towner, 2020]
- Additional tension in the Kaon sector
  - Semi-leptonic $K_{\ell 3}$ decays measure the quantity

$$|V_{us}| f^+(0) = 0.21635(38)(3)$$
$$\Downarrow$$
$$|V_{us}| = 0.2231(6)$$

  - Purely leptonic $K_{\ell 2}$ decays are sensitive to the ratio

$$\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = 0.27600(37)$$
$$\Downarrow$$
$$|V_{us}| = 0.2252(5)$$

Statistically incompatible results!

Why?

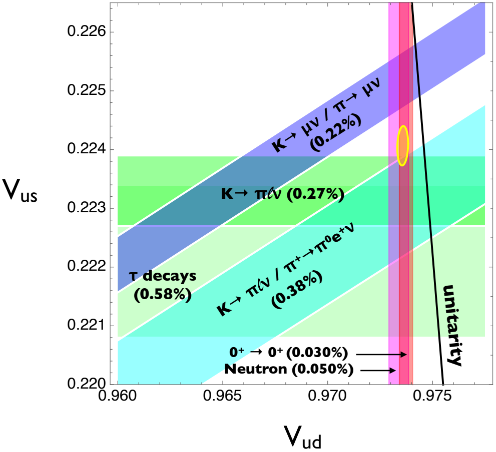


Figure: [Crivellin & Hoferichter, 2020]

Why?

- Possible interpretations from BSM physics of these tensions have been investigated in the literature [Belfatto *et al.*, 2020]
  - Vector-like quarks [Belfatto & Berezhiani, 2021]
  - Modified couplings [Crivellin *et al.*, 2021a]
  - Exploring Lepton Flavour Universality Violation [Crivellin *et al.*, 2021b]
  - Using SMEFT [Cirigliano *et al.*, 2023a]
  - Many more!
- However, more precise theoretical predictions are needed in order to shed light on the nature of the current tensions in the CKM sector
  - Improving conversion factor between $\overline{\mathrm{MS}}$ and Lattice, so far known @ $\mathcal{O}(\alpha)$ [Di Carlo *et al.*, 2019]
  - Higher-order corrections EW at the high-scale $\mu_W \sim M_W$, so far known @ $\mathcal{O}(\alpha)$ [Gambino & Haisch, 2001]
  - Better RGE for the Wilson Coefficient $\rightarrow$ Higher-order corrections to ADM

$\rightarrow$ 1$^{\mathrm{st}}$ Part of the talk

2$^{\mathrm{nd}}$ Part of the talk $\leftarrow$

Outline

1 Introduction

2 Hadronic Matrix Elements
- Lattice QCD
  - Lattice Renormalisation
  - $\overline{\mathrm{MS}}$ Renormalisation
  - $\overline{\mathrm{MS}}$ Matching

3 Short-Distance Contribution
- $\overline{\mathrm{MS}}$

4 Conclusions

Hadronic Matrix Elements

Karlsruher Institut für Technologie

- $\mathrm{O_{sem}(x) = \bar{d}(x)\gamma^\mu P_L u(x) \otimes \bar{\nu}_\ell(x)\gamma_\mu P_L \ell(x), \quad P_L = (1-\gamma^5)/2}$

$$\langle \pi(p)|\bar{\mathrm{d}}\gamma^\mu \mathrm{P_L u}|K(p')\rangle = f_+^{K\pi}(q^2)(p+p')^\mu + f_-^{K\pi}(q^2)(p-p')^\mu$$

Evaluation of long-distance contribution

- χPT [Cirigliano *et al.*, 2023b],[Seng *et al.*, 2020]
- Lattice QCD [Di Carlo *et al.*, 2019],[Carrasco *et al.*, 2015]

$\longrightarrow$ Focus of our work

Lattice QCD

- QED corrections $\rightarrow$ Lattice renormalisation;
- We proposed a new scheme [Gorbahn, Jäger, FM, v. d. Merwe]

$\Downarrow$

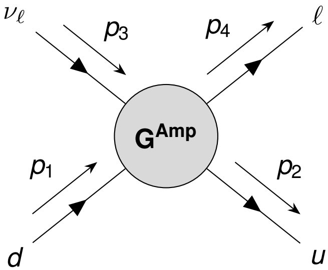
Cancellation of extraneous pure QCD corrections ;

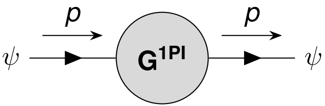
- two-loop $\mathcal{O}(\alpha\alpha_s)$ scheme changing onto the $\overline{\mathrm{MS}}$;

More on this later! $\longleftarrow$

Lattice Renormalisation

Lattice Renormalisation





- $\mathrm{RI}' - \mathrm{MOM}$ [Martinelli *et al.*, 1995]

$$p_1 = p_2 = p_3 = p_4 = p, \quad p^2 = -\mu^2;$$

- $\mathrm{RI} - \mathrm{SMOM}$ [Sturm *et al.*, 2009]

$$p_1 = p_3, \quad p_2 = p_4, \quad p_1^2 = p_2^2 = -\mu^2, \quad p_1 \cdot p_2 = -\frac{1}{2}\mu^2.$$

Lattice Renormalisation

- The RI schemes are defined by imposing the off-shell renormalization conditions on the projected Green's functions

$$\sigma^A \equiv \frac{1}{4\,p^2}\mathrm{Tr}(S_A^{-1}(p)\not{p}) \overset{A=\mathrm{RI}}{=} 1, \quad \lambda^A \equiv \Lambda^A_{\alpha\beta\gamma\delta}\mathcal{P}^{\alpha\beta\gamma\delta} \overset{A=\mathrm{RI}}{=} 1.$$

$\mathcal{P}$ is a constant Dirac tensor satisfying $\Lambda^{(\mathrm{tree})}_{\alpha\beta\gamma\delta}\,\mathcal{P}^{\alpha\beta\gamma\delta} = 1$.

- We define the scheme conversion factors as

$$\mathcal{C}_f^{\overline{\mathrm{MS}}\to RI} = \left(\sigma^{\overline{\mathrm{MS}}}\right)^{-1/2}, \quad \mathcal{C}_O^{\overline{\mathrm{MS}}\to RI} = \lambda^{\overline{\mathrm{MS}}}\left(\sigma_u^{\overline{\mathrm{MS}}}\sigma_d^{\overline{\mathrm{MS}}}\sigma_\ell^{\overline{\mathrm{MS}}}\right)^{1/2}.$$

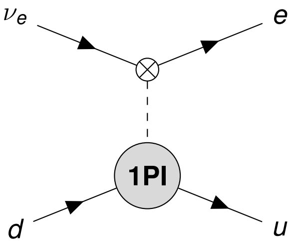
Choice of Projector

- Crucial role of $\mathcal{P}$ $\to$ What is a "good" projector?
- Conventionally [Garron, 2018], $\mathcal{P} = -\frac{1}{16}\left(\gamma^\mu P_R \otimes \gamma_\mu P_R\right)^{\alpha\beta\gamma\delta}$.
- Ward Identity "violation" $\to$ scale dependence of the conversion factor already in pure QCD.

Lattice Renormalisation

Statement Of The Problem

- QCD corrections $\Longrightarrow$



- Neglecting QED $\rightarrow \Lambda^b = \Lambda^{b,\mu}(p) \otimes \gamma_\mu P_L + \mathcal{O}(\alpha)$, where

$$\Lambda^{b,\mu}(p) = F_1(p)\gamma^\mu P_L + F_2(p)\frac{p^\mu \not{p}}{p^2} P_L.$$

Scalar Form Factors

Lorentz Structures

- Conserved current $\rightarrow \underbrace{\Lambda^{b,\mu}(p) = \frac{\partial}{\partial p_\mu} S^b(\not{p})^{-1}}_{F_1(p) = S^{-1}(p^2)}$ (Ward Identity)

- $\mathcal{P}(\Lambda^{b,\mu}) \neq F_1(p)$!

Lattice Renormalisation

Statement Of The Problem

$\overline{\mathrm{MS}}$

- Ward Identity holds in NDR after minimal subtraction;
- Cancellation of loop corrections against the field renormalisation;
- $Z_{OO}^{\overline{\mathrm{MS}}} = 1 + \mathcal{O}(\alpha)$.

$$\not\Updownarrow$$

RI

- Extraneous contribution not matched by field renormalisation;
- $Z_{OO}^{\mathrm{RI}} = 1 + \mathcal{O}(\alpha_s)$;
- Artificial scale dependence dominant at low scales.

- $\overline{\mathrm{RI}}$ scheme defined via Ward Identity;
- Imposing $\begin{cases} \mathcal{P}(\gamma^\mu P_L \otimes \gamma_\mu P_L) = 1 \\ \mathcal{P}(\frac{p^\mu \not{p}}{p^2} P_L \otimes \gamma_\mu P_L) = 0 \end{cases}$

$$\Downarrow$$

$$\mathcal{P}^{\overline{\mathrm{RI}}-\mathrm{MOM}} = -\frac{1}{12\,p^2}\left(\not{p}P_R \otimes \not{p}P_R + \frac{p^2}{2}\gamma^\nu P_R \otimes \gamma_\nu P_R\right).$$

$\overline{\mathrm{RI}}-\mathrm{MOM}$

- $Z_{OO}^{\overline{\mathrm{RI}}-\mathrm{MOM}} = 1 + \mathcal{O}(\alpha)$.

- Similar (yet more complicated) results for $\overline{\mathrm{RI}}-\mathrm{SMOM}$

$\overline{\mathrm{MS}}$

$\overline{\mathrm{MS}}$ Renormalisation

Karlsruher Institut für Technologie

- Naive Dimensional Regularisation (NDR) $\Rightarrow d = 4 - 2\epsilon$;
- Presence of Evanescent Operators [Gorbahn & Haisch, 2005]
  $$\mathrm{E} = (\bar{\mathrm{d}}\gamma^\mu\gamma^\nu\gamma^\lambda \mathrm{P_L u})(\bar{\nu}_\ell\gamma_\mu\gamma_\nu\gamma_\lambda \mathrm{P_L}\ell) - (16 - 4\,\mathrm{a}\epsilon - 4\,\mathrm{b}\epsilon^2)(\bar{\mathrm{d}}\gamma^\mu \mathrm{P_L u})(\bar{\nu}_\ell\gamma_\mu \mathrm{P_L}\ell);$$
- $\psi_f^b = \left(Z_{2,f}^{\overline{\mathrm{MS}}}\right)^{1/2} \psi_f^{\overline{\mathrm{MS}}}$, f = u, d, ℓ;
- $$\begin{pmatrix} \mathrm{O}_{\mathrm{sem}}^{\overline{\mathrm{MS}}} \\ \mathrm{E}^{\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{OO}^{\overline{\mathrm{MS}}} & Z_{OE}^{\overline{\mathrm{MS}}} \\ Z_{EO}^{\overline{\mathrm{MS}}} & Z_{EE}^{\overline{\mathrm{MS}}} \end{pmatrix} \begin{pmatrix} \mathrm{O}_{\mathrm{sem}}^{\mathrm{b}} \\ \mathrm{E}^b \end{pmatrix};$$

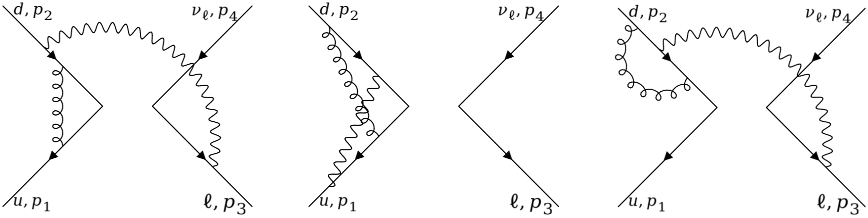
Amputated Green's Function

- $$\Lambda_{\mathrm{O_{sem}}}^{\overline{\mathrm{MS}}} = \left(Z_{2,\mathrm{u}}^{\overline{\mathrm{MS}}}\right)^{1/2}\left(Z_{2,\mathrm{d}}^{\overline{\mathrm{MS}}}\right)^{1/2}\left(Z_{2,\ell}^{\overline{\mathrm{MS}}}\right)^{1/2}\left(Z_{OO}^{\overline{\mathrm{MS}}}\,\Lambda_{\mathrm{O_{sem}}}^b + Z_{OE}^{\overline{\mathrm{MS}}}\,\Lambda_{\mathrm{E}}^b\right).$$

$\overline{\text{MS}}$ Matching

Details of Calculation

- Appearance of tensor integrals$\rightarrow$ Passarino-Veltman decomposition [Passarino & Veltman, 1979].
- Reduction to master integrals using Reduze 2 [von Manteuffel & Studerus, 2012] and FIRE6 [Smirnov & Chukharev, 2020].
- Analytical [Ussyukina & Davydychev, 1994,Ussyukina & Davydychev, 1995,Almeida & Sturm, 2010] and numerical (PySecDec) [Borowka *et al.*, 2018] evaluation of the masters.



Details of Calculation

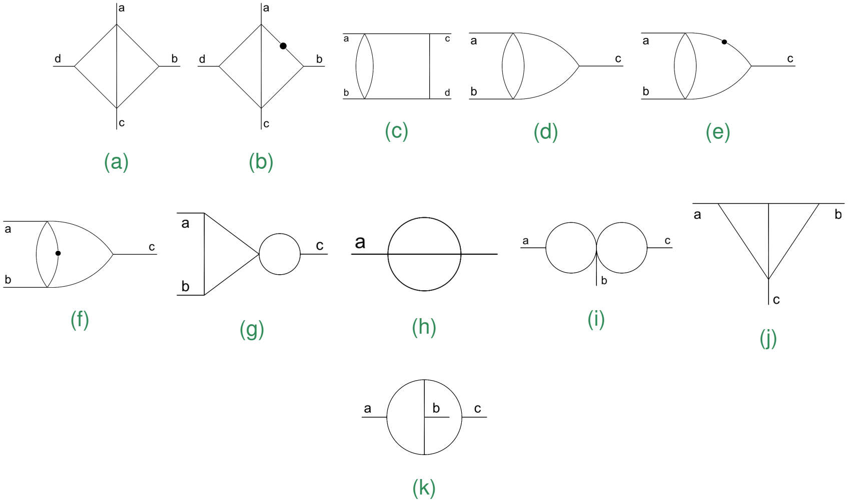


Figure: The topologies for all master integrals. All external momenta are incoming. The dotted lines are squared propagators.

- The expression for the Wilson Coefficient in the RI schemes is given by

$$C_O^{\mathrm{RI}} = \overbrace{\mathcal{C}^{\overline{\mathrm{MS}}\to\mathrm{RI}}(\mu_L, p^2)}^{\text{low-scale}} \, \overbrace{\mathcal{U}^{\overline{\mathrm{MS}}}(\mu_W, \mu_L) C_O^{\overline{\mathrm{MS}}}(\mu_W)}^{\text{high-scale}}$$

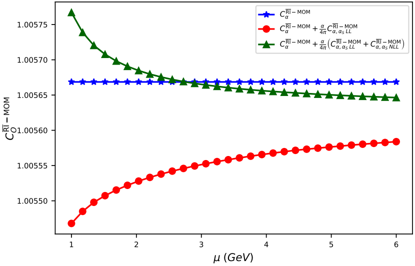
More on this later

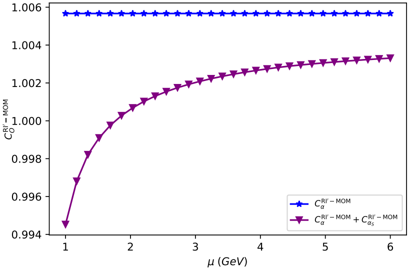
- $C_O^{\mathrm{RI}}(\mu_L, p^2) = C_\alpha^{\mathrm{RI}} + C_{\alpha_s}^{\mathrm{RI}} + \dfrac{\alpha}{4\pi}\left(C^{\mathrm{RI}}_{\alpha,\alpha_s\,LL} + C^{\mathrm{RI}}_{\alpha,\alpha_s\,NLL}\right)$
- C_α^{RI} and $C_{\alpha_s}^{\mathrm{RI}}$ are the resummed QED and leading QCD contributions. Neglecting threshold corrections

$$C^{\mathrm{RI}}_{\alpha,\alpha_s LL} = -\frac{\gamma^{(1)}_{OO}}{2\beta^{(5)}_{(0)}} \ln\left(\frac{\alpha_s(\mu_L)}{\alpha_s(\mu_W)}\right), \quad C^{\mathrm{RI}}_{\alpha,\alpha_s NLL} = \frac{\alpha_s(\mu_L)}{4\pi}\left(\mathcal{C}^{es}_O(-p^2, \mu_L^2) + \bar{\gamma}^{(5)}\right)$$

$$+\frac{\alpha_s(\mu_W)}{4\pi}\left(C^{es}_O(\mu_W, M_Z) - \bar{\gamma}^{(5)}\right), \qquad \bar{\gamma}^{(N_f)} = \frac{1}{2\beta_0^{(N_f)}}\left(\gamma^{(1)}_{OO}\,\frac{\beta_1^{(N_f)}}{\beta_0^{(N_f)}} - \gamma^{(2)}_{OO}\right)$$

Wilson Coefficient in RI schemes





- Cancellation of artificial running at $\mathcal{O}(\alpha_s)\rightarrow$Residual scale dependence suppressed by $\mathcal{O}(\alpha)$
- Reduced uncertainties from higher-order corrections
- Similar results for SMOM kinematics

Outline

1 Introduction

2 Hadronic Matrix Elements
- Lattice QCD
  - Lattice Renormalisation
  - $\overline{\mathrm{MS}}$ Renormalisation
  - $\overline{\mathrm{MS}}$ Matching

3 Short-Distance Contribution
- $\overline{\mathrm{MS}}$

4 Conclusions

$\overline{\text{MS}}$

Short-Distance Contribution

EFT & $\overline{\text{MS}}$

- Clear scale separation thanks to EFT framework

$$\Downarrow$$

- High-scale matching onto the Standard Model $\rightarrow$ Wilson Coefficient C_O
- $C_O = 1 + \frac{\alpha}{4\pi} C_O^e + \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi} \underbrace{C_O^{es}}_{\text{New result}}$
- Resummation of large logarithms via RGE solutions

$$\mu \frac{d}{d\mu} C_O = \gamma_{OO}\, C_O$$

Anomalous Dimension (ADM)

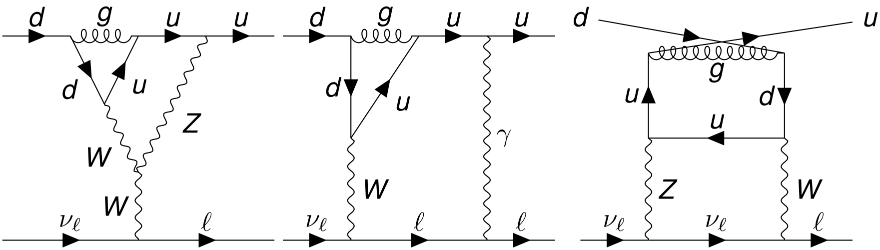
- $\gamma_{OO} = \overbrace{\frac{\alpha}{4\pi}\gamma_{OO}^{e} + \frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}\gamma_{OO}^{es}}^{\text{[Gorbahn, Jäger, FM, v. d. Merwe]}} + \underbrace{\left(\frac{\alpha}{4\pi}\right)^2 \gamma_{OO}^{ee}}_{\text{[Cirigliano } et.\ al\text{, 2023]}} + \frac{\alpha}{4\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \underbrace{\gamma_{OO}^{ess}}_{\text{New result}} .$

Short-Distance Contribution

High-Scale Matching

$\overline{\mathrm{MS}}$ Wilson Coefficient at $\mathcal{O}(\alpha\alpha_s)$

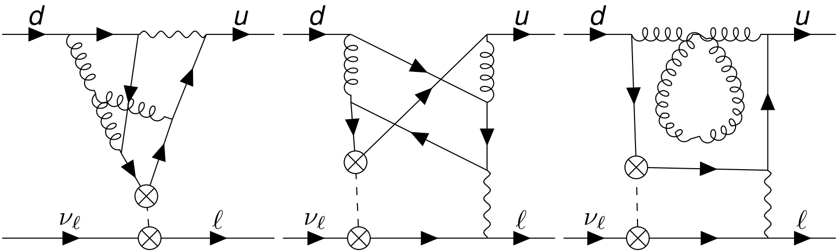
- $$C_O^{es} = C_F\left(-\frac{7a}{6} - \frac{3}{s_W^2}\left(\frac{c_W^2 \log\left(\frac{M_W}{M_Z}\right)}{s_W^2} + 1\right) + 3\log\left(\frac{\mu_W}{M_Z}\right) + \frac{41}{8}\right)$$



- Feynman diagrams generated using the Mathematica package *FeynArts* [Hahn, 2010] $\sim$ 600 diagrams;
- *FeynArts* built-in routines used to create Feynman amplitudes;

$\Downarrow$ Conversion to personal notation

- Personal Mathematica libraries for the final evaluation of amplitudes;
- Employment of R_ξ gauge to check gauge independence of final results.

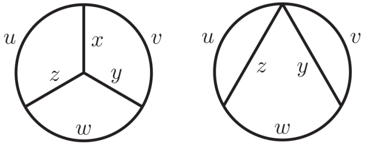


Infra-Red Rearrangement

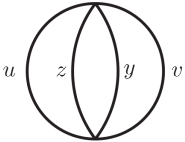
- Isolating the UV poles $\Rightarrow$ zero masses and external momenta;
- $$\frac{1}{(k+p)^2} = \frac{1}{k^2 - M^2} - \frac{p^2 + 2p\cdot k + M^2}{k^2 - M^2}\frac{1}{(k+p)^2}$$ [Chetyrkin *et al.*, 1998]

  IR regulator
- Gauge non-invariant counter-terms: $M^2 G^{\mu\,a} G^a_\mu$.

  "Gluon Mass"



[Broadhurst, 1999]



[Broadhurst, 1992]

RGI

- $$\mathcal{U}^{\overline{\mathrm{MS}}}(\mu,\mu_0) = \overbrace{\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\frac{\gamma^{e}_{OO}}{2\beta_0}}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{\gamma^{es}_{OO}}{2\beta_{0,s}}\frac{\alpha(\mu)}{4\pi}}}^{LL}\Bigg(1+\overbrace{\frac{\gamma^{ee}_{OO}}{2\beta_0}\left(\frac{\alpha(\mu)-\alpha(\mu_0)}{4\pi}\right)}^{NLL_{QED}}+ \\ +\underbrace{\frac{\alpha(\mu)}{2\beta_{0,s}}\left(\gamma^{es}_{OO}\frac{\beta_{1,s}}{\beta_{0,s}}-\gamma^{ess}_{OO}\right)\left(\frac{\alpha_s(\mu)-\alpha_s(\mu_0)}{4\pi}\right)}_{NLL_{QCD}}\Bigg)$$

RGI

- $$\mathcal{U}^{\overline{\mathrm{MS}}}(\mu,\mu_0) = \overbrace{\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\frac{\gamma^{e}_{OO}}{2\beta_0}}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{\gamma^{es}_{OO}}{2\beta_{0,s}}\frac{\alpha(\mu)}{4\pi}}}^{LL}\Bigg(1+\overbrace{\frac{\gamma^{ee}_{OO}}{2\beta_0}\left(\frac{\alpha(\mu)-\alpha(\mu_0)}{4\pi}\right)}^{NLL_{QED}}+$$
$$\underbrace{+\frac{\alpha(\mu)}{2\beta_{0,s}}\left(\gamma^{es}_{OO}\frac{\beta_{1,s}}{\beta_{0,s}}-\gamma^{ess}_{OO}\right)\left(\frac{\alpha_s(\mu)-\alpha_s(\mu_0)}{4\pi}\right)}_{NLL_{QCD}}\Bigg)$$
- Neglecting $\mathcal{O}(\alpha^2\alpha_s) \to \mathcal{U}^{\overline{\mathrm{MS}}}(\mu,\mu_0) = J_f(\mu)u_f(\mu)u_f^{-1}(\mu_0)J_f^{-1}(\mu_0)$
  - $$u_f(\mu) = \left(\frac{\alpha^{(f)}(\mu)}{\alpha(M_Z)}\right)^{\frac{\gamma_e}{2\beta_{0,e}}}\left(\frac{\alpha_s^{(f)}(\mu)}{\alpha_s(M_Z)}\right)^{-\frac{\alpha}{4\pi}\frac{\gamma_{es}}{2\beta_0}}$$
  - $$J_f(\mu) = 1+\frac{\alpha^{(f)}(\mu)}{4\pi}\left[\frac{\gamma_{ee}}{2\beta_{0,e}}-\frac{\beta_{1,e}\gamma_e}{2\beta_{0,e}^2}\right]-\frac{\alpha}{4\pi}\frac{\alpha_s^{(f)}(\mu)}{4\pi}\left[\frac{\gamma_{ess}}{2\beta_0}-\frac{\beta_1\gamma_{es}}{2\beta_0^2}\right]$$

f =Number of active quarks/leptons

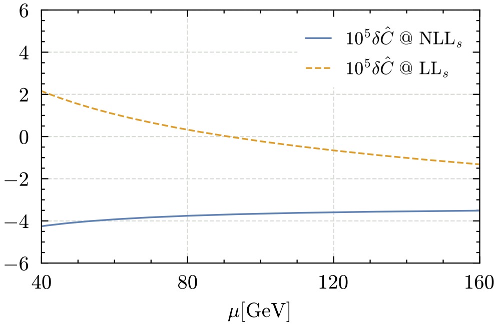
RGI

- $$\mathcal{U}^{\overline{\text{MS}}}(\mu,\mu_0) = \overbrace{\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\frac{\gamma^{e}_{OO}}{2\beta_0}} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\frac{\gamma^{es}_{OO}}{2\beta_{0,s}}\frac{\alpha(\mu)}{4\pi}}}^{LL} \Bigg(1 + \overbrace{\frac{\gamma^{ee}_{OO}}{2\beta_0}\left(\frac{\alpha(\mu)-\alpha(\mu_0)}{4\pi}\right)}^{NLL_{QED}} + \underbrace{+\frac{\alpha(\mu)}{2\beta_{0,s}}\left(\gamma^{es}_{OO}\frac{\beta_{1,s}}{\beta_{0,s}} - \gamma^{ess}_{OO}\right)\left(\frac{\alpha_s(\mu)-\alpha_s(\mu_0)}{4\pi}\right)}_{NLL_{QCD}}\Bigg)$$
- Neglecting $\mathcal{O}(\alpha^2\alpha_s) \to \mathcal{U}^{\overline{\text{MS}}}(\mu,\mu_0) = J_f(\mu)u_f(\mu)u_f^{-1}(\mu_0)J_f^{-1}(\mu_0)$
  - $$u_f(\mu) = \left(\frac{\alpha^{(f)}(\mu)}{\alpha(M_Z)}\right)^{\frac{\gamma_e}{2\beta_{0,e}}} \left(\frac{\alpha_s^{(f)}(\mu)}{\alpha_s(M_Z)}\right)^{-\frac{\alpha}{4\pi}\frac{\gamma_{es}}{2\beta_0}}$$
  - $$J_f(\mu) = 1 + \frac{\alpha^{(f)}(\mu)}{4\pi}\left[\frac{\gamma_{ee}}{2\beta_{0,e}} - \frac{\beta_{1,e}\gamma_e}{2\beta_{0,e}^2}\right] - \frac{\alpha}{4\pi}\frac{\alpha_s^{(f)}(\mu)}{4\pi}\left[\frac{\gamma_{ess}}{2\beta_0} - \frac{\beta_1\gamma_{es}}{2\beta_0^2}\right]$$
- Threshold corrections $\to$ Decoupling operator
  - $\hat{M}_{f\downarrow} = u_{f-1}^{-1}(\mu)J_{f-1}^{-1}(\mu)J_f(\mu)u_f(\mu)$

RGI

- Renormalisation independent Wilson Coefficient
  - $\hat{C}_5 = u_5^{-1}(\mu) J_5^{-1}(\mu) C_5(\mu)$
- Decoupling of heavy flavours
  - $\hat{C}_3 = \hat{M}_{4\downarrow} \hat{M}_{5\downarrow} \hat{C}_5$
- Effects of strong corrections to pure QED results

- $\delta\hat{C} \text{ @ } LL_s = \dfrac{\hat{C} \text{ @ } LL_s}{\hat{C} \text{ @ } NLL_e}$
- $\delta\hat{C} \text{ @ } NLL_s = \dfrac{\hat{C} \text{ @ } NLL_s}{\hat{C} \text{ @ } NLL_e}$



Outline

1 Introduction

2 Hadronic Matrix Elements
- Lattice QCD
  - Lattice Renormalisation
  - $\overline{\text{MS}}$ Renormalisation
  - $\overline{\text{MS}}$ Matching

3 Short-Distance Contribution
- $\overline{\text{MS}}$

4 Conclusions

Conclusions

Karlsruher Institut für Technologie

Summary

- Improved scheme for Lattice evaluation of Hadronic Matrix Elements
- Calculation of the $\mathcal{O}(\alpha\alpha_s)$ $\overline{\mathrm{MS}}$ $\rightarrow$RI conversion factor
- Derivation of the $\mathcal{O}(\alpha\alpha_s)$ EW corrections to the $\overline{\mathrm{MS}}$ Wilson Coefficient
- Evaluation of the $\mathcal{O}\left(\alpha\alpha_s^2\right)$ ADM

What's Next?

- Implementation of our results on V_{ud} extraction $\Rightarrow$ CKM Unitarity
- Evaluation of the two-loop $\mathcal{O}\left(\alpha^2\right)$ EW corrections to the Wilson Coefficient

Conclusions

Karlsruher Institut für Technologie

Summary

- Improved scheme for Lattice evaluation of Hadronic Matrix Elements
- Calculation of the $\mathcal{O}(\alpha\alpha_s)$ $\overline{\mathrm{MS}}$ $\to$RI conversion factor
- Derivation of the $\mathcal{O}(\alpha\alpha_s)$ EW corrections to the $\overline{\mathrm{MS}}$ Wilson Coefficient
- Evaluation of the $\mathcal{O}\left(\alpha\alpha_s^2\right)$ ADM

What's Next?

- Implementation of our results on V_{ud} extraction $\Rightarrow$ CKM Unitarity
- Evaluation of the two-loop $\mathcal{O}\left(\alpha^2\right)$ EW corrections to the Wilson Coefficient

Thank You!

Backup Slides

- $\overline{\mathrm{RI}}$ scheme defined via Ward Identity;
- In SMOM, analogous conditions are imposed, now involving 6 Lorentz Structures

$$\Downarrow$$

- $$\mathcal{P}^{\overline{\mathrm{RI}}-\mathrm{SMOM}} = \frac{1}{4}\Big(- \frac{1}{2}\gamma^{\nu} P_R \otimes \gamma_{\nu} P_R + \frac{1}{p^2} \not{p}_1 P_R \otimes \not{p}_1 P_R + \frac{1}{p^2} \not{p}_2 P_R \otimes \not{p}_2 P_R - \frac{1}{p^2} \not{p}_1 P_R \otimes \not{p}_2 P_R - \frac{1}{p^2} \not{p}_2 P_R \otimes \not{p}_1 P_R \Big).$$

$\overline{\mathrm{RI}} - \mathrm{SMOM}$

- $Z_{OO}^{\overline{\mathrm{RI}}-\mathrm{SMOM}} = 1 + \mathcal{O}(\alpha).$

- The expression for the Wilson Coefficient in the RI schemes is given by

$$C_O^{\text{RI}} = \overbrace{\mathcal{C}^{\overline{\text{MS}}\to\text{RI}}(\mu_L, p^2)}^{\text{low}-\text{scale}}\,\overbrace{\mathcal{U}^{\overline{\text{MS}}}(\mu_W, \mu_L)C_O^{\overline{\text{MS}}}(\mu_W)}^{\text{high}-\text{scale}}$$

More on this later

- $C_O^{\text{RI}}(\mu_L, p^2) = C_\alpha^{\text{RI}} + C_{\alpha_s}^{\text{RI}} + \dfrac{\alpha}{4\pi}\left(C_{\alpha,\alpha_s\,LL}^{\text{RI}} + C_{\alpha,\alpha_s\,NLL}^{\text{RI}}\right)$
- C_α^{RI} and $C_{\alpha_s}^{\text{RI}}$ are the resummed QED and leading QCD contributions. Neglecting threshold corrections

$$C_{\alpha,\alpha_s LL}^{\text{RI}} = -\frac{\gamma_{OO}^{(1)}}{2\beta_{(0)}^{(5)}}\ln\left(\frac{\alpha_s(\mu_L)}{\alpha_s(\mu_W)}\right),\;\; C_{\alpha,\alpha_s NLL}^{\text{RI}} = \frac{\alpha_s(\mu_L)}{4\pi}\left(\mathcal{C}_O^{es}(-p^2, \mu_L^2) + \bar{\gamma}^{(5)}\right)$$
$$+\frac{\alpha_s(\mu_W)}{4\pi}\left(\mathcal{C}_O^{es}(\mu_W, M_Z) - \bar{\gamma}^{(5)}\right), \quad \bar{\gamma}^{(N_f)} = \frac{1}{2\beta_0^{(N_f)}}\left(\gamma_{OO}^{(1)}\frac{\beta_1^{(N_f)}}{\beta_0^{(N_f)}} - \gamma_{OO}^{(2)}\right)$$

Short-Distance Contribution

3-Loop Anomalous Dimension

ADM from Renormalisation Constants

- $\gamma_{\mathrm{ij}} = Z_{\mathrm{ik}} \frac{\mathrm{d}}{\mathrm{d}\ln(\mu)} \left(Z^{-1}\right)_{\mathrm{kj}}$

$$\Big\Downarrow \text{ Mass-Independent Renormalisation Scheme}$$

- $\gamma_{\mathrm{ij}} = 2\beta(\epsilon, \alpha, \alpha_s) Z_{\mathrm{ik}} \frac{\partial}{\partial \alpha_s} \left(Z^{-1}\right)_{\mathrm{kj}} + 2\beta_e(\epsilon, \alpha, \alpha_s) Z_{\mathrm{ik}} \frac{\partial}{\partial \alpha} \left(Z^{-1}\right)_{\mathrm{kj}};$

- $\beta(\epsilon, \alpha, \alpha_s) = \alpha_s(-\epsilon + \beta(\alpha, \alpha_s)) \qquad \beta_e(\epsilon, \alpha, \alpha_s) = \alpha(-\epsilon + \beta_e(\alpha, \alpha_s)).$