Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

N-jettiness soft function at NNLO in QCD

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Introduction

- Substantional progress in providing precise predictions of cross sections at NNLO QCD level in the last few years, especially in handling IR divergences.

- **Slicing Schemes** : Using a suitable variable to split the phase space.

  — q_T subtraction *[Catani, Grazzini '07]*

  — N-jettiness subtraction *[Gaunt et al.'15] [Boughezal et al.'15]*

- **Subtraction Schemes** : Contructing integrable counterterms to cancel divergences.

  — Numerous established subtraction schemes

  e.g. *[Gehrmann et al.'05] [Somogyi et al.'05] [Czakon et al.'11] [Cacciari et al.'15] [Melnikov et al.'17] [Magnea et al.'18]*

N-jettiness Subtraction

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- N-jettiness variable: $\tau = \sum_k \min\left(\frac{2p_i \cdot p_k}{P_i}, \frac{2p_j \cdot p_k}{P_j}, \ldots\right)$ [Stewart, Tackmann, Waalewijn '10]

  (a complicated function)

- τ can be used to slice the phase space in the following way:

$$\sigma = \int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau} + \int_{\tau_o} d\tau \frac{d\sigma}{d\tau}$$

— τ_o : Imposed cut on τ

- The SCET factorization theorem makes the above slicing a convenient choice:

$$\int_0^{\tau_o} d\tau \frac{d\sigma}{d\tau} = \int B_\tau \otimes B_\tau \otimes S_\tau \otimes H_\tau \otimes \prod^N J_{i,\tau} + \mathcal{O}(\tau_o)$$

— Hard Function H_τ

— Beam function B_τ , Jet function $J_{i,\tau}$

— Soft Function S_τ

Soft factorization and Eikonal functions

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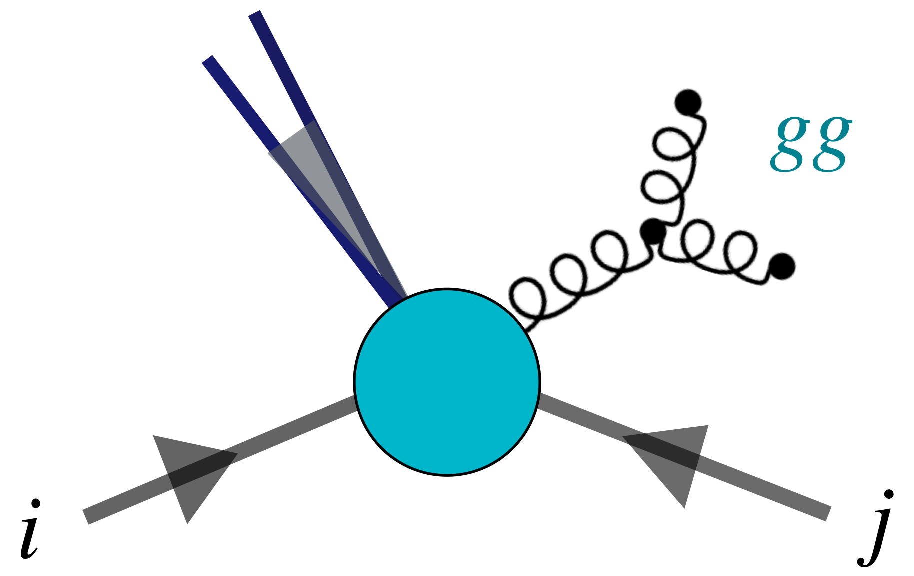
- In color space, we have the soft factorization theorem:

$$|\mathbf{M}_{g,a_1,\ldots,a_n}(m,p_1,\ldots,p_n)|^2 \propto \alpha_s \sum_{i,j=1}^{n} S_{ij}(m)\,|\mathbf{M}^{(i,j)}_{a_1,\ldots,a_n}(p_1,\ldots,p_n)|^2$$

NLO :: Single real emission :

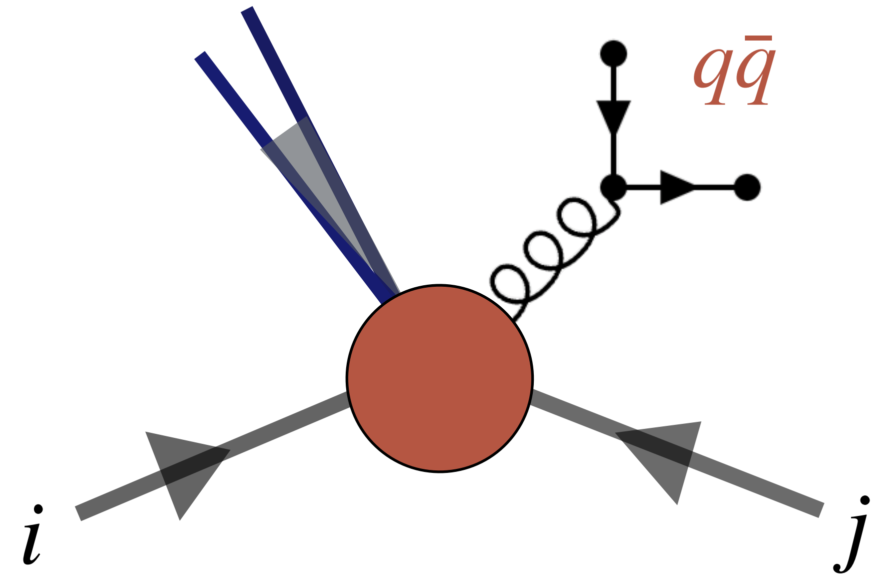
$$S_{ij}(m) = \frac{p_i \cdot p_j}{p_i \cdot p_m p_j \cdot p_m}$$



NNLO ::

- Uncorrelated contribution: Iterated NLO soft eikonal
- Correlated contribution – Gluon emission : $S_{ij}^{gg}(m,n)$

  – Quark emission : $S_{ij}^{q\bar{q}}(m,n)$



Soft Function Calculation

- The Soft function at NNLO was previously available for 0-, 1- and 2-jettiness, *[Boughezal et al.’15] [Campbell et al.’17] [Jin, Liu ’19]* and only recently available for generic N-jettiness. *[Bell et al.’23] [Agarwal et al.’24]*
- It has also been recently calculated at N3LO for 0-jettiness. *[Baranowski et al.’24]*
- Upto N3LO, most of the previous calculations were done by mapping the phase space of soft-gluon emissions to hemispheres using theta functions and integrate numerically within each of them.

$$\sum_{r,s=1}^{N} \mathbf{\Theta}_{rs} = \sum \delta(\tau - q_m . p_r - q_n . p_s) \prod_{k\neq r} \theta(q_m . p_k - q_m . p_r) \prod_{l\neq s} \theta(q_n . p_l - q_n . p_s)$$

Soft Function Calculation

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$$\sum_{r,s=1}^{N} \mathbf{\Theta}_{rs} = \sum \delta(\tau - q_m \cdot p_r - q_n \cdot p_s) \prod_{k \neq r} \theta(q_m \cdot p_k - q_m \cdot p_r) \prod_{l \neq s} \theta(q_n \cdot p_l - q_n \cdot p_s)$$

Another way: Handle the N-jettiness functions analytically using subtractions which helps in generalizations.

Soft Function Calculation

Motivation:

- Recent developments in understanding the NNLO subtraction schemes for a generic N-jet problem.
- Exploiting the connection between modern slicing schemes and established subtraction schemes.

We use established NNLO subtraction methods and show the analytic cancellation of divergences present in the N-jettiness soft function against the renormalization matrix, treating N as a generic parameter.

Renormalization

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- Infrared divergences manifest themselves through ϵ poles. It is useful to renormalize in Laplace space:

$$S(u) = \int_0^\infty d\tau \, S_\tau(\tau) e^{-u\tau} .$$

- Then the renormalization matrix Z in color space is multiplicative: $S = Z\tilde{S}Z^+$

- We expand S, Z and Z^+ in powers of α_s :

Use:

$$S_2 = \frac{1}{2} S_1 S_1 + S_{2,r}$$

$$Z_2 = \frac{1}{2} Z_1 Z_1 + Z_{2,r}$$

$\rightarrow$

$$\tilde{S}_1 = S_1 - Z_1 - Z_1^+$$

$$\tilde{S}_2 = \frac{1}{2}\tilde{S}_1\tilde{S}_1 + \frac{1}{2}[Z_1, Z_1^+] + \frac{1}{2}\left[S_1, Z_1 - Z_1^+\right] + S_{2,r} - Z_{2,r} - Z_{2,r}^+$$

NLO Soft Function

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- Unresolved gluon { m }, $P_i = 2E_i$; We have: $\tau(m) = E_m \psi_m$ $\quad \psi_m = min(\rho_{im}, \rho_{jm}, \ldots)$ and

  $\rho_{ij} = 1 - n_i . n_j$ such that $\rho_{ij} = 0$ when $i || j$

- NLO soft function:

$$S_1(\tau) = -\sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, [\alpha_s] \, [d\Omega_m^{(d-1)}] \, \frac{dE_m}{E_m^{2\epsilon-1}} \, \delta(\tau - E_m \psi_m) \, S_{ij}(m) \qquad \text{where } S_{ij} = \frac{1}{E_m^2} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}}$$

– Integrate over the energy and use Laplace transform. We then use $\lim_{m||i} \psi_m = \rho_{im}$:

Trick:

$$\psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} = \boxed{\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right)^{2\epsilon}} \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} = \left(1 + 2\epsilon g_{ij,m}^{(2)}\right) \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}}$$

NLO Soft Function

- We can now integrate over m ($\eta_{ij} = \rho_{ij}/2$),

$$\left\langle \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} \right\rangle_m = \frac{2\eta_{ij}^{\epsilon}}{\epsilon}\frac{\Gamma(1+\epsilon)^2}{\Gamma(1+2\epsilon)}\, {}_2F_1\left(\epsilon,\epsilon,1-\epsilon,1-\eta_{ij}\right) = \frac{2\eta_{ij}^{\epsilon}}{\epsilon}K_{ij}^{(2)}$$

- Using the matrices Z and Z^{+}, we get the renormalized soft function:

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right)\frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m + \mathcal{O}(\epsilon) \right]$$

(A simple representation)

NNLO Soft Function

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- The bare NNLO soft function is given as: $S_2 = S_{2,RR} + S_{2,RV} - a_s \frac{\beta_0}{\epsilon} S_1$

- Our calculation is organized as follows :

$$\tilde{S}_2 = \tilde{S}_{2,ucorr} + \tilde{S}_{2,corr} + \tilde{S}_{2,tcc}$$

$$\tilde{S}_{2,uncorr} = S_{2,RR,T^4} = \frac{1}{2}\tilde{S}_1\tilde{S}_1$$

$$\tilde{S}_{2,tcc} = S_{RV,tcc} + \frac{1}{2}\left[Z_1, Z_1^+\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^+\right]$$

$$\tilde{S}_{2,corr} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^+ - \frac{a_s\beta_0}{\epsilon}S_1$$

Uncorrelated Emissions

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$$S_{2,RR,\tau,T^4} = \frac{1}{2} \sum_{(ij),(k,l)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\}\, I_{T^4,ij,kl}$$

where $$I_{T^4,ij,kl} = \frac{[\alpha_s]^2}{2} \left\langle \int_0^\infty \frac{dE_m}{E_m^{1+2\epsilon}} \frac{dE_n}{E_n^{1+2\epsilon}}\, \delta(\tau - E_m\psi_m - E_n\psi_n) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_{mn}$$

- Integrate over both the energies
- Apply Laplace transform

} **Helps us identify the iterated NLO contribution**

$$S_{2,RR,T^4} = \frac{[\alpha_s]^2}{4} \sum_{(ij),(kl)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\} \left(\frac{u^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon} \right)^2 \times \left\langle \psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m \left\langle \psi_n^{2\epsilon} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_n = \frac{1}{2} S_1 S_1$$

Renormalization

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- Infrared divergences manifest themselves through ϵ poles. It is useful to renormalize in Laplace space:

$$S(u) = \int_0^\infty d\tau \, S_\tau(\tau) e^{-u\tau}.$$

- Then the renormalization matrix Z in color space is multiplicative: $S = Z\tilde{S}Z^+$
- We expand S, Z and Z^+ in powers of α_s :

Use:

$$S_2 = \frac{1}{2}S_1S_1 + S_{2,r}$$

$$Z_2 = \frac{1}{2}Z_1Z_1 + Z_{2,r}$$

→

$$\tilde{S}_1 = S_1 - Z_1 - Z_1^+$$

$$\tilde{S}_2 = \frac{1}{2}\tilde{S}_1\tilde{S}_1 + \frac{1}{2}[Z_1, Z_1^+] + \frac{1}{2}\left[S_1, Z_1 - Z_1^+\right] + S_{2,r} - Z_{2,r} - Z_{2,r}^+$$

Triple Color Correlated Terms

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$$\tilde{S}_{2,tcc} = S_{RV,tcc} + \frac{1}{2}\left[Z_1, Z_1^{\dagger}\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^{\dagger}\right]$$

- The triple color terms only contribute when the process has two or higher number of jets. :: *Due to color conservation*
- The commutator terms having the triple color structure can be calculated as shown in *[Devoto et al.'23]*

$$\frac{1}{2}[Z_1, Z_1^{\dagger}] = -\frac{\pi a_s^2}{\epsilon^2} \sum_{(kij)} \lambda_{kj} \ln \eta_{ij}\, F^{kij} \qquad \frac{1}{2}[S_1, Z_1 - Z_1^{\dagger}] \propto -\frac{a_s^2 \pi}{\epsilon^2} \sum_{(kij)} \kappa_{kj} \left\langle \psi_m^{2\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \right\rangle_m F^{kij}$$

$$S_{RV,tcc} \propto \frac{a_s^2 \pi}{2\epsilon^2} \sum_{(kij)} \kappa_{kj} \left\langle \psi_m^{4\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}}\right)^{\epsilon} \right\rangle_m F^{kij}$$

where $F^{kij} = f_{abc} T_k^a T_i^b T_j^c$ and $\kappa_{kj} = \lambda_{ij} - \lambda_{im} - \lambda_{jm}$ $\qquad \lambda_{ij} = 1$ if i and j are incoming/outgoing otherwise zero.

Triple Color Correlated Terms

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- Using the same trick as in the NLO case:

$$\frac{1}{2}[S_1, Z_1 - Z_1^{+}] \propto -\frac{a_s^2 \pi}{\epsilon^2} \sum_{(kij)} \kappa_{kj} \left\langle \left(1 + 2\epsilon g^{(2)}_{kl,m}\right) \frac{\rho_{ki}^{1-2\epsilon}}{\rho_{km}^{1-2\epsilon} \rho_{im}^{1-2\epsilon}} \right\rangle_m F^{kij}$$

$$S_{RV,tcc} \propto \frac{a_s^2 \pi}{2\epsilon^2} \sum_{(kij)} \kappa_{kj} \left\langle \left(1 + 4\epsilon g^{(4)}_{ki,m}\right) \frac{\rho_{ki}^{1-4\epsilon}}{\rho_{km}^{1-4\epsilon} \rho_{im}^{1-4\epsilon}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}}\right)^{\epsilon} \right\rangle_m F^{kij}$$

- Combining the contributions, the divergent terms containing the N-jettiness function cancel.

$$\Rightarrow \quad -\frac{2a_s^2 \pi}{\epsilon} \sum_{(kij)} \kappa_{kj} \left\langle \frac{\rho_{ik}}{\rho_{im}\rho_{km}} \left(g^{(2)}_{ki,m} - g^{(4)}_{ki,m}\right) \right\rangle_m F^{kij} \quad = \quad \mathcal{O}(\epsilon^0)$$

We obtain a finite remainder containing the N-jettiness function.

Triple Color Correlated Terms

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- **Remaining poles** : Extracted using the idea

$$\Rightarrow \quad \left\langle \frac{\rho_{ki}^{1-4\epsilon}}{\rho_{km}^{1-4\epsilon}\rho_{im}^{1-4\epsilon}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m$$

Triple Color Correlated Terms

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- **Remaining poles** : Extracted using the idea

$$\Rightarrow \underbrace{\left\langle \frac{\rho_{ki}^{1-4\epsilon}}{\rho_{km}^{1-4\epsilon}\rho_{im}^{1-4\epsilon}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m - \left\langle \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m}$$

Calculated in [Devoto et al.'23]

$$\Rightarrow \quad \mathcal{O}(\epsilon^0)$$

- All poles cancel and we get a finite remainder:

$$\tilde{S}_{2,tcc} = a_s^2 \pi \sum_{(kij)} F^{kij} \, \kappa_{kj} G_{kij}^{triple}$$

Correlated Emissions

$$\tilde{S}_{2,corr} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z^{+}_{2,r} - \frac{a_s\beta_0}{\epsilon} S_1$$

Real-Virtual Contribution :

$$S_{2,RV,T^2} \propto \frac{[\alpha_s]^2}{\epsilon^3} C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left\langle \psi_m^{4\epsilon} \left(\frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right)^{1+\epsilon} \right\rangle_m \Bigg\}$$ Use the same NLO trick

Double Real Contribution : Double emission eikonals $S_{ij}^{gg}(m,n)$ and $S_{ij}^{q\bar{q}}(m,n)$

$$S_{2,RR,T^2,\tau} = -\frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \frac{g_s^4}{2} \underbrace{\int [dp_m][dp_n] \; \delta\left(\tau - E_m\psi_m - E_n\psi_n\right) \tilde{S}_{ij}^{gg}(m,n)}_{I_{ij}(m,n)}$$

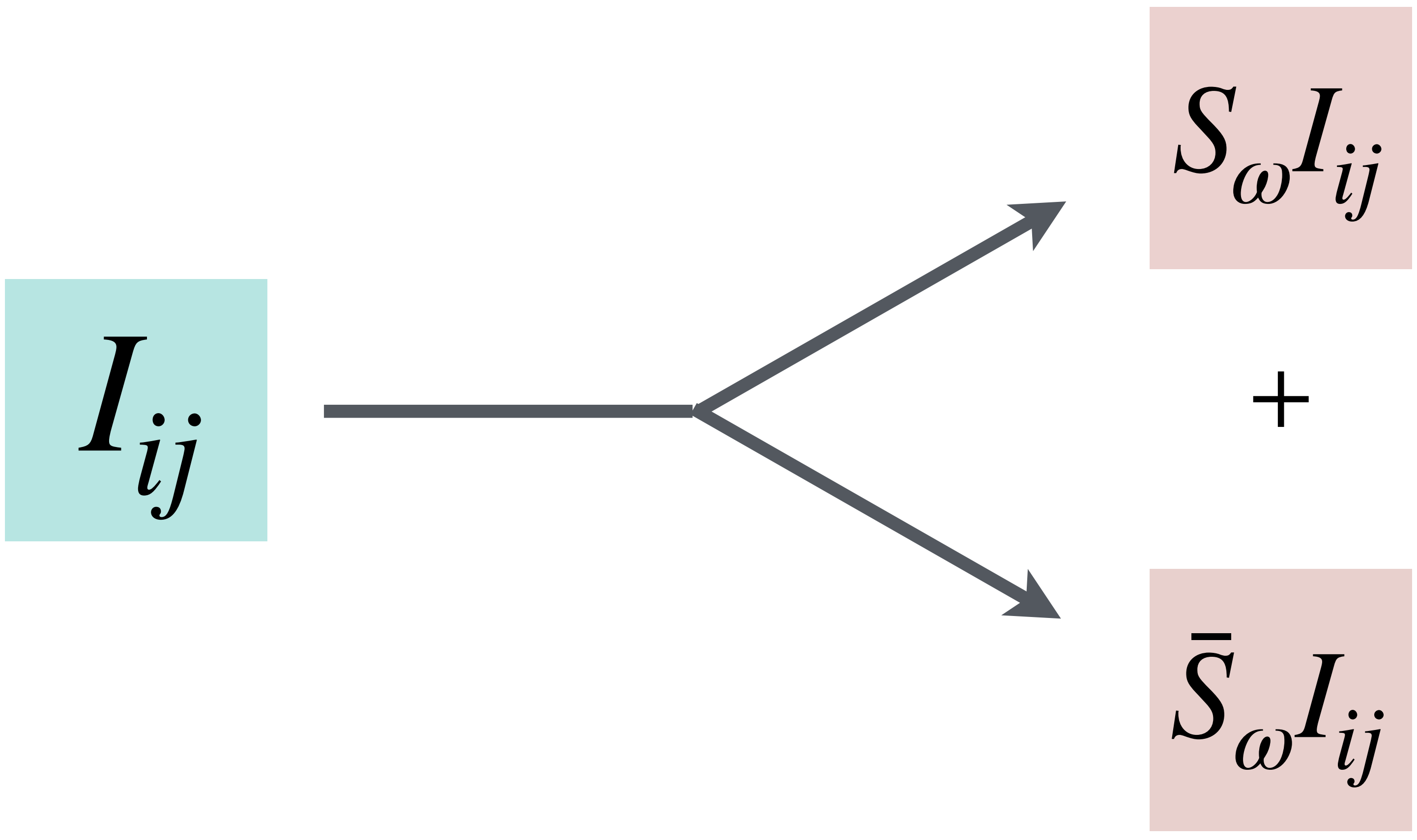
Challenging! Many Soft and Collinear divergences. $\Rightarrow$ Nested Subtractions

Correlated Emissions: Nested Subtractions

$$I_{ij} \longrightarrow \begin{cases} S_\omega I_{ij} \\ + \\ \bar{S}_\omega I_{ij} \end{cases}$$

$$(\bar{S}_\omega = I - S_\omega)$$

Correlated Emissions: Nested Subtractions

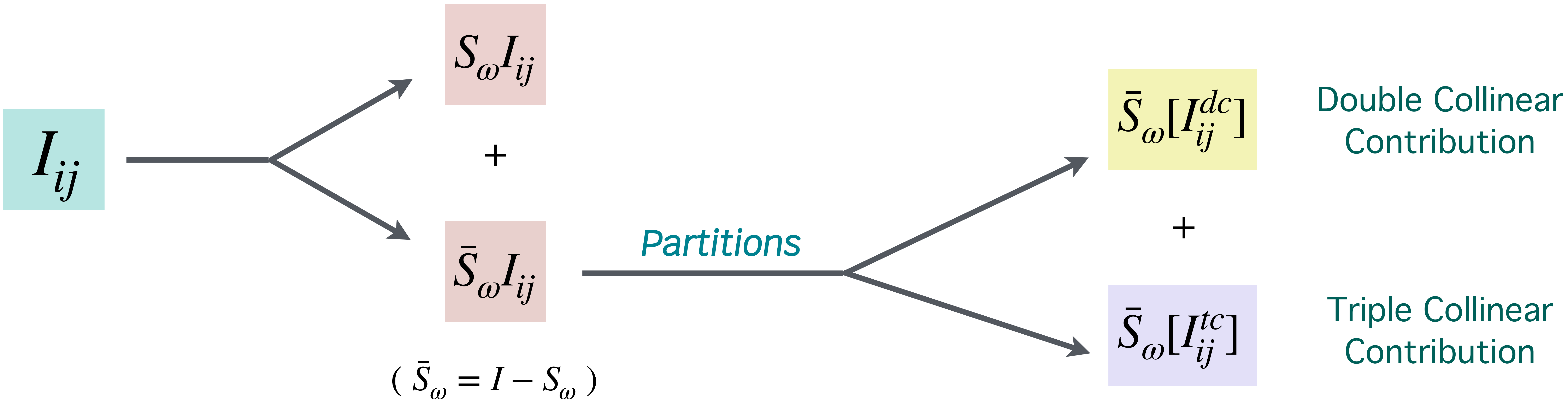


($\bar{S}_\omega = I - S_\omega$)

(Contains collinear divergences only)

- S_ω: Strongly ordered operator (double soft limit with energy ordering ω)
- The poles present in $S_\omega I_{ij}$ are extracted analytically using subtractions.
- The extracted N-jettiness poles cancel with the ones obtained from the S_{2,RV,T^2} .

Correlated Emissions: Nested Subtractions



Correlated Emissions: Nested Subtractions

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I_{ij} → $S_\omega I_{ij}$ + $\bar{S}_\omega I_{ij}$

$(\bar{S}_\omega = I - S_\omega)$

$\bar{S}_\omega I_{ij}$ —Partitions→ $\bar{S}_\omega[I_{ij}^{dc}]$ + $\bar{S}_\omega[I_{ij}^{tc}]$

$\bar{S}_\omega[I_{ij}^{dc}]$: Double Collinear Contribution

$\bar{S}_\omega[I_{ij}^{tc}]$: Triple Collinear Contribution

$$\bar{S}_\omega[I_{ij}^{dc}] = \frac{N_u}{\epsilon}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle \psi_{mn}^{4\epsilon}\left(w^{mi,nj} + w^{ni,mj}\right) \bar{S}_\omega \left[\omega^2 \tilde{S}_{ij}^{gg}(m,n)\right]\right\rangle_{mn} \Rightarrow$$

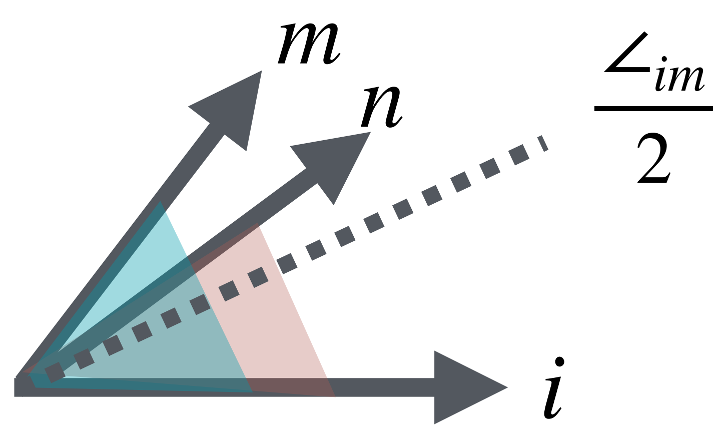
Poles independent of N-jettiness function

- The divergent poles of this contribution can then be extracted from *[Delto et al. '18]*

Correlated Emissions: Nested Subtractions

Triple Collinear Contribution:

$$\bar{S}_\omega[I_{ij}^{tc}] = \frac{N_u}{\epsilon}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle \psi_{mn}^{4\epsilon}\,(w^{mi,ni} + w^{mj,nj})\,\bar{S}_\omega\left[\omega^2 \tilde{S}_{ij}^{gg}(m,n)\right]\right\rangle_{mn}$$



- Contains $m||n$, $m||n||i$ and $m||n||j$ singularities.

Strategy :
- — Introduce sectors to handle the $m||n$ singularity. [Devoto et al.'23]
- — Use the triple collinear limits of the integral as subtraction terms and calculate the poles analytically.
- — Identify the terms corresponding to calculations without the N-jettiness constraint to avoid complex calculations.

Correlated Emissions: Nested Subtractions

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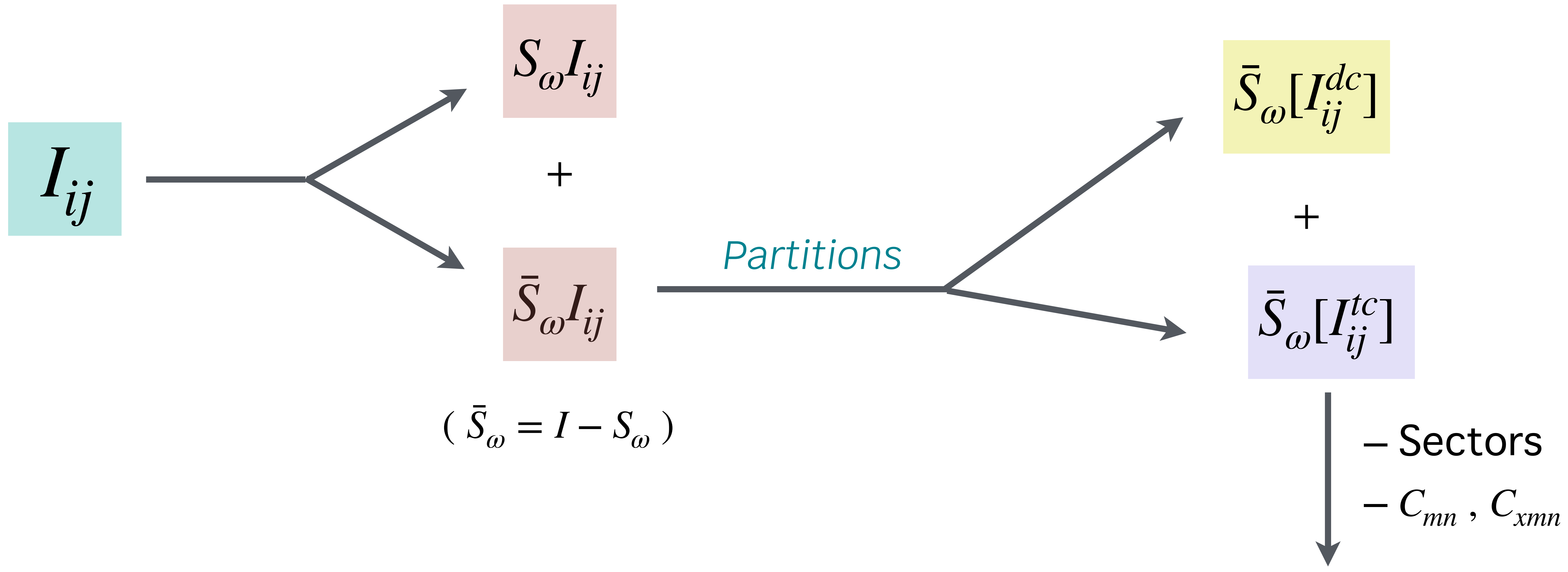

$$\frac{N_u}{\epsilon} \sum_{x\in\{i,j\}} \int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle (1-\theta^{b+d} C_{mn}) \left[d\Omega_{mn}\right] \bar{C}_{xmn} \, w^{mx,nx} \, \psi_{mn}^{4\epsilon} \bar{S}_\omega \left[\omega^2 \tilde{S}_{ij}^{gg}(m,n)\right] \right\rangle_{mn}$$

Fully subtracted term

Final Renormalized Soft function

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- We obtain a compact finite representation of the renormalized soft function at NNLO:

$$\tilde{S}_2 = \frac{1}{2}\,\tilde{S}_1^2 + a_s^2 C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, G_{ij} + a_s^2\, n_f\, T_R \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j\, Q_{ij} + a_s^2 \pi \sum_{(kij)} F^{kij}\, \kappa_{kj} G_{kij}^{triple}\,.$$

- G_{ij} , Q_{ij} and G_{kij}^{triple} are finite functions containing:
  — *Analytic functions of* η_{ij}
  — *A low number of numerical integrations* over one- and two-particle phase space in 4 dimensions.

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m + \mathcal{O}(\epsilon) \right]$$

Numerical Checks

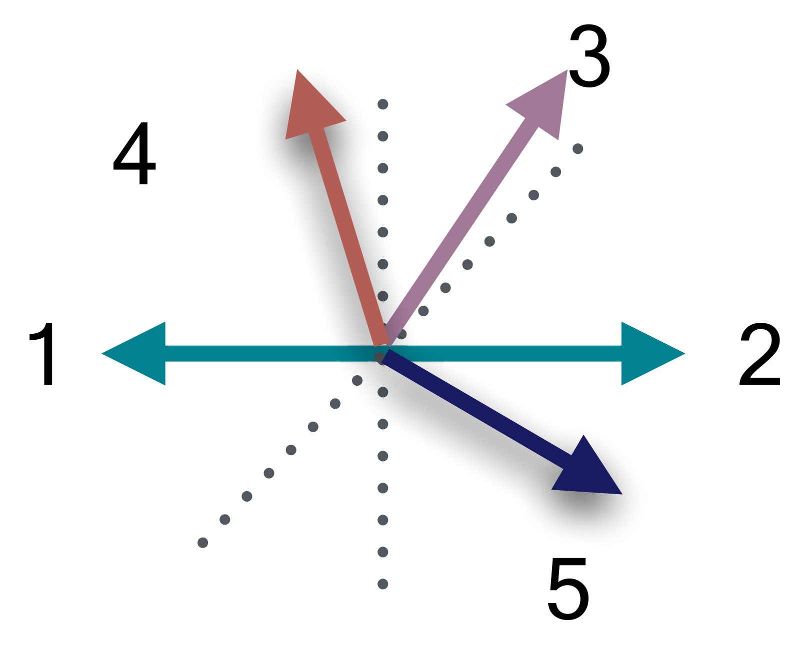
- We compare our results for the renormalized N-jettiness soft function with those presented in previous calculations, and find excellent agreement for N = 1, 2 and 3.

  [Boughezal et al.'15] [Campbell et al.'17] [Bell et al.'23]

- We present the comparison of our results for the 3-jettiness case with the benchmark result provided in [Bell et al.'23] . For N jets, we will have $\binom{N+2}{2}$ dipoles.

  For the back to back configuration, we need 5 angles to parametrize the phase space. Consider the phase space point :

$$\theta_{13} = \frac{3\pi}{10}, \quad \theta_{14} = \frac{6\pi}{10}, \quad \theta_{15} = \frac{9\pi}{10}, \quad \phi_4 = \frac{3\pi}{5}, \quad \phi_5 = \frac{6\pi}{5}$$



Numerical Checks

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Dipole Configurations for N=3

Dipoles	Gluons		Quarks	
	G_{ij}	[Bell et. al.'23]	Q_{ij}	[Bell et. al.'23]
12	116.20 ± 0.01	116.20 ± 0.16	-36.249 ± 0.001	-36.244 ± 0.009
13	38.13 ± 0.03	37.63 ± 0.03	-21.717 ± 0.007	-21.732 ± 0.005
14	63.63 ± 0.01	63.66 ± 0.06	-25.189 ± 0.003	-25.192 ± 0.006
15	107.17 ± 0.01	106.99 ± 0.12	-35.268 ± 0.001	-35.256 ± 0.009
23	97.11 ± 0.01	96.97 ± 0.10	-32.875 ± 0.002	-32.872 ± 0.008
24	67.36 ± 0.02	67.51 ± 0.08	-26.821 ± 0.003	-26.815 ± 0.007
25	30.87 ± 0.03	30.73 ± 0.04	-21.561 ± 0.009	-21.561 ± 0.005
34	69.43 ± 0.01	69.24 ± 0.07	-25.854 ± 0.002	-25.861 ± 0.006
35	106.13 ± 0.02	105.97 ± 0.13	-34.799 ± 0.002	-34.796 ± 0.008
45	74.45 ± 0.02	74.36 ± 0.09	-28.247 ± 0.004	-28.251 ± 0.007

Tripole Configurations

	$\tilde{c}_{\text{tripoles}}$	[Bell et. al.'23]
$\tilde{c}^{(2,124)}_{\text{tripoles}}$	-683.25 ± 0.01	-683.23 ± 0.04
$\tilde{c}^{(2,125)}_{\text{tripoles}}$	-2203.3 ± 0.2	-2203.5 ± 0.1
$\tilde{c}^{(2,145)}_{\text{tripoles}}$	-6.324 ± 0.004	-6.325 ± 0.04
$\tilde{c}^{(2,245)}_{\text{tripoles}}$	-0.837 ± 0.008	-0.830 ± 0.039

These are the 4 independent tripole configurations mentioned in *[Bell et al.'23]*.

Conclusions

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- We derive a compact finite result for the N-jettiness soft function (for a generic N), which allows for faster numerical implementations especially for higher number of jets.

- We demonstrate the analytic cancellation of divergences against the renormalization matrix.

- We successfully show the benefits of using subtraction based methods to derive representations for building blocks of modern slicing methods.

- We find excellent agreement while comparing our results for N=1,2 and 3 with previous calculations. *[Campbell et al. '17] [Bell et al. '23] [Boughezal et al. '15]*

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- We find excellent agreement while comparing our results for N=1,2 and 3 with previous calculations. *[Campbell et al. '17] [Bell et al. '23] [Boughezal et al. '15]*

Thank you for listening!

Backup Slides

Backup : Renormalization

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- Expanding in α_s and putting in $S = Z\tilde{S}Z^{+}$

$$S = 1 + S_1 + S_2$$
$$Z = 1 + Z_1 + Z_2$$
$$Z^{+} = 1 + Z_1^{+} + Z_2^{+}$$

→

$$\tilde{S}_1 = S_1 - Z_1 - Z_1^{+},$$
$$\tilde{S}_2 = S_2 - Z_2 - Z_2^{+} + Z_1 Z_1 + Z_1^{+} Z_1^{+} - Z_1 S_1 - S_1 Z_1^{+} + Z_1 Z_1^{+}.$$

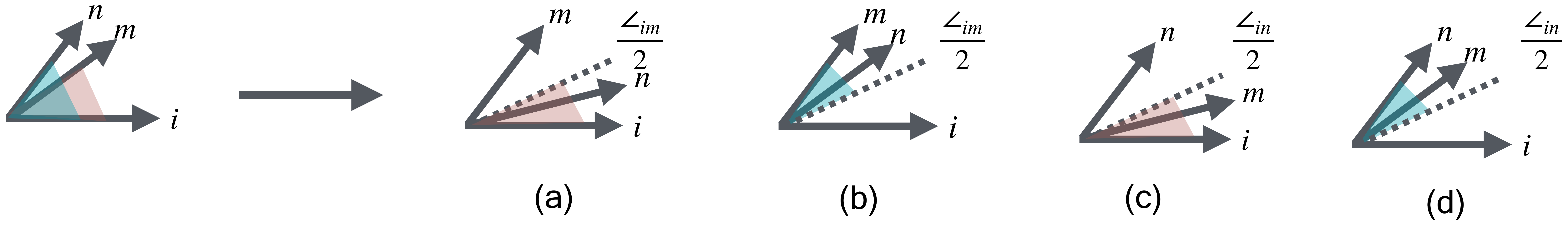
Use:

$$S_2 = \frac{1}{2} S_1 S_1 + S_{2,r}$$
$$Z_2 = \frac{1}{2} Z_1 Z_1 + Z_{2,r}$$

→

$$\tilde{S}_1 = S_1 - Z_1 - Z_1^{+}$$
$$\tilde{S}_2 = \frac{1}{2}\tilde{S}_1\tilde{S}_1 + \frac{1}{2}[Z_1, Z_1^{+}] + \frac{1}{2}\left[S_1, Z_1 - Z_1^{+}\right] + S_{2,r} - Z_{2,r} - Z_{2,r}^{+}$$

Backup: Sectors and TC contributions



Triple Collinear Contribution

$$\bar{S}_\omega[I_{ij}^{tc}] = \frac{N_u}{\epsilon}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle C_{mn}\left[d\Omega_{mn}\right]\theta^{b+d} w^{tc}\psi_{mn}^{4\epsilon}\bar{S}_\omega\left[w^2\tilde{S}_{ij}^{gg}(m,n)\right]\right\rangle_{mn}$$

$$+\frac{N_u}{\epsilon}\sum_{x\in\{i,j\}}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}}\left\langle(1-\theta^{b+d}C_{mn})\left[\mathrm{d}\Omega_{mn}\right]C_{xmn}w^{tc}\psi_{mn}^{4\epsilon}\bar{S}_\omega\left[w^2\tilde{S}_{ij}^{gg}(m,n)\right]\right\rangle_{mn}$$

$$+\frac{N_u}{\epsilon}\sum_{x\in\{i,j\}}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}}\left\langle(1-\theta^{b+d}C_{mn})\left[\mathrm{d}\Omega_{mn}\right]\bar{C}_{xmn}\,w^{mx,nx}\psi_{mn}^{4\epsilon}\bar{S}_\omega\left[w^2\tilde{S}_{ij}^{gg}(m,n)\right]\right\rangle_{mn}$$

Backup : Renormalized function Q_{ij}

$$
\begin{aligned}
Q_{ij} \; &= -\frac{8}{9}L_{ij}^3 - \frac{20}{9}L_{ij}^2 - L_{ij}\left(\frac{4}{3}\left\langle L^{\psi}_{ij,m}\frac{\rho_{ij}}{\rho_{im}\rho_{jm}}\right\rangle_m + \frac{4}{3}\, Li_2(1-\eta_{ij}) + \frac{56}{27}\right) - \left\langle \frac{\rho_{ij}}{\rho_{im}\rho_{jm}}\left(\frac{2}{3}\left(L^{\psi}_{ij,m}\right)^2 - L^{\psi}_{ij,m}\left(\frac{2}{3}\ln\left(\frac{\eta_{ij}^2}{4\eta_{im}\eta_{jm}}\right) - \frac{26}{9} + \frac{2(\rho_{im}+\rho_{jm})}{3\rho_{ij}}\right)\right)\right\rangle_m \\
&+\left\langle A^{fin}_{ij,m}\left(-\frac{2}{3}L^{\psi}_{ij,m} + \frac{2}{3}\ln\left(\frac{\eta_{ij}}{\eta_{im}\eta_{jm}}\right) - \frac{23}{36}\right)\right\rangle_m + \left\langle B^{fin}_{ij,m}\left(\frac{1}{3}L^{\psi}_{ij,m} - \frac{1}{3}\ln\left(\frac{\eta_{ij}}{\eta_{im}\eta_{jm}}\right) + \frac{13}{36}\right)\right\rangle_m + \frac{2}{3}Li_3(\eta_{ij}) \\
&+Li_2(1-\eta_{ij})\left(\frac{2\ln(\eta_{ij})}{3} - \frac{7}{2} - 4\ln 2\right) + \ln^2 2\left(-\frac{2}{3} + \frac{4}{3}\ln(\eta_{ij})\right) + \ln 2\left(-\frac{4}{3}\ln^2(\eta_{ij}) - \frac{20}{9}\ln(\eta_{ij}) + \frac{4\pi^2}{9} + \frac{4}{9}\right) \\
&-\ln^2(\eta_{ij})\left(\frac{10}{9} - \frac{1}{3}\ln(1-\eta_{ij})\right) + \ln(\eta_{ij})\left(\frac{\pi^2}{9} - \frac{335}{54}\right) + \frac{4\zeta_3}{9} + \frac{122}{81} - \frac{47\pi^2}{108} + \frac{4}{3}Ci_3(2\delta_{ij}) + \frac{1}{3\tan(\delta_{ij})}Si_2(2\delta_{ij}) \\
&- \sum_{x\in\{i,j\}}\int_0^1 \frac{d\omega}{\omega}\left\langle (1-\theta^{b+d}C_{mn})\left[d\Omega_{mn}\right]\bar{C}_{xmn}\, w^{xm,xn}\,\ln\psi_{mn}\,[\omega^2\tilde{S}^{q\bar{q}}_{ij}(m,n)]\right\rangle_{mn} - \int_0^1 \frac{d\omega}{\omega}\left\langle (w^{im,jn}+w^{jm,in})\,\ln\psi_{mn}\,[\omega^2\tilde{S}^{q\bar{q}}_{ij}(m,n)]\right\rangle_{mn},
\end{aligned}
$$