Lifting degenerate neutrino masses, threshold corrections and maximal mixing

Young Scientists Workshop — Waldhotel Zollernblick, Freudenstadt (Black Forest)

Wolfgang Gregor Hollik | Oct 31, 2014

INSTITUT FOR THEORETICAL PARTICLE PHYSICS — KARLSRUHE INSTITUTE OF TECHNOLOGY (KIT)

The issue of nearly degenerate neutrinos

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The neutrino mass spectrum

$$m_1 = m_0\,, \quad m_2 = \sqrt{m_0^2 + \Delta m_{21}^2}\,, \quad m_3 = \sqrt{m_0^2 + \Delta m_{31}^2}$$

- Oscillations: [νfit: www.nu-fit.org]
  - $\Delta m_{21}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\text{eV}^2$
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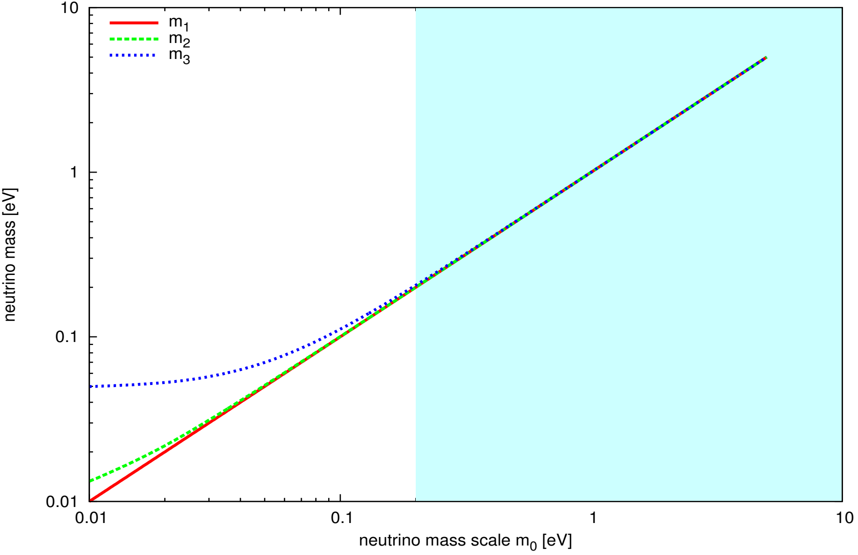
- Unknown: Absolute neutrino mass scale $\rightarrow$ KATRIN ?
- new limit $(2016 + x?)$: **0.2 eV**, discovery $(5\,\sigma)$: **0.35 eV**

Quasi-Degeneration



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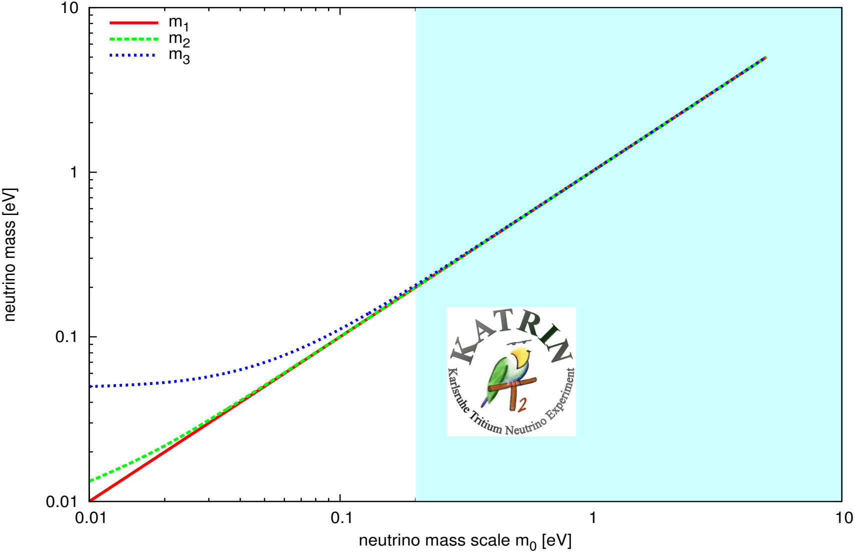

The history of degenerate neutrinos

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- degenerate patterns follow from S_3 symmetries ("flavour democracy") [Fitzsch and Xing since 1996]
- Renormalization Group effects on degenerate neutrinos [Ellis, Lola 1999; Casas, Espinosa, Ibarra, Navarro 1999; Balaji, Dighe, Mohapatra, Parida 2000]
- Quantum corrections: fixed point solutions [Chankowski, Krolikowski, Pokorski 2000; Chankowski, Pokorski 2002]
- Quantum corrections: threshold effects [Chun, Pokorski 2000; Chun 2001; Chankowski, Ioannisian, Pokorski, Valle 2001]
- Issues on degenerate Majorana neutrinos: Majorana phases [Branco, Rebelo, Silva-Marcos 1998; Haba, Matsui, Okamura 2000; Branco, Rebelo, Silva-Marcos, Wegman 2014]

RGE Corrections

[Chankowski, Pokorski 2002]

Renormalization Group Equation for ν masses and mixing

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{C} = -K\boldsymbol{C} - \kappa\left[\left(\boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right)^T \boldsymbol{C} + \boldsymbol{C}\left(\boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right)\right]$$

$$t = \frac{1}{16\pi^2}\ln\left(\frac{Q}{M_Z}\right)$$

SM: $\kappa = -\frac{3}{2}$ and $K = -3g_2^2 + 2\,\mathrm{Tr}\left(3\boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u + 3\boldsymbol{Y}_d^\dagger \boldsymbol{Y}_d + \boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right) + 2\lambda$

MSSM: $\kappa = +1$ and $K = -6g_2^2 - 2g_Y^2 + 2\,\mathrm{Tr}\left(3\boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u\right)$

Solving the RGE

$$\boldsymbol{C}(t) = I_K \mathcal{I}\boldsymbol{C}(0)\mathcal{I},$$

where $\mathcal{I} = \mathrm{diag}(I_e, I_\mu, I_\tau)$ and

$$I_K = \exp\left(-\int_0^t K(t')\mathrm{d}t'\right), \quad I_{e_A} = \exp\left(-\kappa\int_0^t y_{e_A}^2(t')\mathrm{d}t'\right).$$

Threshold Corrections

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Generic treatment

$$m^{\nu}_{AB} = m^{(0)}_{AB} + m^{(0)}_{AC} I_{CB} + I_{AC} m^{(0)}_{CB}$$

I: threshold correction $I \sim \frac{g^2}{16\pi^2} f\left(\ln(M^2/Q^2)\right)$ (in the SM diagonal).

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- tree-level rotation matrix $\boldsymbol{U}^{(0)}$: ${\boldsymbol{U}^{(0)}}^T \, \boldsymbol{m}^{(0)} \, \boldsymbol{U}^{(0)} =$ diagonal

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Mass basis

$$m^{\nu}_{ab} = m^{(0)}_{a} \delta_{ab} + \left(m^{(0)}_{a} + m^{(0)}_{b}\right) I_{ab}$$

$I_{ab} = \sum_{AB} I_{AB} U^{(0)}_{Aa} U^{(0)}_{Bb}$

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- assumption: degenerate tree-level masses, $|m^{(0)}_1| = |m^{(0)}_2| = |m^{(0)}_3|$

Comments on degenerate masses

- if $\boldsymbol{m}^{(0)} = m_0\,\mathbb{1}$: ${\boldsymbol{U}^{(0)}}^T\,\boldsymbol{m}^{(0)}\,\boldsymbol{U}^{(0)} = m_0\,\mathbb{1}$ for any (real) $\boldsymbol{U}^{(0)}$

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  - taking CP as good symmetry: $\alpha_{1,2} \in \{0, \pm\frac{\pi}{2}\}$
- choice: $m_1 = -m_2 = m_3$:

$$\boldsymbol{m}^\nu = m_0 \begin{pmatrix} 1 + 2U_{\alpha 1}U_{\beta 1}I_{\alpha\beta} & 0 & 2U_{\alpha 1}U_{\beta 3}I_{\alpha\beta} \\ 0 & -1 - 2U_{\alpha 2}U_{\beta 2}I_{\alpha\beta} & 0 \\ 2U_{\alpha 1}U_{\beta 3}I_{\alpha\beta} & 0 & 1 + 2U_{\alpha 3}U_{\beta 3}I_{\alpha\beta} \end{pmatrix}$$

Same CP

Trivial mixing @ tree-level

$$\boldsymbol{m}^\nu = m_0\,\mathbb{1} + m_0 \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{pmatrix}$$

- requirements for I_{ij}:
  1. $\theta_{23} \approx \pi/4$

$$\boldsymbol{U}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

  2. $I_{12} = I_{13} \hookrightarrow \theta_{13} = 0$
  3. $\theta_{12} = \frac{1}{2} \arctan\left(\frac{2\sqrt{2} I_{12}}{2I_{22} - I_{11}}\right)$
- get $m_{1,2}$ in terms of I_{ij}, $m_3 = m_0$

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$$\boldsymbol{I}' = \boldsymbol{U}_{23}^T \boldsymbol{I} \boldsymbol{U}_{23} = \begin{pmatrix} I_{11} & \frac{I_{12}+I_{13}}{\sqrt{2}} & -\frac{I_{12}-I_{13}}{\sqrt{2}} \\ \frac{I_{12}+I_{13}}{\sqrt{2}} & 2I_{22} & 0 \\ -\frac{I_{12}-I_{13}}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

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Nonzero θ_{13}

Deviations

- $\theta_{13} \neq 0 \qquad \hookrightarrow I_{13} \neq I_{12}$
- $\theta_{23} \lesssim \frac{\pi}{2}$

$$I_{33} = I_{22} + \varepsilon$$
$$I_{13} = I_{12} + \delta$$

$$\begin{pmatrix} m_0 & & \\ & \sqrt{m_0^2 + \Delta m_{21}^2} & \\ & & \sqrt{m_0^2 + \Delta m_{31}^2} \end{pmatrix}$$

$$= m\,\boldsymbol{U}(\theta_{12}, \theta_{13}, \theta_{23})^T \begin{pmatrix} 1 + I_{11} & I_{12} & I_{12} + \delta \\ I_{12} & 1 + I_{22} & I_{22} \\ I_{12} + \delta & I_{22} & 1 + I_{22} + \varepsilon \end{pmatrix} \boldsymbol{U}(\theta_{12}, \theta_{13}, \theta_{23})$$

Numerical example

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Inputs (central values, “old” numbers”), $m_0 = 0.35\,\mathrm{eV}$

$$\theta_{12} \approx 31.8^\circ\ ,\ \theta_{13} \approx 8.5^\circ\ ,\ \theta_{23} \approx 39.2^\circ\ ,$$
$$\Delta m_{21}^2 \approx 7.5 \times 10^{-5}\,\mathrm{eV}^2\ ,\ \Delta m_{31}^2 \approx 2.458 \times 10^{-3}\,\mathrm{eV}^2$$

Outputs

$$m \approx 0.35\,\mathrm{eV}\ ,$$
$$I_{11} \approx 0.98 \times 10^{-3}\ , \quad I_{22} \approx 0.47 \times 10^{-2}\ , \quad I_{12} \approx 0.10 \times 10^{-2}\ ,$$
$$\delta \approx 0.19 \times 10^{-4}\ , \quad \varepsilon \approx 0.20 \times 10^{-2}$$

$$\boldsymbol{I} = \begin{pmatrix} 0.000976 & 0.00103 & 0.00105 \\ 0.00103 & 0.00475 & 0.00475 \\ 0.00105 & 0.00475 & 0.00674 \end{pmatrix}$$

Non-diagonal threshold corrections

MSSM with righthanded neutrinos

$$\mathcal{W} \supset \mu H_1 \cdot H_2 + Y^\nu_{ij} \, H_2 \cdot L_{L,i} N_{R,j} - Y^\ell_{ij} \, H_1 \cdot L_{L,i} E_{R,j} + \frac{1}{2} M^R_{ij} N_{R,i} N_{R,j}$$

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New soft SUSY breaking terms

$$V^{\tilde\nu}_{\text{soft}} = \left(\boldsymbol{m}^2_{\tilde L}\right)_{ij} \tilde\nu^*_{L,i} \tilde\nu_{L,j} + \left(\boldsymbol{m}^2_{\tilde R}\right)_{ij} \tilde\nu_{R,i} \tilde\nu^*_{R,j} + \left(A^\nu_{ij}\, h^0_2\, \tilde\nu_{L,i} \tilde\nu^*_{R,j} + \left(\boldsymbol{B}^2\right)_{ij} \tilde\nu^*_{R,i} \tilde\nu^*_{R,j} + \text{h.c.} \right)$$

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- seesaw type I:

$$\boldsymbol{m}^{(0)}_\nu = -v_u^2 \boldsymbol{Y}^T_\nu \boldsymbol{M}^{-1}_R \boldsymbol{Y}_\nu + \mathcal{O}(v_u^4/M_R^3)$$

- adding SUSY 1-loop [Dedes, Haber, Rosiek 2007]

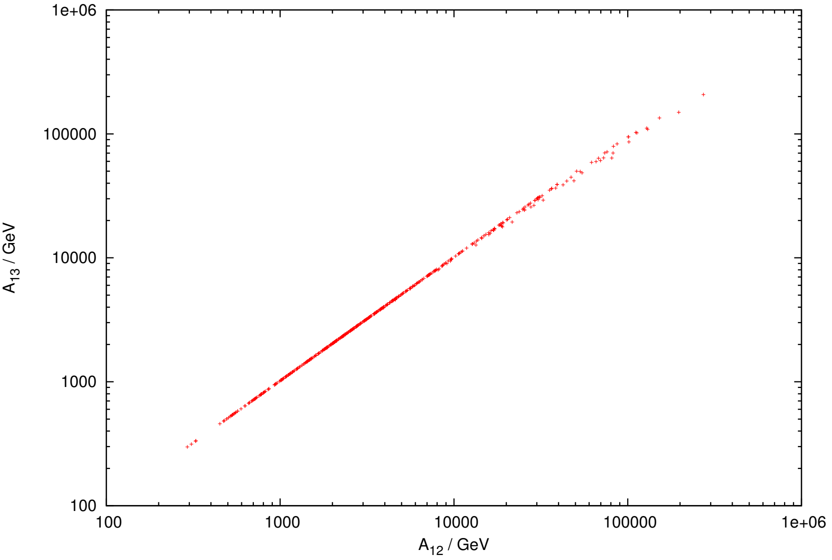
$$\left(\boldsymbol{m}^{1-\text{loop}}_\nu\right)_{ij} = (\boldsymbol{m}_\nu)_{ij} + \text{Re}\left[\Sigma^{(\nu),S}_{ij} + \frac{m_{\nu_i}}{2}\Sigma^{(\nu),V}_{ij} + \frac{m_{\nu_j}}{2}\Sigma^{(\nu),V}_{ji}\right]$$

Constraints on A^ν

Random scan

$$M_{\text{SUSY}} \in [500, 5000]\,\text{GeV}$$
$$M_1 \in [0.3, 3]\,M_{\text{SUSY}}$$
$$M_2 \in [1, 5]\,M_{\text{SUSY}}$$
$$\mu \in [-15, 15]\,\text{TeV}$$
$$\tan\beta \in [10, 60]$$

Constraints on A^ν



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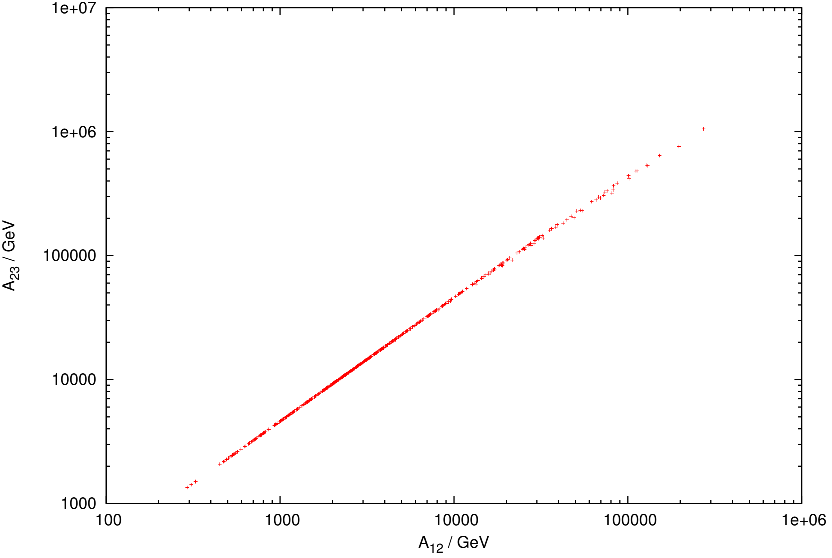

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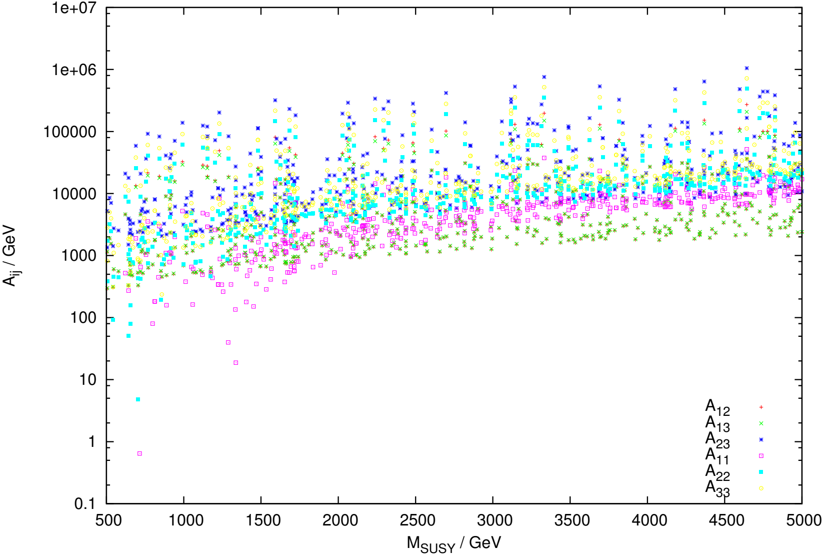

Conclusions

- if tritium decay measures a neutrino mass: spectrum quasidegenerate
- large effects from renormalization group running
- threshold effects become important for degenerate masses
- update on "$m_1 = -m_2 = m_3$": $\sin\theta_{13}$ ok, but not Δm^2_{ij}
- same CP eigenvalues: threshold corrections have the power to completely determine the mixing and deviations from degenerate masses
- non-diagonal corrections needed: e.g. supersymmetric
- requirement for off-diagonal A^ν

Backup

Slides

Side Note

the same follows from a special tree-level mass matrix

$$\boldsymbol{m}^{\nu}_{\text{tree}} = x \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + z \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} x & y & y \\ y & x+z & z \\ y & z & x+z \end{pmatrix},$$

which can be diagonalized by

$$\boldsymbol{U}_{\text{tree}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

with $s_{12} = \sin\theta_{12}, c_{12} = \cos\theta_{12}$,
$\tan 2\theta_{12} = \sqrt{2}\frac{y}{z}$

Side Note (cont'd)

can be inverted

$$\boldsymbol{m}^{\nu}_{\text{tree}} =$$

$$\begin{pmatrix} m_1 & \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} \\ \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \frac{m_2+m_3}{2} & \frac{1}{2}\left(\sum_i m_i - 3m_1\right) \\ \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \frac{1}{2}\left(\sum_i m_i - 3m_1\right) & \frac{m_2+m_3}{2} \end{pmatrix}$$

The philosophy behind threshold corrections

- exact degeneracy @ tree-level: trivial mass matrix
- $m^{(1)} = m^{(0)} + m^{(0)} I, \qquad I \sim \frac{1}{16\pi^2} \approx \frac{1}{100}$
- small perturbation

Update on the $+-+$ scenario

Brief review of [Chankowski, Pokorski 2002]

- degeneracy leaves freedom of rotation $U^{(0)} \to U^{(0)} R_{13}$

$$\sum_{\alpha\beta} U^{(0)}_{\alpha 1} U^{(0)}_{\beta 3} I_{\alpha\beta} = 0$$

- flavour diagonal corrections: $I_{\alpha\beta} = I_\alpha \delta_{\alpha\beta}$
- explain deviation from (tri-)bi-maximal mixing: $s_{13} = \sin\theta_{13} \approx 0$

$$s_{13} = -\frac{s_{12}}{c_{12}} s_{23} c_{23} \frac{I_\tau}{I_e}, \qquad \text{where } I_\mu = 0 \text{ and } I_e \gg I_\tau$$

Update on the $+-+$ scenario

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$I_\mu \neq 0$

try to accommodate $s_{13} \approx 0.15$ and $\Delta m^2_{31}/\Delta m^2_{21} \approx 33$

$$s_{13} = c_{23} s_{23} \frac{s_{12}}{c_{12}} \frac{I_\mu - I_\tau}{I_e - s^2_{23} I_\mu - c^2_{23} I_\tau}$$

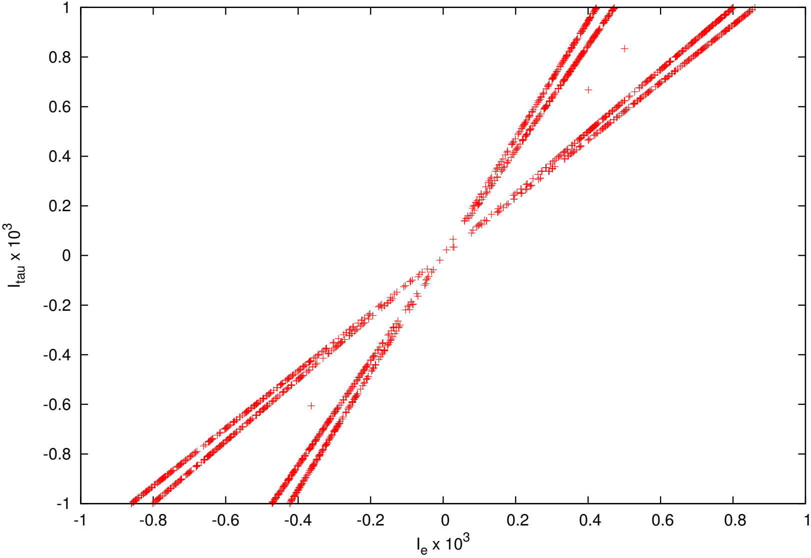
Update on the $+-+$ scenario (cont'd)

$$\Delta m^2_{ab} = m^2 \left([1 + 2U^2_{\alpha a} I_\alpha]^2 - [1 + 2U^2_{\alpha b} I_\alpha]^2 \right)$$

- m^2 overall scale
- use relation for s_{13} to get correlation between I_e and I_μ, I_τ
- try to fit

$$\Delta m^2_{31} / \Delta^2_{m21} = \frac{\left([1 + 2U^2_{\alpha 3} I_\alpha]^2 - [1 + 2U^2_{\alpha 1} I_\alpha]^2 \right)}{\left([1 + 2U^2_{\alpha 2} I_\alpha]^2 - [1 + 2U^2_{\alpha 1} I_\alpha]^2 \right)}$$

Update on the $+-+$ scenario (cont'd)



Update on the $+-+$ scenario (cont'd)