Lepton Flavor Violation in the SM with general Dimension-6 Operators

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Analysis in terms of effective higher dimension operators

- . Physical observables calculation:
  - radiative lepton decays $l \to l'\gamma'$.
  - charged lepton EDMs and $g-2$ anomaly.
  - 3-body LFV charged lepton decays $l \to l'l''l'''$.
  - $Z^0 \to ll'$ decays.

Neutrino oscillation

- $P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \neq 0$ only if neutrino is massive
- SM: neutrino is massless, No LFV - leptonic flavor strictly conserved.

  $\Rightarrow$ need new physics

  - Seasaw mechanism
  - R-Parity violation
  - Extra dimensions

- Neutrino oscillation $\rightarrow$ neutral lepton flavor violation

  Charged lepton flavor violation(cLFV)?!

  -Clear evidence of physics at a higher scale

  -Understand the origin of flavors more deeply

cLFV in the ν SM

The ν SM give rise to radiative charged lepton decay and 3-body charged leptons,
Still very small - always GIM-suppressed by $m_\nu^2/M_W^2 \sim 10^{-25}$.
e.g.(Cheng-Lee textbook)$\mu \to e\gamma$ decay(U is PMNS matrix):

$$BR(\mu \to e\gamma) = \frac{3\alpha_{em}}{32\pi} \sum_{i=1}^{3} U_{ei}^* U_{\mu i} (\frac{m_{\nu_i}^2}{M_W^2}) \cong (2.5 - 3.9) \times 10^{-55}$$

Effective approach:

Parameterize New Physics effects in terms of higher dimension operators. Express LFV observables in terms of Wilson coefficients. Need to be done just once - model independent analysis.

Then, only the values Wilson coefficients of new operators need to be calculated within a model of NP-This part of analysis is always model dependent.

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) .$$

- 59 independent operators
- Only few are important in a specific case.

Full list of dimension 5- an 6-operators given in Buchmiller-Wyler 1986, reduced in Grzadkowski, Iskrzynski, Misiak and Rosiek, JHEP **1010**

Unique dimension-5 term

The Weinberg operator:

$$Q_{\nu\nu} = \varepsilon_{ab}\varepsilon_{cd}\varphi^a\varphi^c(\ell_i^b)^T C\ell_j^d\,.$$

Generates neutrino masses and mixing angles, does not contribute directly to cLFV processes - can be neglected.

$\ell\ell\ell\ell$		$\ell\ell X\varphi$		$\ell\ell\varphi^2 D$ and $\ell\ell\varphi^3$	
$Q_{\ell\ell}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar\ell_k\gamma^\mu\ell_l)$	Q_{eW}	$(\bar\ell_o\sigma^{\mu\nu}e_j)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar\ell_i\gamma^\mu\ell_j)$
Q_{ee}	$(\bar e_i\gamma_\mu e_j)(\bar e_k\gamma^\mu e_l)$	Q_{eB}	$(\bar\ell_i\sigma^{\mu\nu}e_j)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar\ell_i\tau^I\gamma^\mu\ell_j)$
$Q_{\ell e}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar e_k\gamma^\mu e_l)$			$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_i\gamma^\mu e_j)$
				$Q_{e\varphi 3}$	$(\varphi^\dagger\varphi)(\bar\ell_i e_j\varphi)$
$\ell\ell qq$					
$Q^{(1)}_{\ell q}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar q_k\gamma^\mu q_l)$	$Q_{\ell d}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar d_k\gamma^\mu d_l)$	$Q_{\ell u}$	$(\bar\ell_i\gamma_\mu l_j)(\bar u_k\gamma^\mu u_l)$
$Q^{(3)}_{\ell q}$	$(\bar\ell_i\gamma_\mu\tau^I\ell_j)(\bar q_k\gamma^\mu\tau^I q_l)$	Q_{ed}	$(\bar e_i\gamma_\mu e_j)(\bar d_k\gamma^\mu d_l)$	Q_{eu}	$(\bar e_i\gamma_\mu e_j)(\bar u_k\gamma^\mu u_l)$
Q_{eq}	$(\bar e_i\gamma^\mu e_j)(\bar q_k\gamma_\mu q_l)$	$Q_{\ell edq}$	$(\bar\ell^a_i e_j)(\bar d_k q^a_l)$	$Q^{(1)}_{\ell equ}$	$(\bar\ell^a_i e_j)\varepsilon_{ab}(\bar q^b_k u_l)$
				$Q^{(3)}_{\ell equ}$	$(\bar\ell^a_i\sigma_{\mu\nu}e_a)\varepsilon_{ab}(\bar q^b_k\sigma^{\mu\nu}u_l)$

After simplifications: **only 9 operators remain in LFV:**

1. **2** $(\ell\ell\varphi X)$ **operators:** $(\bar{\ell}_i\sigma^{\mu\nu}e_j)\tau^I\varphi W^I_{\mu\nu}$, $(\bar{\ell}_i\sigma^{\mu\nu}e_j)\varphi B_{\mu\nu}$
2. **3** $(\ell\ell)(\varphi D\varphi)$ **operators:**
$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{\ell}_i\gamma^\mu\ell_j)$, $(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{\ell}_i\tau^I\gamma^\mu\ell_j)$, $(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_i\gamma^\mu e_j)$
3. **3 four-lepton contact couplings:**
$(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{\ell}_k\gamma^\mu\ell_l)$, $(\bar{e}_i\gamma_\mu e_j)(\bar{e}_k\gamma^\mu e_l)$, $(\bar{\ell}_i\gamma_\mu\ell_j)(\bar{e}_k\gamma^\mu e_l)$
4. **1 two-lepton two-quark coupling:** $(\bar{\ell}^a_i e_j)(\bar{d}_k q^a_l)$

$\ell_i \to \ell_j\gamma$ decay rate

The general form of lepton-photon vertex can be written as:

$$V_{\ell\ell\gamma}^{IJ\,\mu} = \frac{i}{\Lambda^2}[\gamma^\mu(F_{VL}^{IJ}P_L + F_{VR}^{IJ}P_R) + (F_{SL}^{IJ}P_L + F_{SR}^{IJ}P_R)q^\mu + i(F_{TL}^{IJ}\sigma^{\mu\nu}P_L + F_{TR}^{IJ}\sigma^{\mu\nu}P_R)q_\nu]$$

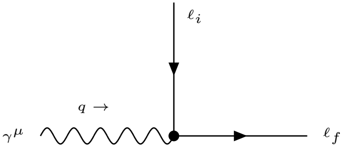
The general form of the branching ratio can be expressed in terms of F_{TL}^{IJ} and F_{TR}^{IJ} as follows:

$$\mathcal{B}\left(\ell^J \to \ell^I\gamma\right) = \frac{m_{\ell_J}^3}{16\pi\Lambda^4\,\Gamma_{\ell_J}}\left(\left|F_{TR}^{IJ}\right|^2 + \left|F_{TL}^{IJ}\right|^2\right),$$

where Γ_{ℓ_J} is the total decay width of decaying lepton.

Effective lepton-photon coupling

Tree level LFV contribution exist:



$$i\left(e\gamma^\mu\delta^{fi} + i\sigma^{\mu\nu}\left[C^{fi}_{\gamma L}P_L + C^{fi}_{\gamma R}P_R\right]q_\nu\right)$$

$$C^{\gamma R}_{fi} = C^{\gamma L\star}_{fi} = \frac{v\sqrt{2}}{\Lambda^2}\left(c_W C^{fi}_{eB} - s_W C^{fi}_{eW}\right)$$

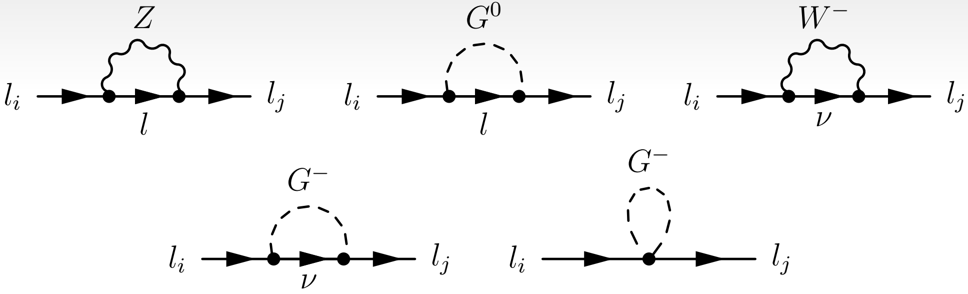
If they are dominated by tree-level contributions from Q_{eB} and Q_{eW}, one gets:

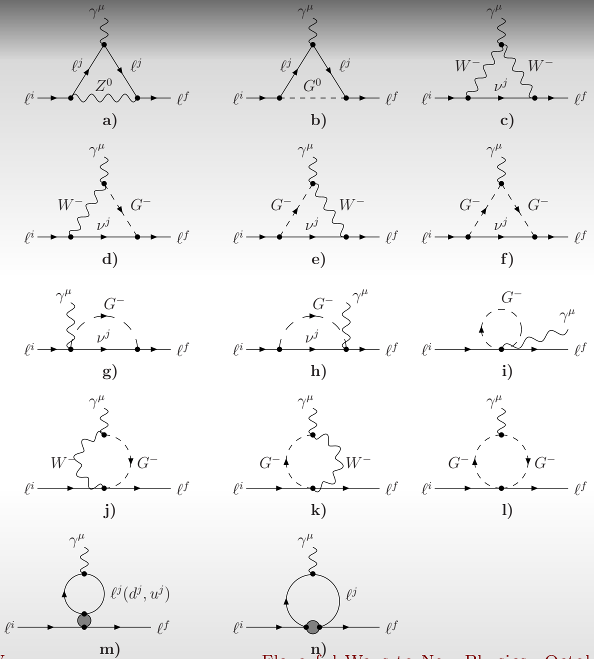
$$\sqrt{\left|C_\gamma^{12}\right|^2 + \left|C_\gamma^{21}\right|^2} \leq 2.45 \times 10^{-10} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\mu \to e\gamma\right]}{5.7 \times 10^{-13}}},$$

$$\sqrt{\left|C_\gamma^{13}\right|^2 + \left|C_\gamma^{31}\right|^2} \leq 2.35 \times 10^{-6} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to e\gamma\right]}{3.3 \times 10^{-8}}},$$

$$\sqrt{\left|C_\gamma^{23}\right|^2 + \left|C_\gamma^{32}\right|^2} \leq 2.71 \times 10^{-6} \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[\tau \to \mu\gamma\right]}{4.4 \times 10^{-8}}}.$$

Lepton self-energy diagrams




a) b) c) d) e) f) g) h) i) j) k) l) m) n)
γ^μ
ℓ^i ℓ^f ℓ^j
Z^0 G^0
W^- G^- ν^j
$\ell^j(d^j, u^j)$

1-loop results for F_{TL}, F_{TR} form-factors

$$F^{IJ}_{TL} =$$

$$\frac{4e}{(4\pi)^2}\left[\frac{C^{(1)IJ}_{\phi l} m_I(1+s^2_W)-(C^{(3)IJ}_{\phi l} m_I + C^{IJ}_{\phi e} m_J)(\frac{3}{2}-s^2_W)}{3} + \sum_{K=1}^{3} C^{IKKJ}_{\ell e} m_K\right]$$

$$F^{IJ}_{TR} =$$

$$\frac{4e}{(4\pi)^2}\left[\frac{C^{(1)IJ}_{\phi l} m_J(1+s^2_W)-(C^{(3)IJ}_{\phi l} m_J + C^{IJ}_{\phi e} m_I)(\frac{3}{2}-s^2_W)}{3} + \sum_{K=1}^{3} C^{KJIK}_{\ell e} m_K\right]$$

general expression for EDM can be obtained from effective lepton-photon interaction:

$$d_{\ell_i} = \frac{-1}{\Lambda^2}\,\mathrm{Im}\left[F_{TR}^{ii}\right]\,, \tag{1}$$

numerical expressions for the bounds on Wilson coefficients:

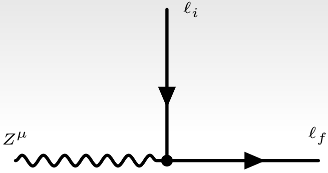
$$\begin{aligned}
d_e &= -2.08\times 10^{-18}\ \mathrm{Im}\left[2\times 10^{-5}\,C_{\ell e}^{3113} + C_\gamma^{11}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,,\\
d_\mu &= -2.08\times 10^{-18}\ \mathrm{Im}\left[2\times 10^{-5}\,C_{\ell e}^{3223} + C_\gamma^{22}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,,\\
d_\tau &= -2.08\times 10^{-18}\ \mathrm{Im}\left[C_\gamma^{33}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 e\,\mathrm{cm}\,,
\end{aligned}$$

anomalous magnetic moments of charged leptons:

$$a_{\ell_i} = \frac{2m_{\ell_i}}{e\Lambda^2}\,\mathrm{Re}\left[F_{TR}^{ii}\right] \,.$$

numerical expressions for the bounds on Wilson coefficients:

$$\begin{aligned}
a_e &= 1.17\times 10^{-6}\ \mathrm{Re}\left[2\times 10^{-5}\, C_{\ell e}^{3113} + C_\gamma^{11}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \,,\\
a_\mu &= 2.43\times 10^{-4}\ \mathrm{Re}\left[2\times 10^{-5}\, C_{\ell e}^{3223} + C_\gamma^{22}\right]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \,,\\
a_\tau &= 4.1\times 10^{-3}\ \mathrm{Re}\Big[10^{-5}\times\Big(1.6\, C_{\varphi\ell}^{(1)33} + 2.0\, C_{\ell e}^{3333}\\
&- 1.7\left(C_{\varphi\ell}^{(3)33} + C_{\varphi e}^{33}\right)\Big) + C_\gamma^{33}\Big]\left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2 \,.
\end{aligned}$$



$$i(\gamma^\mu \left[\Gamma^{ZL}_{fi} P_L + \Gamma^{ZR}_{fi} P_R\right] + i\sigma^{\mu\nu}[C^{ZL}_{fi} P_L + C^{ZR}_{fi} P_R] q_\nu)$$

$$\Gamma^{ZL}_{fi} = \frac{e}{2 s_W c_W}\left(\frac{v^2}{\Lambda^2}(C^{(1)fi}_{\phi l} + C^{(3)fi}_{\phi l}) + (1 - 2s^2_W)\delta_{fi}\right.$$

$$\Gamma^{ZR}_{fi} = \frac{e}{2 s_W c_W}\left(\frac{v^2}{\Lambda^2} C^{fi}_{\phi e} - 2s^2_W \delta_{fi}\right)$$

$$C^{ZR}_{fi} = C^{ZL*}_{if} = \frac{-v\sqrt{2}}{\Lambda^2}(s_W C^{fi}_{eB} + c_W C^{fi}_{eW}) \equiv \frac{-v\sqrt{2}}{\Lambda^2} C^{fi}_Z$$

$$\sqrt{\left|C_{\varphi\ell}^{(1)12} + C_{\varphi\ell}^{(3)12}\right|^2 + \left|C_{\varphi e}^{12}\right|^2 + \left|C_{Z}^{12}\right|^2 + \left|C_{Z}^{21}\right|^2} \leq 0.06 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \mu^{\pm} e^{\mp}\right]}{1.7 \times 10^{-6}}},$$

$$\sqrt{\left|C_{\varphi\ell}^{(1)13} + C_{\varphi\ell}^{(3)13}\right|^2 + \left|C_{\varphi e}^{13}\right|^2 + \left|C_{Z}^{13}\right|^2 + \left|C_{Z}^{31}\right|^2} \leq 0.14 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^{\pm} e^{\mp}\right]}{9.8 \times 10^{-6}}}, \qquad (2)$$

$$\sqrt{\left|C_{\varphi\ell}^{(1)23} + C_{\varphi\ell}^{(3)23}\right|^2 + \left|C_{\varphi e}^{23}\right|^2 + \left|C_{Z}^{23}\right|^2 + \left|C_{Z}^{32}\right|^2} \leq 0.16 \left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \sqrt{\frac{\text{Br}\left[Z^0 \to \tau^{\pm} \mu^{\mp}\right]}{1.2 \times 10^{-5}}}.$$

Three-body $l \to l'l''l'''$ decays

3 groups of decays, depending on composition of the final state leptons:

- Three leptons of the same flavor: $\mu^\pm \to e^\pm e^+ e^-$, $\tau^\pm \to e^\pm e^+ e^-$ and $\tau^\pm \to \mu^\pm \mu^+ \mu^-$.
- Three distinguishable leptons: $\tau^\pm \to e^\pm \mu^+ \mu^-$ and $\tau^\pm \to \mu^\pm e^+ e^-$.
- Two lepton of the same flavor and charge and one with different flavor and opposite charge: $\tau^\pm \to e^\mp \mu^\pm \mu^\pm$ and $\tau^\pm \to \mu^\mp e^\pm e^\pm$.

$$C_{\mu eee} \leq 3.29 \times 10^{-5} \left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2 \sqrt{\frac{\mathrm{Br}[\mu \to eee]}{1\times 10^{-12}}}\,,$$
$$C_{\tau eee} \leq 1.28 \times 10^{-2} \left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2 \sqrt{\frac{\mathrm{Br}[\tau \to eee]}{2.7\times 10^{-8}}}\,,$$
$$C_{\tau \mu\mu\mu} \leq 1.13 \times 10^{-2} \left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2 \sqrt{\frac{\mathrm{Br}[\tau \to \mu\mu\mu]}{2.1\times 10^{-8}}}\,,$$

with $C_{\ell_i \ell_f \ell_f \ell_f}$ given by

$$C_{\ell_i \ell_f \ell_f \ell_f} = \Big\{ \Big| 0.46 \left(\mathrm{C}^{(1)fi}_{\phi\ell} + \mathrm{C}^{(3)fi}_{\phi\ell} \right) + \mathrm{C}^{fiff}_{\ell e} \Big|^2 + 2 \Big| \mathrm{C}^{fiff}_{\ell e} - 0.54 \left(\mathrm{C}^{(1)fi}_{\phi\ell} + \mathrm{C}^{(3)fi}_{\phi\ell} \right) \Big|^2$$
$$+ \Big| \mathrm{C}^{fffi}_{\ell e} - 0.54 \mathrm{C}^{fi}_{\phi e} \Big|^2 + 2 \Big| \mathrm{C}^{fiff}_{ee} + 0.46 \mathrm{C}^{fi}_{\phi e} \Big|^2 \Big\}\,.$$

Conclusions

- We calculated the expresion for several theoretically important and experimentally well constrained lepton flavor observables, giving for them the impact results in term of Wilson coefficients of all effective operators which could give contribution to such processes.
- Clear evidence of physics at higher scale
- Understand the origin of flavors more deeply