NNLO corrections to the decay $B \to D\pi$

Susanne Kränkl

Universität Siegen
Theoretische Physik I

in collaboration with
Tobias Huber

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table of contents

1. Motivation for NNLO calculation
2. QCD factorization
3. Calculation methods
4. Outlook

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Decay rate in QCD factorization

$$\Gamma(\bar{B}_0 \to D^+\pi^-) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{q}|}{16\pi m_B^2} |V_{ud}^* V_{cb}| \, |a_1(D\pi)|^2 \, f_\pi^2 \, F_0^2(m_\pi^2)$$

Theory predictions

BBNS, 2000

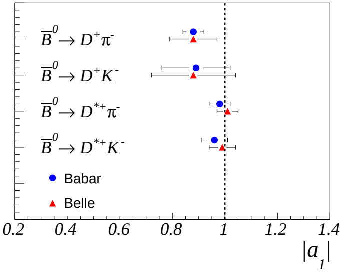
$$|a_1(\bar{B}_0 \to DL)_{NLO}| = (1.055^{+0.019}_{-0.013}) - (0.013^{+0.011}_{-0.006})\alpha_1^K$$

$$|a_1(\bar{B}_0 \to D^* L)_{NLO}| = (1.054^{+0.018}_{-0.017}) - (0.015^{+0.013}_{-0.007})\alpha_1^{K*}$$

$L = \{\pi, \rho, K\}$ and $|\alpha_1^{K(*)}| < 1$

Experimental prediction

Fleischer et al, 2011



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Decay rate in QCD factorization

$$\Gamma(\bar{B}_0 \to D^+\pi^-) = \frac{G_F^2(m_B^2 - m_D^2)^2|\vec{q}|}{16\pi m_B^2}|V_{ud}^* V_{cb}|\,|a_1(D\pi)|^2\, f_\pi^2\, F_0^2(m_\pi^2) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

Theory predictions

BBNS, 2000

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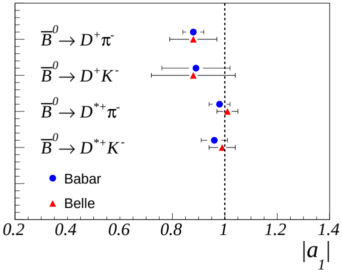
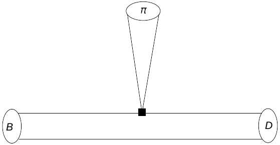


Main motivation for NNLO correction

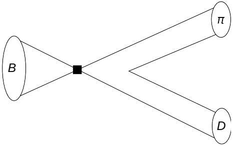
Decay rate in QCD factorization

$$\Gamma(\bar{B}_0 \to D^+\pi^-) = \frac{G_F^2(m_B^2 - m_D^2)^2|\vec{q}|}{16\pi m_B^2}|V_{ud}^* V_{cb}|\, |a_1(D\pi)|^2\, f_\pi^2\, F_0^2(m_\pi^2) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

- Estimate power corrections to QCD factorization
- NLO correction rather small:
  - color suppression
  - small Wilson coefficient



Tree topology



Weak annihilation, power suppressed

m_c heavy

m_c/m_b fixed as $m_b \to \infty$

Tree topology can be factorized in the heavy quark limit:

$$\langle D^+\pi^-|\mathcal{Q}_i|\bar{B}_0\rangle = \sum_j F_j^{B\to D}(m_\pi^2)\int_0^1 du\, T_{ij}(u)\phi_\pi(u) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

BBNS, 2000

$F_j^{B\to D}$: $B \to D$ form factor

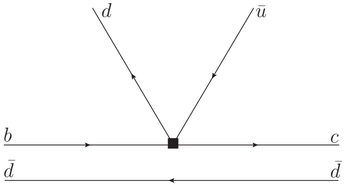
ϕ_π : light-cone distribution amplitude of the pion

} soft part, non-perturbative

T_{ij} : hard scattering kernel   hard part, perturbative

$$T_{ij} = T_{ij}^0 + \alpha_s T_{ij}^1 + \alpha_s^2 T_{ij}^2 + O(\alpha_s^3)$$

Tree topology



Interactions involving the spectator quark are power suppressed

BBNS, 2000

Effective Hamiltonian:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left(C_0 \mathcal{Q}_0 + C_8 \mathcal{Q}_8 \right)$$

$$\left.\begin{aligned} \mathcal{Q}_0 &= \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b \\ \mathcal{Q}_8 &= \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b \end{aligned}\right\} \text{Chetyrkin-Misiak-Münz basis}$$

Calculation of NNLO diagrams

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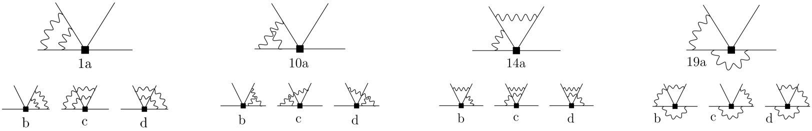
Around 70 diagrams, sample diagrams:



Dimensional regularization in $d = 4 - 2\epsilon$ dimensions

Renormalization of strong coupling α_s in $\overline{\text{MS}}$ scheme

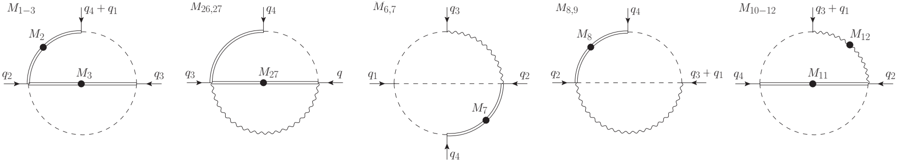
Calculation divided in two parts:

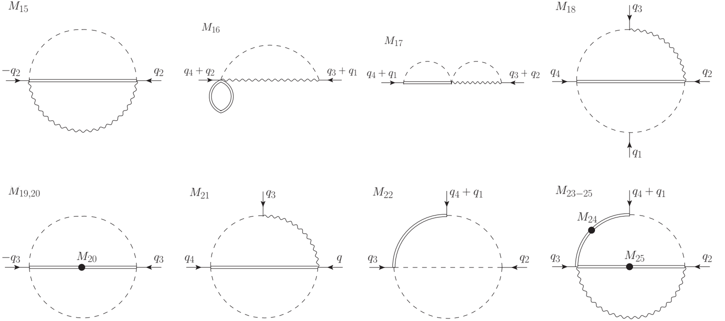
(1) Strings of Dirac matrices: reduced to set of operators by inhouse mathematica (MMA) routine

(2) 2-loop integrals: Laporta algorithm (MMA package: FIRE) — Laporta, Remiddi 1996; Smirnov, 2008

reduces huge number of complicated 2-loop integrals to a few rather simple master integrals

We found: 25 unknown master integrals



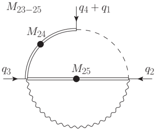


dashed line: massless quark

curly line: c quark

double line: b quark

dot: squared propagator



Master integrals depend on two arguments:

momentum fraction: u

ratio of heavy quark masses: m_c^2/m_b^2

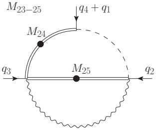
- Analytical methods
  - Feynman parameters (MMA package: HypExp) Huber, Maitre, 2005
  - Differential equation Kotikov, 1991, Caffo et al, 1998, Argeri, Mastrolia 2007
- Semi-analytical method
  - Mellin Barnes representation (MMA packages: Ambre, MB) used for numerical evaluation Gluza et al, 2007 Czakon, 2006

All masters calculated and double cross-checked ✓

New method for calculating loop integrals Henn, 2013

- Construct differential equation, find canonical basis work in progress
- Obtain analytic expression for remaining master integrals
- Hope: Convolution of $T^{(2)}(u)$ with $\Phi(u)$ may be easier

Calculation methods for master integrals



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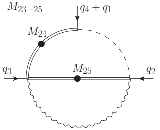

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Next steps towards two-loop result $a_1{}^{NNLO}$

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Towards hard scattering kernel $T^{(2)}$

- Bare two-loop amplitude contains UV and IR divergencies, up to $1/\epsilon^4$ poles
- Perform UV renormalization
- Kinematics allow matching to Soft Collinear Effective Theory
- Final expression for $T^{(2)}$ free of poles in ϵ

Result for colour singlet kernel

$$T_2^{(2)} = \sum_a \left(A_{2a}^{(2),\mathrm{nf}} + Z_{2j}^{(1)} A_{ja}^{(1)} + Z_{2j}^{(2)} A_{ja}^{(0)} \right) , \quad T_1^{(2)}$$

work in progress

Evaluate amplitude

$$\langle D^+ \pi^- | \mathcal{Q}_i | \bar{B}_0 \rangle = \sum_j F_j^{B \to D}(m_\pi^2) \int_0^1 du T_{ij}(u) \phi_\pi(u)$$

Analytical result feasible?

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- Calculation of a_1 and $BR(\bar{B}_d^0 \to D^+\pi^-)_{\text{HQET}}$ @ NNLO accuracy
- Estimation of power corrections by comparison of $BR(\bar{B}_d^0 \to D^+\pi^-)_{\text{HQET}}$ to experimental branching ratio
- Check whether calculation is applicable to other decays like $\bar{B}_d^0 \to D^{*+}\pi^-$, $\bar{B}_d^0 \to D^+\rho^-$ or even $\Lambda_d^0 \to \Lambda_c^+\pi^-$
- NNLO correction "last word" on perturbative side of theory prediction