Strong couplings from Light-cone sum rules

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Introduction

We study strong coupling constants of heavy-light mesons with a pion

Definition

$\langle H^*(p)\pi(q)|H(p+q)\rangle = g_{H^*H\pi} q_\mu \epsilon^\mu, \quad H = \{B, D\}$ and $H^* = \{B^*, D^*\}$

Why are strong couplings important (for definiteness $g_{B^*B\pi}$)?

- Determine the B^* pole of $B \to \pi$ formfactor, important for flavour changing semileptonic B decays
- Key parameter $\hat{g}$ of $H\chi PT$ in $m_b \to \infty$ limit, $\quad g_{B^*B\pi} = \frac{2m_B}{f_\pi}\hat{g}(1 + \mathcal{O}(\frac{1}{m_B}))$
- Previous result from LCSR lacks accuracy

Overview of experiment and QCD calculations

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	Exp.	Lattice	LCSR '99 [5]
$g_{D^*D\pi}$	17.9 ± 2.2 [1] $16.92 \pm 0.13 \pm 0.14$ [2]	$15.9^{+0.7+0.2}_{-0.7-0.4}$ [3]	10.5 ± 3
$g_{B^*B\pi}$	-	$35.7^{+2.4+5.6}_{-2.4-0}$ [4]	22 ± 7

[1]: CLEO Collaboration
[2]: BaBar Collaboration '13
[3]: D. Bećirević, F. Sanfilippo '13
[4]: D. Bećirević, B. Blossier, E. Chang, B. Haas '09 (static heavy quark limit)
[5]: A. Khodjamirian, R. Rückl, S. Weinzierl and O. I. Yakovlev '99

LCSR result

Since '99 no major update!

QCD sum rules

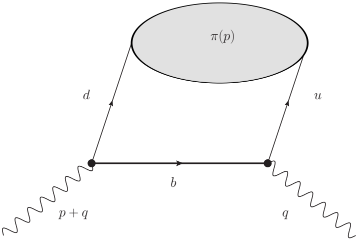
A (very) brief reminder about QCD sum rules

- Start with correlation function
- $q^2 \ll m_Q^2 \Rightarrow$ OPE representation: calculable, but no hadrons around
- $q^2 > m_Q^2 \Rightarrow$ hadronic representation: hadronic matrix elements occure, but unknown ρ^h
- Write them both in form of a dispersion relation (Cauchys theorem $\rightarrow$ Analyticity)
- Match them via Quark Hadron Duality (QHD) [1]

[1]: E.C. Poggio, H.R. Quinn and S. Weinberg '76

Light cone sum rules (LCSR)

The strong coupling $B^* \to B\pi$ is accesible via the same correlation function as for $B \to \pi$ form factors [Belyaev, Braun, Khodjamirian, Rückl ' 94]



Correlation function

$$F_\mu(p,q) = i \int d^4x\, e^{iqx} \langle \pi(p) | \mathcal{T}\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i\gamma_5 u(0)\} |0\rangle$$
$$= F(q^2,(p+q)^2) p_\mu + F'(q^2,(p+q)^2) q_\mu$$

Light-cone expansion & twists

Expand nonlocal matrix elements in terms of DAs - ordered in twists

Operator Twist definition

Twist := Dimension - Spin

Example: $\langle \pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0\rangle$ contains lowest twist 2

$$\langle \pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0\rangle = -iq_\mu f_\pi \int_0^1 du\ e^{iuqx}(\varphi_\pi(u) + x^2 g_1(u) + \mathcal{O}(x^4)) + f_\pi\left(x_\mu - \frac{x^2 q_\mu}{q\cdot x}\right)\int_0^1 du\ e^{iuqx} g_2(u) + \ldots$$

DAs: Twist 2 and Twist 4 , u: quark (antiquark) momentum fraction

The sum rule

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- Hadronic representation of F in a form of double dispersion integral:

Hadronic representation

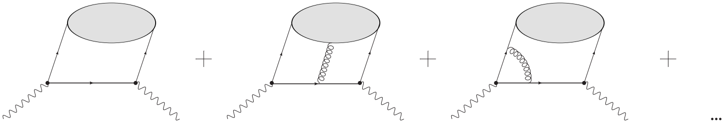
$$F(q^2,(p+q)^2) = \frac{m_B^2 m_{B^*} f_B f_{B^*} g_{B^* B\pi}}{m_b(q^2-m_{B^*}^2)((p+q)^2-m_B^2)} + \int \frac{\rho^h(s_1,s_2) ds_1 ds_2}{(s_1-q^2)(s_2-(p+q)^2)}$$

Decay constants

$$\langle 0|J_\mu|B^*(q)\rangle = m_{B^*}\epsilon_\mu f_{B^*}, \quad \langle 0|J_5|B(p)\rangle = m_B^2 f_B$$

update by [PG, Khodjamirian, Pivovarov, Rosenthal '13]

- Calculate light cone OPE in terms of a twist expansion



Twist 2,3,4 LO Twist 3,4 LO Twist 2,3 NLO

The final LCSR

Hadronic and OPE representation of correlation function:

The OPE representation (double dispersion relation)

$$F^{OPE}(p^2,(p+q)^2) = \int_{m_b^2}^{\infty} \frac{ds_1}{s_1-q^2} \int_{m_b^2}^{\infty} \frac{ds_2}{s_2-(p+q)^2} \rho^{OPE}(s_1,s_2)$$

The hadronic representation

$$F^{had}(p^2,(p+q)^2) = \frac{m_B^2 m_{B^*} f_B f_{B^*} g_{B^*B\pi}}{m_b(q^2-m_{B^*}^2)((p+q)^2-m_B^2)} + \int_{s_h}^{\infty} \frac{ds_1}{s_1-q^2} \int_{s_h}^{\infty} \frac{ds_2}{s_2-(p+q)^2} \rho^{had}(s_1,s_2)$$

match them and apply Quark Hadron Duality (QHD)

The final sum rule

$$f_B f_{B^*} g_{B^*B\pi} = \frac{1}{m_B^2 m_{B^*}} \int \int_R ds_1 ds_2 exp(\frac{m_{H^*}^2 - s_1}{M_1^2} + \frac{m_H^2 - s_2}{M_2^2}) \rho^{OPE}(s_1,s_2)$$

R : Region of $\{(m_Q+m_q)^2 \leq s_{1,2} \leq s_{1,2}^0\}$, $M_1 \sim M_2$: Borel parameters

Calculating $\rho^{OPE}(s_1, s_2)$

Main Task: Calculate the double spectral density from the diagrams

QHD approximation of $\rho^{had}(s_1, s_2)$

$$\rho^{had}(s_1, s_2) = \theta(s_1 - s_1^0)\theta(s_2 - s_2^0)\rho^{OPE}(s_1, s_2)$$

Example: Leading Order Twist 2 Contribution

$$F(q^2, (p+q)^2) = m_b f_\pi \int_0^1 \frac{du\varphi_\pi(u)}{m_b^2 - (q+up)^2}, \quad \varphi_\pi(u) = \Sigma_k a_k (1-u)^k$$

Applying $u \to \frac{m_b^2 - q^2}{s - q^2}$, then $s = s_1 + s_2$ and $v = \frac{s_1}{v}$ this yields

$$\rho^{OPE}(s_1, s_2) = m_b f \pi \Sigma_k \frac{(-1)^k a_k}{\Gamma(k+1)} (s_1 - m_b^2)^k \delta^{(k)}(s_1 - s_2)$$

Calculating $\rho^{OPE}(s_1, s_2)$

Spectral density of NLO terms $\Rightarrow$ Take double imaginary part!

Reparametrisation

$$r = \frac{s_1 - m^2}{s_2 - m^2}, \quad \sigma = \frac{s_1}{m^2} + \frac{s_2}{m^2} - 2$$

Use f.e. results from $B \to \pi$ analysis [1] and take second imaginary part

Twist 2 NLO

$$\mathrm{Im}\, T^{Tw2}_{NLO} = \mathrm{Im}_r \left[\frac{1}{(r-1)^3}\right] f_1(r,\sigma) + \mathrm{Im}_r \left[\frac{r \ln(-r)}{(r-1)^3}\right] f_2(r,\sigma) + \mathrm{Im}_r \left[\frac{r \ln^2(-r)}{(r-1)^3}\right] f_3(r,\sigma)$$

Explicitly:

$$\frac{1}{\pi}\mathrm{Im}_r \left[\frac{r \ln^2(-r)}{(r-1)^3}\right] = (\pi^2 - 1) r \delta(1-r) + 2\pi^2 r^2 \frac{d}{dr}\delta(1-r) + \frac{\pi^2}{2} r^3 \frac{d^2}{dr^2}\delta(1-r) - \frac{2r \ln(r)}{(r-1)^3}$$

[1]: G. Duplančić, A. Khodjamirian, Th. Mannel, B. Melić, N. Offen '08

Why Twist 3 NLO?

Analysis only including LO terms:

	Lattice	LCSR '99 [3]	LCSR '94 [4]
$g_{D^* D\pi}$	$15.9^{+0.7+0.2}_{-0.7-0.4}$ [1]	10.5 ± 3	12.5 ± 1
$g_{B^* B\pi}$	$35.7^{+2.4+5.6}_{-2.4-0}$ [2]	22 ± 7	29 ± 3

$\Rightarrow$ Tw. 2 NLO pushes result away from lattice & experiment ☹

From $B \to \pi$ analysis

$\mathcal{O}$(Tw. 3 NLO) $\sim \mathcal{O}$(Tw. 2 NLO) and of opposite sign!

$\Rightarrow$ Including Tw. 3 NLO might close the gap ☺

[1]: D. Bećirević, F. Sanfilippo '13

[2]: D. Bećirević, B. Blossier, E. Chang, B. Haas '09 (static heavy quark limit)

[3]: A. Khodjamirian, R. Rückl, S. Weinzierl and O. I. Yakovlev '99 (Tw. 2 NLO)

[4]: V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Rückl '94 (Only LO)

Conclusion

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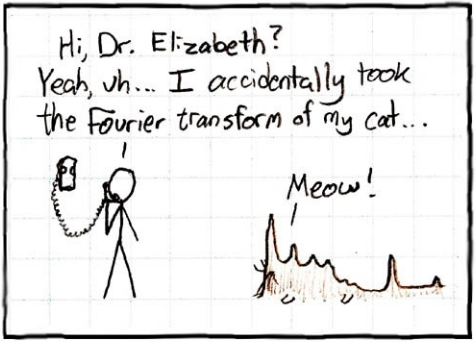
So what happened?

- LCSRs can provide numerical estimates for strong couplings $g_{B^*B\pi}$ and $g_{D^*D\pi}$
- In the previous calculation important parts of OPE (NLO twist 3) were missing - need a major update
- Work in progress: Calculate double spectral density for NLO (twist 2 and 3), update input, try different QHD ansätze, estimate uncertainties
- Investigate also coupling of heavy mesons including s quark by adding light mass corrections
- Go for it ☺

Thank you for your Attention!



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