The bottom quark mass from non-relativistic sum rules at NNLO

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based on work in collaboration with

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Flavorful Ways to New Physics

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Motivation

- Motivation for a precise determination of the bottom quark mass
  - Conceptual: One of only a handful of QCD parameters
  - Flavor physics: Bottom decay rates scale as $\Gamma_b \propto m_b^5$
  - Higgs physics: Dominant uncertainty in $\Gamma(h \to b\bar{b})$
    $\Rightarrow$ 1% parametric uncertainty for all Higgs branching ratios
    Together with m_H this is the dominant source of uncertainties for most channels

The sum rule

- Consider the $b\bar{b}$ production cross section

$$R_b \equiv \frac{\sigma(e^+e^- \ \rightarrow \ b\bar{b} + X)}{\sigma(e^+e^- \ \rightarrow \ \mu^+\mu^-)} = 12\pi \ \mathrm{Im}\Pi_b$$

  and define its moments as

$$\mathcal{M}_n \equiv \int_0^\infty ds \frac{R_b(s)}{s^{n+1}} = \frac{12\pi^2}{n!} \left(\frac{d}{dq^2}\right)^n \Pi_b(q^2)\Big|_{q^2=0}. \qquad (1)$$

- The left-hand side is an experimental observable.
- The right-hand side is IR safe due to $q^2 = 0$ being far off-shell. Thus, assuming quark-hadron duality, it can be computed as an OPE in $\Lambda_{\mathrm{QCD}}/(m_b/n)$, where m_b/n is the effective smearing range.
- Determine the bottom quark mass from equation (1).
- For large $n \approx 10$ the moments are dominated by the threshold region.
- For still higher n the OPE parameter becomes $\mathcal{O}(1)$ and the OPE breaks down.

Experimental moments

- The moments receive contributions from $\Upsilon(NS)$ bound states and the open $B\bar{B}$ continuum

$$\mathcal{M}_n^{\text{exp}} = 9\pi \sum_{N=1}^{4} \frac{1}{\alpha(M_{\Upsilon(NS)})^2} \frac{\Gamma_{\Upsilon(NS)\to l^+l^-}}{M_{\Upsilon(NS)}^{2n+1}} + \int_{s_{\text{cont}}}^{\infty} ds \frac{R_b(s)}{s^{n+1}}.$$

  - $\Upsilon(NS)$ resonance masses and leptonic widths are well-known.
  - Between $\sqrt{s} = 10.62$ GeV and 11.21 GeV R_b was measured by the BaBar collaboration.
  - For higher energies, there is no experimental data and we assume $R_b = 0.3 \pm 0.2$.

- For large n the resonances dominate $\Rightarrow$ very little sensitivity to poorly known continuum

Cross section near threshold

- The characteristic bottom quark velocity for the nth moment is $v \sim 1/\sqrt{n}$.
  - For our choice $n \approx 10$ this implies that $v \sim 1/\sqrt{n} \sim \alpha_s$.
  - Near threshold, multiple scales are relevant to the problem.
    In terms of the bottom quark mass m_b and velocity v:

hard scale	m_b	mass
soft scale	$m_b v$	momentum
ultrasoft scale	$m_b v^2$	energy

  - Conventional perturbation theory in α_s fails.
  - Coulomb singularities $(\alpha_s/v)^k$ have to be summed to all orders.
  - This is achieved by the use of PNRQCD (potential non-relativistic QCD), an effective field theory with the power counting

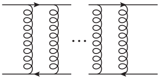
$$R_b \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases}$$

Effective field theory setup

- Use EFTs that subsequently integrate out the hard and soft scale

QCD	Full theory
↓	Integrated out hard modes $k \sim m_b$ Hard subgraphs become point-like vertices
NRQCD	Contains nonrelativistic modes
↓	Integrated out soft modes $k \sim m_b v$ Soft subgraphs become non-local vertices
PNRQCD	Contains potential heavy quarks and ultrasoft gluons

- PNRQCD [Pineda, Soto] is a spatially non-local theory, where the LO Coulomb potential is part of the LO Lagrangian. The heavy-quark pair propagator in PNRQCD is given by the sum of ladder diagrams involving arbitrary numbers of potential gluon exchanges. Higher corrections follow from Rayleigh-Schrödinger perturbation theory.



- For a detailed account of the EFT setup see [Beneke, Kiyo, Schuller: 1312.4791]

PNRQCD cross section

- In PNRQCD the cross section takes the form

$$R_b = 12\pi e_b^2 \,\mathrm{Im}\left[\frac{N_c}{2m_b^2}\left(c_v\left[c_v - \frac{E_N}{m_b}\left(c_v + \frac{d_v}{3}\right)\right]G(E) + \ldots\right)\right].$$

- Below threshold, the Green function contains bound-state poles

$$G(E) \overset{E\to E_N}{\longrightarrow} \frac{|\psi_N(0)|^2}{E_N - E - i\epsilon}$$

- Ingredients for the NNNLO cross section:
  - hard matching coefficients c_v at order α_s^3 [Marquard, Piclum, Seidel, Steinhauser] and d_v at order α_s [Luke, Savage]
  - Green function $G(E)$, energy levels E_N and wave functions at the origin $|\psi_N(0)|^2$ at NNNLO [Beneke, Kiyo, Schuller; Kniehl, Penin, Smirnov, Steinhauser; Schröder; Anzai, Kiyo, Sumino; Smirnov, Smirnov, Steinhauser]
  - Ultrasoft corrections [Beneke, Kiyo, Penin; Beneke, Kiyo]

Theory Moments

- To compute the theory moments we use (1) as a dispersion relation

$$\widetilde{\mathcal{M}_n^{\text{th}}} = 48\pi^2 N_c e_b^2 \sum_{N=1}^{\infty} \frac{\widetilde{Z}_N}{(E_N + 2m_b)^{2n+3}} + \int_{4m_b^2}^{\infty} ds \frac{\widetilde{R}_b(s)}{s^{n+1}},$$

where

$$\widetilde{Z}_N = c_v \left[c_v - \frac{E_N}{m_b} \frac{d_v}{3} \right] |\psi_N(0)|^2 .$$

Mass schemes

- The quark pole mass exhibits an infrared renormalon ambiguity of order Λ_{QCD} [Bigi, Shifman, Uraltsev, Vainshtein; Beneke, Braun]
  - The cross section is less infrared dependent than pole mass.
  - The spurious ambiguity can be removed by using proper short distance mass, e.g. PS mass [Beneke]
  - The $\overline{\text{MS}}$ mass is renormalon free, but not adequate for threshold problems.

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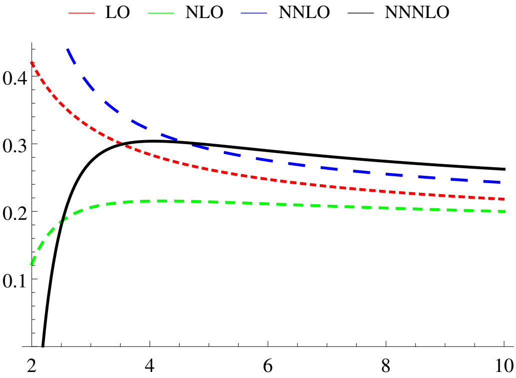
  - The $\overline{\text{MS}}$ mass is renormalon free, but not adequate for threshold problems.
- We determine the PS mass from the moments.
  - This considerably improves the convergence.
- The result is then converted to the $\overline{\text{MS}}$ mass.

Charm mass effects

- Charm mass effects are potentially large, since $m_c \sim m_b v$ and the moments can not be expanded in m_c.
- They have been computed at NNLO for non-relativistically expanded moments. [Hoang]
- We have done the computation at the cross section level, i.e. repeated large parts of the NNLO computation with an additional scale. The required contributions at NNLO are
  - Corrections to Coulomb potential [Melles]
  - Energy levels and wave functions
  - Continuum cross section (only done at NLO, affects m_b by less than 0.1 MeV)
  - PS-pole mass relation
  - $\overline{\text{MS}}$-pole mass relation [Bekavac, Grozin, Seidel, Steinhauser]
- Result for m_b^{PS} $\left[m_b^{\overline{\text{MS}}}\right]$ is affected by ~ 1 MeV [~ 3 MeV]
- Due to the missing NNNLO we assign a 100% uncertainty to charm mass effects.

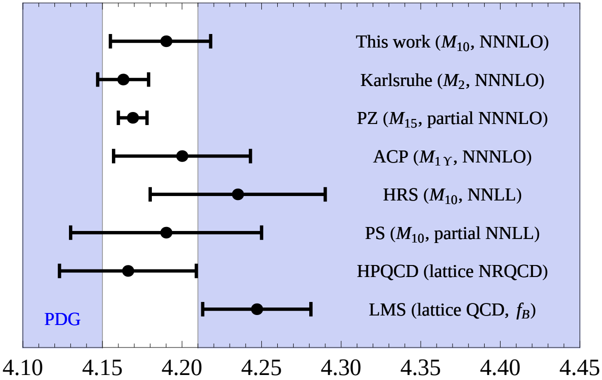
Scale dependence of the moments

- Scale dependence of $\widetilde{\mathcal{M}}_{10}^{\text{th}} \cdot (10\text{ GeV})^{20}$



- For $\mu \lesssim 3$ GeV no convergence is observed. Therefore we vary $\mu \in [3, 10]$ GeV, c.f. $\Gamma(\Upsilon(1S) \to l^+l^-)$ [Beneke, Kiyo, Marquard, Penin, Piclum, Seidel, Steinhauser]

Preliminary result and comparison to other works



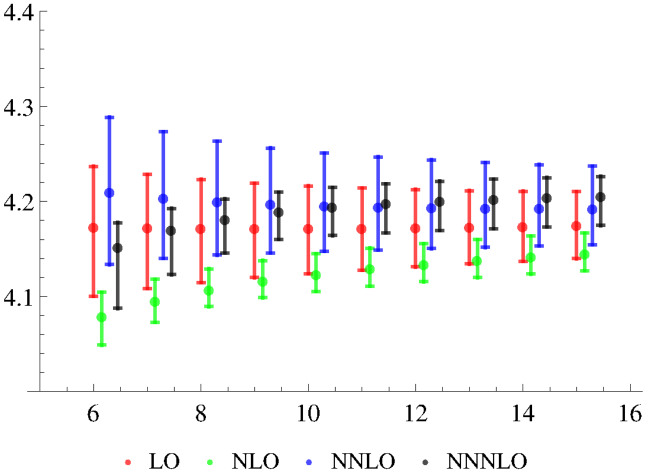
[Chetyrkin et al.; Penin, Zerf; Ayala, Cvetic, Pineda; Hoang, Ruiz-Femenia, Stahlhofen; Pineda, Signer; Lee et al.; Lucha, Melikhov, Simula]

Summary

- Moments of the $b\bar{b}$ production cross section provide a reliable method for precision determinations of the bottom quark mass
- For large n the moments are saturated by the threshold region.
- Coulomb singularities are resummed automatically in PNRQCD.
- The renormalon ambiguity of the pole mass is removed by the use of the PS mass.
- Charm mass effects are tiny at NNLO, 100% uncertainty assigned for missing NNNLO.
- First full NNNLO analysis for m_b from large n moments.
- The preliminary is result in good agreement with other recent precision determinations.

Backup: Results from different moments

- Values of $m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}})$ obtained from m_b^{PS} for different moments $\widetilde{\mathcal{M}}_n$.



Backup: Higgs branching ratios

	Δ_{m_t}	Δ_{m_H}	$\Delta_{\alpha(M_Z)}$	$\Delta_{\alpha_S(M_Z)}$	Δ_{m_b}	Δ_{M_Z}	Δ_{m_c}	Δ_{m_τ}	Δ_{G_F}
gg	0.07	0.46 (0.12)	0.01	1.77	1.00	0.01	0.15	-	-
$\gamma\gamma$	-	0.01 (-)	0.03	0.31	0.94	-	0.15	-	-
$b\bar{b}$	0.02	1.13 (0.28)	0.01	0.36	0.74	0.01	0.15	-	-
$c\bar{c}$	0.01	1.13 (0.28)	0.01	1.53	0.95	0.01	5.08	-	-
$\tau^+\tau^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	0.02	-
WW^*	0.04	2.97 (0.74)	0.04	0.30	0.95	0.02	0.15	-	-
ZZ^*	0.03	3.48 (0.87)	0.02	0.30	0.95	0.02	0.15	-	-
$Z\gamma$	0.01	2.14 (0.53)	-	0.30	0.96	-	0.15	-	-
$\mu^+\mu^-$	0.04	1.07 (0.27)	0.01	0.30	0.95	0.01	0.15	-	-

Table 15: Table of percentage uncertainties of branching fractions due to uncertainties in each of the input observables, as calculated by eq. 14. The input parameters for this computation are from Table 1. In addition we also compute the branching ratio uncertainties due to $\Delta m_h = 0.1$ GeV, the expected uncertainty after LHC run. These values are in parenthesis in the Δ_{m_H} column. Percentages less than 0.1% are listed as $-$. These results were computed using $\overline{MS}$ m_b and m_c inputs (see Table 10) rather than their pole mass inputs (see Table 1). Compare results with the pole mass input results of Table 6.

	$P^{\pm}_{\rm BR}$(par.-add.)	$P^{\pm}_{\rm BR}$(par.-quad.)	$(P^+_{\rm BR}, P^-_{\rm BR})(\mu)$
gg	3.47 (3.12)	2.09 (2.04)	(0.03,1.38)
$\gamma\gamma$	1.45 (1.44)	1.01 (1.01)	(1.81,1.83)
$b\bar{b}$	2.43 (1.58)	1.41 (0.89)	(0.21,0.)
$c\bar{c}$	8.72 (7.87)	5.51 (5.40)	(0.54,0.44)
$\tau^+\tau^-$	2.55 (1.75)	1.47 (1.04)	(0.09,0.07)
WW^*	4.48 (2.26)	3.13 (1.25)	(0.10,0.08)
ZZ^*	4.96 (2.34)	3.63 (1.33)	(0.10,0.08)
$Z\gamma$	3.56 (1.96)	2.36 (1.15)	(0.83,0.80)
$\mu^+\mu^-$	2.53 (1.73)	1.47 (1.04)	(0.07,0.06)

[Almeida, Lee, Pokorski, Wells]