A Solution to Singularities Arising at NLO in the Calculation of ϵ'/ϵ

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29.10.2014

Flavorful Ways to New Physics, Freudenstadt

What is this?

- Direct CP violation in the Kaon sector
  $\rightarrow$ (ϵ'/ϵ) NLO calculations done by Munich[1] and Rome[2] Groups roughly 20 years ago
- Hadronic Matrix Elements still contain large uncertainties
- A well known problem at NLO causing singularities exists

[1] e.g. Buras, Jamin, Lautenbacher: The Anatomy of epsilon-prime / epsilon beyond leading logarithms with improved hadronic matrix elements (1993)

[2] e.g. Ciuchini, Franco, Martinelli, Reina: The Delta S $= 1$ effective Hamiltonian including next-to-leading order QCD and QED corrections (1994)

Brief Review of the Framework[3]

[3] Andrzej J. Buras: Weak Hamiltonian, CP Violation and Rare Decays (1998); hep-ph/9806471

RGE of the Wilson Coefficients

The RGE for the Wilson Coefficients is

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g(\mu))\frac{\partial}{\partial g}\right]\vec{C}(\mu) = \gamma^T(g)\vec{C}(\mu)$$

The anomalous dimension matrix $\gamma(g)$ can be expanded in the couplings with coefficient matrices

$$\gamma(g) \overset{(QCDNLO)}{=} \frac{g^2}{16\pi^2}\gamma_s^{(0)} + \frac{g^4}{(16\pi^2)^2}\gamma_s^{(1)}$$

We work with a Basis of 10 operators, thus each is a 10×10 matrix

Evolution of the Wilson Coefficients

Define the evolution matrix

$$\vec{C}(\mu_1) = U(g(\mu_1), g(\mu_2))\vec{C}(\mu_2)$$

and plug it into the RGE for the Wilson Coefficients yields a g-ordered exponential solution

$$U(g_1, g_2) = T_g exp \int_{g_2}^{g_1} dg' \frac{\gamma(g')^T}{\beta(g')}$$

Evolution of the Wilson Coefficients

Solution at LO:

$$U_0(g_1, g_2) = \exp\left[-\frac{\gamma_s^{(0)\,T}}{\beta_0} \ln\left(\frac{g_1}{g_2}\right)\right]$$

Ansatz for NLO:

$$U(g_1, g_2) = K(g_1) U_0(g_1, g_2) K^{-1}(g_2)$$

with

$$K(g) = 1 + \frac{g^2}{16\pi^2} J_0$$

with a constant matrix J_0

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NLO evolution

Inserting this Ansatz into the evolution matrix, we can derive a matrix equation for J_0

$$J_0 - \left[J_0, \frac{\gamma_s^{(0)\,T}}{2\beta_0} \right] = \frac{\beta_1}{\beta_0} \frac{\gamma_s^{(0)\,T}}{2\beta_0} - \frac{\gamma_s^{(1)\,T}}{2\beta_0}$$

which is singular in the case of three active flavours

The Singularity

Following Buras et al, we choose a basis which diagonalizes the LO anomalous dimension matrix

$$V^{-1}\gamma_s^{(0)T}V = \gamma_{s,D}^{(0)T}$$

leads to the matrix equation for J_0 taking on the form

$$\left(VJ_0V^{-1}\right)_{ij} = \frac{\frac{\beta_1}{\beta_0}\left(\gamma_{s,D}^{(0)T}\right)_{ij} - \left(V\gamma_s^{(1)T}V^{-1}\right)_{ij}}{\left(2\beta_0 - \left(\gamma_{s,D}^{(0)T}\right)_{jj} + \left(\gamma_{s,D}^{(0)T}\right)_{ii}\right)}$$

and the singularity arises because

$$2\beta_0 \overset{(f=3)}{=} 18 \qquad \{-16, 2\} \in \mathrm{EV}(\gamma_s^{(0)T})$$

The Solution

We add a logarithmic dependency to the ansatz for $K(g)$

$$K(g) = 1 + \frac{g^2}{16\pi^2}\left(J_0 + \log\left(\frac{g^2}{4\pi}\right) J_1\right)$$

The evolution matrix then yields two matrix equations

$$J_1 - \left[J_1, \frac{\gamma_s^{(0)T}}{2\beta_0}\right] = 0$$

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- Solving the homogeneous equation wrt J_1
  $\longrightarrow$ J_1 w one free parameter t_1
- Solving the inhomogeneous equation wrt J_0 and t_1
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Numerics

$B_6^{(1/2)}$	ϵ'/ϵ
1.25	2.30572×10^{-3}
1.20	2.17727×10^{-3}
1.15	2.04881×10^{-3}
1.10	1.92036×10^{-3}
1.05	1.79191×10^{-3}
1.00	1.66345×10^{-3}
0.95	1.53500×10^{-3}
0.90	1.40655×10^{-3}
0.85	1.27809×10^{-3}
0.80	1.14964×10^{-3}
0.75	1.02118×10^{-3}

Tabelle: Values of ϵ'/ϵ depending on the parameter $B_6^{(1/2)}$. $m_s = 0.128$ GeV

$$(\epsilon'/\epsilon)_{exp}$$
$$= (1.66 \pm 0.23) \times 10^{-3}$$

(PDG 2014)