# QCD Factorization

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New Physics (?) $\quad \mathcal{L}_{\text{unknown}}$

Electroweak scale $M_W$ $\quad \underset{d=4}{\mathcal{L}_{SM}}$ + higher-dim operators

Higgs, top, W, Z integrated out

Flavour change only at EW scale

Heavy-quark scale $m_b$

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_i \overset{d=6}{C_i Q_i} + \text{higher dim ops.}$$

$$+ \mathcal{L}_{\text{QCD+QED}}$$

QCDF deals with this step

$$\langle f | Q_i | \bar{B} \rangle = ?$$

short-distance fluctuations integrated out

QCD scale $\Lambda$

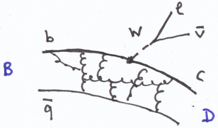
$\mathcal{L}_{\text{eff}}$ depends on process

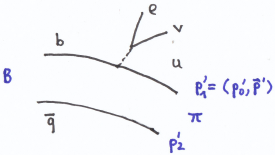
[ Heavy quarks still present as external lines ]

Still multiple scales :

$m_b, \sqrt{m_b \Lambda}, m_c, \Lambda$

# When do we need QCDF ?




$m_b, m_c \to \infty$
no large boost

massless fields + heavy quark
fluctuations soft $k^\mu \sim \Lambda$

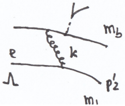
$$\mathcal{L} = \sum_{Q=b,c} \bar{h}_{Qv_Q} iv_Q \cdot D_s h_{Qv_Q} + \mathcal{L}_{gluon} + \sum_{q\ light} \bar{q} i \not{D}_s q \qquad \text{(HQET)}$$

[ → HQ symmetries ]

- $p_1' \sim \Lambda \quad \longrightarrow \quad$ soft pion
  heavy-to-light transitions in HQET

- $p_0' \sim m_b$ , but $p'^2 = m_\pi^2 \ll m_b^2$
  $\longrightarrow$ energetic pion
  no such field in HQET

$p_1' \sim (m_b, \Lambda, \Lambda, m_b)$ "collinear" (nearly light-like)




$k^2 = (p_2' + \ell)^2 \sim m_b \Lambda$
$k^0 \sim m_b$
"hard-collinear"

QCDF deals with B decays that involve energetic light particles (hadrons)

$B \to \pi \ell \nu$

$B \to \gamma \ell \nu$

$\pi, \gamma$ energetic

do not involve four-quark operators
$\to$ QCD form factors

$B \to D\pi$

$B \to \pi\pi, \ldots$ [charmless]

$B \to K^* \gamma^{(*)}$
$\hookrightarrow e^+e^-$

involve four quark operators
(strong final state interactions)

Modern language

QCDF $\hat{=}$ Soft-collinear effective theory for heavy quark physics

[ extends HQET by (hard-) collinear modes ]

- Example : $B \to D\pi$

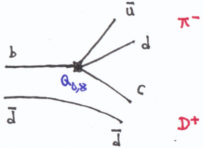
  Diagrammatic , hadronic wave functions

- Soft-collinear effective theory

- The general case – hard spectator interactions

- Some results from factorization

## $\bar{B}_d \to D^+\pi^-$




Want to show that

$$\langle D^+\pi^- | Q_i | \bar{B}_d\rangle = F_{B\to D}(m_\pi^2) \int_0^1 du\, T_i^{I}(u)\, \phi_\pi(u) + \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

form factor
long-distance / soft

short-distance ; perturbation theory in $\alpha_s(m_b)$

LCDA
long-distance / collinear
Probability to find quark with mom. fraction $u$ in pion

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \Big( C_0 [\bar{c}b]_{V-A} [\bar{d}u]_{V-A} + C_8 [\bar{c}T^A b]_{V-A} [\bar{d}T^A u]_{V-A} \Big)$$

$b \to c\bar{u}d$

### Naive factorization

$$\langle D^+\pi^- | Q_0 | \bar{B}_d\rangle \to \underbrace{\langle D^+ | [\bar{c}b] | \bar{B}_d\rangle}_{F^+_{B\to D}\,(p+p')_\mu + \ldots} \underbrace{\langle \pi^- | [\bar{d}u] | 0\rangle}_{i f_\pi q^\mu} = i f_\pi F_{B\to D}(m_\pi^2)(m_B^2 - m_D^2)$$

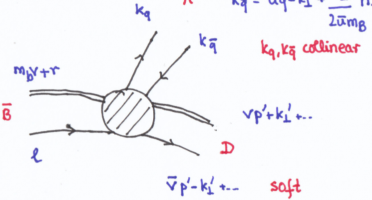
$$\langle D^+\pi^- | Q_8 | \bar{B}_d\rangle \to 0$$

corresponds to $T_0^{I}(u) \equiv 1$ $\quad T_8^{I}(u) \equiv 0$

scale dependence of $Q_{0,8}$ not cancelled

$$k_q = uq + k_\perp + \frac{k_\perp^2}{2u m_B}\, n_-$$

$$k_{\bar q} = \bar u q - k_\perp + \frac{k_\perp^2}{2\bar u m_B}\, n_-$$




$n_\pm = (1,0,0,\pm 1)$

$n_\pm^2 = 0\,,\ n_+ n_- = 2$

$k_q^2 = k_{\bar q}^2 = 0$

$q = \pi$ momentum $= \frac{m_B}{2} n_+$

$A(p_A', \ell, r\,;\ u, k_\perp; v, k_\perp')$
amplitude

(1) No leading contribution from endpoints $u \sim \Lambda/m_b$, $\bar u = 1-u \sim \Lambda/m_b$

- otherwise $k_q, k_{\bar q}$ not collinear; no $\phi_\pi(u)$

(2) Can expand $A$ in $k_\perp$ ($k_\perp \to 0$ in LO). Otherwise $\Psi_\pi(u, k_\perp)$ or sth. more complicated

(3) Leading contributions only from $\bar v p' \sim \Lambda$, i.e. $\bar v \sim \Lambda/m_b$. Otherwise could have

and no $F_{B\to D}$.

(4)  dominated by hard interactions / virtualities $\gg \Lambda^2$ $\Rightarrow$ calculate in PT (quark+ gluon lines) $\Rightarrow$ $T_i^I(u)$

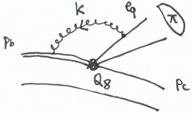
(5) Higher-Fock states are non-leading. Otherwise could have

## Assumptions on hadronic wave functions [(1),(3)]

- $k_\perp \sim \Lambda$
- Heavy mesons in non-boosted frame are made of soft stuff ($\phi_D(\bar{v} \sim 1) \approx 0$)
- $\phi_\pi(u,\mu) \xrightarrow{\mu\to\infty} 6u\bar{u}$ so $\phi_\pi(\text{endpoint}) \sim \Lambda/m_b$

  ↳ Need to exclude $\sim \int du\, \phi_\pi(u)\, 1/u^2$ (for $u \to 0$)

## Dominance of short-distance fluctuations [(4)]

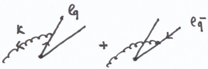



$$= i g_s^2 \frac{C_F}{2} \int_0^1 du \int \frac{d^2\ell_\perp}{16\pi^3} \frac{\Psi^*_\pi(u,\ell_\perp)}{\sqrt{2N_c}} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}\, \mathrm{tr}\left(\gamma_5 \not{q}\, \frac{\gamma_\lambda(\not{\ell}_q+\not{k})\Gamma}{k^2+2\ell_q\cdot k}\right) \frac{\bar{u}_c \Gamma(\not{p}_b+\not{k}+m_b)\gamma^\lambda u_b}{k^2+2p_b\cdot k}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $k$ | hard | $m_b^4$ | $\frac{1}{m_b^2}$ | $\frac{1}{m_b}$ | $\frac{1}{m_b}$ | $\sim 1$ |
| | soft | $\Lambda^4$ | $\frac{1}{\Lambda^2}$ | $\frac{1}{\Lambda}$ | $\frac{1}{\Lambda}$ | $\sim 1$ |
| | collinear | $\Lambda^4$ | $\frac{1}{\Lambda^2}$ | $\frac{1}{\Lambda^2}$ | $1$ | $\sim 1$ |

all same size

# Soft cancellations

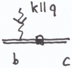


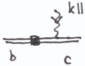

$$\frac{\gamma_\lambda(\not{\ell}_q+\not{k})\Gamma}{k^2+2\ell_q\cdot k}+\frac{\Gamma(-\not{\ell}_{\bar q}-\not{k})\gamma_\lambda}{k^2+2\ell_{\bar q}\cdot k} \overset{k\,\text{soft}}{\approx} \frac{\gamma_\lambda u\not{q}\Gamma}{2uq\cdot k}-\frac{\Gamma\bar u\not{q}\gamma_\lambda}{2\bar u q\cdot k}$$

$$\approx \frac{q_\lambda\Gamma}{q\cdot k}-\frac{\Gamma q_\lambda}{q\cdot k}=0$$ up to higher orders in $\Lambda/m_b$

"colour transparency"

Similar collinear cancellations



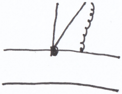
$+$


$= 0$

$\Rightarrow$ only hard survives (in calculations: IR poles cancel)

Not true for  or  $\Rightarrow$ $F_{B\to D}$, $\phi_{\pi}(u)$

Higher Fock states



more internal lines $\rightarrow$ extra suppression

# Simple recipe for calculations

$$\langle c(p')\, d(uq)\, \bar{u}(\bar{u}q) \,|\, Q_i \,|\, b(p)\rangle \;=$$

$$= \;\bar{u}_{a\alpha}\, \Gamma(u,\dots)_{\alpha\beta,ab}\, v_{\beta b}(\bar{u}q)\;\; \bar{u}(p')\,\Gamma'(\dots)\, u(p)$$

quark matrix
element (hard dominance)

$$\longrightarrow \quad \frac{i f_\pi}{4N_c} \int_0^1 du\, \phi_\pi(u)\, (\not{q}\gamma_5)_{\beta\alpha}\, \Gamma(u,\dots)_{\alpha\beta,aa} \;\times\; \bar{u}(p')\,\Gamma'(\dots)\, u(p)$$

$$\longrightarrow \quad i f_\pi \int_0^1 du\, \phi_\pi(u)\, T_i^{I}(u) \;\; \underbrace{\langle c(p')|\Gamma'|b(p)\rangle}_{\to\, F_{B\to D}}$$

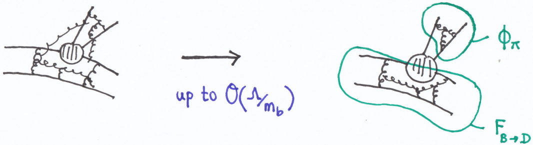
$$T_0^{I} = 1 + \mathcal{O}(\alpha_s^2)$$

$$T_8^{I} = \frac{\alpha_s}{4\pi}\,\frac{C_F}{2N_c}\left(-6\ln\frac{\mu^2}{m_b^2} - 11_{NDR} + F\!\left(u,\frac{m_c}{m_b}\right)\right)$$

logs, dilogs, ..
IR divergences cancel in sum of diagrams

consistent cancellation of scale / scheme dependence

# All-order factorization and SCET




Refs.

QCDF: MB, Buchalla, Neubert, Sachrajda (1999)
2loop: BBNS (2000)

SCET: Bauer, Fleming, Pirjol, Stewart (2000, 2001)
MB, Chapovsky, Diehl, Feldmann (2002)

$B\to D\pi$: all orders / SCET
Bauer, Pirjol, Stewart (2001)

Want to show the above for fluctuations with virtuality $\ll m_b^2$
Exploit properties of an effective Lagrangian

$$\text{soft} \qquad k \sim m_b\,(\underset{n_+p}{\lambda},\ \underset{n_-p}{\lambda},\ \underset{p_\perp}{\lambda}) \qquad\qquad \lambda \equiv \Lambda/m_b$$

$$\text{collinear} \qquad k \sim m_b\,(1, \lambda^2, \lambda)$$

$$\mathcal{L}_{\text{eff}}^{\text{SCET}} = \sum_{Q=b,c} \bar{h}_{v_Q}\, i v_Q\cdot D_s\, h_{v_Q} + \sum_{q\ \text{light}} \bar{\xi}_q \left( i n_-D + i\not{D}_{\perp c} \frac{1}{i n_+ D_c} i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\xi_q + \sum_{q\ \text{light}} \bar{q}\, i\not{D}_s\, q + \mathcal{L}_{\text{pure gluon},\, c\,\&\,s}$$

(1) (3) (2)

collinear quark field

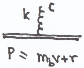
soft quark field

$$iD_\mu = i\partial_\mu + g_s A_{\mu c} + g_s A_{\mu s}$$

$\mathcal{L}_{\text{eff}}$ reproduces QCD Feynman rules for soft & collinear interactions / propagators <u>at leading power</u>

① $\bar{h}_v\, iv\cdot D_s\, h_v$ No interactions of heavy quarks with collinear fields

$iD_s = i\partial + g_s A_s$




$p+k \sim (1,1,\lambda)$ $\Rightarrow$ $(p+k)^2 \sim m_b^2$ already integrated out

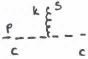
② $\bar{\xi}\, i\not{D}_{\perp c} \frac{1}{in_+ D_c}\, i\not{D}_{\perp c} \frac{\not{n}_+}{2}\, \xi$ Collinear interactions are non-local. Vertices with any number of $n_+ A_c$ fields

Non-local because

$n_+ k_c \sim m_b$ [hard]


③ $\bar{\xi}(x)\left(in_- D_c(x) + g_s n_- A_s(x_-)\right)\frac{\not{n}_+}{2}\xi$

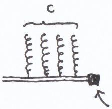
$x_-^\mu \equiv n_+ x\, \frac{n_-^\mu}{2}$




$p + n_- k\, \frac{n_+^\mu}{2}$ since $n_+ k \ll n_+ p,\ k_\perp \ll p_\perp$

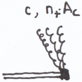
momentum expansion $\Rightarrow$ no translation invariance

Where do the collinear interactions with heavy quarks in full QCD go?




line ends on weak vertex [operator $Q_i$]

up to $\mathcal{O}(\Lambda/m_b)$ $\longrightarrow$




$b \longrightarrow W_c h_v$

collinear Wilson line

$$W_c(x) \equiv P\exp\left(ig_s \int_{-\infty}^{0} ds\, n_+ \cdot A_c(x+sn_+)\right)$$

(1) $W_c W_c^\dagger = \mathbb{1}$

(2) $W_c^\dagger f(in_+ \cdot D_c) W_c = f(in_+ \cdot \partial)$

## Operator matching

$$Q_i = \sum_k \int d\hat{t}\; \tilde{T}^{I}_{ik}(\hat{t})\, O^{I}_{k}(t)$$

$$\left[\hat{t} = m_B t\ ; \quad T^{I}_{ik}(u) = \int d\hat{t}\, e^{iu\hat{t}}\, \tilde{T}^{I}_{ik}(\hat{t})\right]$$

$$O^{I}_{0}(t) = (\bar{\xi} W_c)(tn_+) \frac{\not{n}_+}{2}(1-\gamma_5)\, (W_c^\dagger \xi)(0)\ \bar{h}_{v_c}(0)\, \Gamma\, h_{v_b}(0)$$

$$O^{I}_{8}(t) = \quad \ldots \quad T^A \ldots \quad T^A \ldots$$

Matching to non-local operators in $n_+^\mu$ direction, local in transverse, since $n_+ \cdot k_c \sim m_b$ but $k_{\perp c} \ll m_b$.
[ compare : parton distributions ]

$$\langle \pi^- D^+ |\, (\bar{\xi} W_c)(tn_+)[\ldots]\, (W_c^\dagger \xi)(0)\ \bar{h}_{v_c}(0)\, \Gamma [\ldots]\, h_{v_b}(0)\, | \bar{B}_d \rangle_{\mathcal{L}_{SCET}}$$

$\bar{\xi}\, in_- \cdot A_s \frac{\not{n}_+}{2} \xi$ $\qquad$ $h_{v_Q}\, iv \cdot A_s\, h_{v_Q}$

## Soft-gluon decoupling

$$\xi(x) \equiv Y(x)\, \xi^{(0)}(x) \qquad Y(x) \equiv P\, e^{ig_s \int_{-\infty}^{0} ds\, n_- A_s(x_- + s n_-)} \qquad \text{soft Wilson line}$$

$$A_c(x) \equiv Y(x)\, A_c^{(0)}(x)\, Y^\dagger(x)$$

$$\mathcal{L}_{eff}^{SCET} = \ldots + \sum_{q\ \text{light}} \bar{\xi}_q \left( i n_- D_c + i \not{D}_{\perp c} \frac{1}{i n_+ D_c} i \not{D}_{\perp c} \right) \frac{\not{n}_+}{2} + \ldots$$

No couplings between collinear and soft fields [up to $\mathcal{O}(\Lambda/m_b)$]

$$\langle \pi^- D^+ | \, (\bar{\xi}^{(0)} W_c^{(0)})(t n_+)\, Y^\dagger_{(0)} [\ldots] \left\{ \begin{matrix} 1 \\ T^A \end{matrix} \right\} Y(0)\, (W_c^{(0)\dagger} \xi^{(0)})(0)\, \bar{h}_{v_c}(0) [\ldots] \left\{ \begin{matrix} 1 \\ T^A \end{matrix} \right\} h_{v_b}(0) \, | \bar{B}_d \rangle_{\mathcal{L}_{SCET}}$$

$$= \langle \pi^- | (\bar{\xi}^{(0)} W_c^{(0)})(t n_+) [\ldots] \left\{ \begin{matrix} 1 \\ T^A \end{matrix} \right\} (W_c^{(0)\dagger} \xi^{(0)})(0) | 0 \rangle \langle D^+ | h_{v_c}(0) [\ldots] \left\{ \begin{matrix} Y^\dagger Y \\ Y^\dagger T^A Y \end{matrix} \right\} h_{v_b}(0) | \bar{B}_d \rangle_{\mathcal{L}_{SCET}}$$

$\equiv 1$

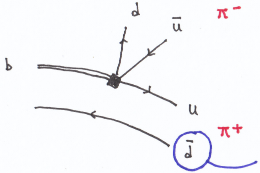
0 for $T^A$ ($\pi^-$, 0 colour singlet)

Fourier trafo of $\phi_\pi(u)$ for 1

For singlet $Y^\dagger Y = 1 \rightarrow$ <u>all soft effects cancel</u>.

Matrix element is $F_{B\to D}$

# Charmless decays

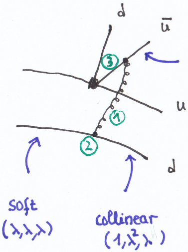



$$\langle \pi^-\pi^+ | Q_i | \bar B_d \rangle \overset{?}{=} F^{B\to\pi}_{(m_\pi^2)}\; i f_\pi \int_0^1 du\, T_i^{I}(u)\, \phi_\pi(u)$$

remains soft. $\pi^+$ produced in an atypical configuration $\Rightarrow$ suppression $F^{B\to\pi}_{(m_\pi^2)} \propto \left(\frac{\Lambda}{m_b}\right)^{3/2}$

[vs. $F^{B\to D}_{(m_\pi^2)} \sim O(1)$]

## Competing process




$q$, $n_- \cdot q$ large

anti-collinear $(\lambda^2, 1, \lambda)$

$p'$, $n_+ \cdot p'$ large

1. hard-collinear virtuality $m_B^2 \lambda$ – new scale
   momentum $(1, \lambda, \sqrt{\lambda})$

2. $\mathcal{L}^{(1)}_{SCET} \supset \bar q\, W_c^\dagger\, i \not{D}_{\perp c}\, \xi + h.c.$
   power-suppressed interaction to convert soft into collinear $\bar q$

3. hard virtuality $m_B^2$

## Hard-spectator interaction

- same order in $\Lambda/m_b$
- $\alpha_s(\sqrt{m_B\Lambda})$ suppressed

Refs

BBNS (1999) tree-level

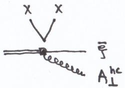
Chay, Kim; MB, Feldmann, Bauer et. al (2003), Neubert, Lange
factorization properties

MB, Kiyo, Yang; Hill et al., MB, Yang; MB, Jäger
1-loop matching calculations (2004–2006)

## Two-step factorization in SCET

(1) hard-scale $m_B$

$$Q_i = \int d\hat t\ \underbrace{[\bar\chi^{(0)}_{(tn_-)}\chi^{(0)}]}_{\text{anticollinear } \phi_\pi(u)} \times \Big\{ T^{I}_i(\hat t)\ \underbrace{[\bar\xi h_v]}_{\substack{\text{soft part of } B\to\pi \\ \text{form factor} \\ F_{B\to\pi}(m_\pi^2)}} + \int d\hat s\ H^{II}(\hat t,\hat s)\ [\bar\xi_{(0)} A^{hc}_\perp(sn_+)\, h_{v(0)}] \Big\}$$

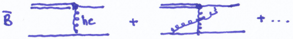



share momentum

non-local form factor $\Xi_{B\to\pi}(\tau, m_\pi^2)$

(2) hard-collinear scale $\sqrt{m_B\Lambda}$

$$\int d\hat s\, e^{-i\tau\hat s} \langle\pi^+|\,\bar\xi_{(0)}A^{hc}_\perp(sn_+)\,h_{v(0)}\,|\bar B_d\rangle = \int_0^\infty d\omega \int_0^1 dv\ \underbrace{J(\tau;v,\omega)}_{\text{hc}}\ \underbrace{f_{\hat B}\phi_B(\omega)}_{\text{soft}}\ \underbrace{f_\pi\phi_\pi(v)}_{\text{collinear}}$$



# QCD factorization formula for charmless decays

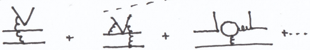
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = i m_B^2 F_+^{B\to M_1}(0) \int_0^1 du\, T_i^{I}\, f_{M_2} \phi_{M_2}(u) + \left( \begin{matrix} M_1 \leftrightarrow M_2 \\ \text{if applicable} \end{matrix} \right)$$

tree operators $(\bar{u}b)(\bar{D}b)$

penguin operators $(\bar{D}b)\sum_q (\bar{q}q)$

+ ....

$$+\, i m_B^2 \int_0^\infty d\omega \int_0^1 du\,dv\, f_{\hat{B}}\, \phi_B(\omega)\, \phi_{M_1}(v)\, \phi_{M_2}(u) \times \left[ \int d\tau\, H^{II}_{(u,\tau)}\, J(\tau;\omega,v) \right]$$



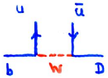
$$+\ {}^{1}/_{m_b}\ \text{power corrections}$$

- precise prescription for higher-order corrections
- summation of logs with RGE
- Imaginary parts ($\rightarrow$ strong phases) only from hard loops ($\propto \alpha_s(m_b)$)

# Applications of factorization

# Colour-suppressed tree amplitude

- Relevant for $B_d \to \pi^0\pi^0, \ldots$
  Naive factorization




$$C \propto a_2(\pi\pi) = 0.220$$

- NNLO factorization

$$\begin{aligned} C \propto a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\ &\quad + \left[\frac{r_{\text{sp}}}{0.485}\right] \Big\{[0.123]_{\text{LOsp}} + [0.053 + 0.054i]_{\text{NLOsp}} + [0.072]_{\text{tw3}}\Big\} \\ &= 0.26 - 0.07i \quad \to \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}}) \end{aligned}$$

$$r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B} \qquad\qquad \frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\Phi_{B+}(\omega,\mu)$$

No colour suppression at NLO. Large cancellation of one-loop and tree.
Amplitude dominated by spectator-scattering

Sizeable correction to imaginary part (phases), but cancellation between vertex and spectator-scattering.

# Branching fractions (tree-dominated decays) [MB, Huber, Li, 2009]

| | Theory I | | Theory II | | Experiment |
|---|---|---|---|---|---|
| $B^- \to \pi^-\pi^0$ | $5.43\,{}^{+0.06}_{-0.06}\,{}^{+1.45}_{-0.84}$ | $(\star)$ | $5.82\,{}^{+0.07}_{-0.06}\,{}^{+1.42}_{-1.35}$ | $(\star)$ | $5.59^{+0.41}_{-0.40}$ |
| $\bar{B}^0_d \to \pi^+\pi^-$ | $7.37\,{}^{+0.86}_{-0.69}\,{}^{+1.22}_{-0.97}$ | $(\star)$ | $5.70\,{}^{+0.70}_{-0.55}\,{}^{+1.16}_{-0.97}$ | $(\star)$ | $5.16 \pm 0.22$ |
| $\bar{B}^0_d \to \pi^0\pi^0$ | $0.33\,{}^{+0.11}_{-0.08}\,{}^{+0.42}_{-0.17}$ | | $0.63\,{}^{+0.12}_{-0.10}\,{}^{+0.64}_{-0.42}$ | | $1.55 \pm 0.19$ |
| | | | BELLE CKM 14: | | $0.90 \pm 0.16$ |
| $B^- \to \pi^-\rho^0$ | $8.68\,{}^{+0.42}_{-0.41}\,{}^{+2.71}_{-1.56}$ | $(\star\star)$ | $9.84\,{}^{+0.41}_{-0.40}\,{}^{+2.54}_{-2.52}$ | $(\star\star)$ | $8.3^{+1.2}_{-1.3}$ |
| $B^- \to \pi^0\rho^-$ | $12.38\,{}^{+0.90}_{-0.77}\,{}^{+2.18}_{-1.41}$ | $(\star)$ | $12.13\,{}^{+0.85}_{-0.73}\,{}^{+2.23}_{-2.17}$ | $(\star)$ | $10.9^{+1.4}_{-1.5}$ |
| $\bar{B}^0 \to \pi^+\rho^-$ | $17.80\,{}^{+0.62}_{-0.56}\,{}^{+1.76}_{-2.10}$ | $(\star)$ | $13.76\,{}^{+0.49}_{-0.44}\,{}^{+1.77}_{-2.18}$ | $(\star)$ | $15.7 \pm 1.8$ |
| $\bar{B}^0 \to \pi^-\rho^+$ | $10.28\,{}^{+0.39}_{-0.39}\,{}^{+1.37}_{-1.42}$ | $(\star\star)$ | $8.14\,{}^{+0.34}_{-0.33}\,{}^{+1.35}_{-1.49}$ | $(\star\star)$ | $7.3 \pm 1.2$ |
| $\bar{B}^0 \to \pi^\pm\rho^\mp$ | $28.08\,{}^{+0.27}_{-0.19}\,{}^{+3.82}_{-3.50}$ | $(\dagger)$ | $21.90\,{}^{+0.20}_{-0.12}\,{}^{+3.06}_{-3.55}$ | $(\dagger)$ | $23.0 \pm 2.3$ |
| $\bar{B}^0 \to \pi^0\rho^0$ | $0.52\,{}^{+0.04}_{-0.03}\,{}^{+1.11}_{-0.43}$ | | $1.49\,{}^{+0.07}_{-0.07}\,{}^{+1.77}_{-1.29}$ | | $2.0 \pm 0.5$ |
| $B^- \to \rho^-_L\,\rho^0_L$ | $18.42\,{}^{+0.23}_{-0.21}\,{}^{+3.92}_{-2.55}$ | $(\star\star)$ | $19.06\,{}^{+0.24}_{-0.22}\,{}^{+4.59}_{-4.22}$ | $(\star\star)$ | $22.8^{+1.8}_{-1.9}$ |
| $\bar{B}^0_d \to \rho^+_L\,\rho^-_L$ | $25.98\,{}^{+0.85}_{-0.77}\,{}^{+2.93}_{-3.43}$ | $(\star\star)$ | $20.66\,{}^{+0.68}_{-0.62}\,{}^{+2.99}_{-3.75}$ | $(\star\star)$ | $23.7^{+3.1}_{-3.2}$ |
| $\bar{B}^0_d \to \rho^0_L\rho^0_L$ | $0.39\,{}^{+0.03}_{-0.03}\,{}^{+0.83}_{-0.36}$ | | $1.05\,{}^{+0.05}_{-0.04}\,{}^{+1.62}_{-1.04}$ | | $0.55^{+0.22}_{-0.24}$ |

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

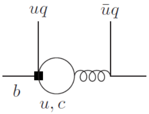
First error $\gamma$, $|V_{cb}|$. $|V_{ub}|$ uncertainty *not* included. Second error from hadronic inputs.
Brackets: form factor uncertainty not included.

# Penguin amplitudes and (direct) CP violation

- Interference of QCD penguin is main source of direct CP violation.

$$\left[\frac{P^c}{T}\right]_{\pi\pi}, \quad \left[\frac{T}{P^c}\right]_{\pi K}, \quad \left[\frac{P^u}{P^c}\right]_{\phi K}$$

- $$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \left( C_1 \mathcal{O}_1^p + C_2 \mathcal{O}_2^p + \sum_{i=\text{pen}} C_i \mathcal{O}_{i,\text{pen}} \right)$$




Two amplitudes $P^{u,c}$. Dominant contribution beyond tree-level from tree operators $\mathcal{O}^p_{1,2}$.

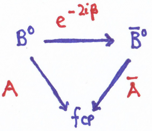
- Non-singlet amplitude $P^{u,c} \sim \lambda_{u,c}^{(D)} \sum_q [\bar{q}_s q][\bar{q} D]$

  Very little known (experimentally) for singlet penguin $S^{u,c} \sim \lambda_{u,c}^{(D)} \sum_q [\bar{q} q][\bar{q}_s D]$.

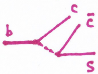
  ($B \to \pi\phi$ in the absence of $\omega - \phi$ mixing.)

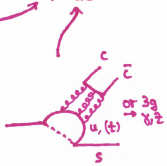
# Mixing-induced CP asymmetry and penguin contributions

Interference of mixing & decay




$$\lambda \equiv e^{-2i\beta}\,\frac{\bar{A}}{A} \overset{SM}{=} e^{-2i\beta}\,\frac{1+a e^{i\theta} e^{-i\gamma}}{1+a e^{i\theta} e^{+i\gamma}}$$




Time-dependent CP asymmetry

$$A_{CP}(t) \equiv \frac{\Gamma(\bar{B}^0(t)\to f_{CP}) - \Gamma(B^0(t)\to f_{CP})}{\Gamma(\bar{B}^0(t)\to f_{CP}) + \Gamma(B^0(t)\to f_{CP})}$$

$|\lambda| \neq 1$
CP in decay
(direct)

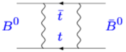
$$= \frac{2\,\mathrm{Im}\,\lambda}{1+|\lambda|^2}\,\sin(\Delta M_{B_d} t) - \frac{1-|\lambda|^2}{1+|\lambda|^2}\,\cos(\Delta M_{B_d} t)$$

$$\approx \eta_{CP}\,\sin(2\beta)\,\sin(\Delta M_{B_d} t)$$

$(=-1)$

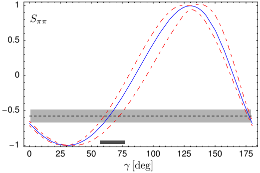
$a \approx 0$

$B\bar{B}$ mixing through top box with phase $\phi_d = 2\beta$



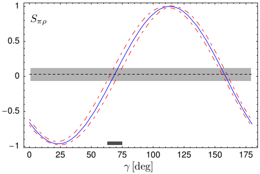

$$= M_{12} - i\,\frac{\Gamma_{12}}{2} \approx (V_{td}^* V_{tb})^2 \hat{M}_{12}$$

# $\gamma$ determination from time-dependent CP asymmetry



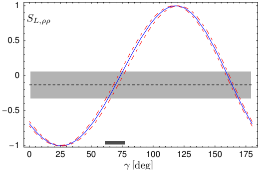

$$S_{\pi\pi} = -0.58 \pm 0.09$$

$$\Rightarrow \qquad \gamma = (65\,^{+12}_{-8})^\circ$$




$$S_{\pi\rho} = 0.03 \pm 0.09$$

$$\Rightarrow \qquad \gamma = (69\,^{+6}_{-6})^\circ$$




$$S_{\rho\rho} = -0.13 \pm 0.19$$

$$\Rightarrow \qquad \gamma = (69\,^{+8}_{-8})^\circ$$

Mutually consistent

$$\gamma = (68 \pm 4)^\circ$$

and consistent with the global unitarity triangle fit (CKMfitter, 2014):

$$\gamma = (66^{+1.3}_{-2.5})^\circ$$

# Penguin amplitudes – Comparison of $P/T$ to data

Final state dependence in good agreement with data.

$$\mathrm{PP} \sim \underbrace{a_4}_{\mathrm{V}\mp\mathrm{A}} + \underbrace{r_\chi a_6}_{\mathrm{S+P}}$$

$$\mathrm{PV} \sim a_4 \approx \frac{\mathrm{PP}}{3}$$

$$\mathrm{VP} \sim a_4 - r_\chi a_6 \sim -\mathrm{PV}$$

$$\mathrm{VV} \sim a_4 \sim \mathrm{PV}$$

Small phases ($\rightarrow$ CP asymmetries)

(Small weak annihilation error for VV unrealistic - similar to VP, PV)

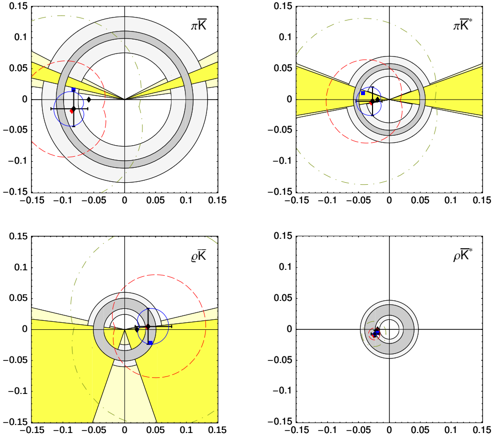
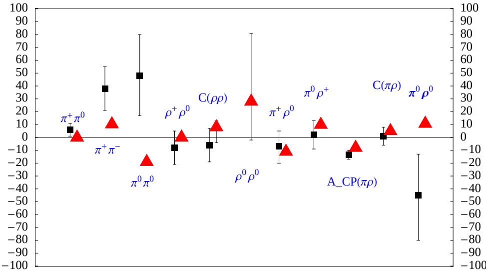
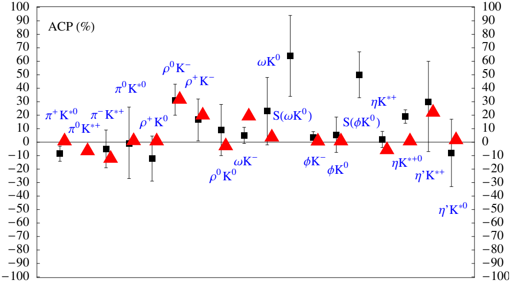



Figure from (MB, Jäger, 2006)




Comparison of direct CP violation in $\Delta D = 1$ (upper plot) and PV $\Delta S = 1$ decays (lower plot).

(Triangles: theory [MB, Neubert, 2003; MB, Rohrer, Yang, 2006])

# Summary/Outlook

I. Mature theory at leading power (SCET).
Similiarities and difference from collider physics. Soft initial state. Power-suppressed interactions relevant at LP.

$$\mathcal{L}_{\rm eik} = \bar{\xi} i n_- D_s \frac{\not{n}_+}{2} \xi \qquad \mathcal{L}^{(1)}_{\xi q} = \bar{q}_s W_c^\dagger i \not{D}_{\perp c} \xi - \bar{\xi} i \overleftarrow{\not{D}}_{\perp c} W_c q_s$$

II. Qualitative features of factorization evident in data (hierarchy of penguin amplitudes, size of direct CP asymmetries and strong phases)

At the quantitative level, mixed conclusions. Often not clear whether $\mathcal{O}(\alpha_s)$ [known] or $\mathcal{O}(\Lambda/m_b)$ effects [unknown] are more important.

III. NNLO computation for charmless decays (nearly) completed

Soon ready for a major improvement of QCDF predictions (excluding polarisation):

- NLO $\rightarrow$ NNLO
- Improved input parameters

Belle-II start-up 2016, B2TiP effort started.
Still many unmeasured, but predicted observables.