

# End-to-End Multi-Track Reconstruction using Graph Neural Networks at Belle II

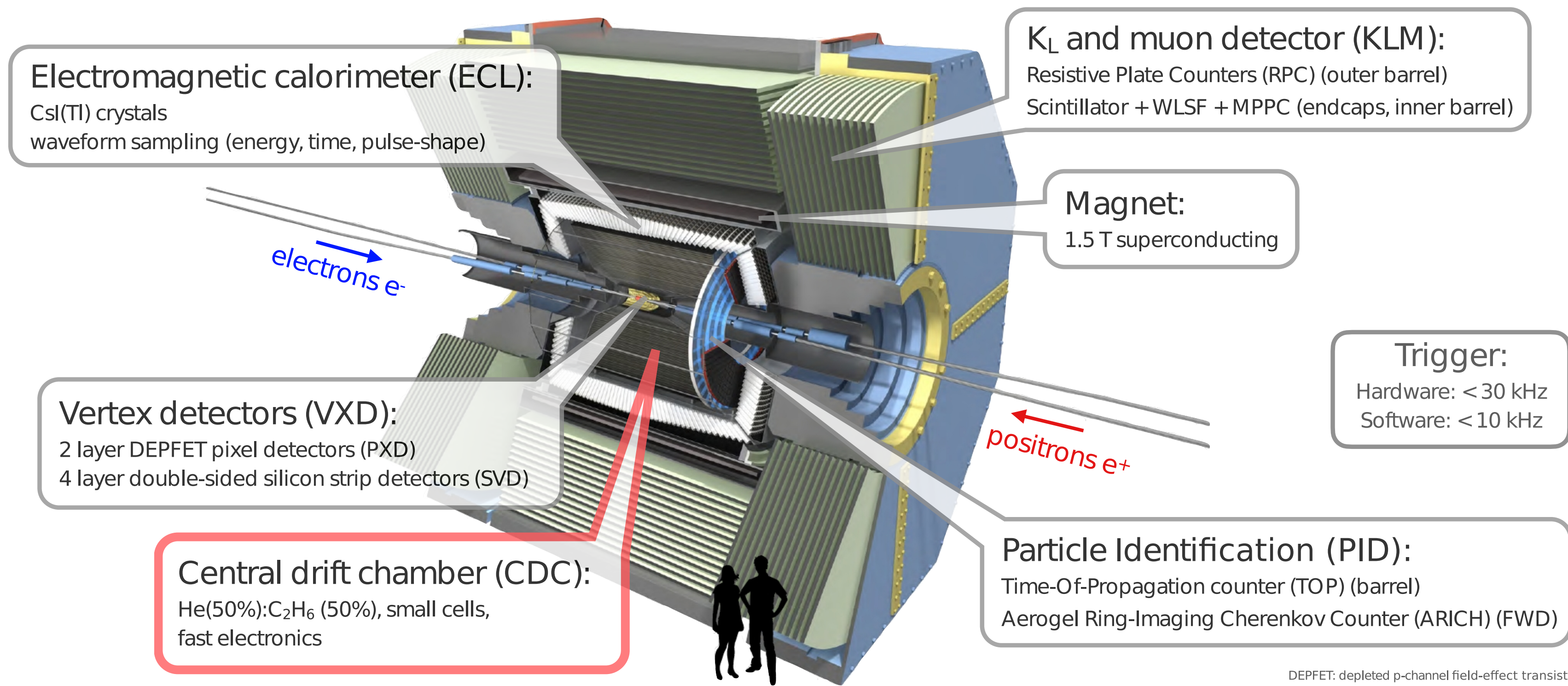
## Glühwein-Workshop 2024 KIT

Lea Reuter, Giacomo De Pietro, Slavomira Stefkova, Torben Ferber





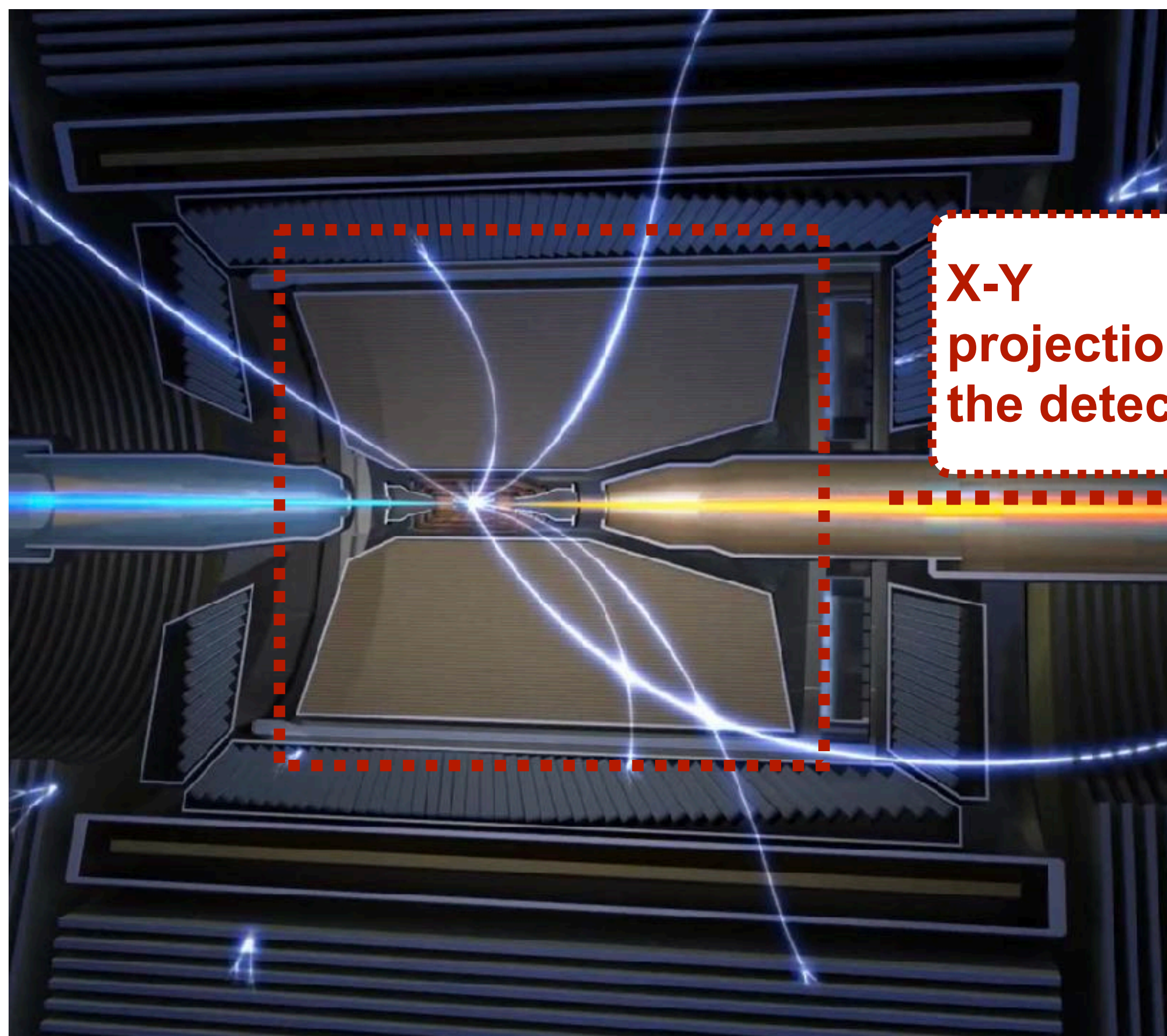
# Belle II Experiment



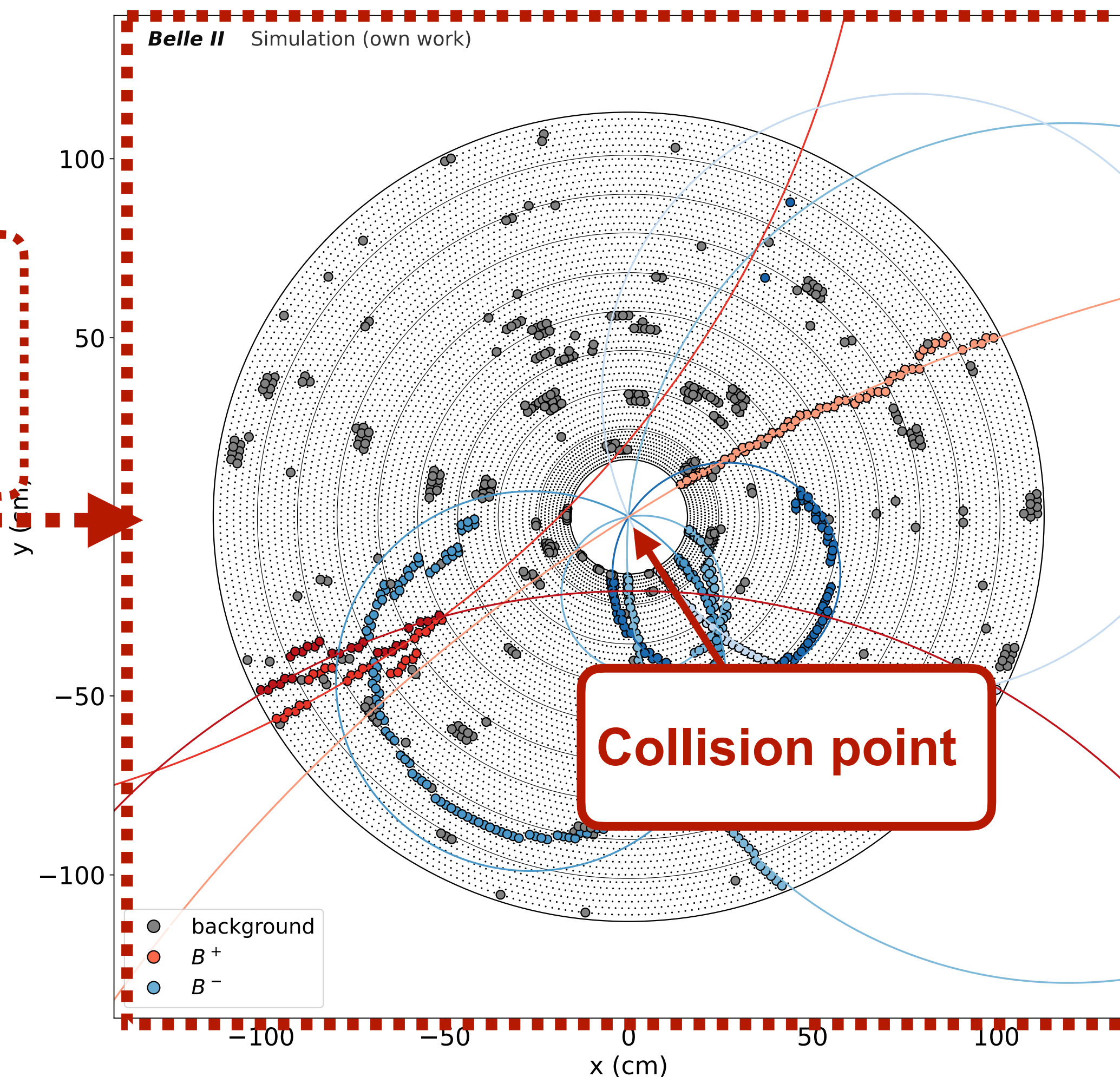
DEPFET: depleted p-channel field-effect transistor  
WLSF: wavelength-shifting fiber  
MPPC: multi-pixel photon counter



# Main Tracking Detector

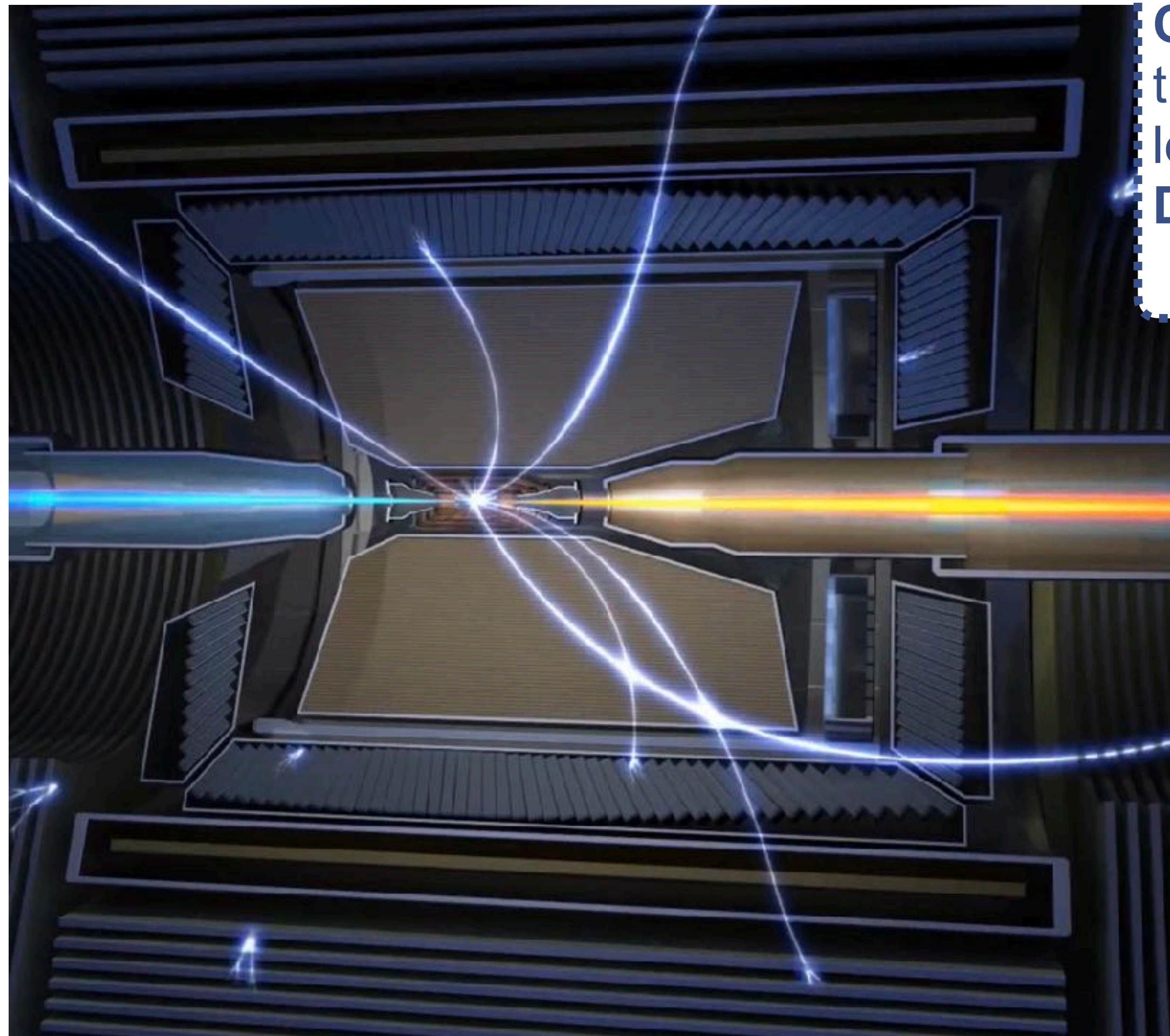


X-Y  
projection of  
the detector

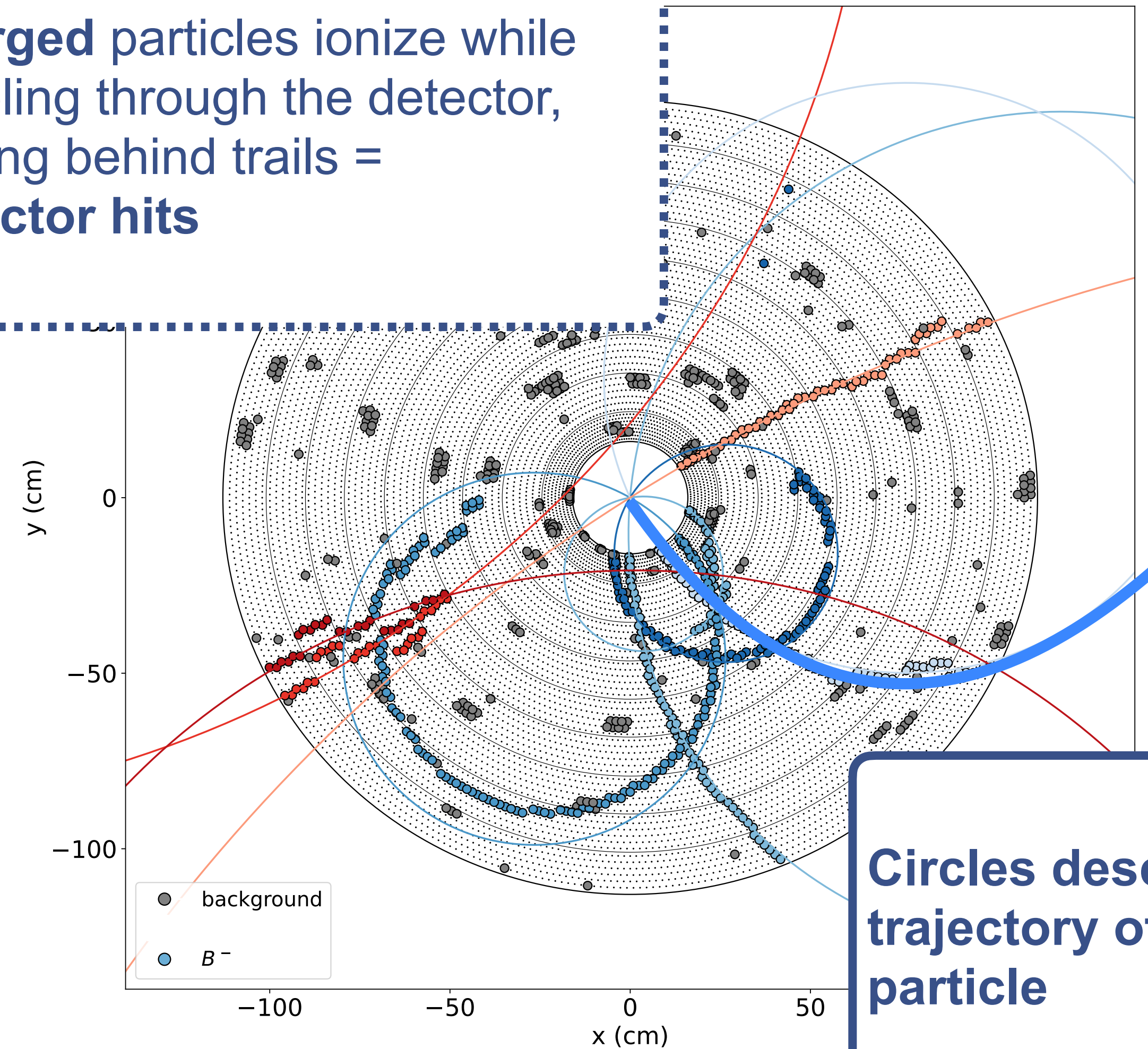




# Main Tracking Detector



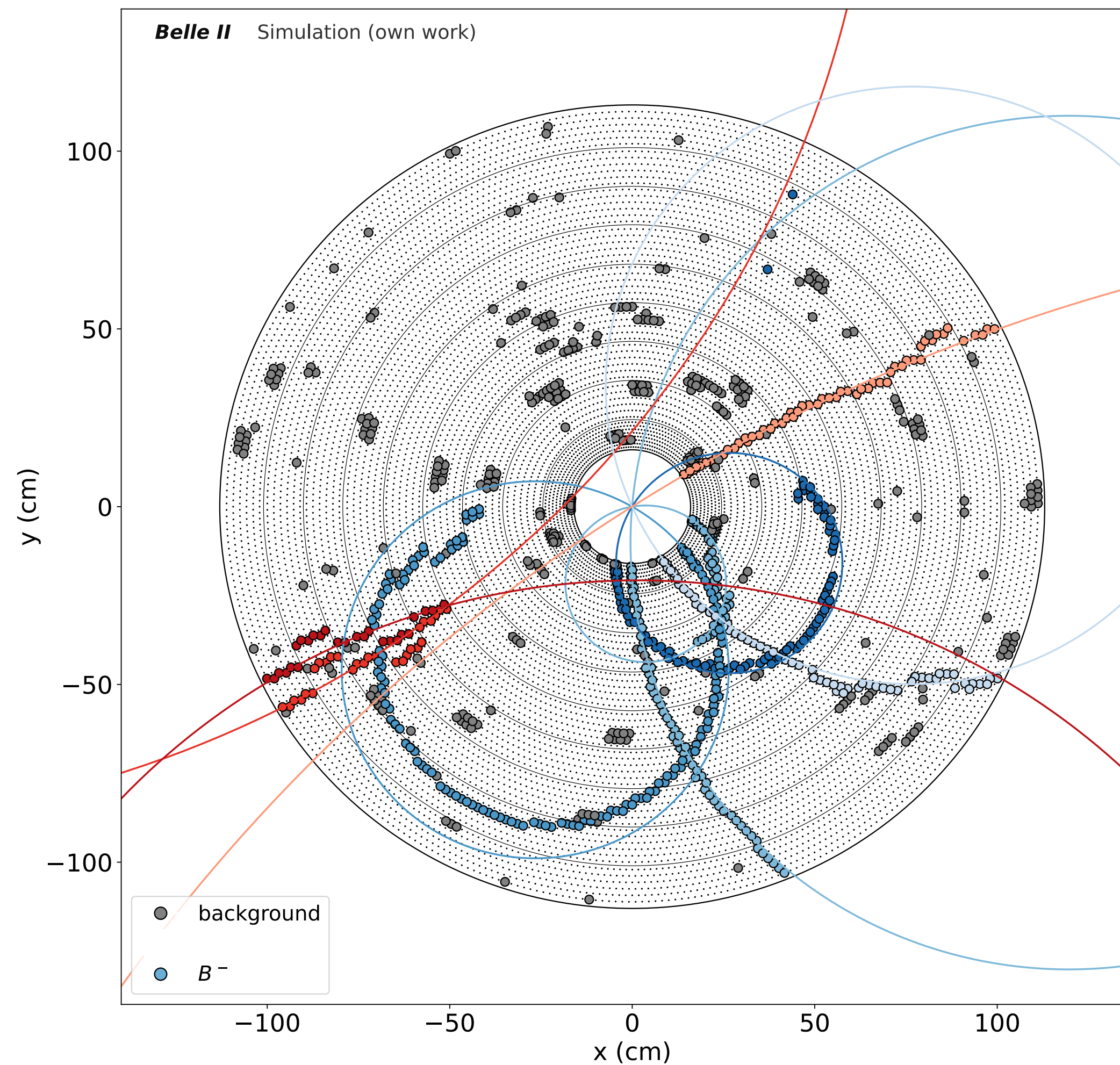
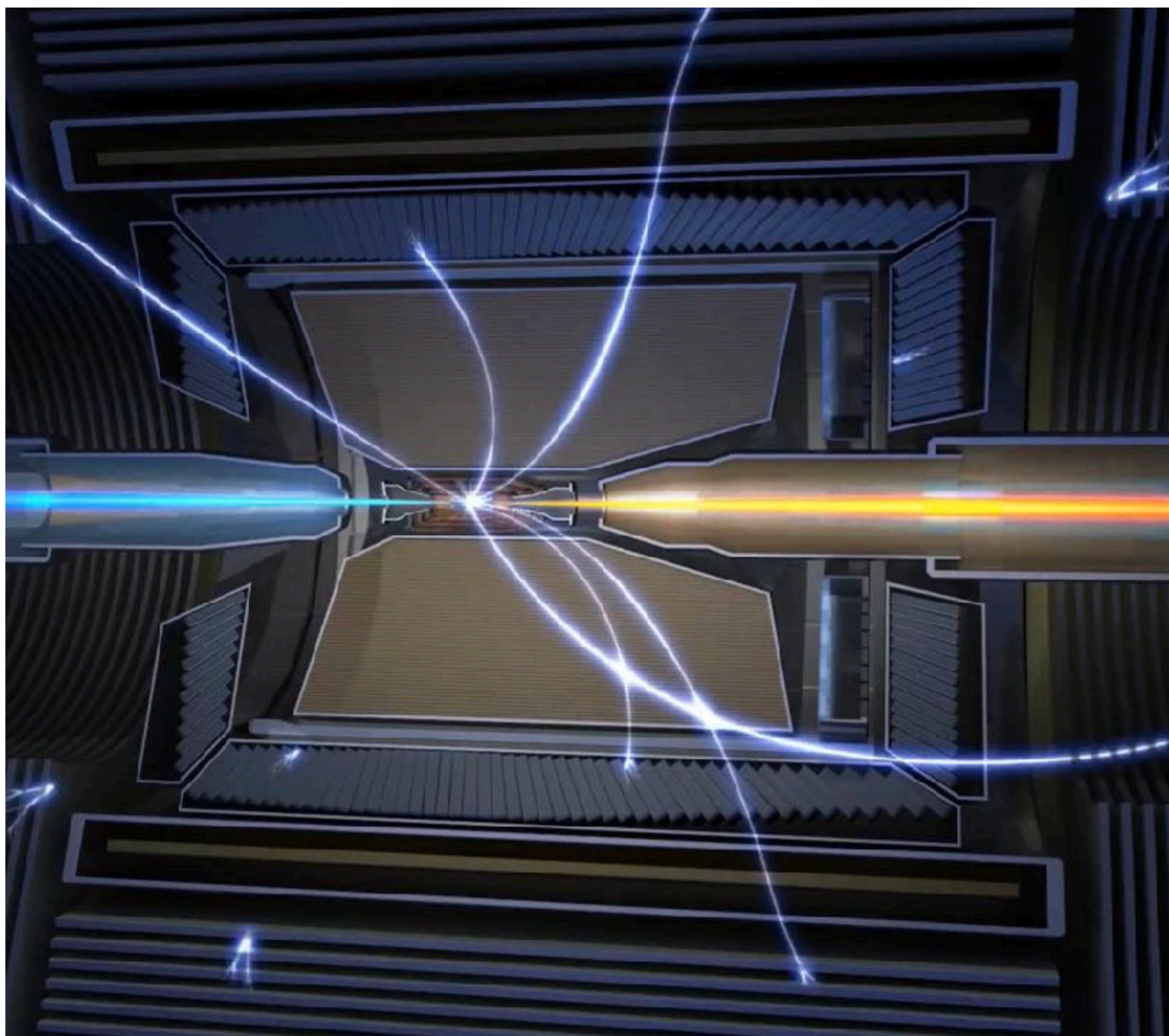
Charged particles ionize while traveling through the detector, leaving behind trails = **Detector hits**



**Circles describe trajectory of the particle**

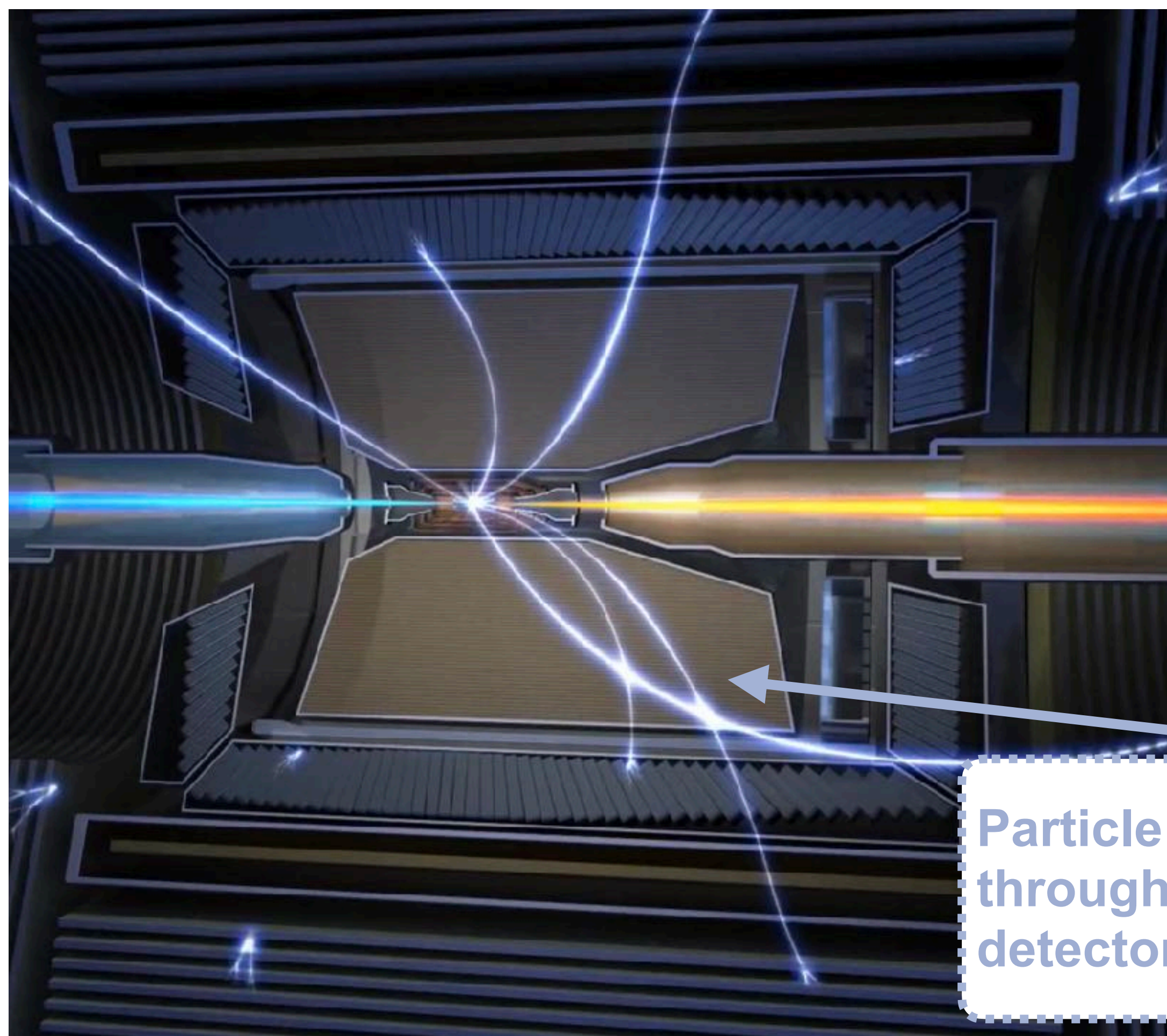


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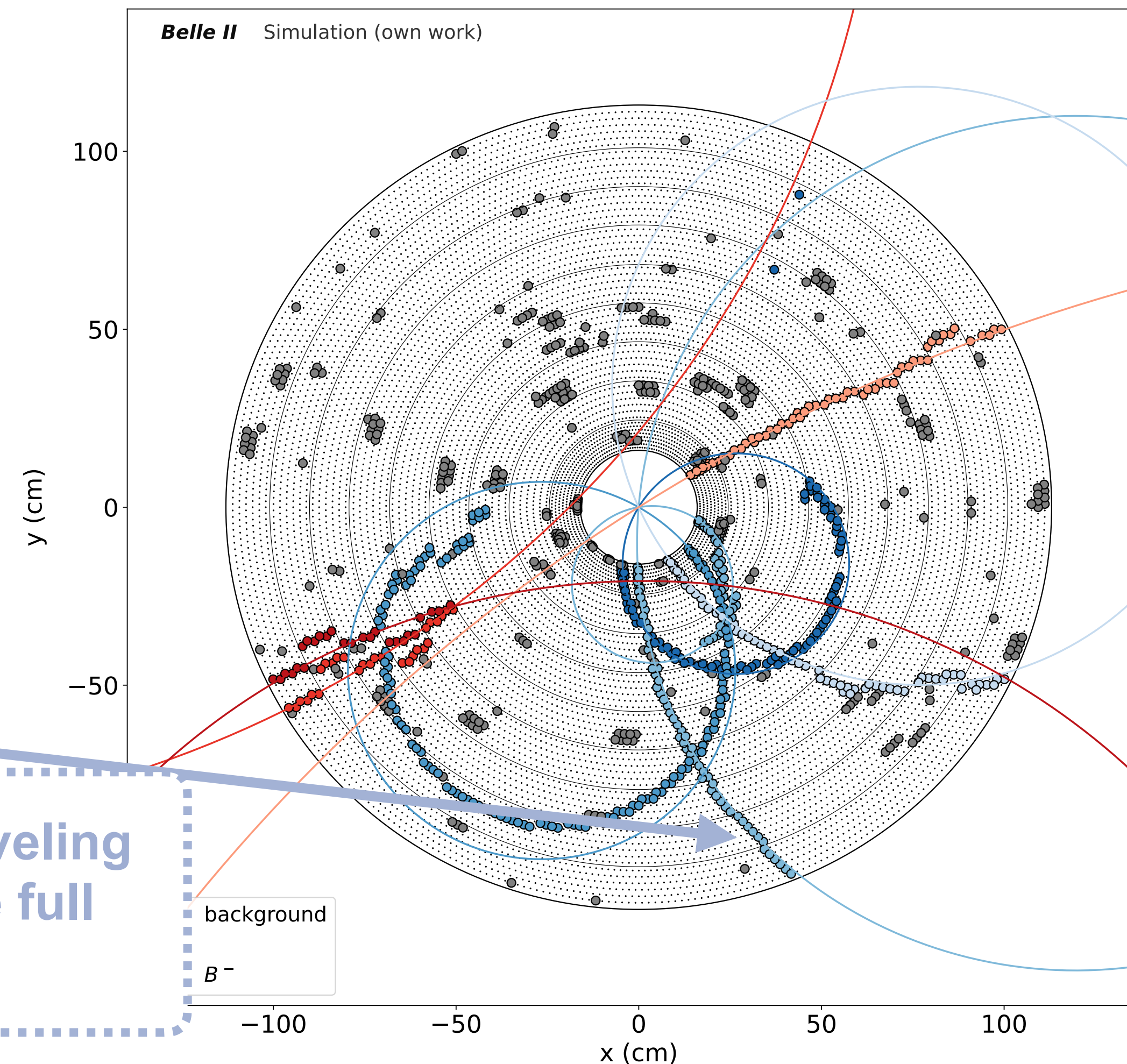




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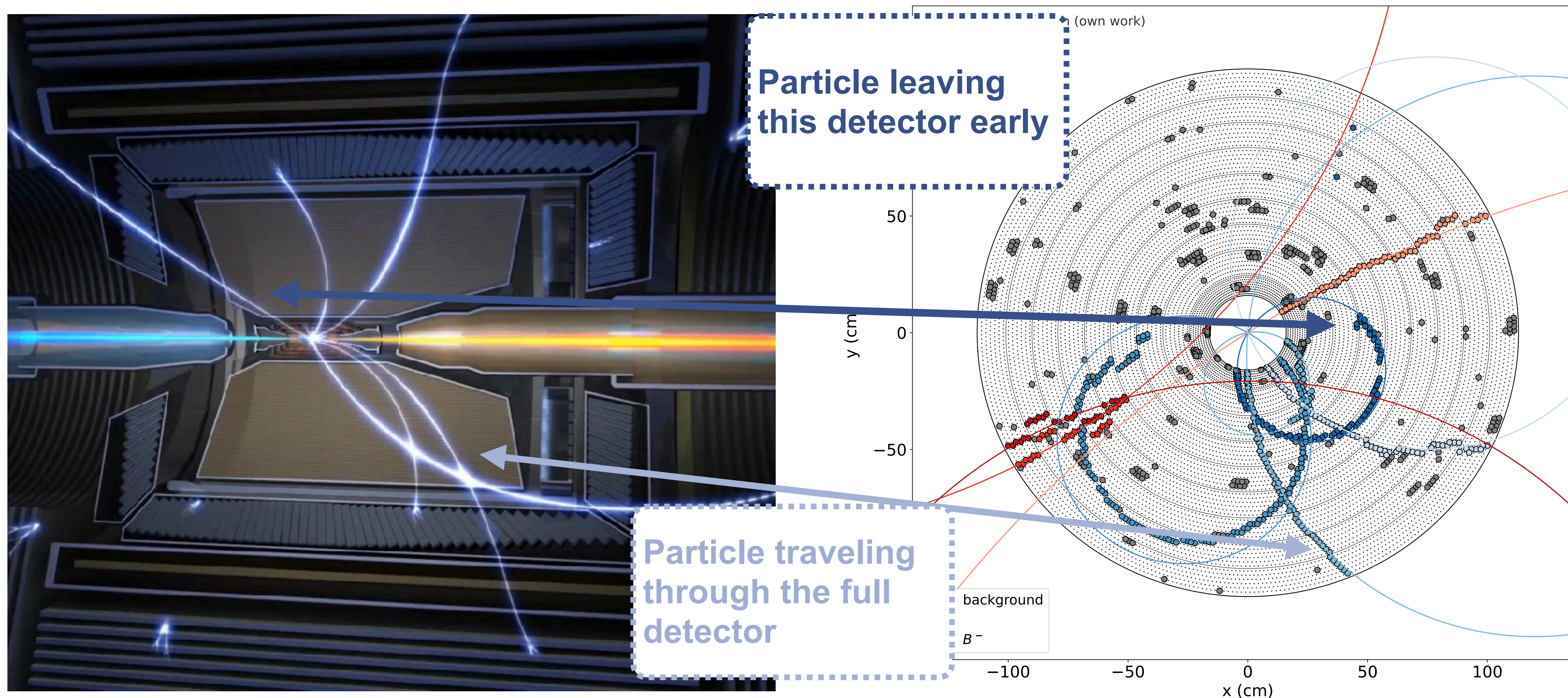


Particle traveling through the full detector



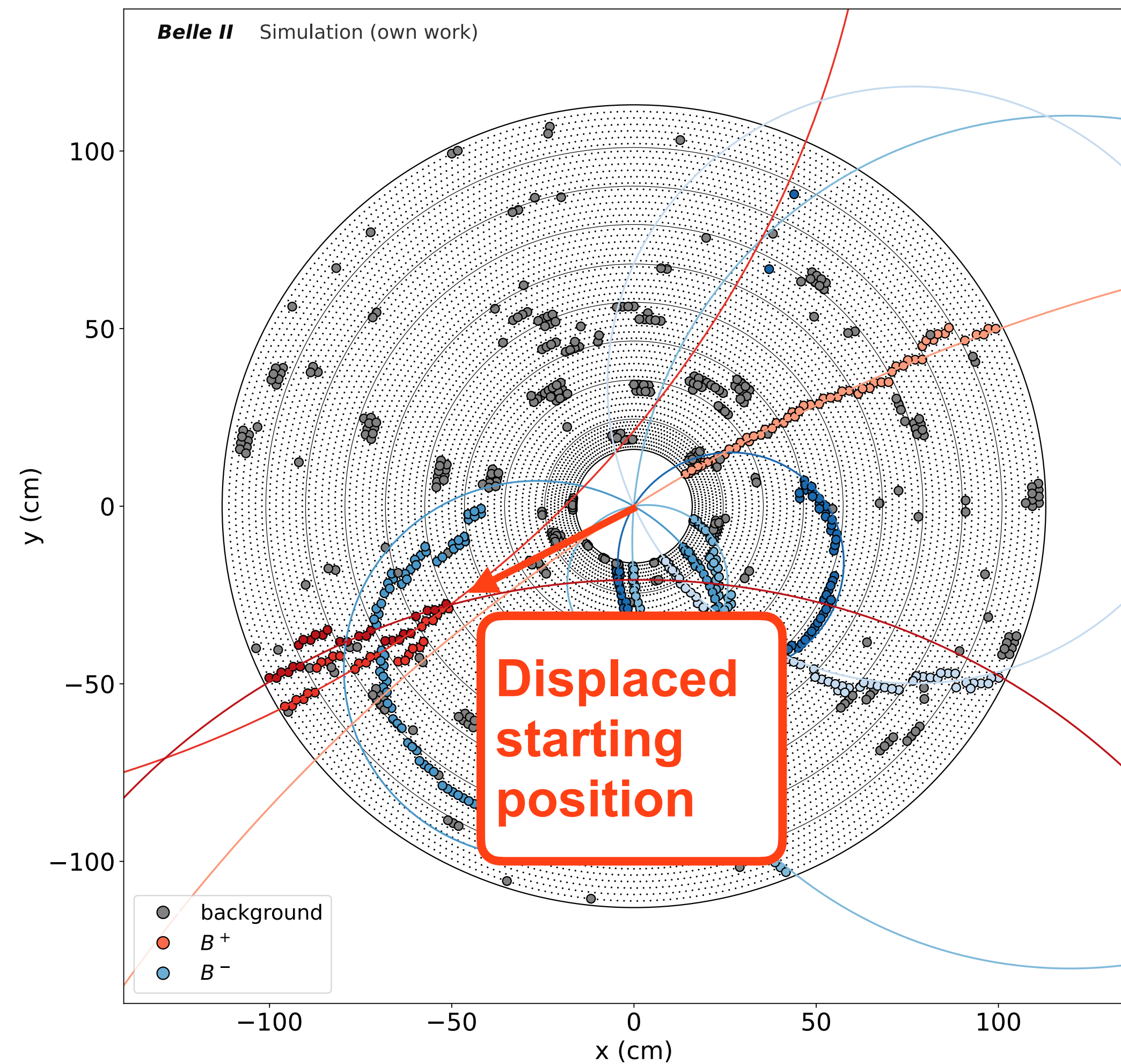
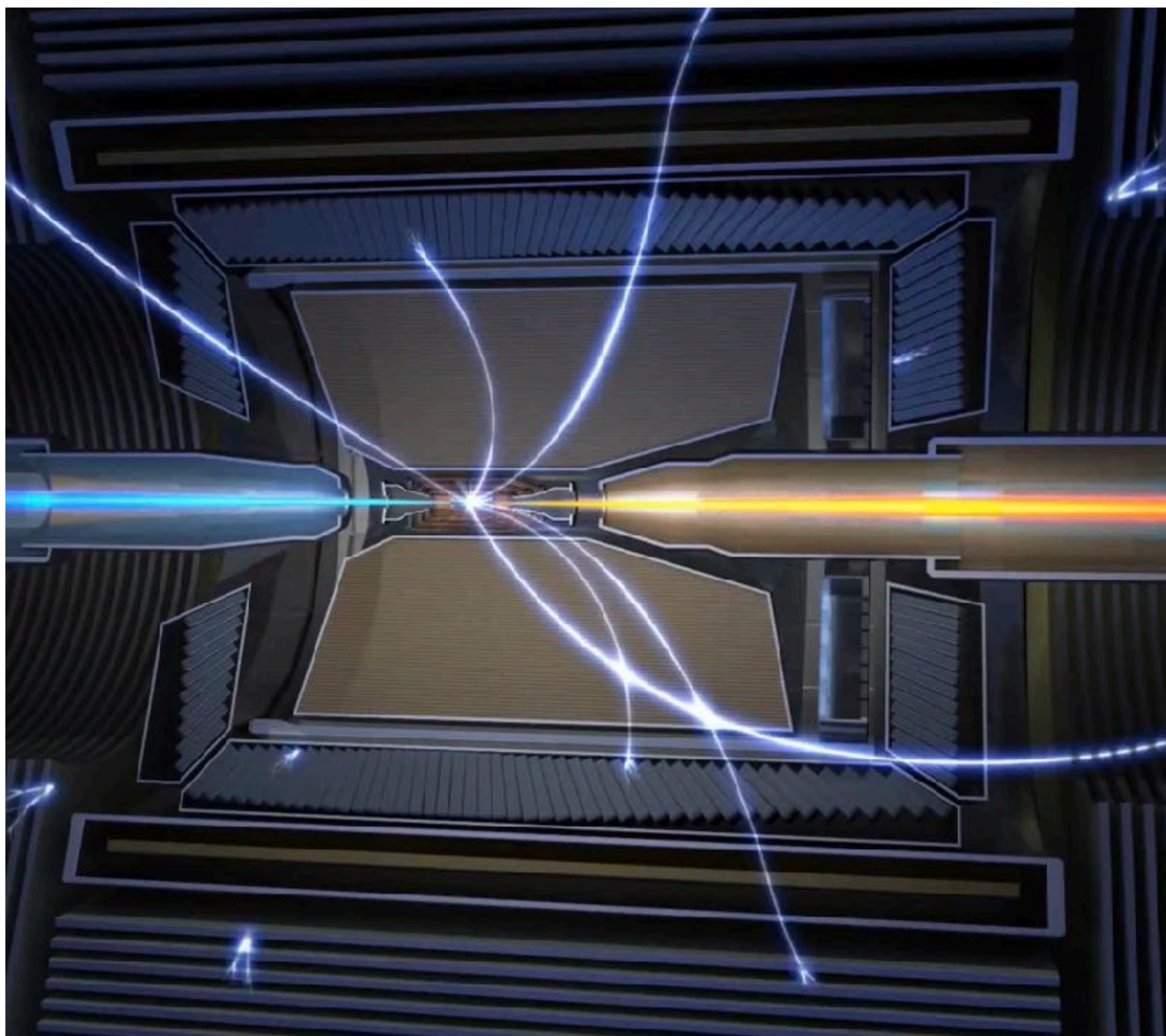


# Main Tracking Detector





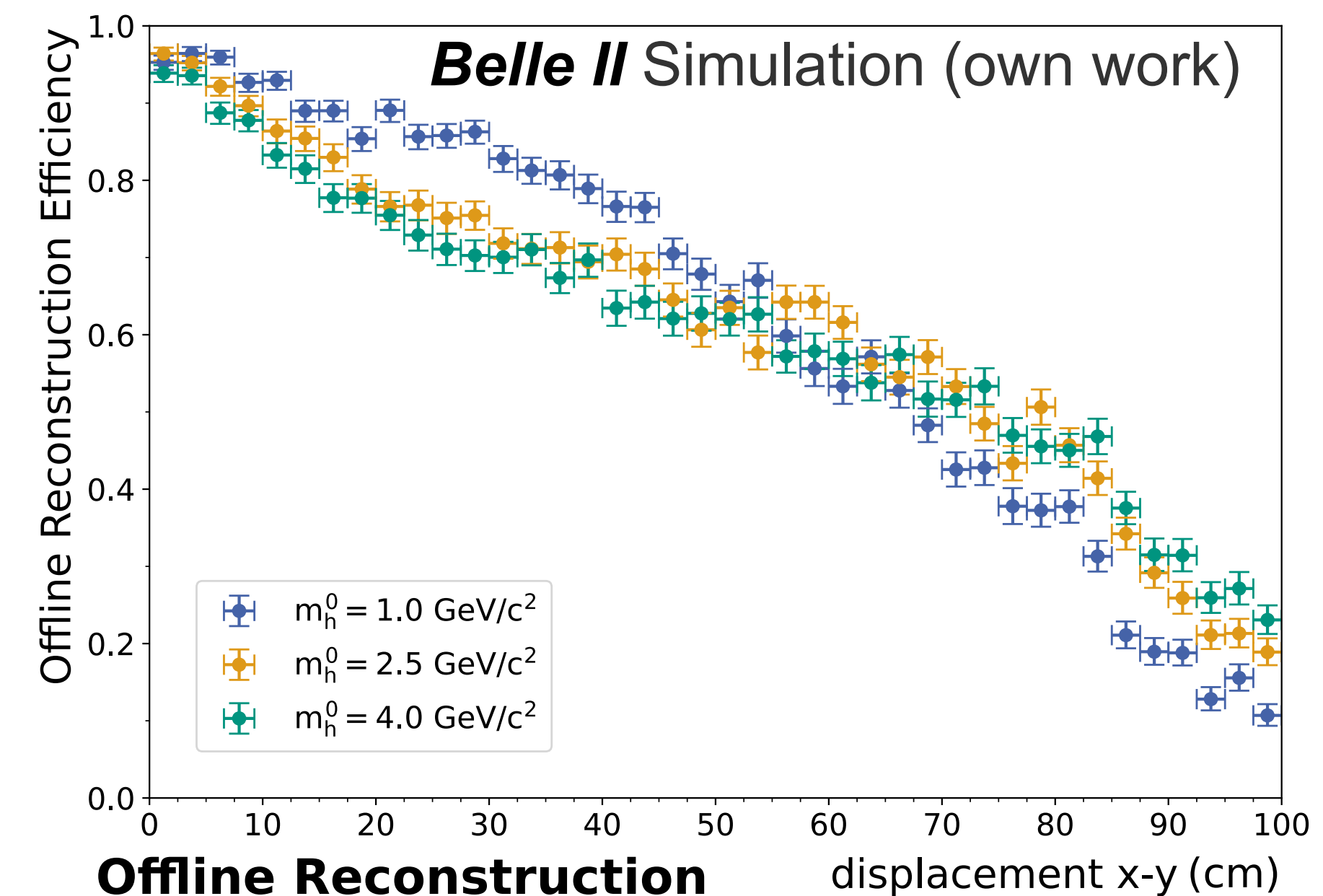
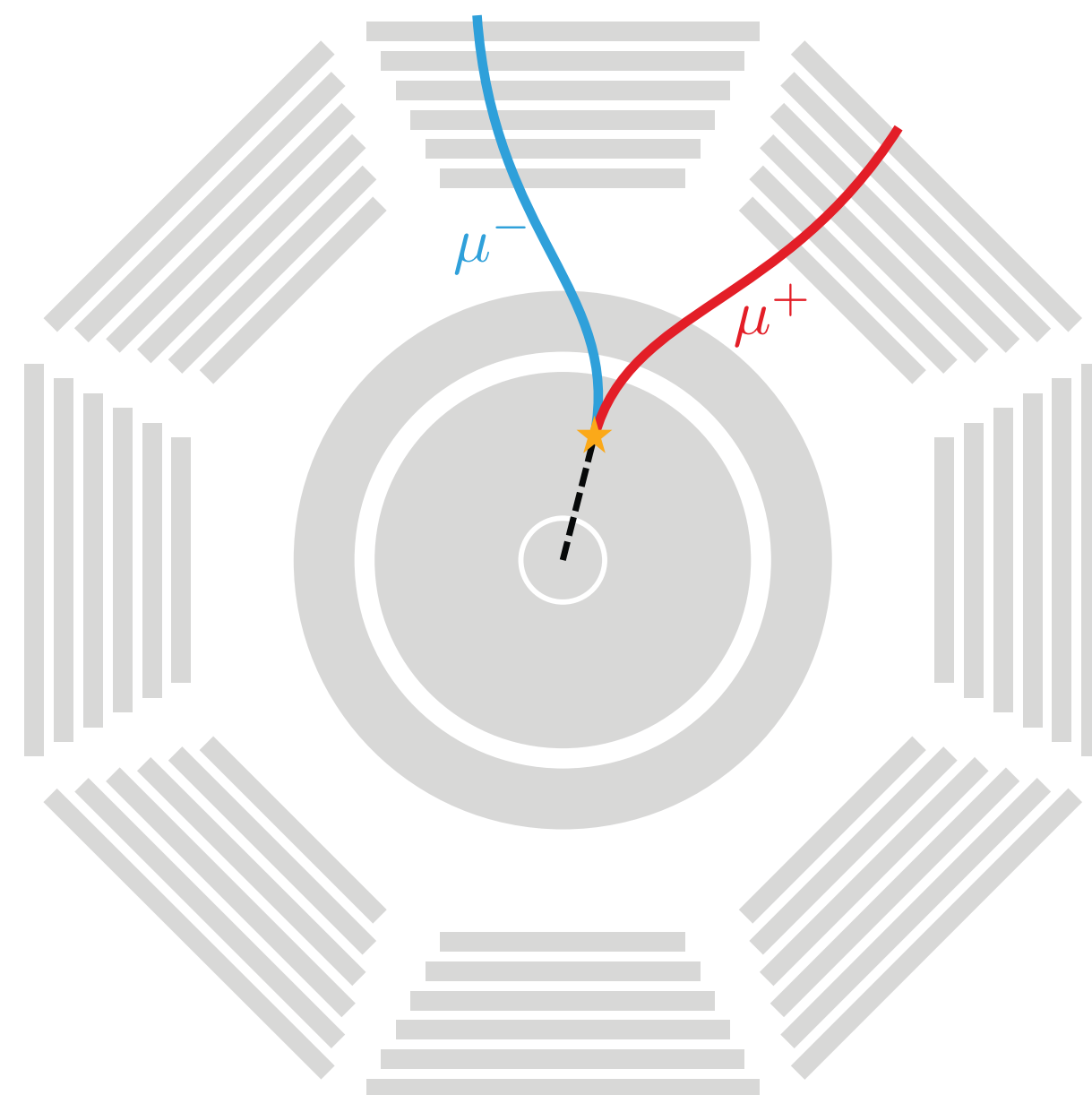
# Main Tracking Detector





# Motivation - Displaced Vertices

Displaced vertices important signature in searches for new physics  
 (for example <sup>1 2</sup> and [arXiv:2012.08595](https://arxiv.org/abs/2012.08595), [arXiv:202.03452](https://arxiv.org/abs/202.03452), [arXiv:1911.03176](https://arxiv.org/abs/1911.03176))



Efficiency decreases depending on displacement ( $K_S^0$ ,  $\Lambda^0$ , Dark Sector searches)

<sup>1</sup>[Search for a long-lived spin-0 mediator in  \$b \rightarrow s\$  transitions at the Belle II experiment \(arXiv:2306.02830\)](https://arxiv.org/abs/2306.02830)

<sup>2</sup>[Search for Inelastic Dark Matter produced in association with a Dark Higgs \(LHC DM WG workshop 2024\)](#)

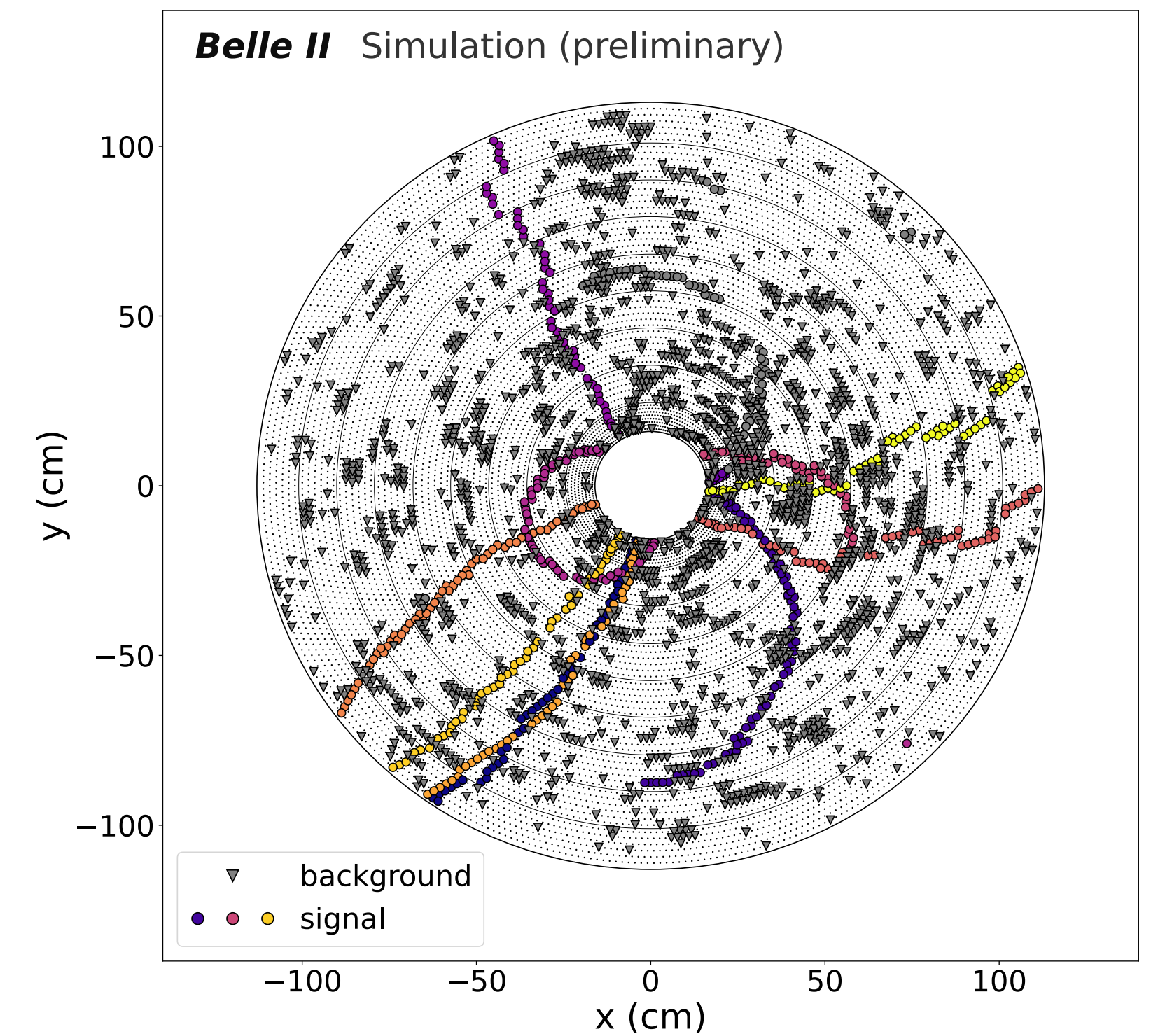
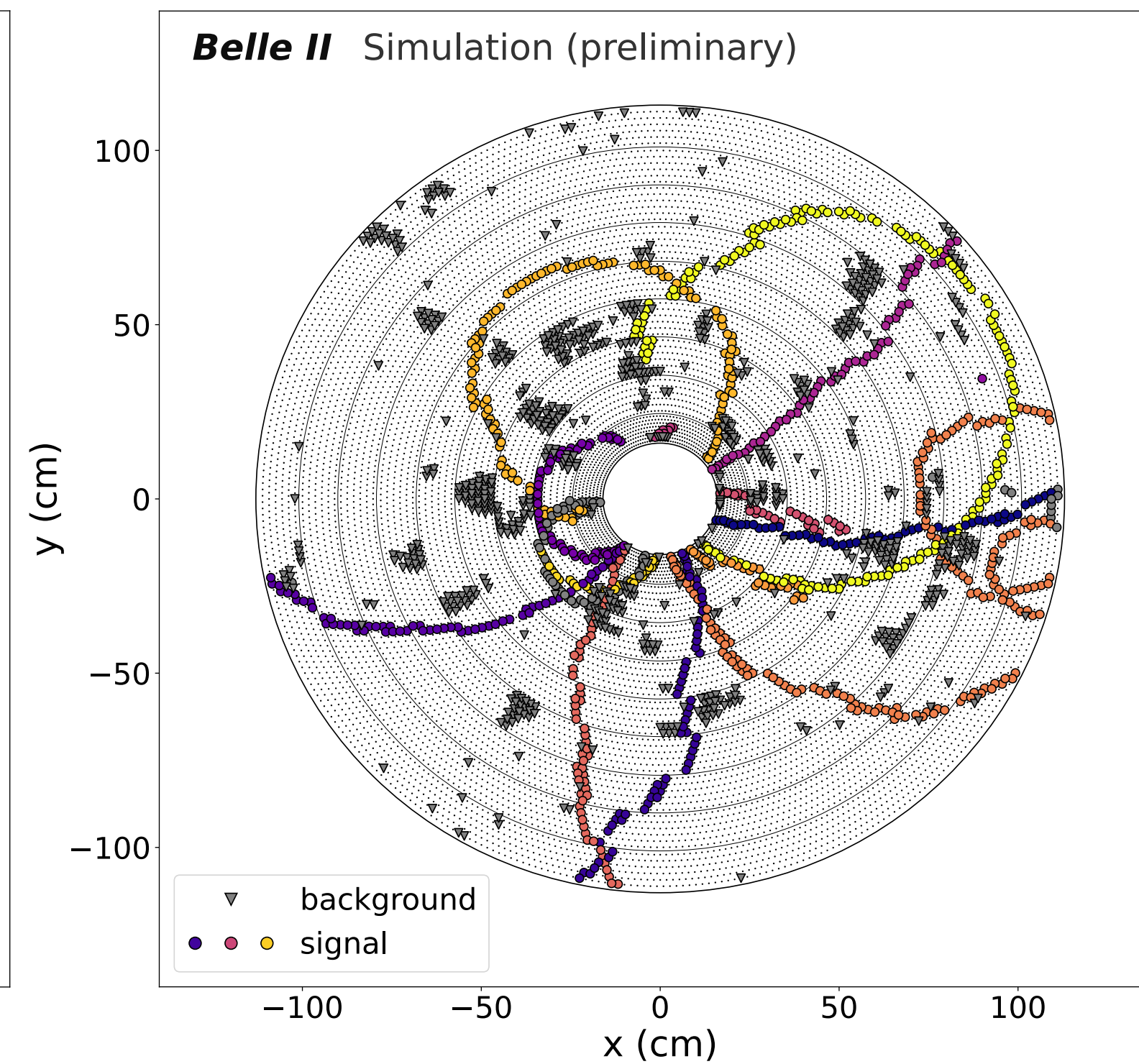
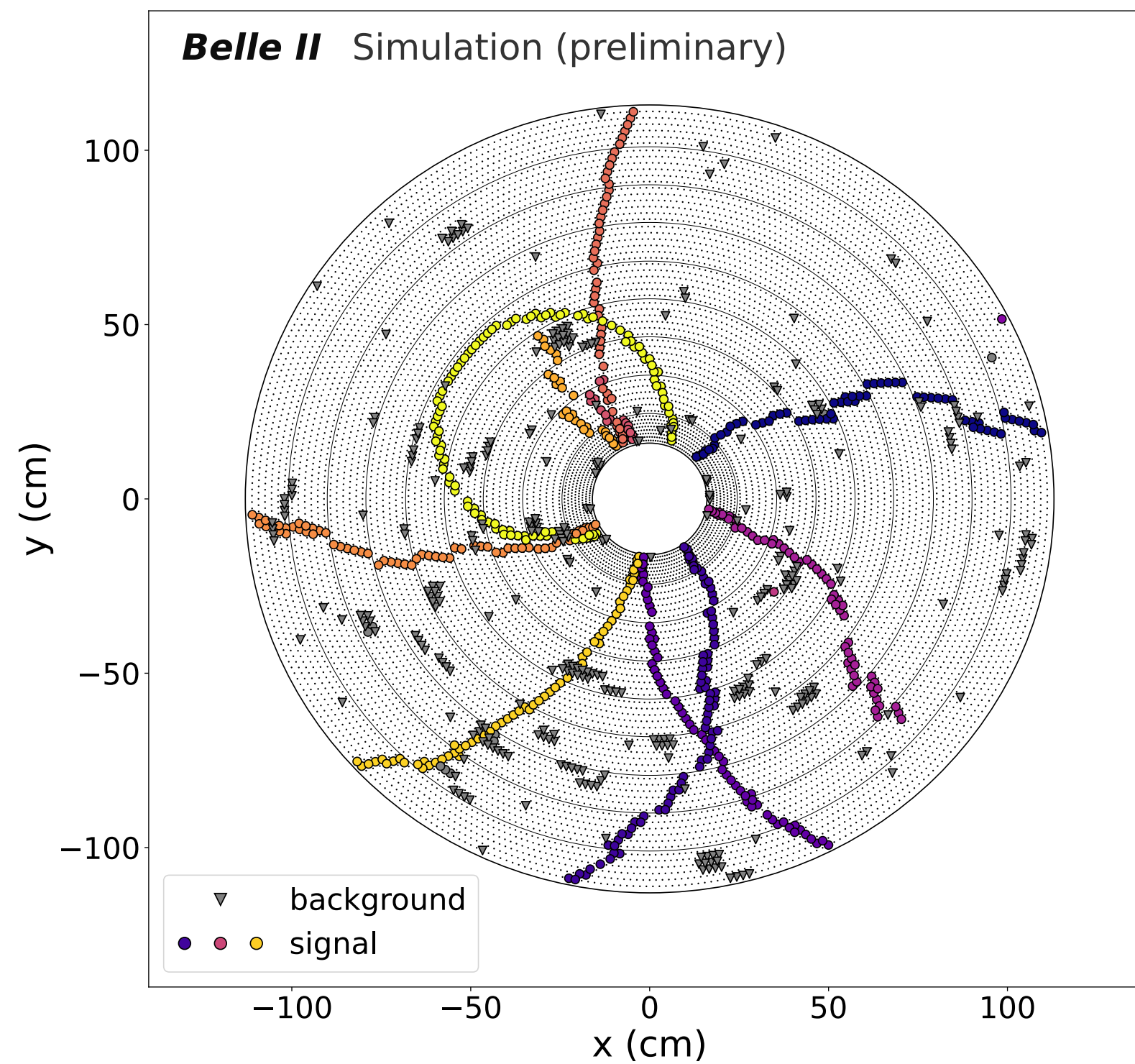


# Motivation - High Backgrounds

Beginning of 2021  
 360 background CDC hits on average

End of 2022  
 1280 background CDC hits on average

Expected 2030  
 3000 background CDC hits on average



Backgrounds are getting higher, harder for tracking



# Motivation - CDC Wire Inefficiencies

Beginning of 2021

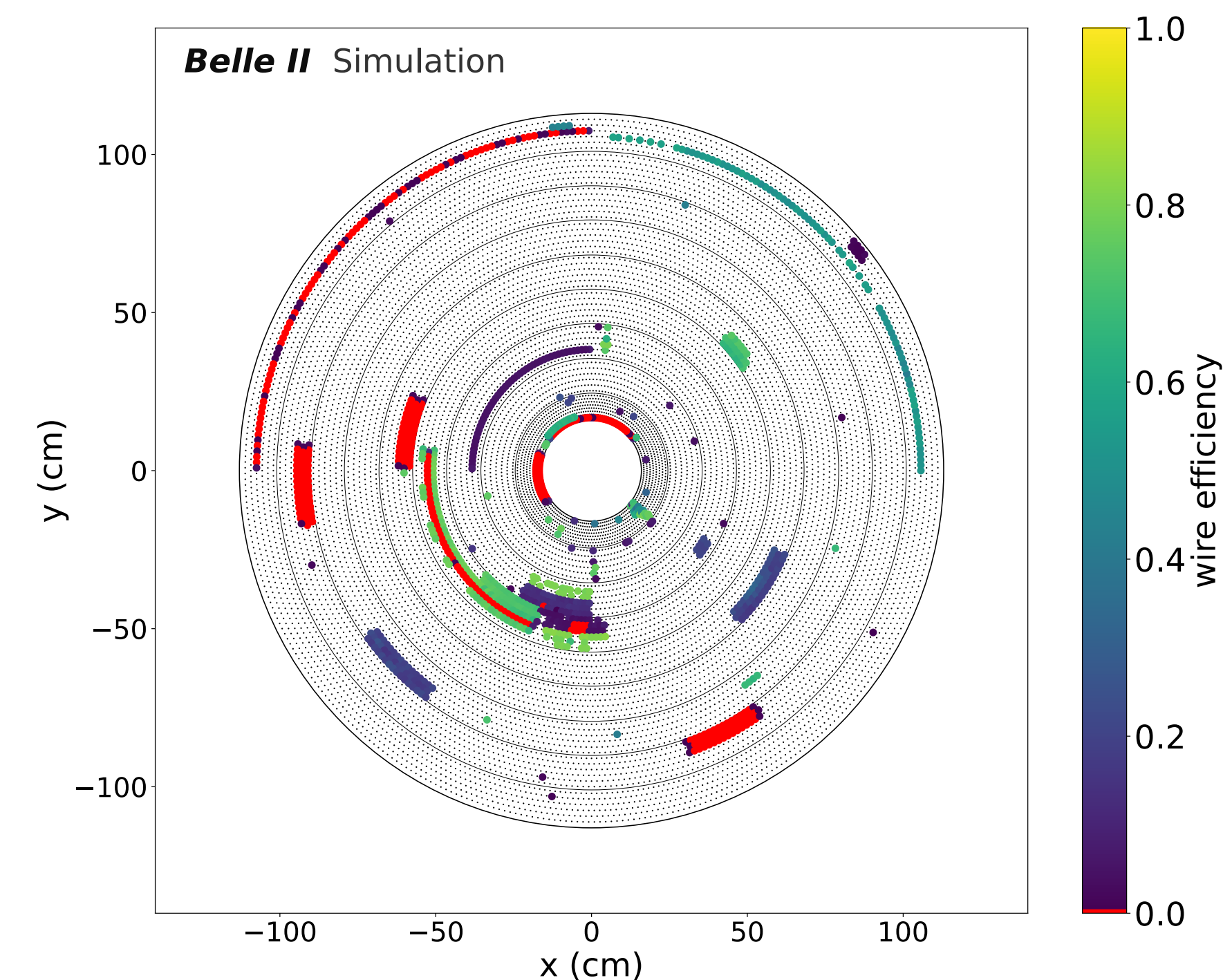
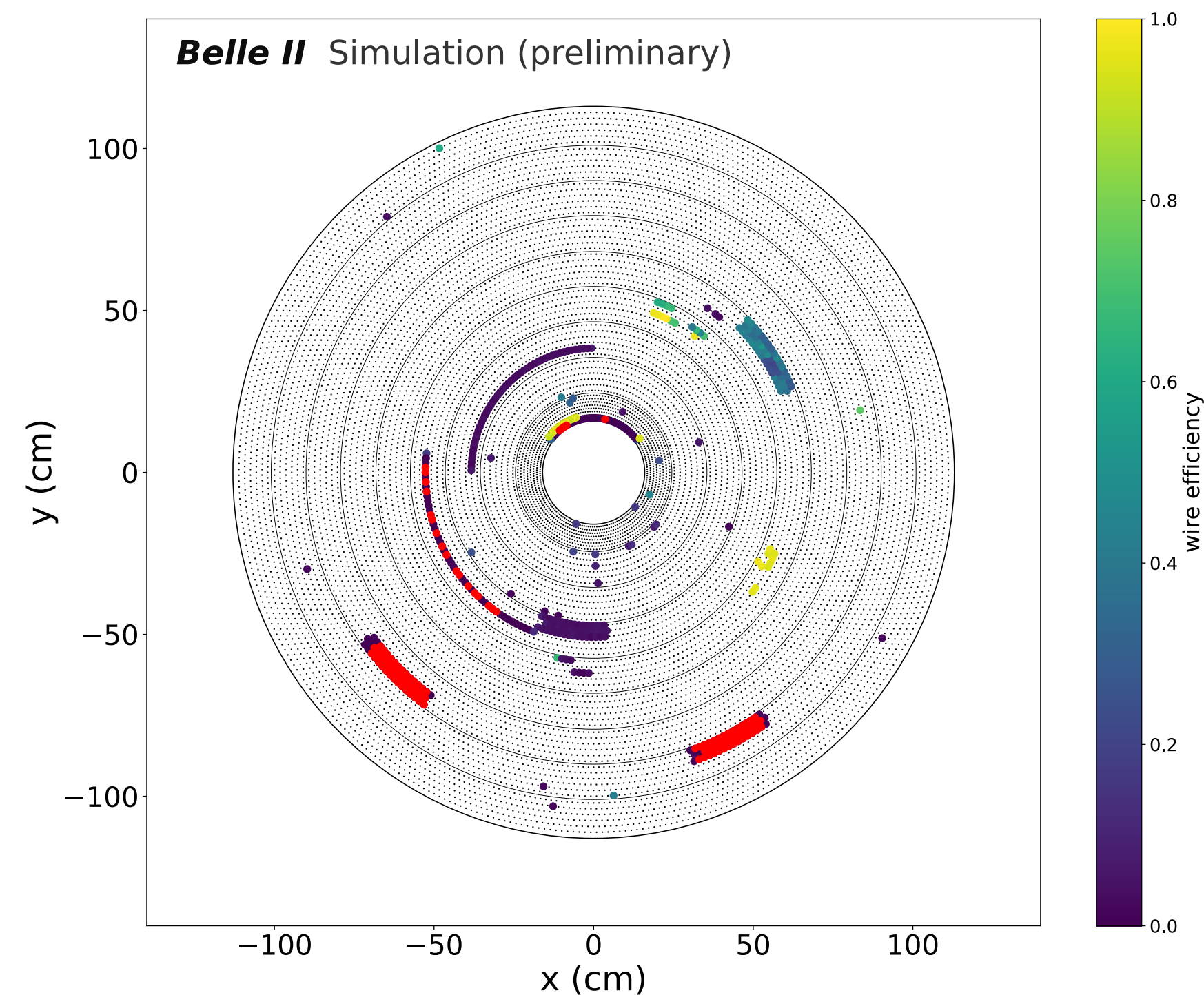
50 wires off, 368 decreased efficiency

Total of 3% of the CDC wires

End of 2022

168 wires off, 809 decreased efficiency

Total of 7% of the CDC wires





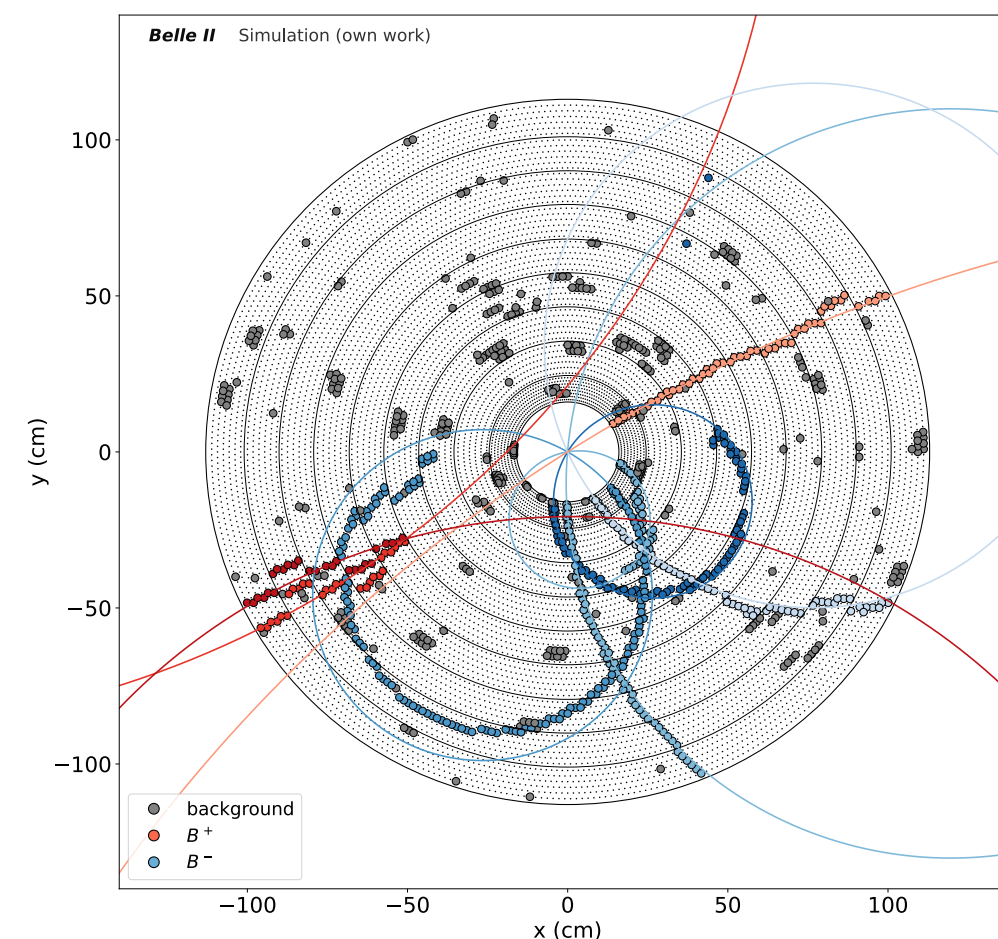
# New Machine Learning Tracking Algorithm

## Framing the Problem:

Have: Hits in our tracking detector

Want: Tracks

- Starting position
- Momentum (starting direction and curvature)
- Charge
- All hits belonging to the track





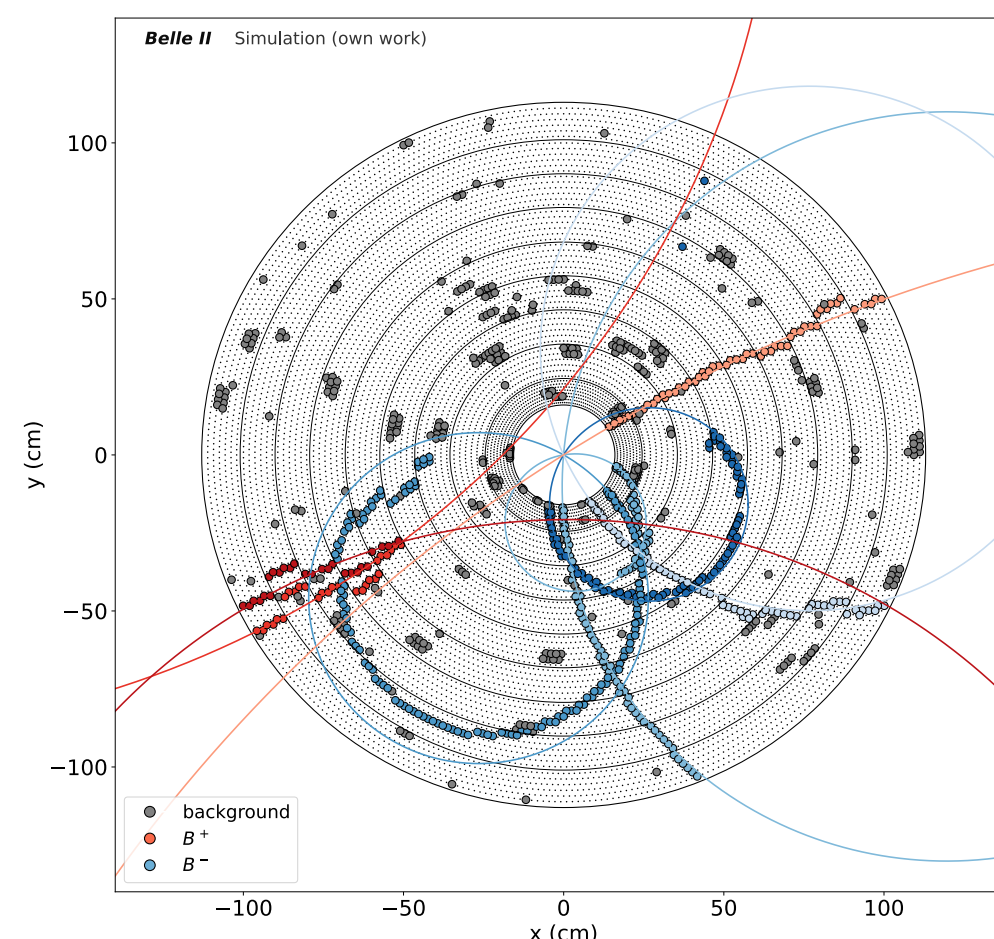
# New Machine Learning Tracking Algorithm

## Framing the Problem:

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## Challenges

- Unknown number of tracks
  - Very high event efficiency and purity required (find all tracks in each event)
  - Very large dynamic range of momenta (curvature)
  - Overlapping objects
- Large and varying number of sense wire inputs (15000 sense wires with up to 30% occupancy)
  - Varying background conditions
- High event rate leads to a lot of data, computing resource and time constraint



# Our Approach

## Graph Neural Network based Object Condensation

- Applicable to overlapping objects without clear spatial boundaries
- No assumption on the object size or sorting is needed

Kieseler, Object Condensation: one-stage grid-free multi-object reconstruction in physics detectors, graph and image data ([arXiv:2002.03605](https://arxiv.org/abs/2002.03605))



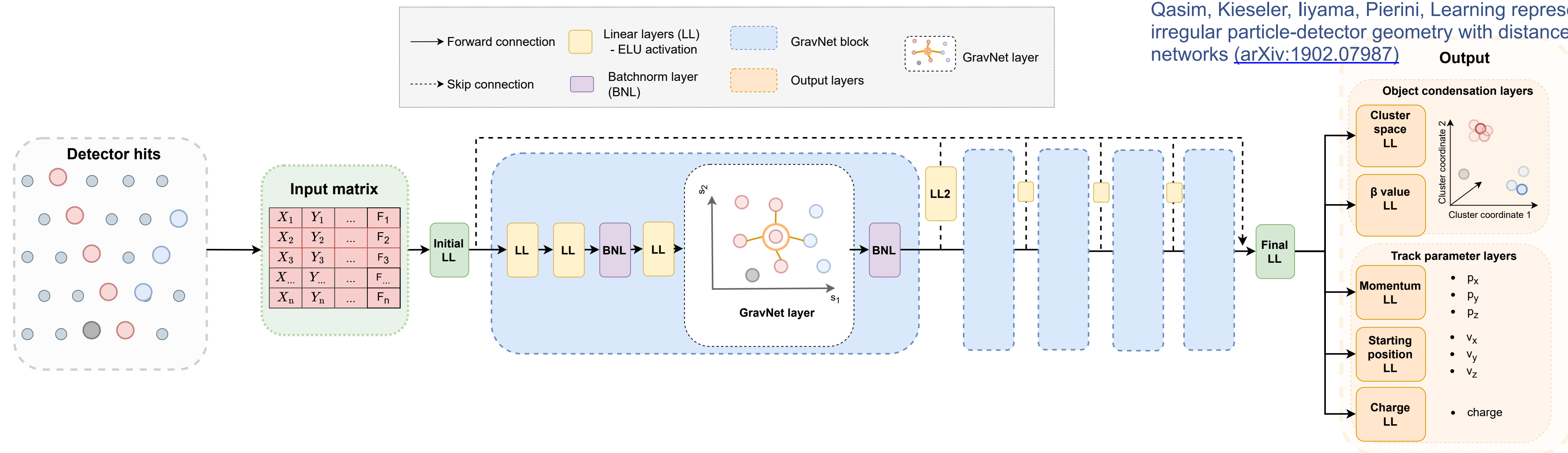
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Qasim, Kieseler, Iiyama, Pierini, Learning representations of irregular particle-detector geometry with distance-weighted graph networks ([arXiv:1902.07987](https://arxiv.org/abs/1902.07987))





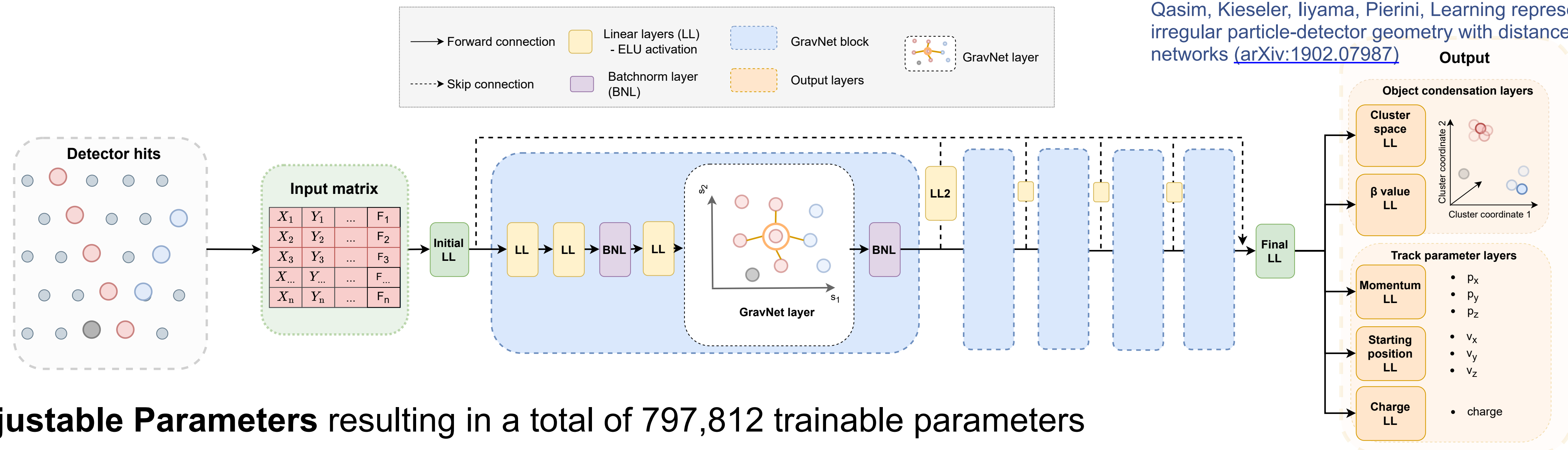
# Our Approach

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**Adjustable Parameters** resulting in a total of 797,812 trainable parameters

### General Parameters

- Dimension of Linear Layers
- Number of GravNet Blocks

### GravNet Parameters

- Number of k-nearest neighbours in GravNet
- GravNet space dimensions

### Output Parameters

- Dimension of Cluster Coordinates
- Number of output layers according to track parameter predictions



# Training samples - Input Features

- Cartesian coordinates for the CDC hits:

- X - position (We don't want polar coordinates due to the flip at  $2\pi$  and because the radius diverges for high momentum)
- Y - position

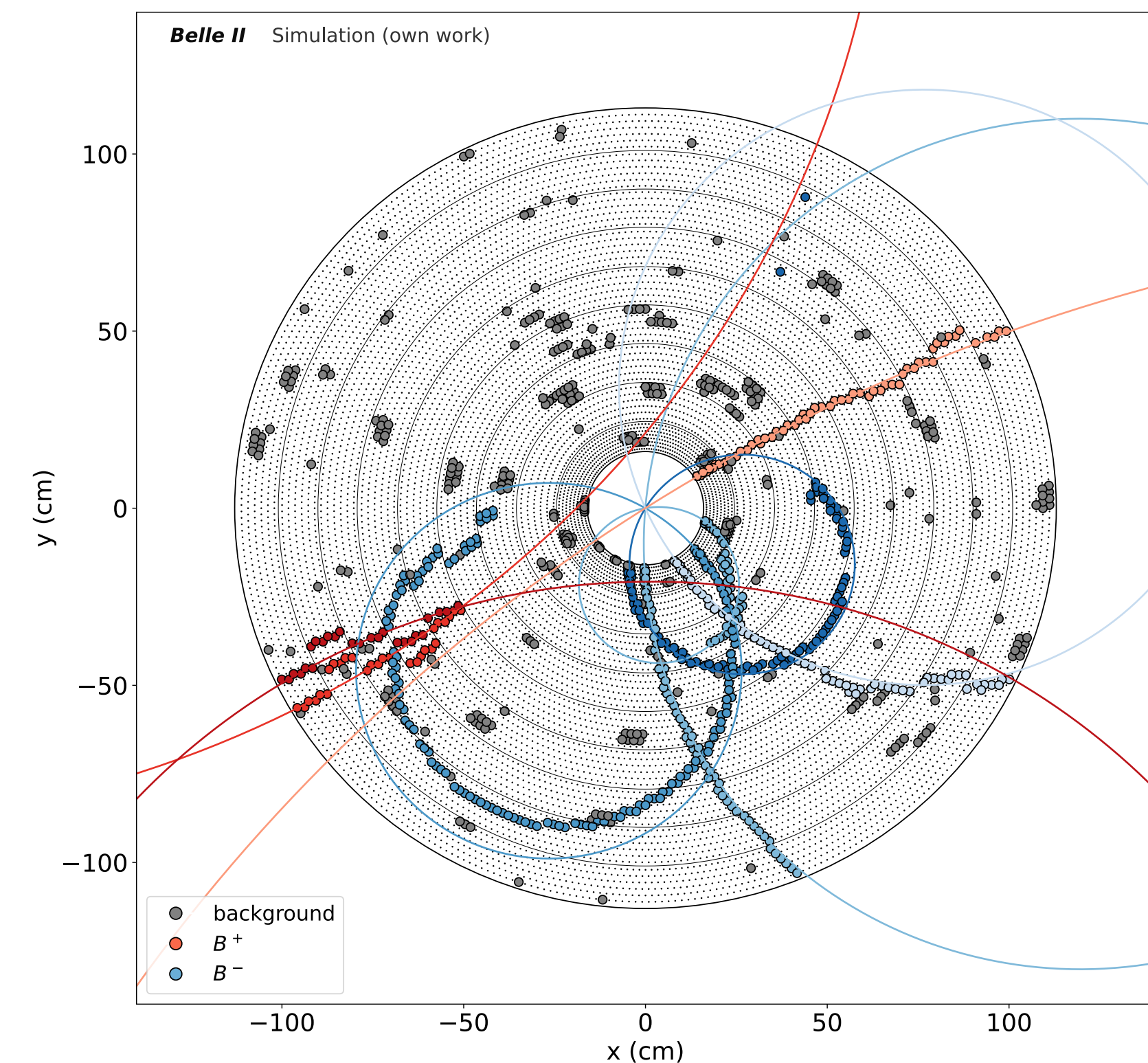
- Wire Signal Measurements

- Signal height
- Signal timing
- Signal time over threshold

- CDC layer information

- Superlayer
- Layer within superlayer
- Total layer

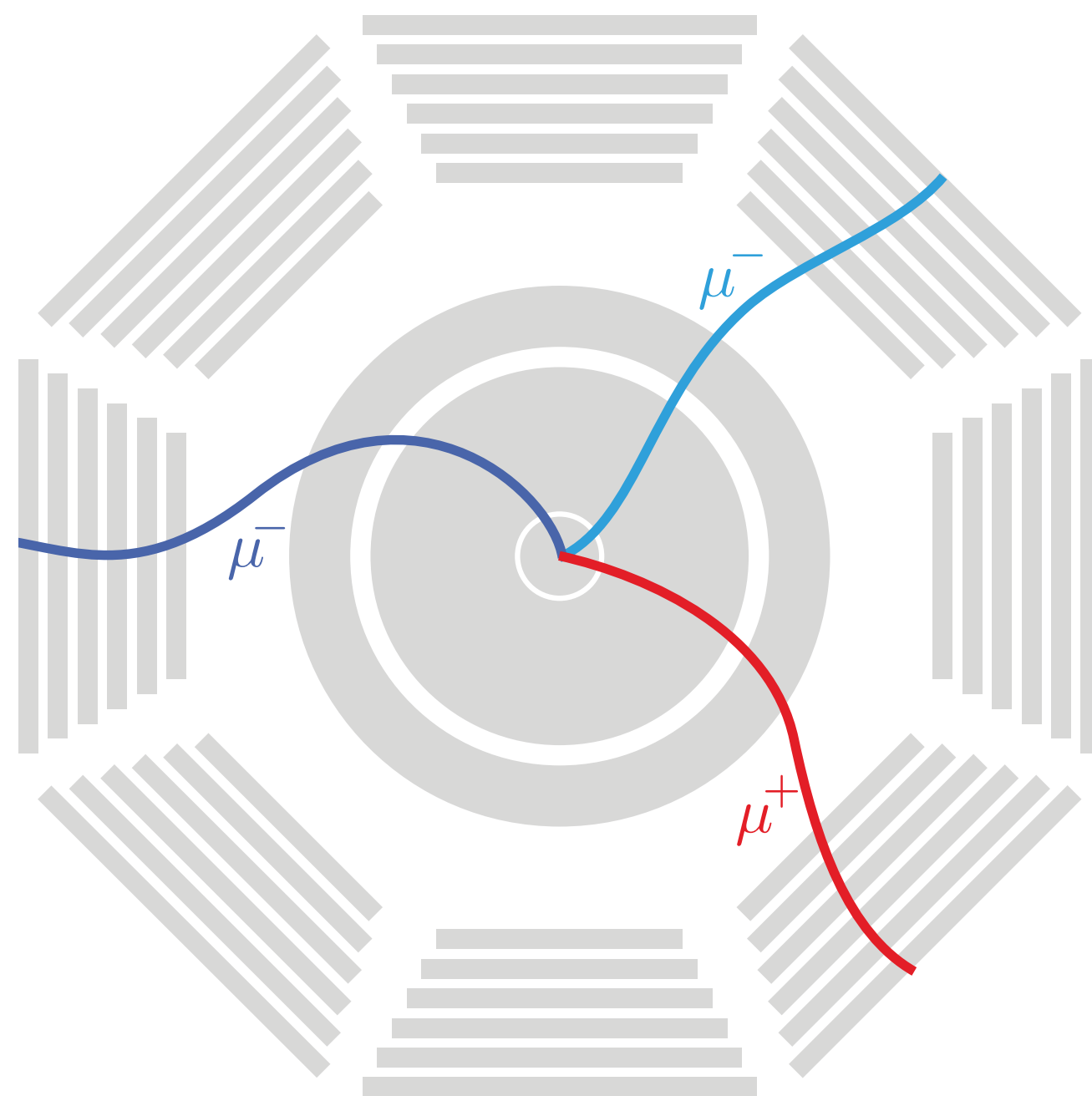
[Bachelor thesis](#) about the input feature optimization



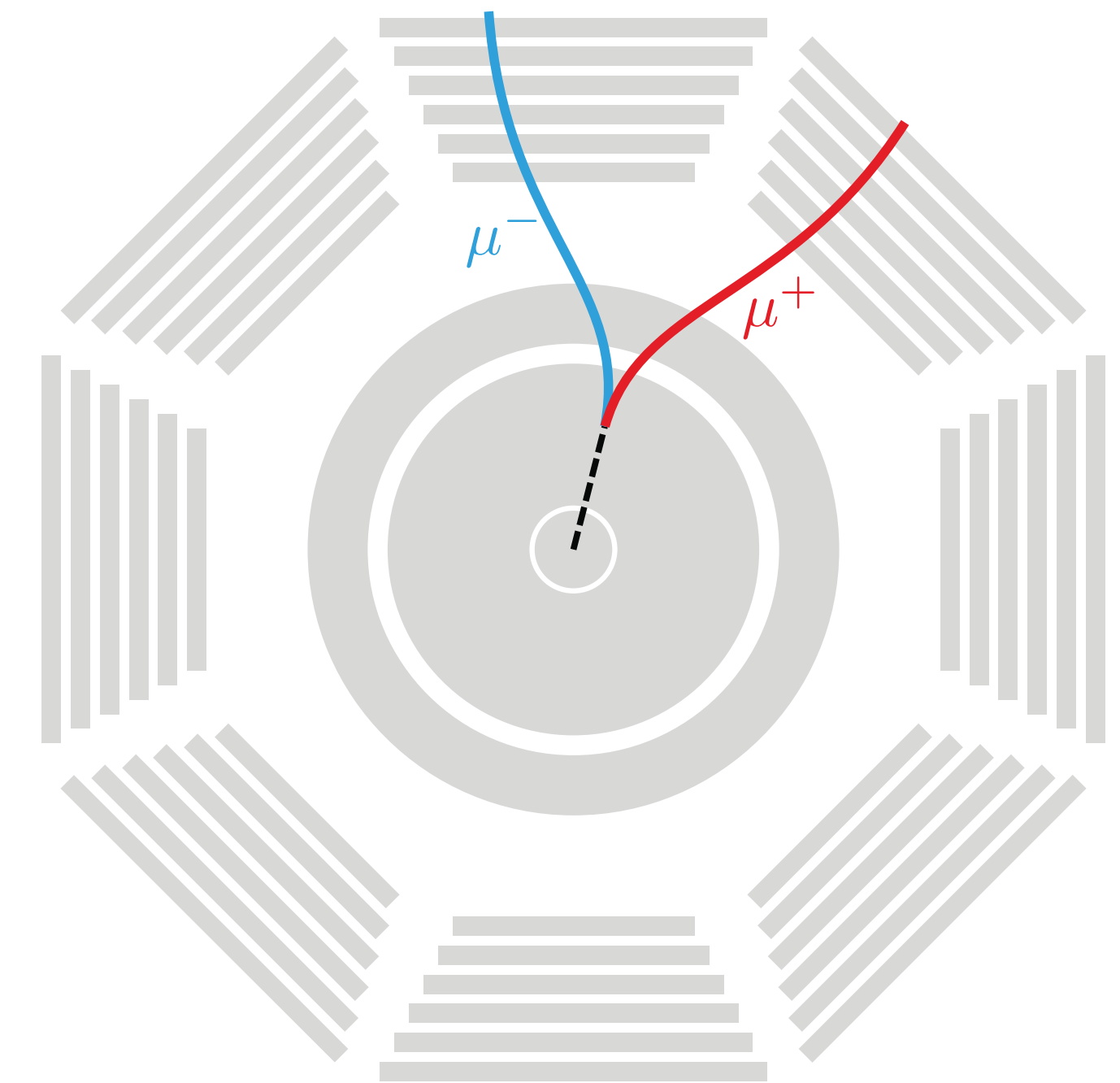


# Training Samples - Topologies

Prompt particles from the interaction point



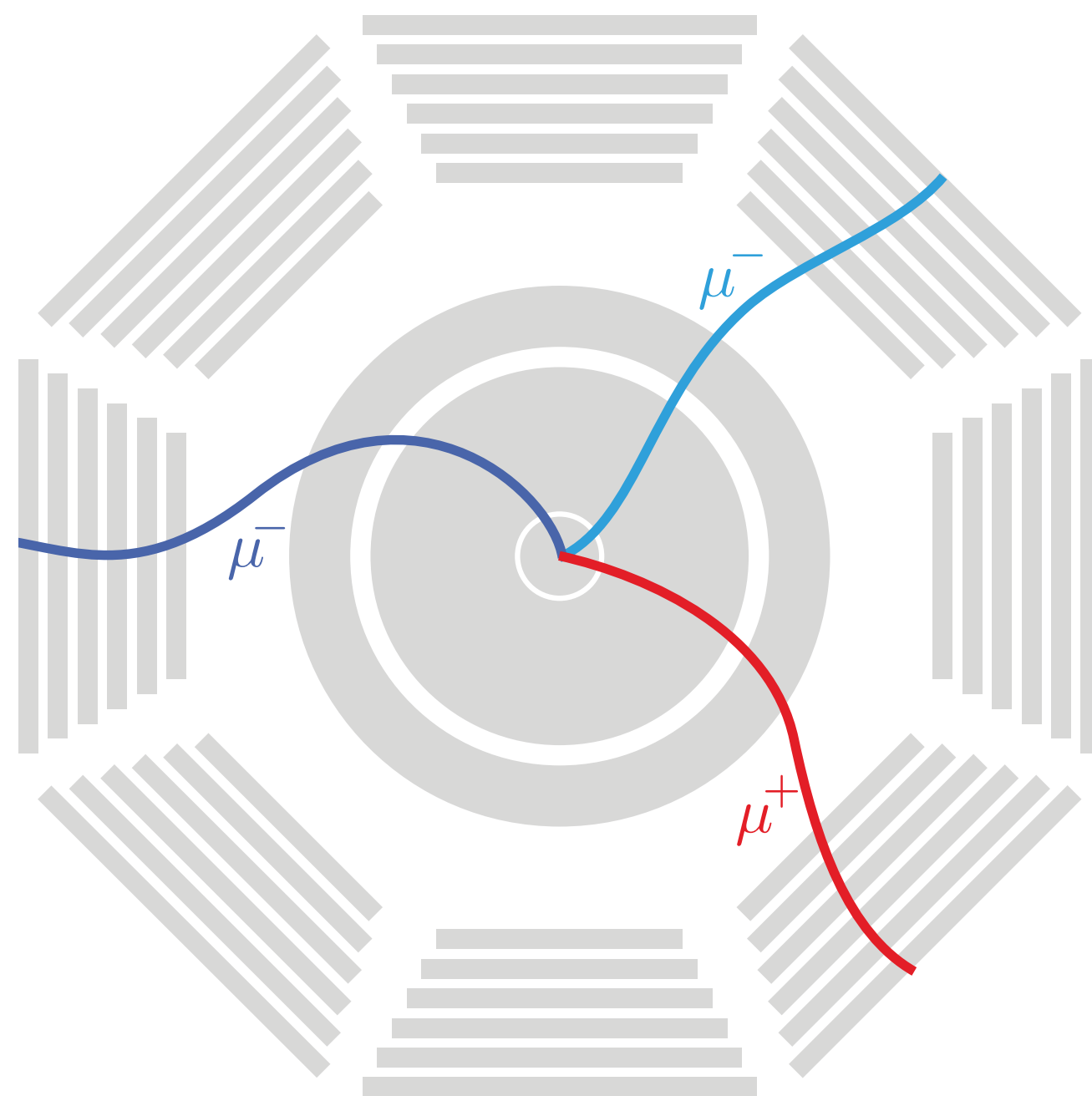
Particles from displaced vertices





# Training Samples - Topologies

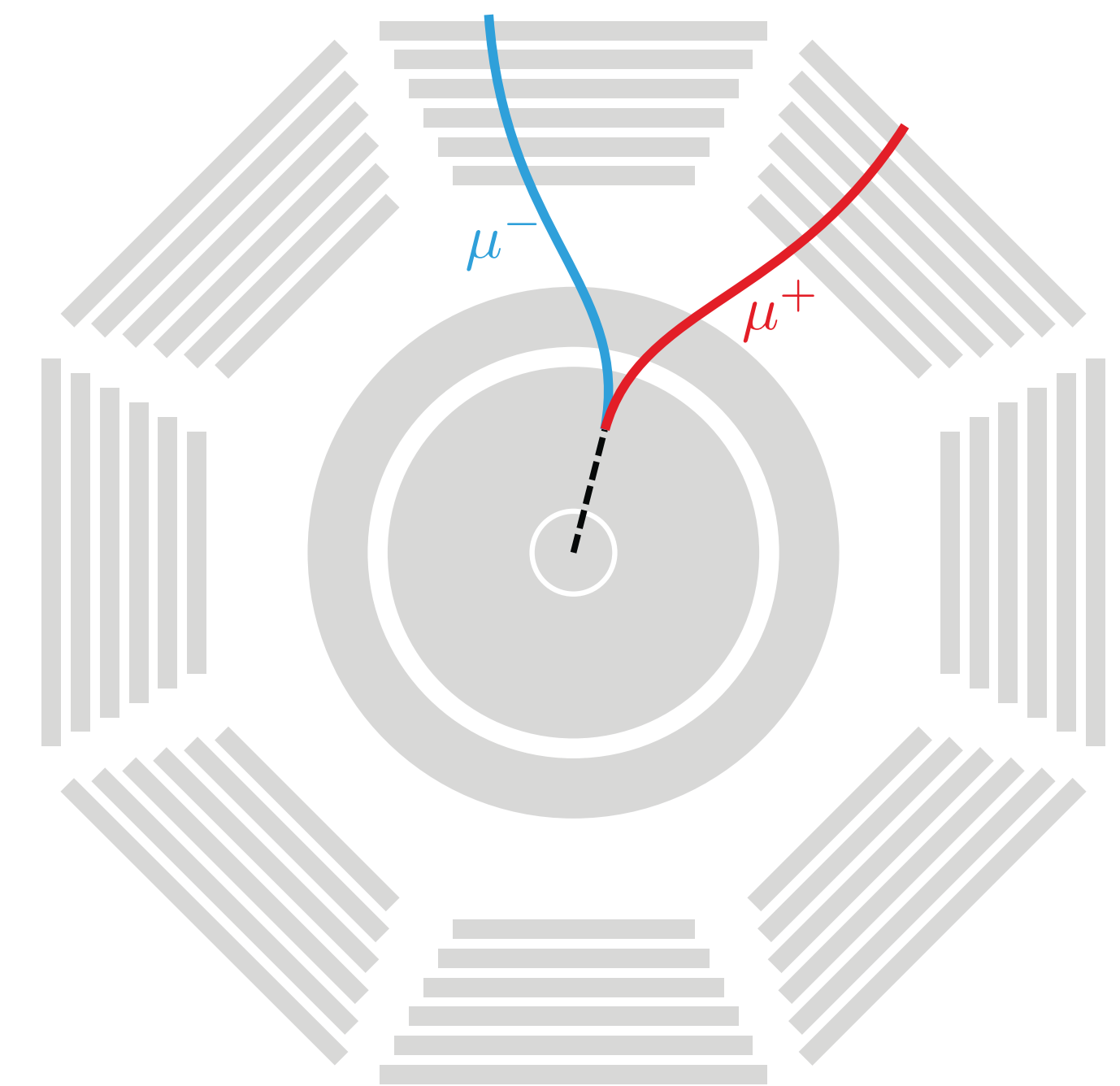
Prompt particles from the interaction point



**Goal: general model that can be used for every possible physics event**

- The model proved to be very good in learning physics dependencies
- To not bias the model towards any physics events, we chose particle gun samples where we sample
  - random positive or negative charge
  - random momentum within detector acceptance (no energy or momentum conservation)
  - random displacement
  - random number of particles per event

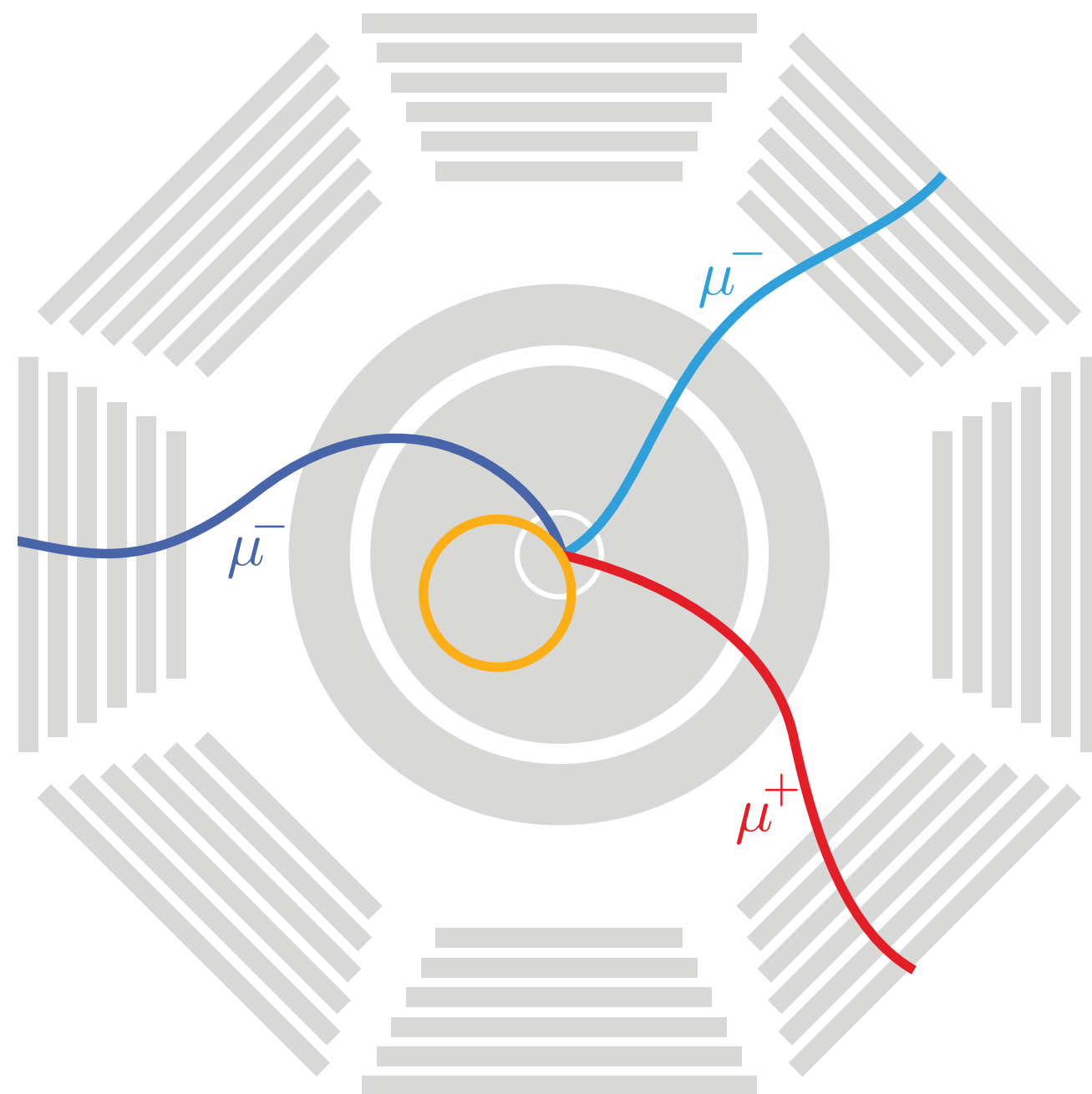
Particles from displaced vertices





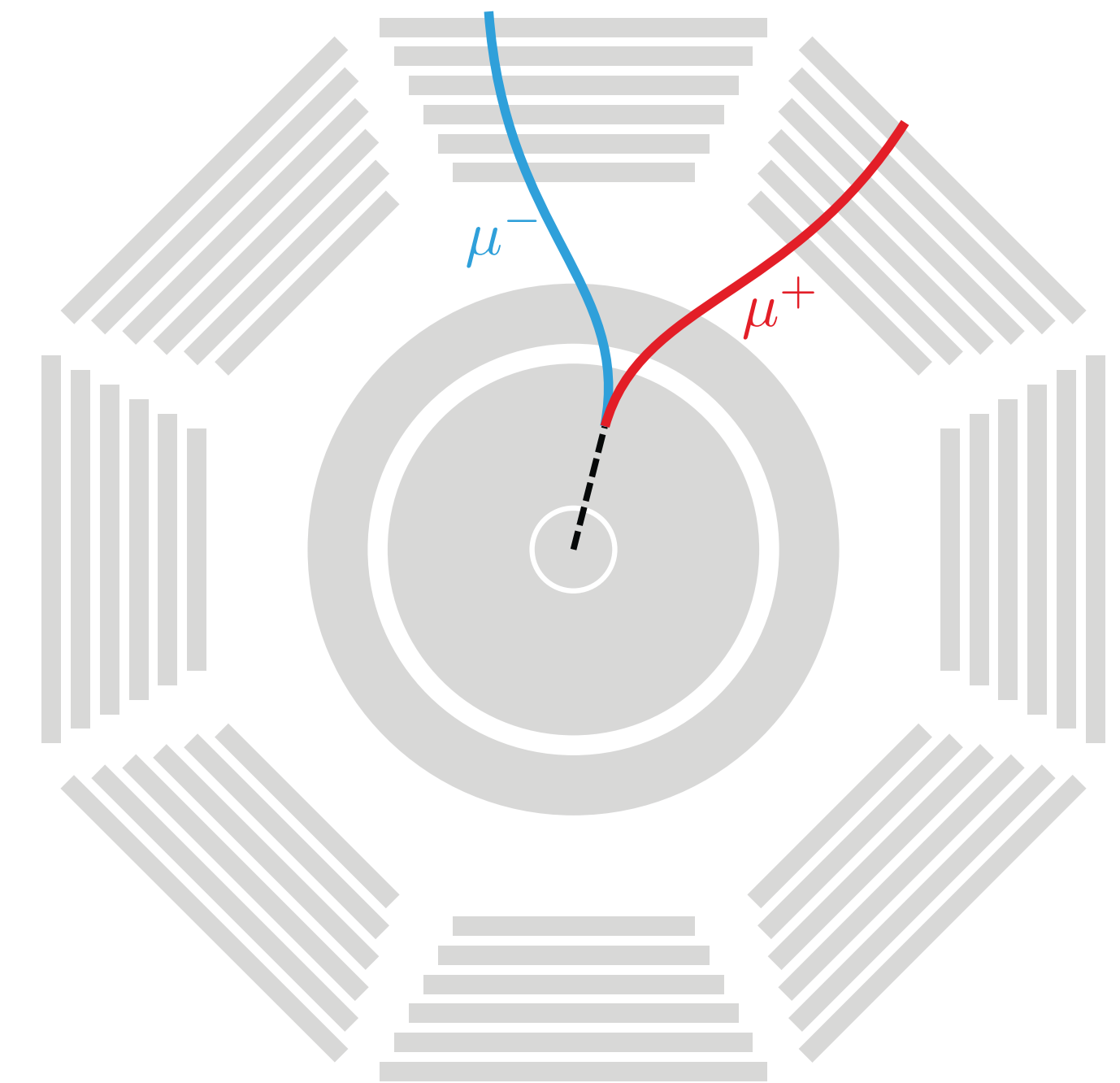
# Training Samples - Topologies

Prompt particles from the interaction point



Enrich trainings samples with low momentum particles

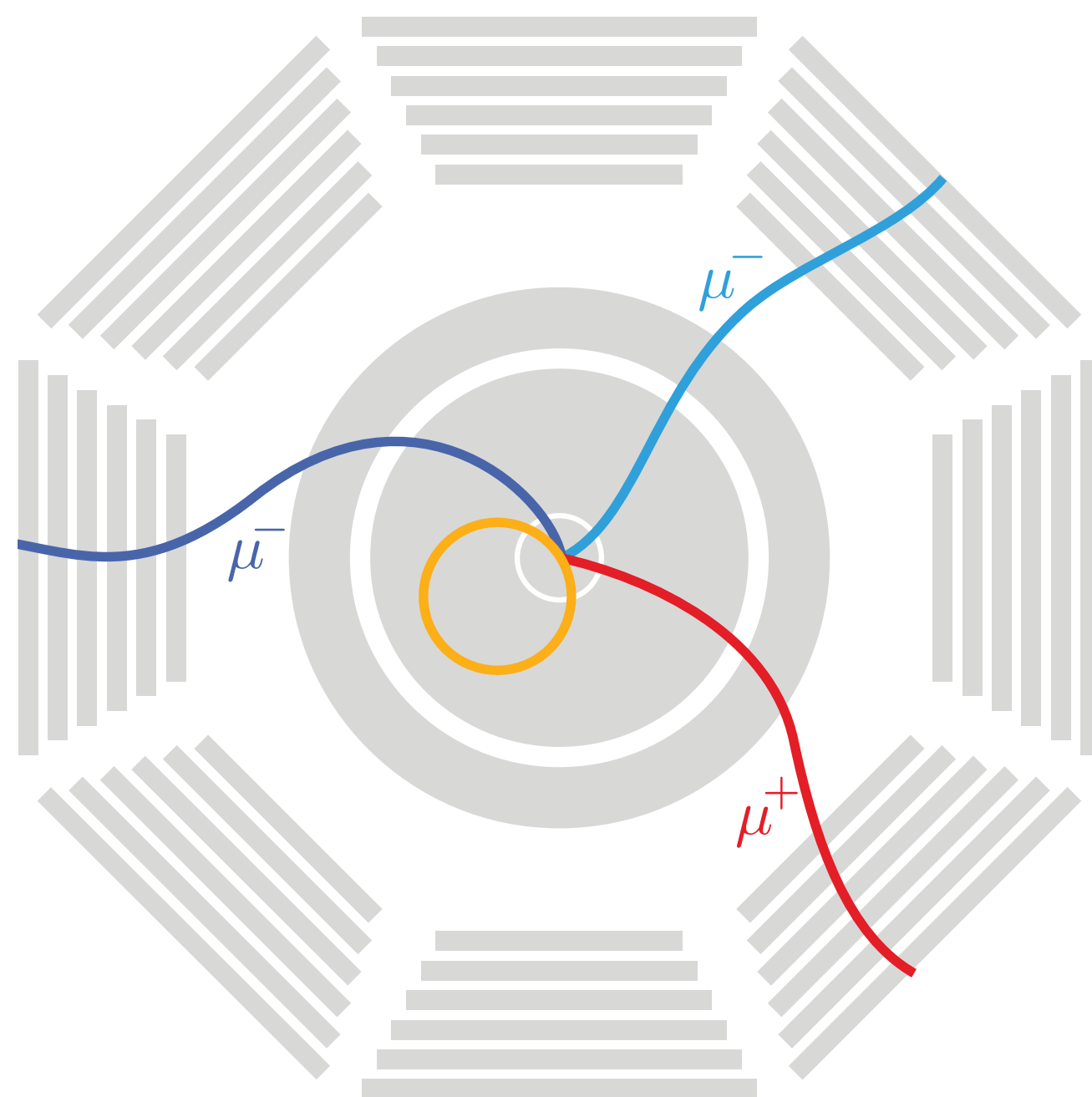
Particles from displaced vertices



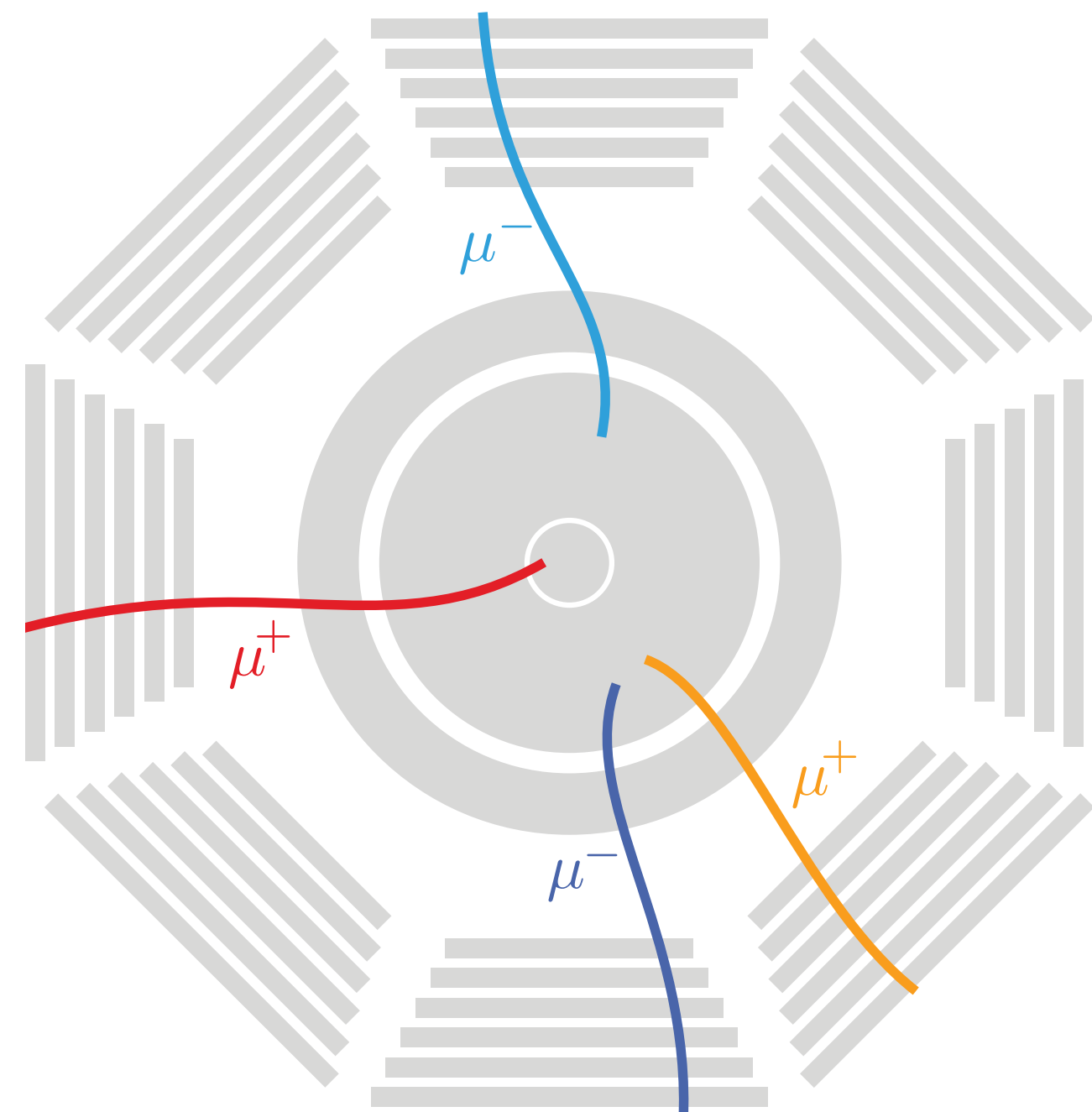


# Training Samples - Topologies

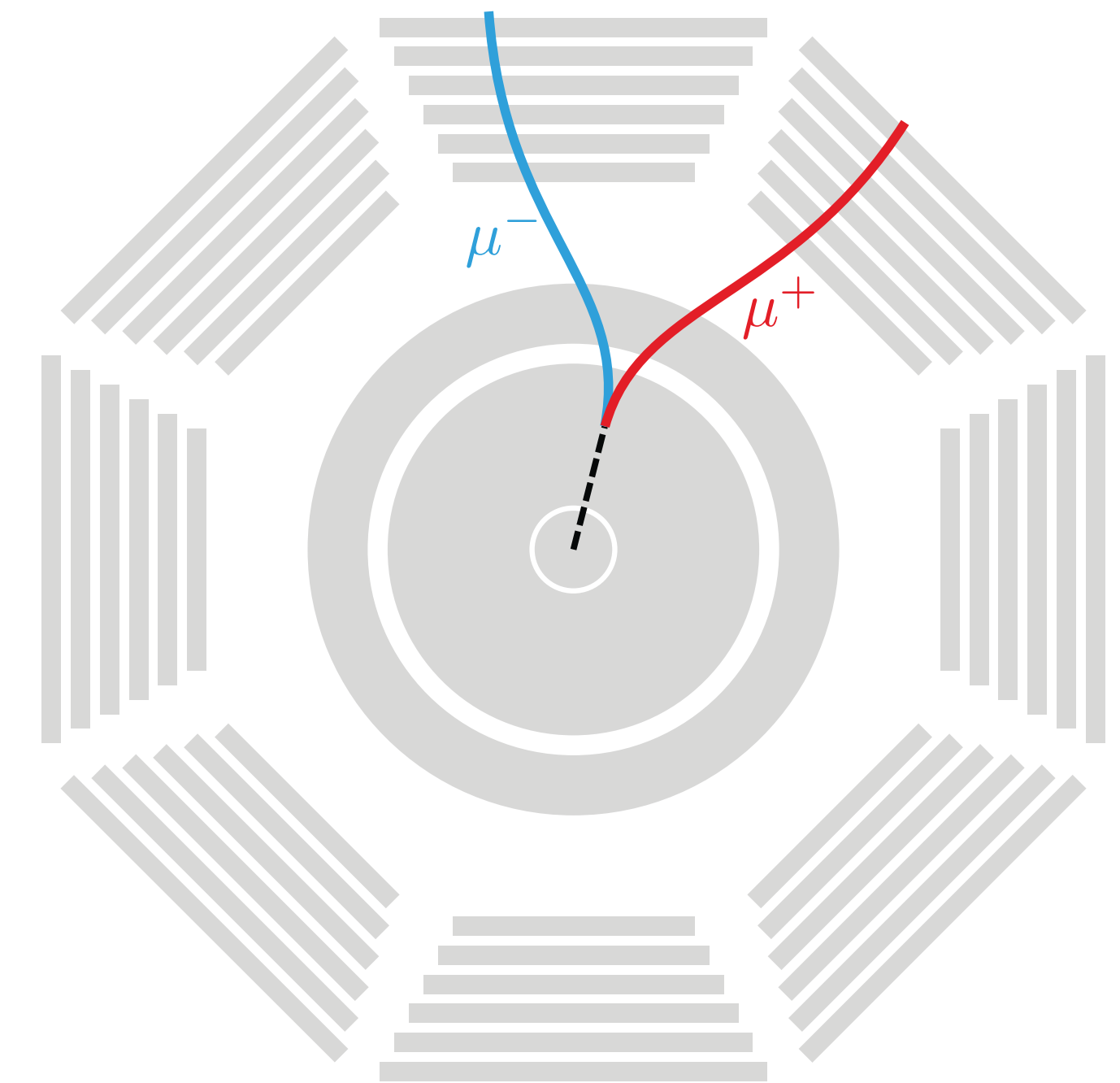
Prompt particles from the interaction point



Adding the transition sample of single displaced particles helped training



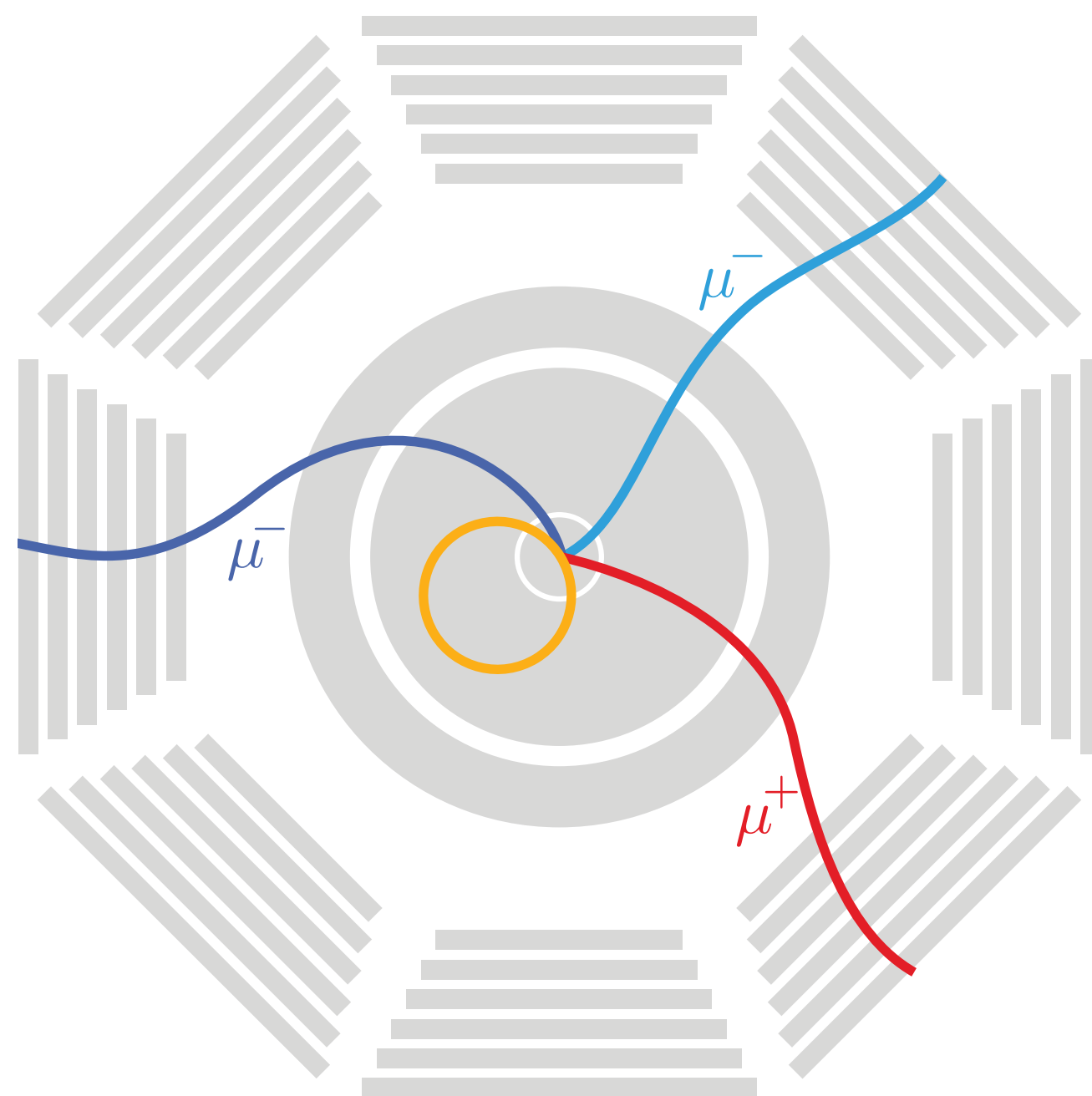
Particles from displaced vertices



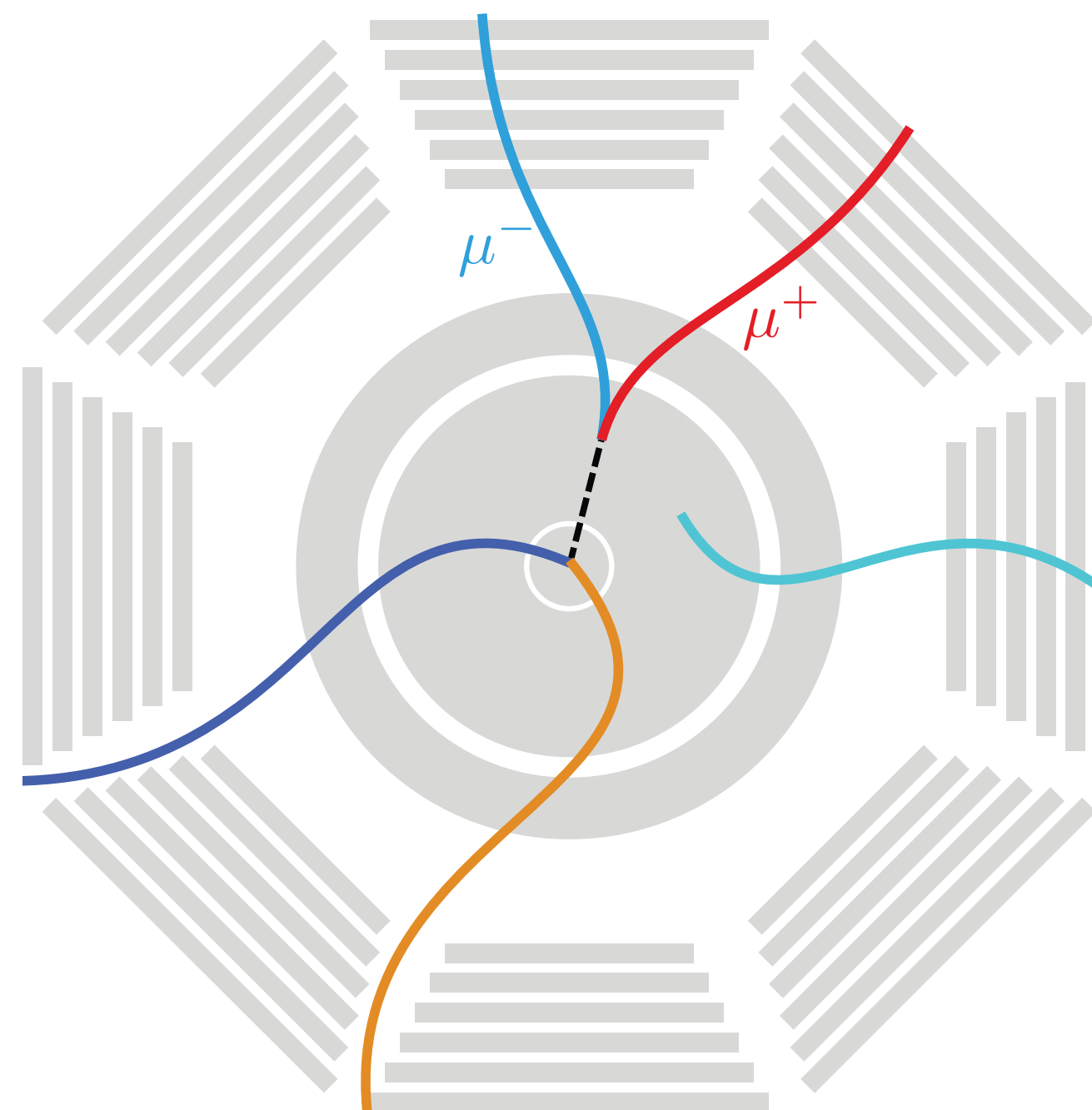


# Training Samples - Topologies

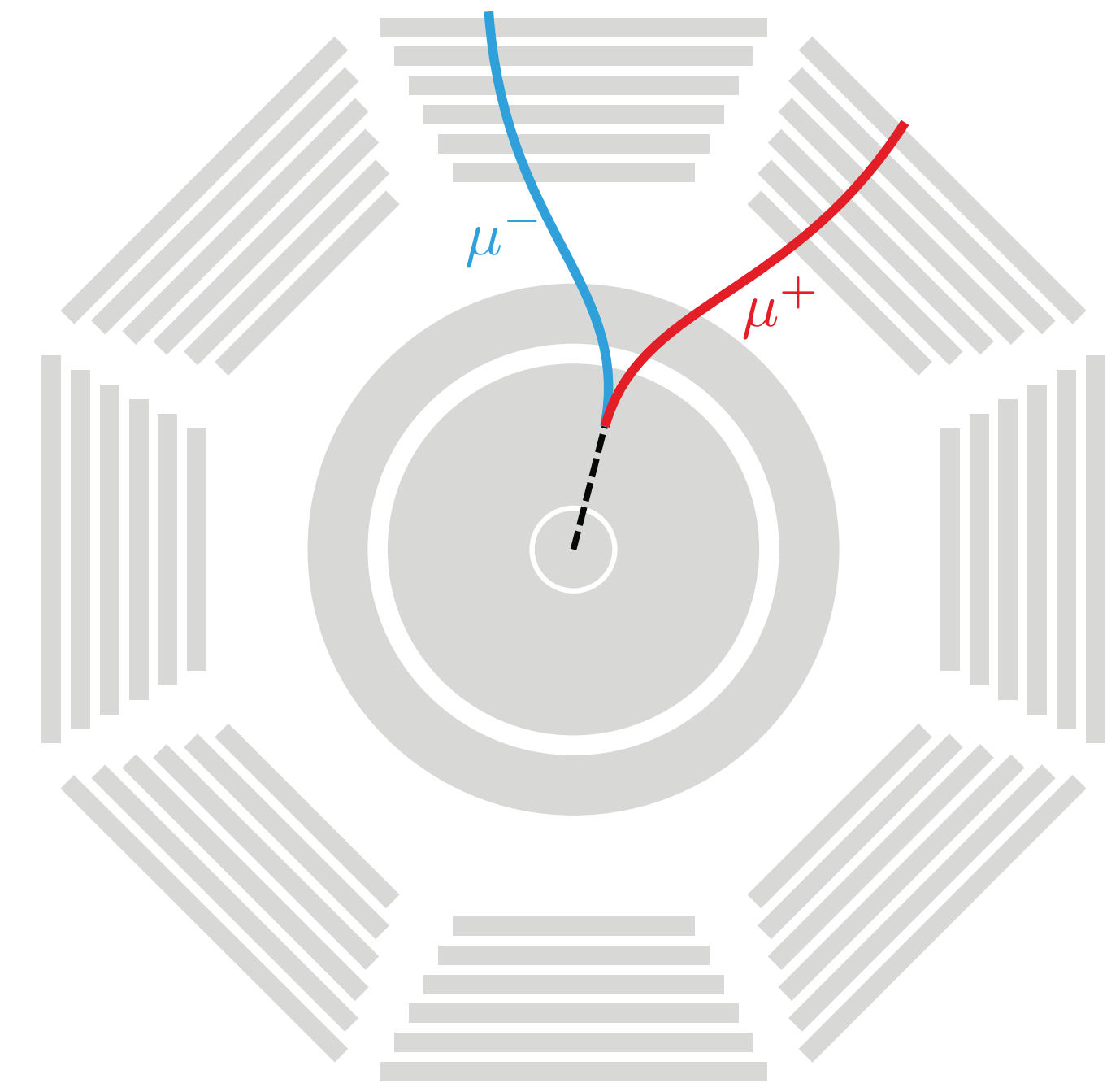
Prompt particles from the interaction point



Mix everything so that the model does not learn topology dependency



Particles from displaced vertices



1 million samples to train on, split 80/20 in training and validation dataset

Equal distribution between all training topologies (prompt, displaced, displaced vertex and mix)



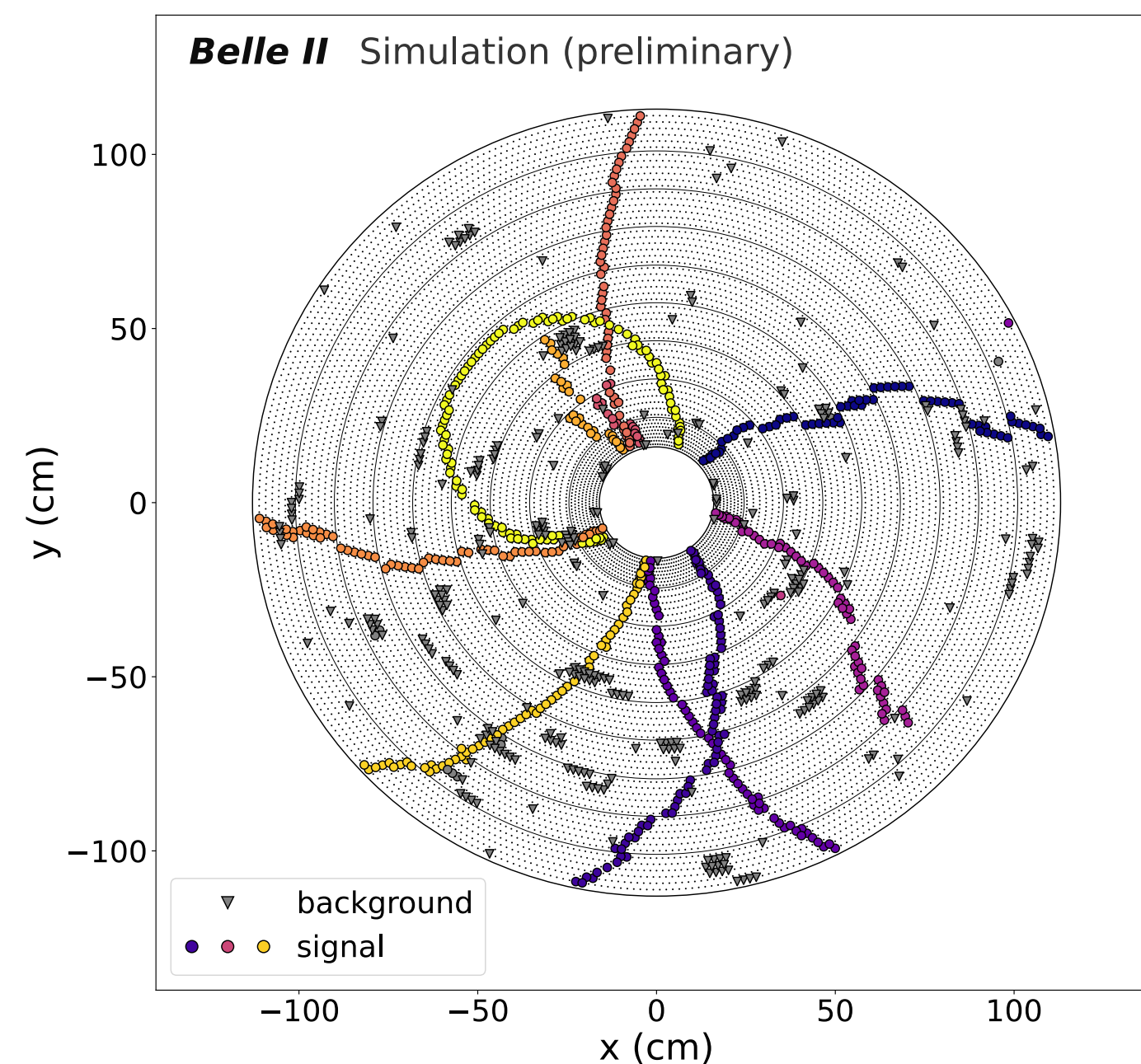
# Training Samples - Backgrounds

Extensive background simulations, correlated over multiple detector components, also including noise from read-out chips

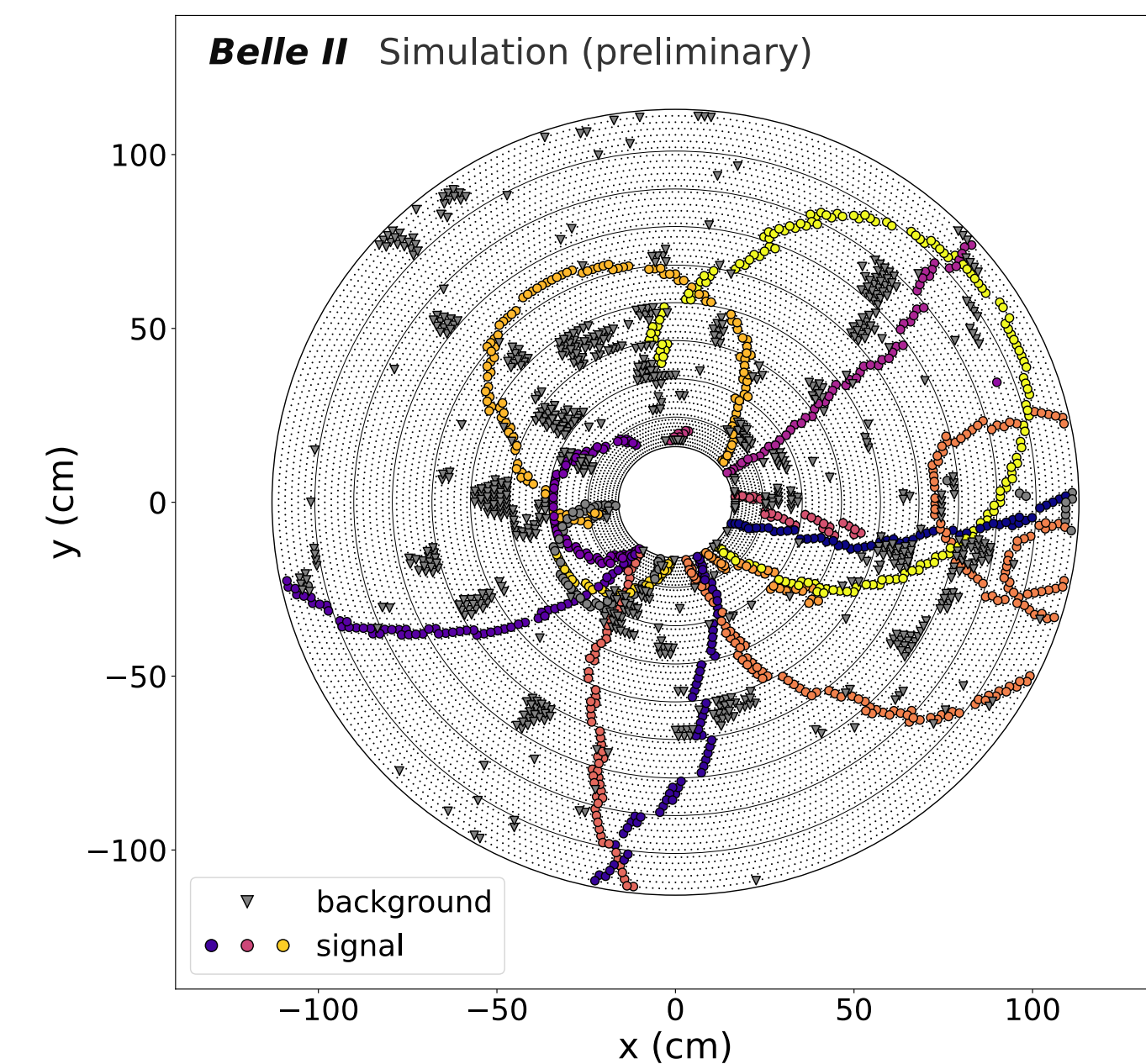
Natochii et al., Beam background expectations for Belle II at SuperKEKB ([arXiv:2203.05731](https://arxiv.org/abs/2203.05731))

Liptak et al., Measurements of Beam Backgrounds in SuperKEKB Phase 2 ([arXiv:2112.14537](https://arxiv.org/abs/2112.14537))

Pretrain model on simulated, low background conditions

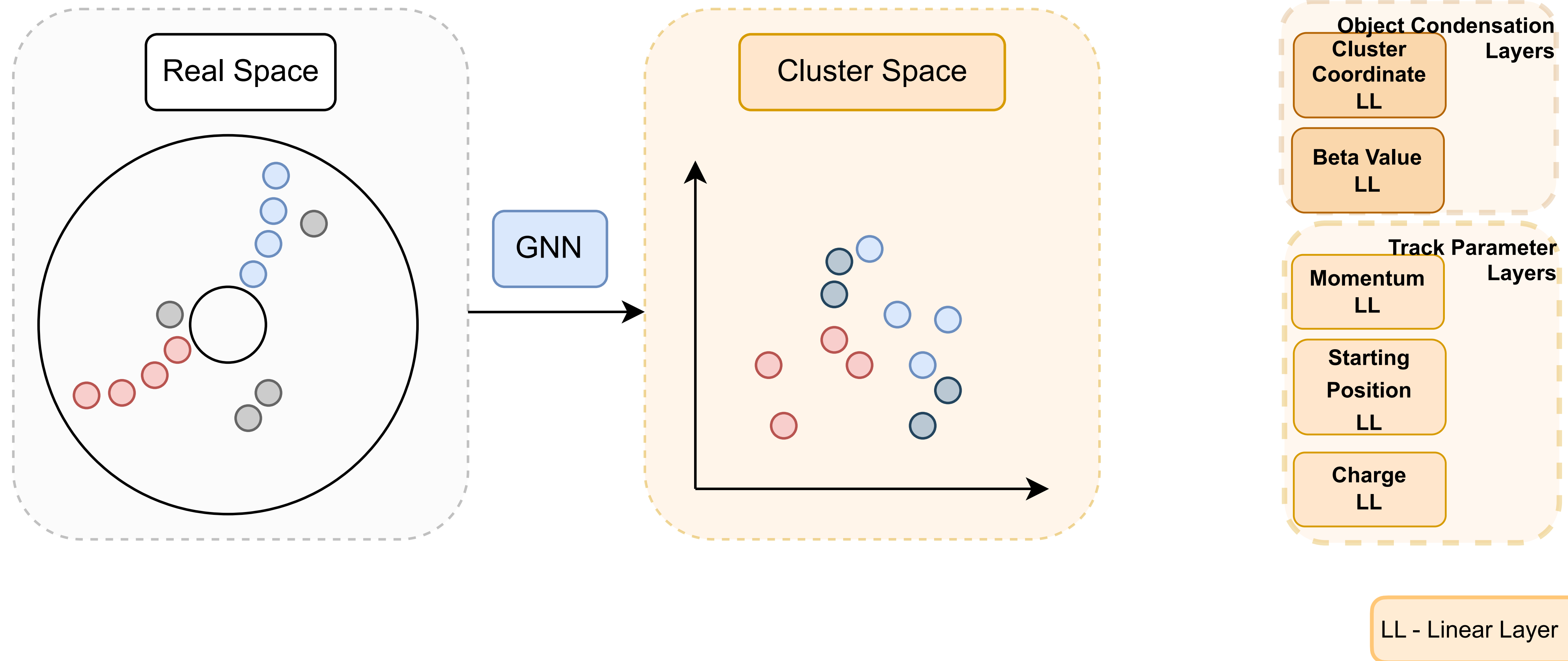


Retrain model on high background conditions taken from data during end of run 1 in 2022





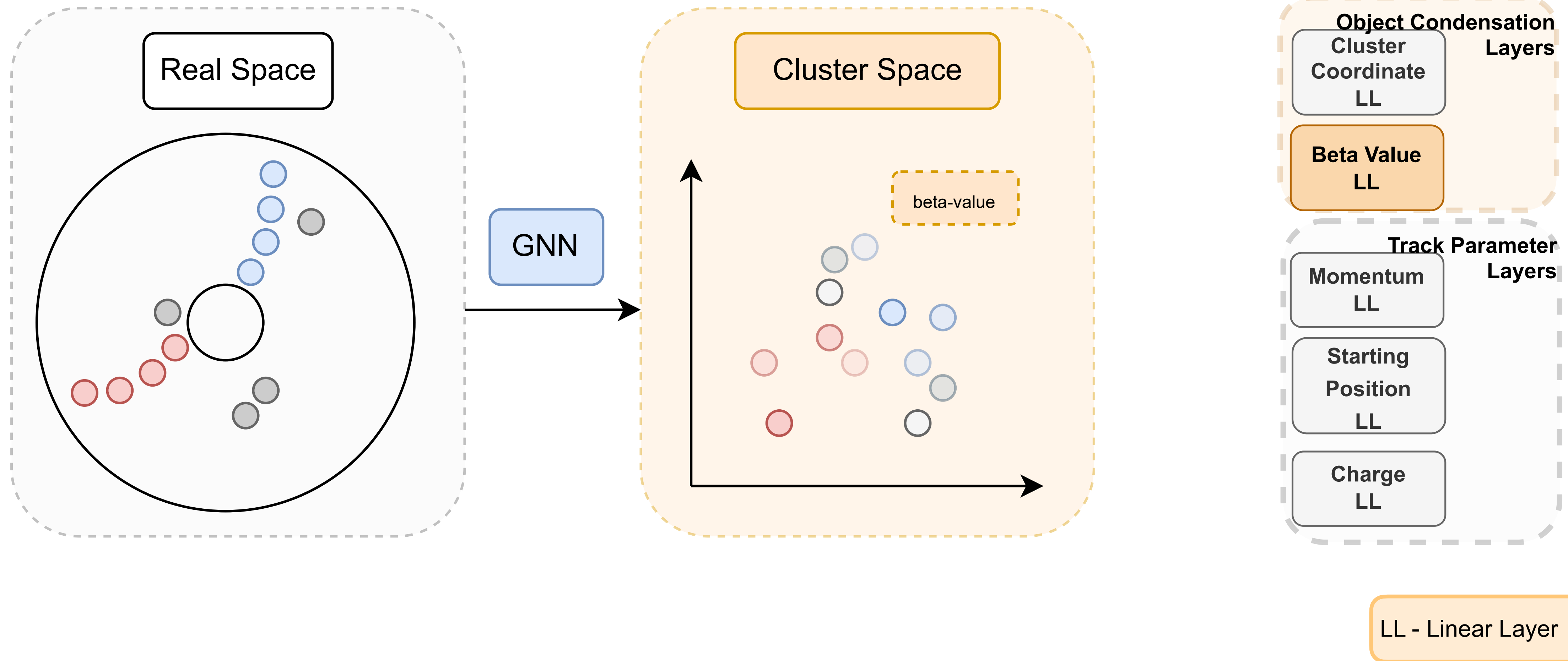
# Object Condensation - Training Loss





# Object Condensation - Training Loss

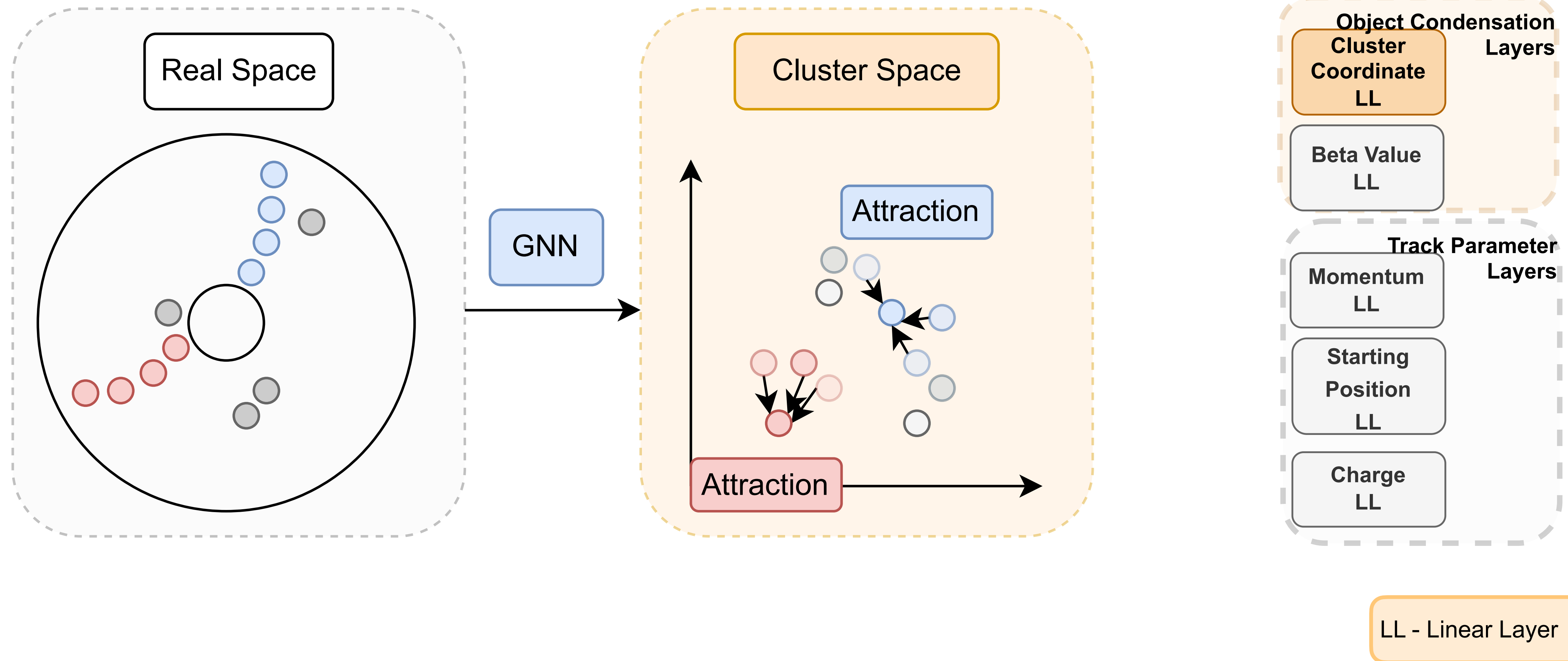
$\beta$ -loss





# Object Condensation - Training Loss

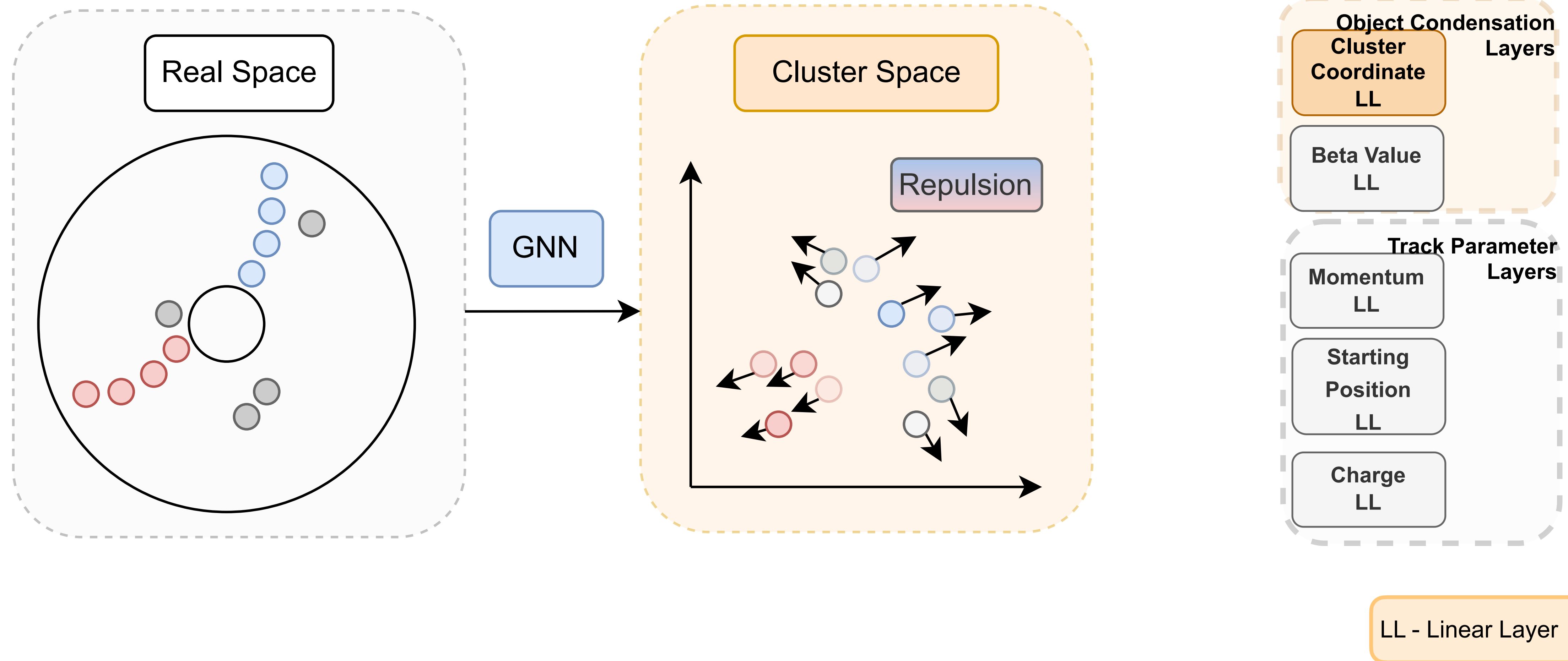
## Attraction loss





# Object Condensation - Training Loss

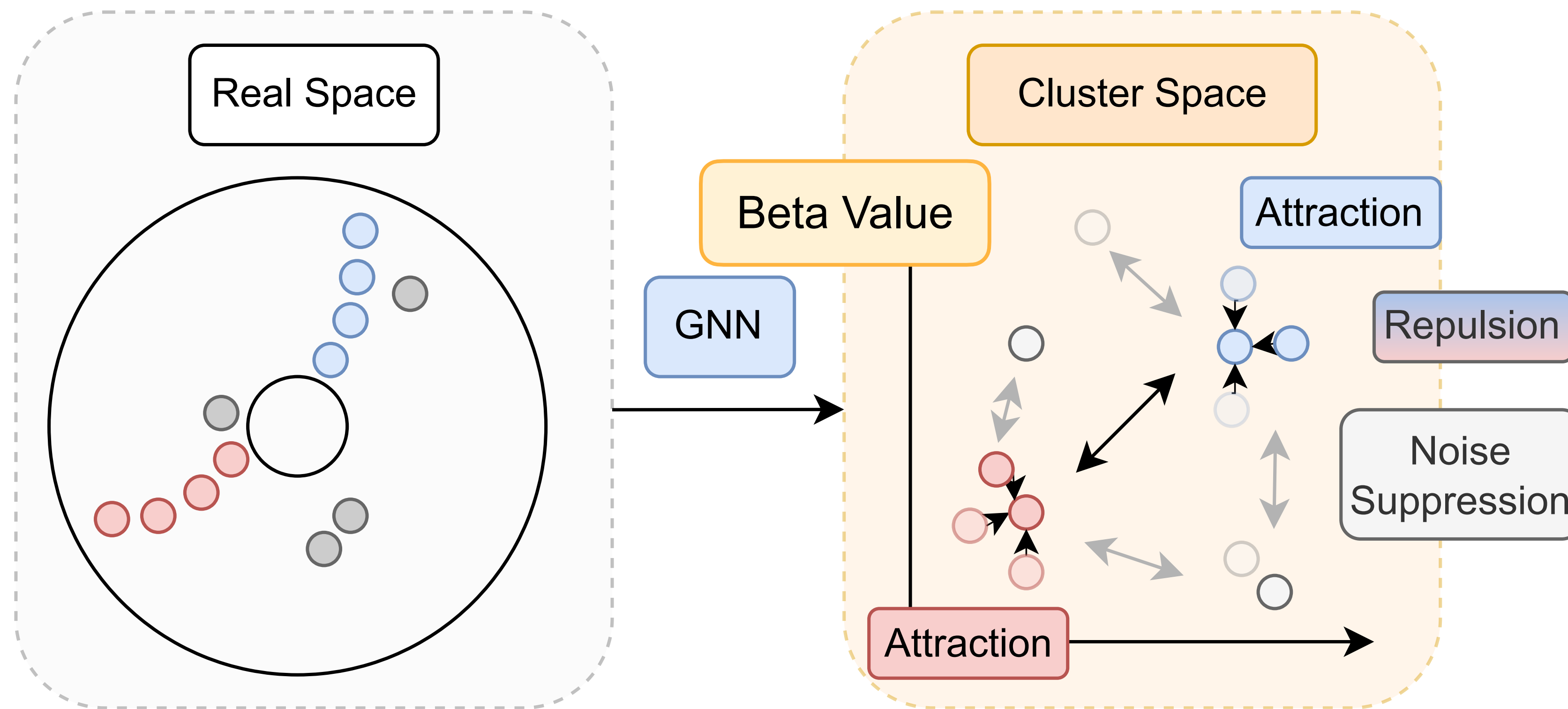
## Repulsion loss





# Object Condensation - Training Loss

## Noise suppression

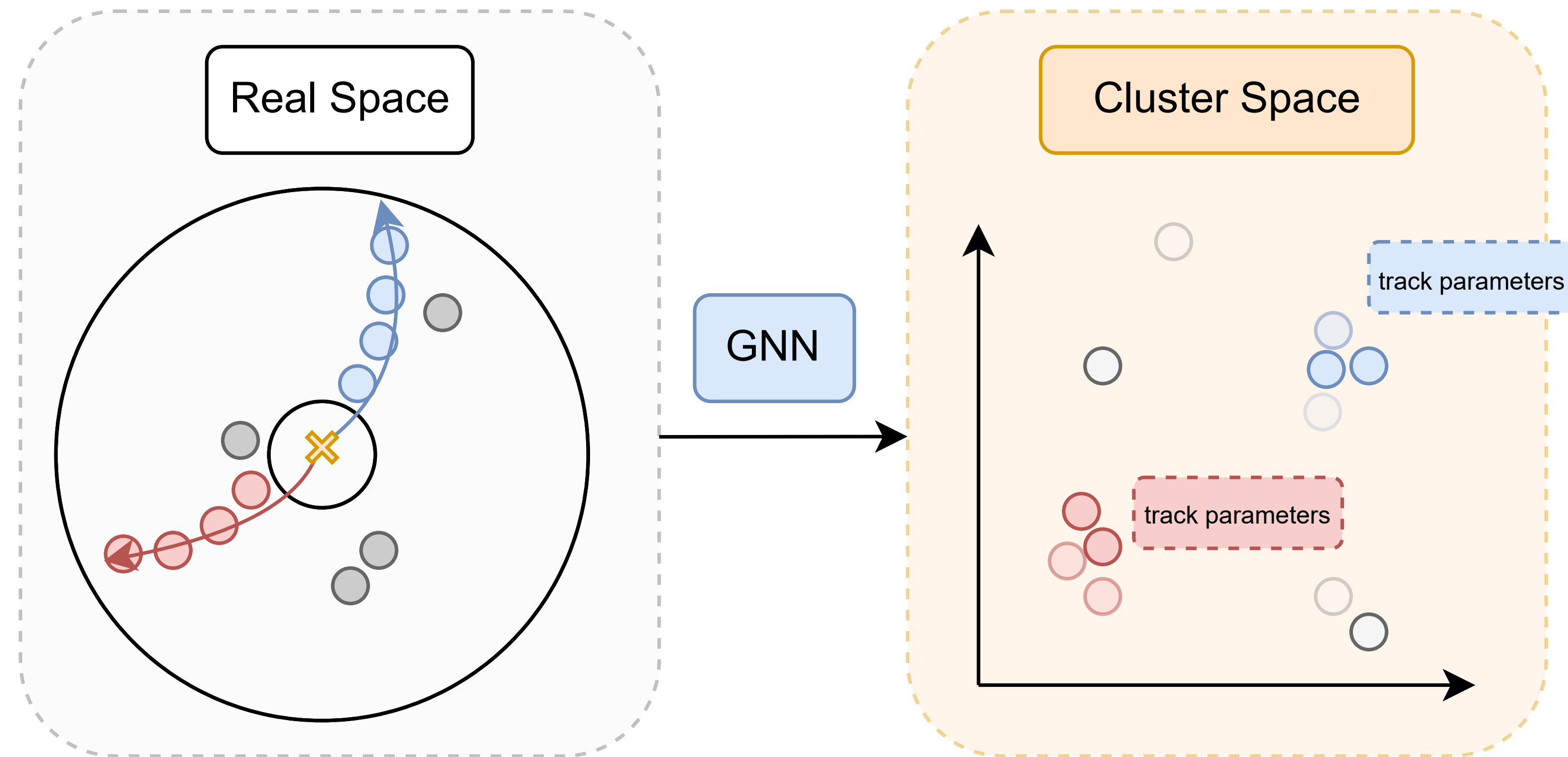


4 loss terms for object condensation



# Object Condensation - Training Loss

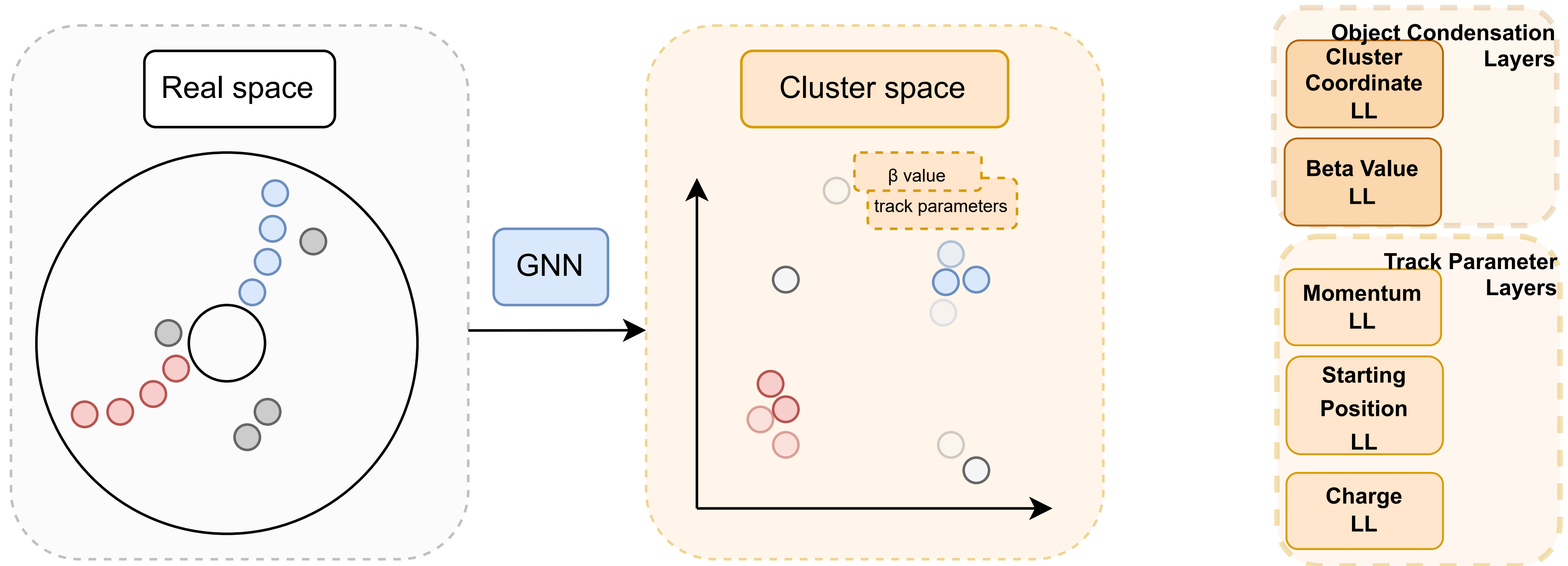
## Parameter loss



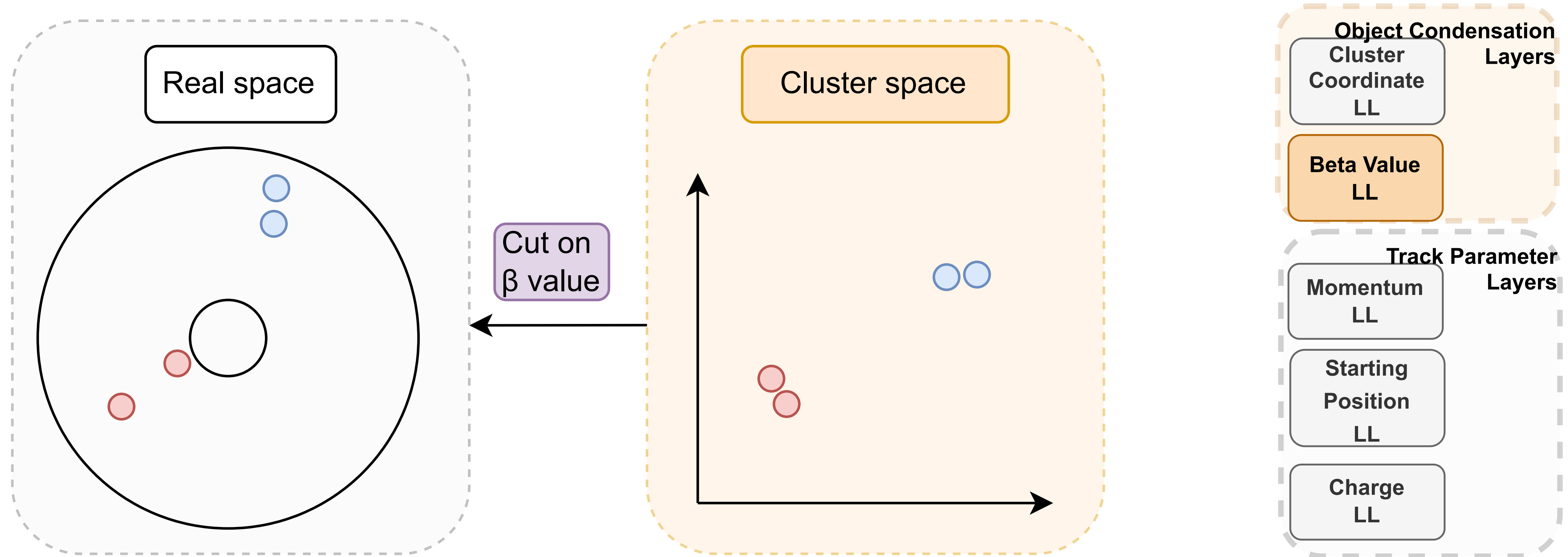
4 loss terms for object condensation  
Additional 3 loss terms according to parameter predictions



# Inference - Translation from real space to cluster space

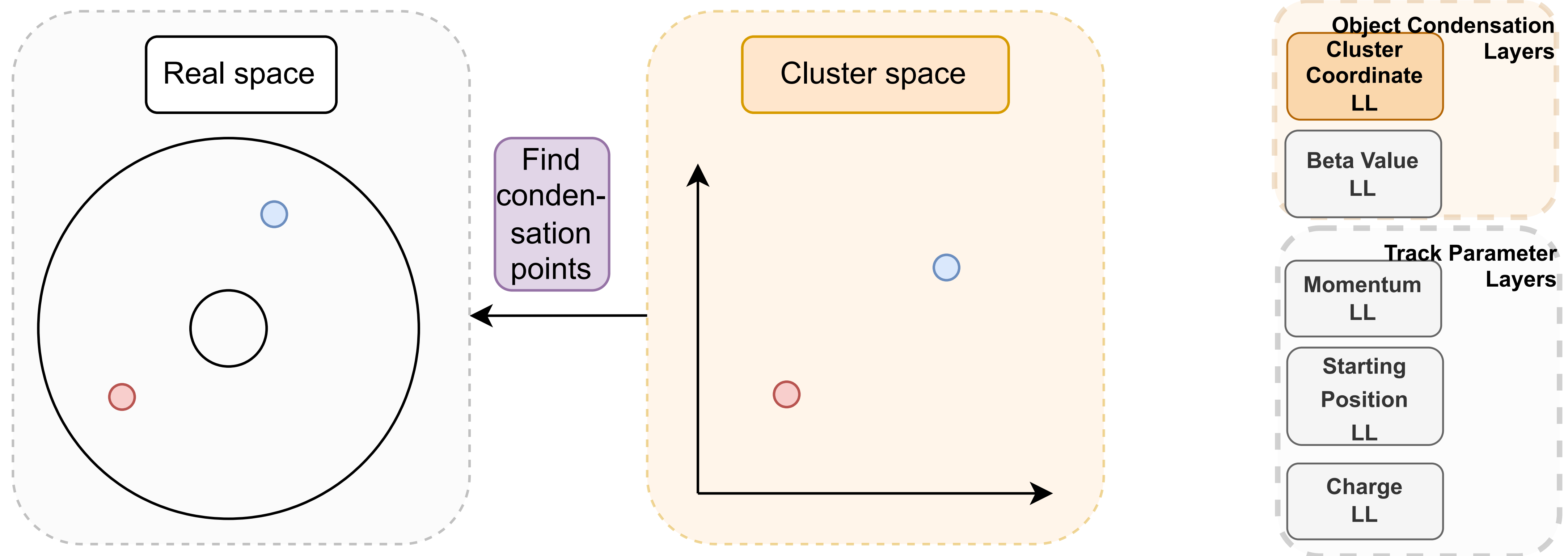


# Inference - Condensation point candidate selection based on $\beta$ threshold

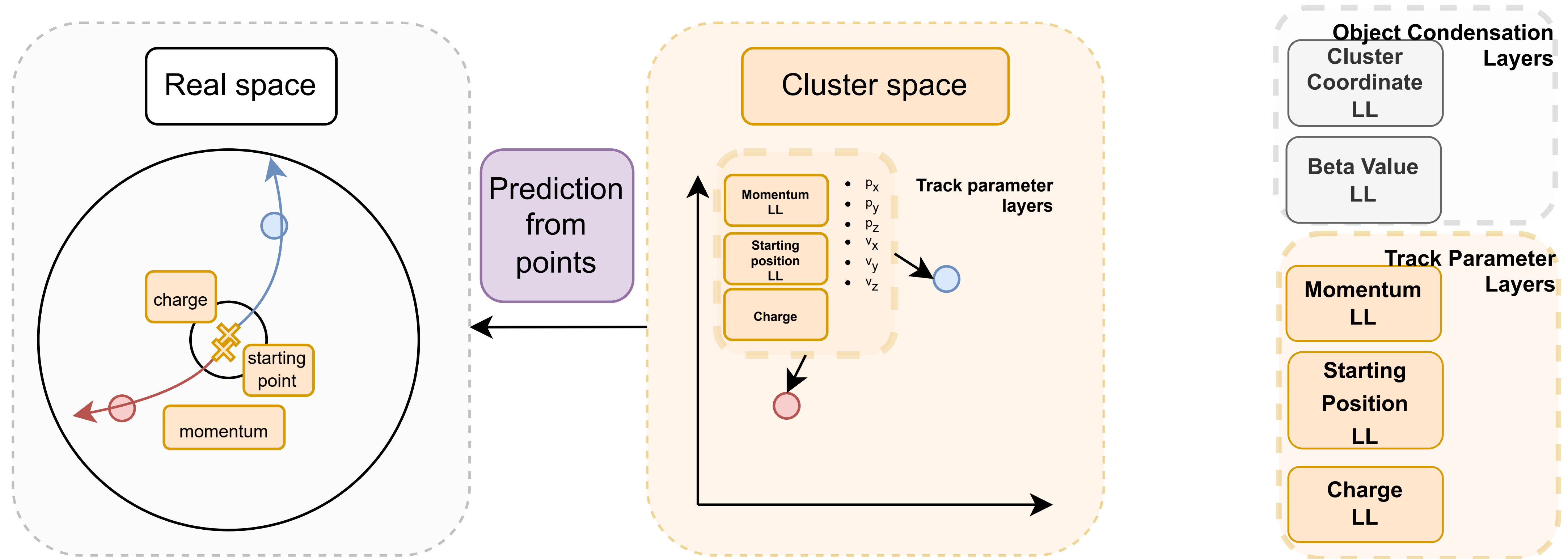




# Inference - Condensation point selection based on isolation criteria

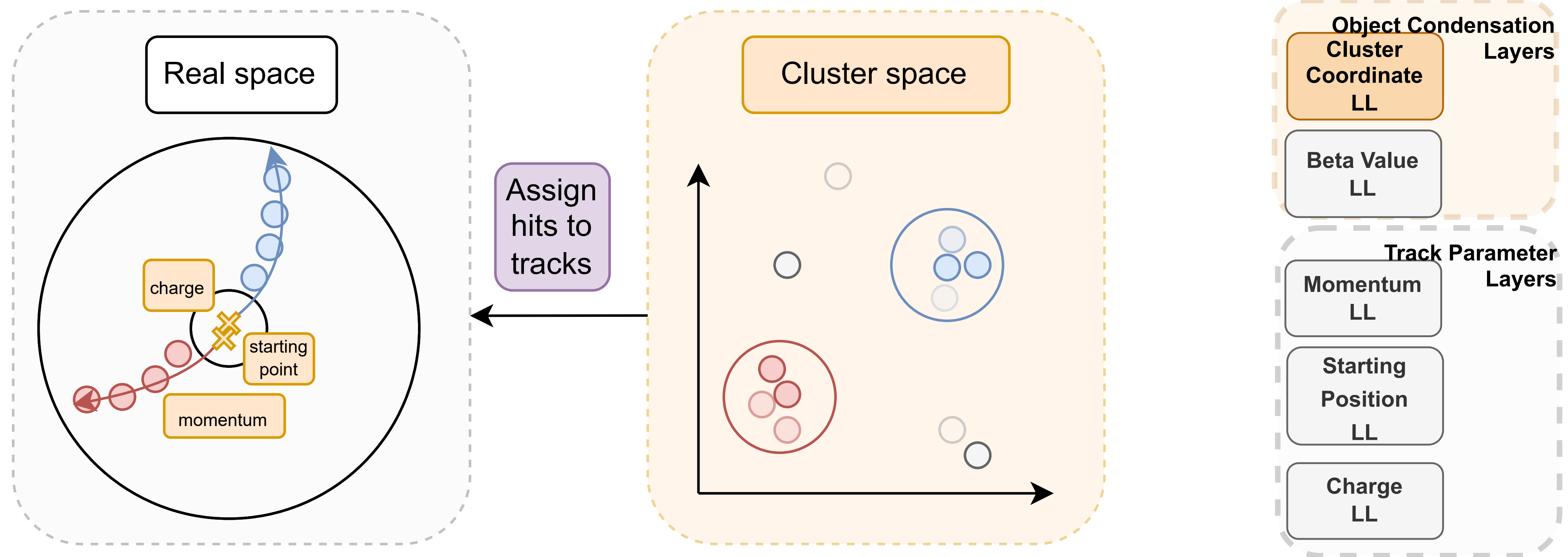


# Inference - Parameter extraction from condensation points



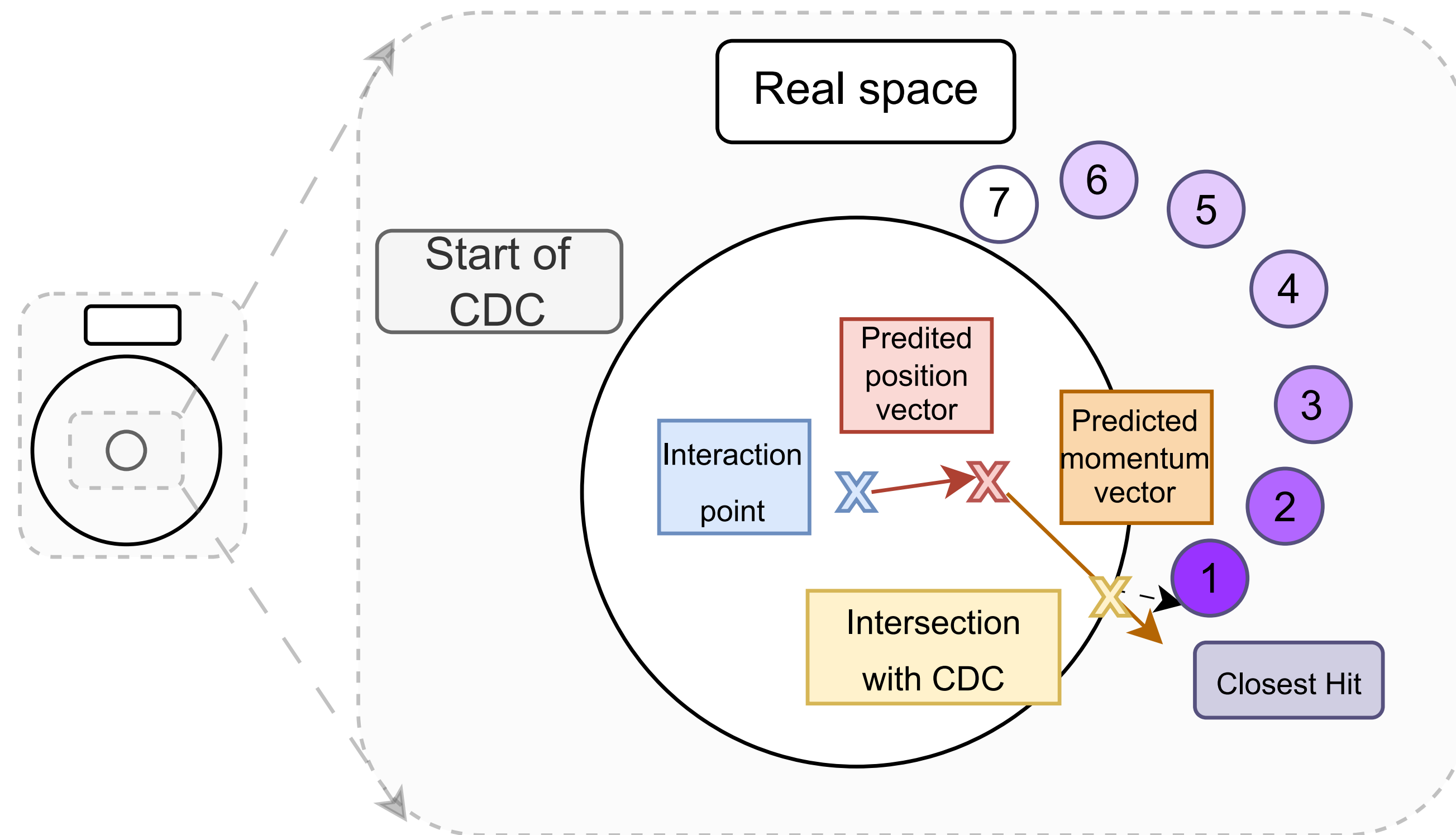


# Inference - Clusterisation based on condensation points with hit assignment



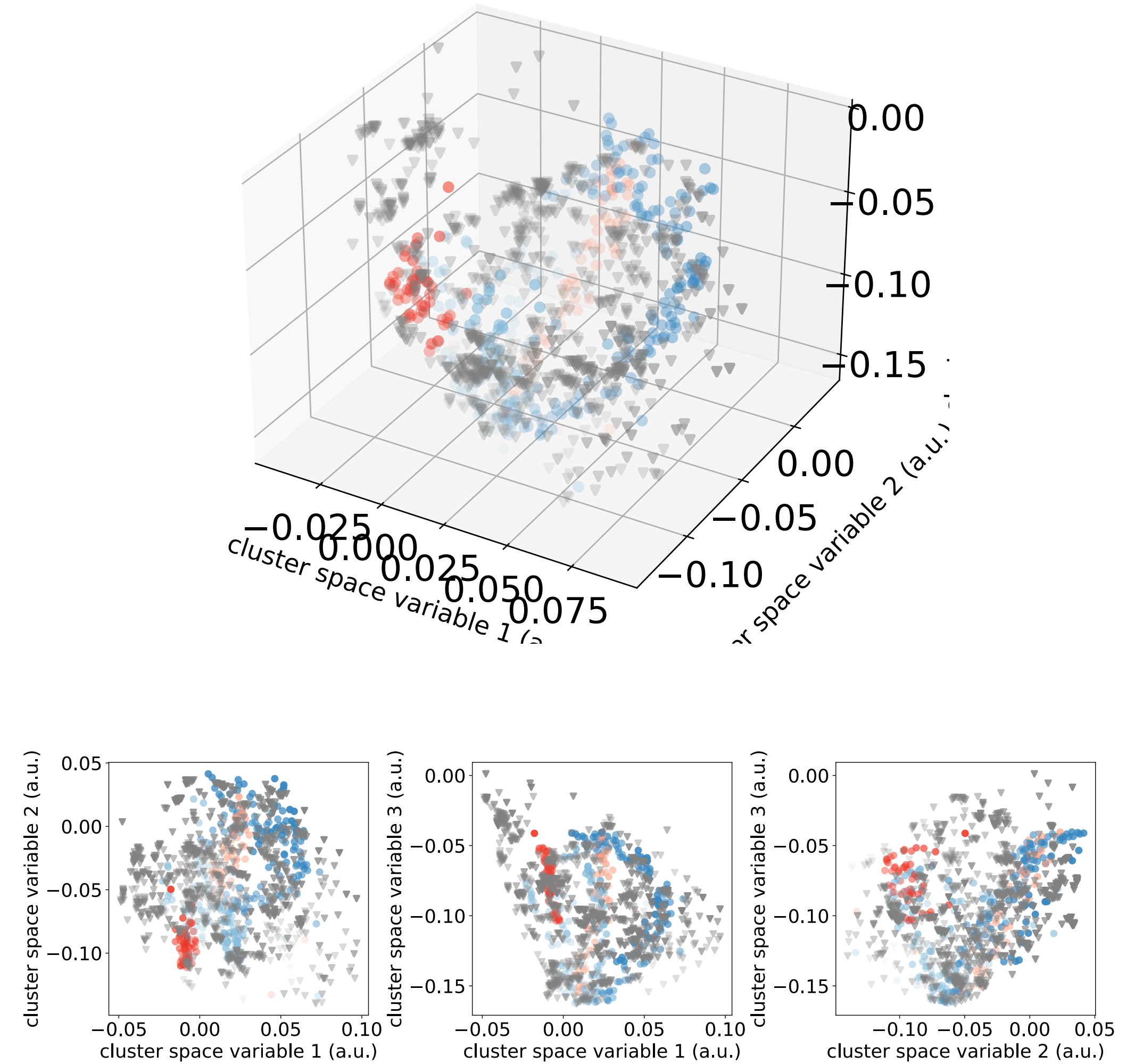
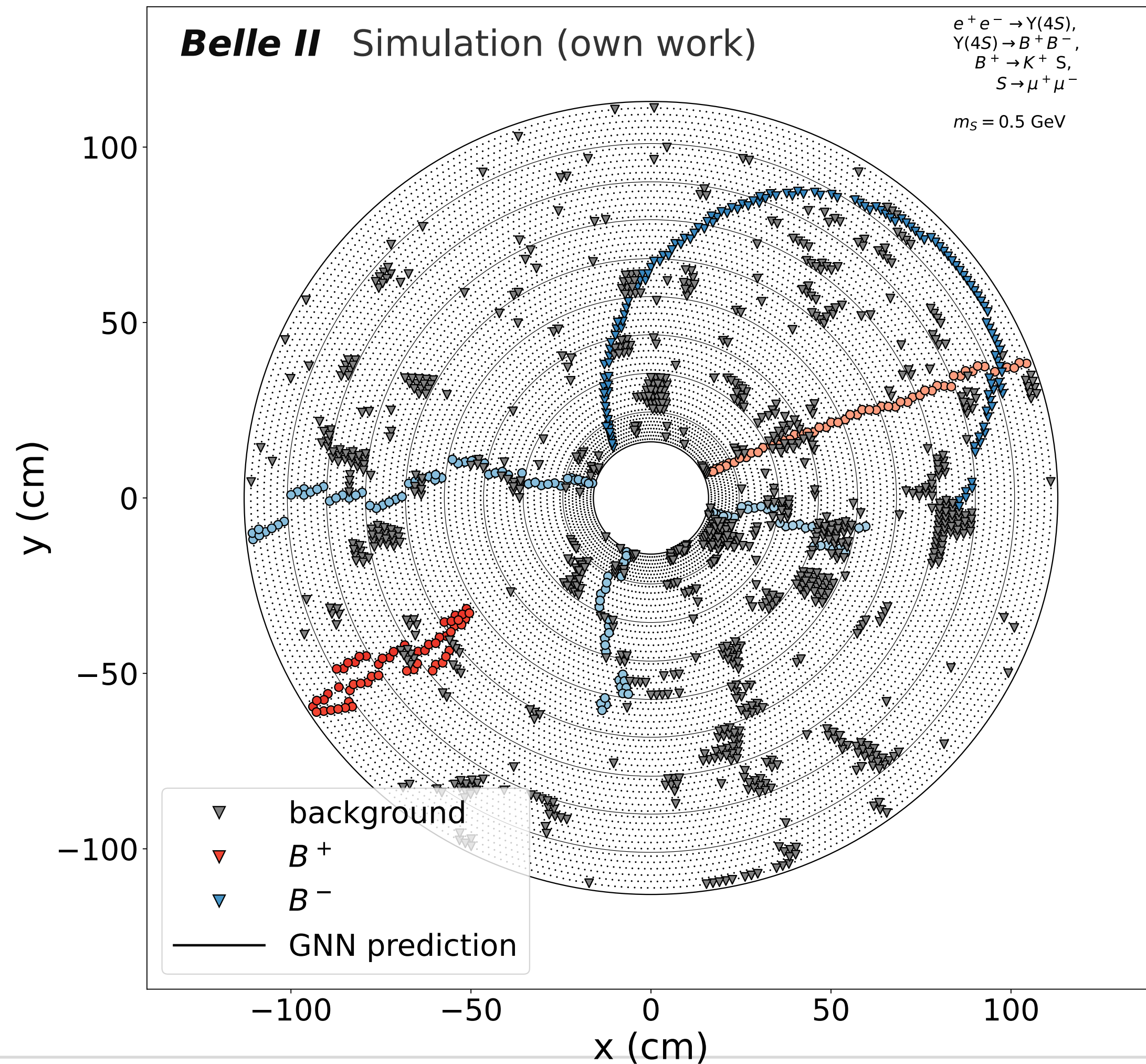
3 additional hyperparameters to tune

# Inference - Hit ordering in real space



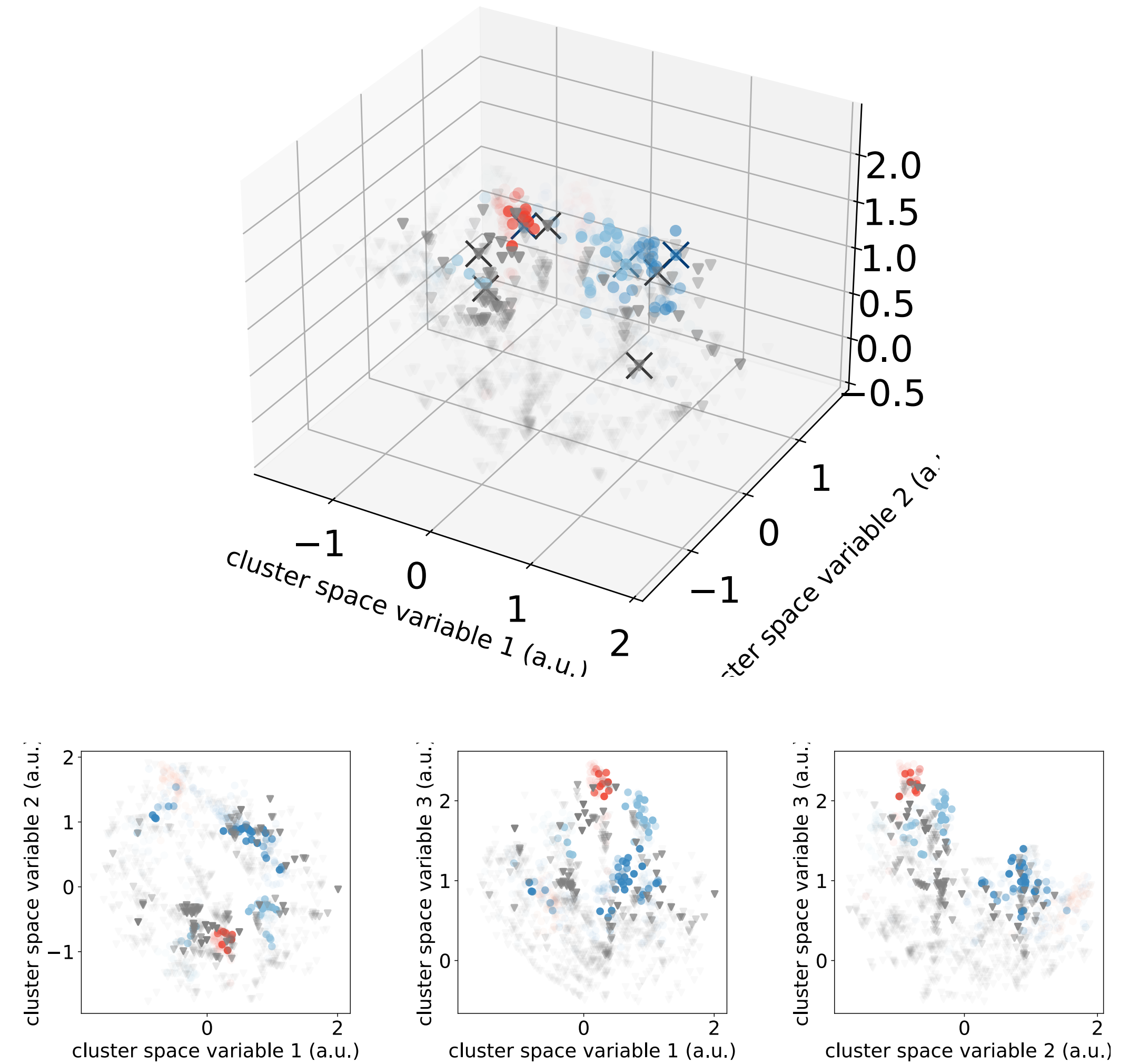
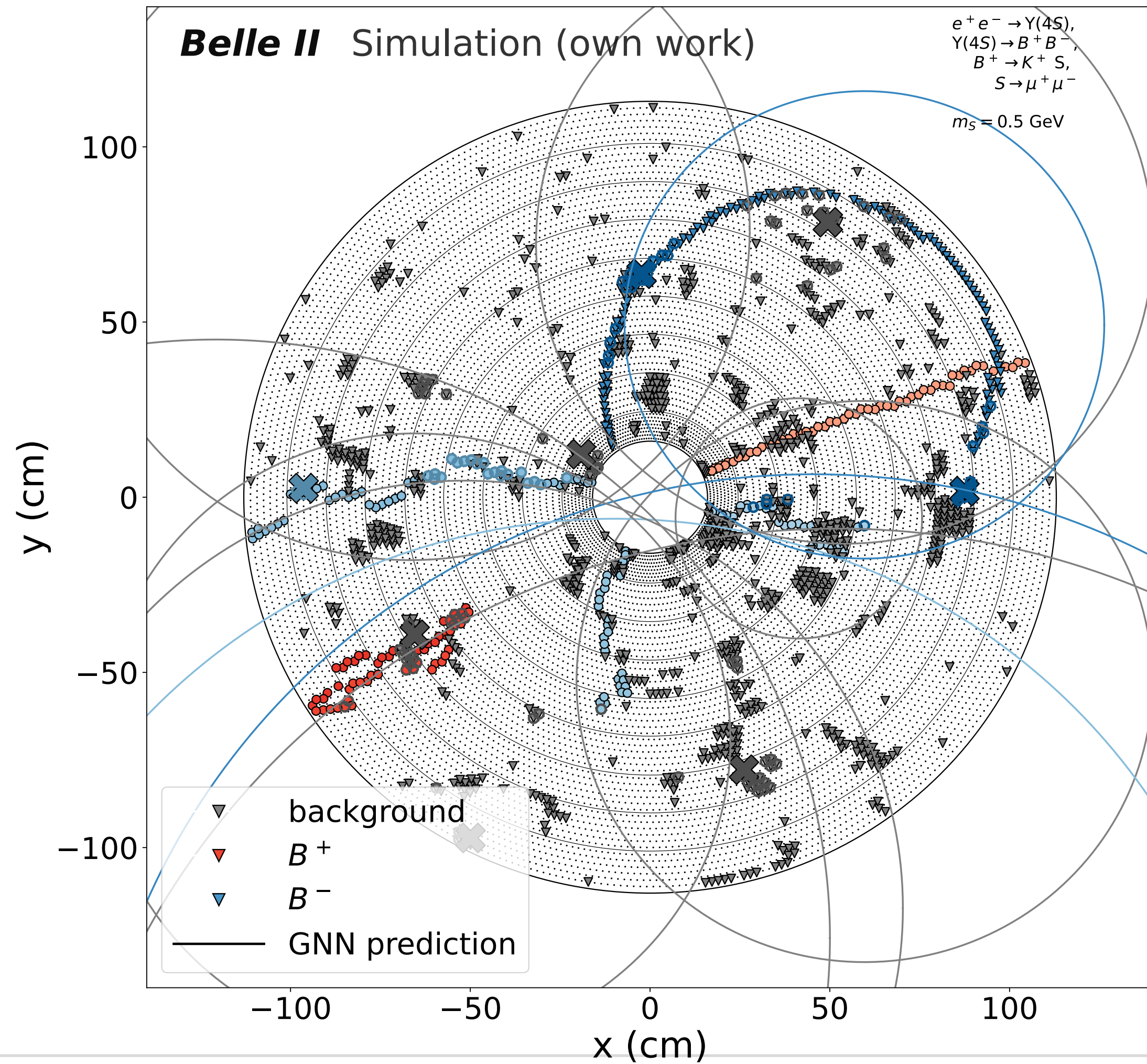


# Model Learning



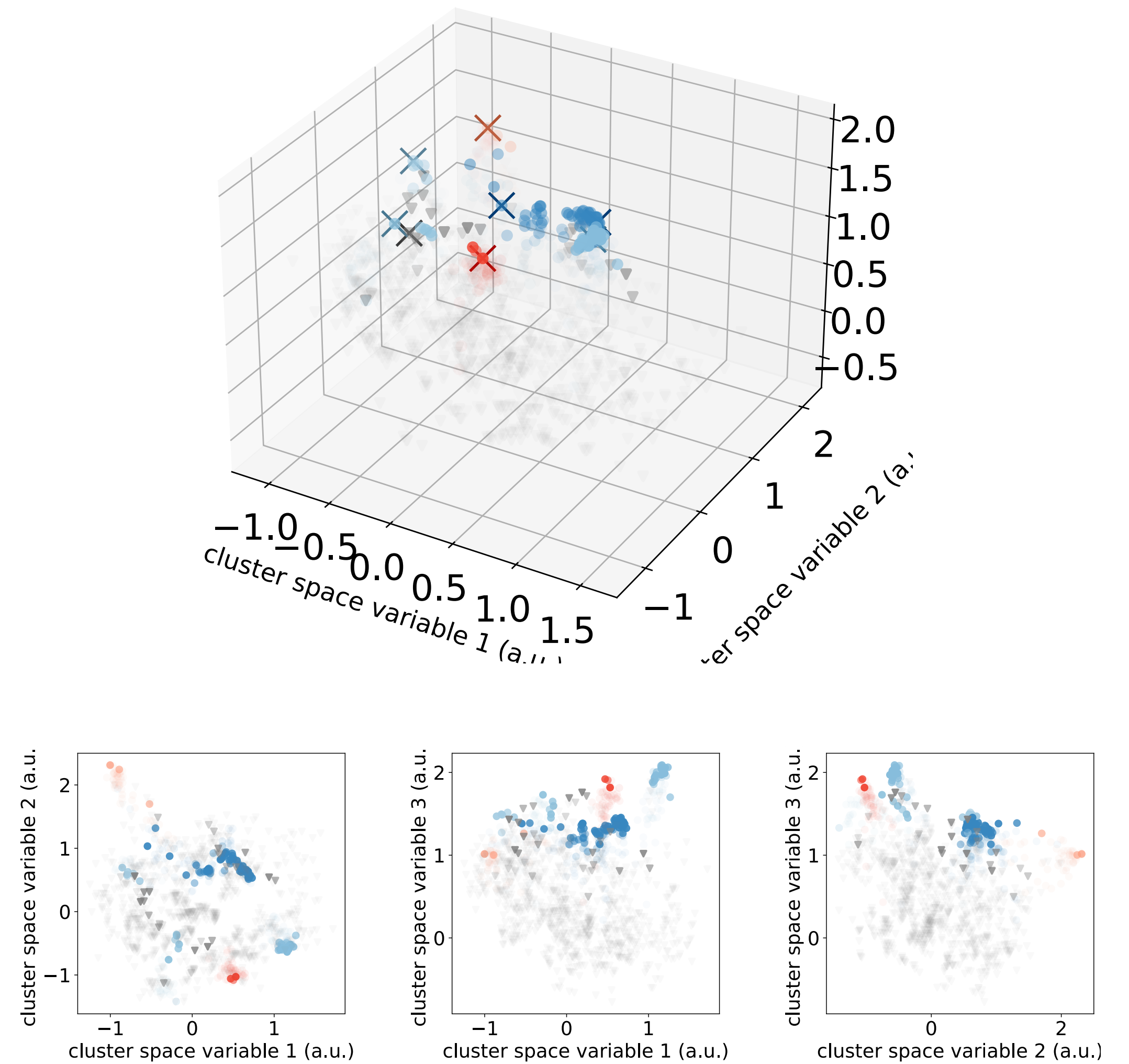
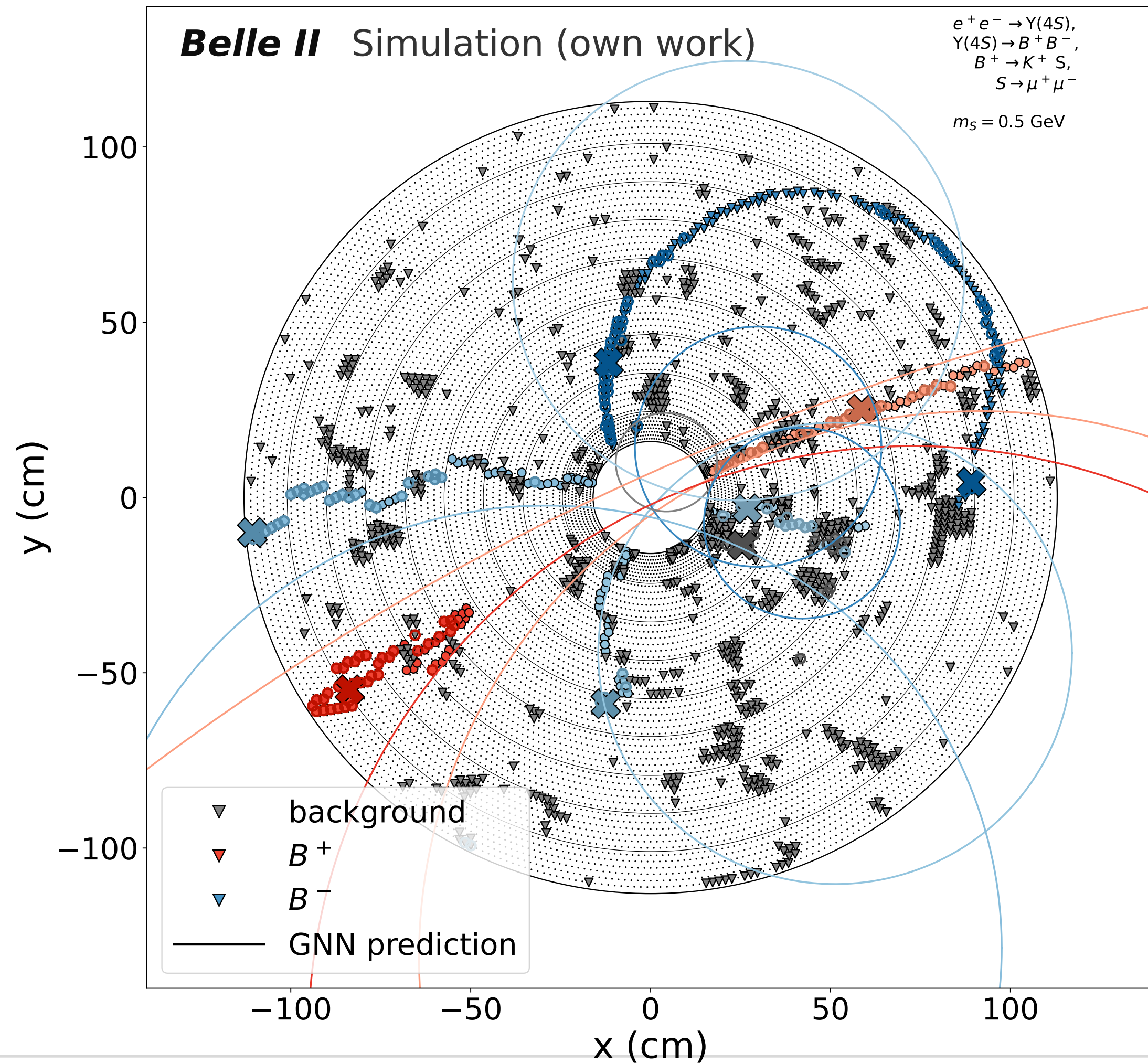


# Model Learning



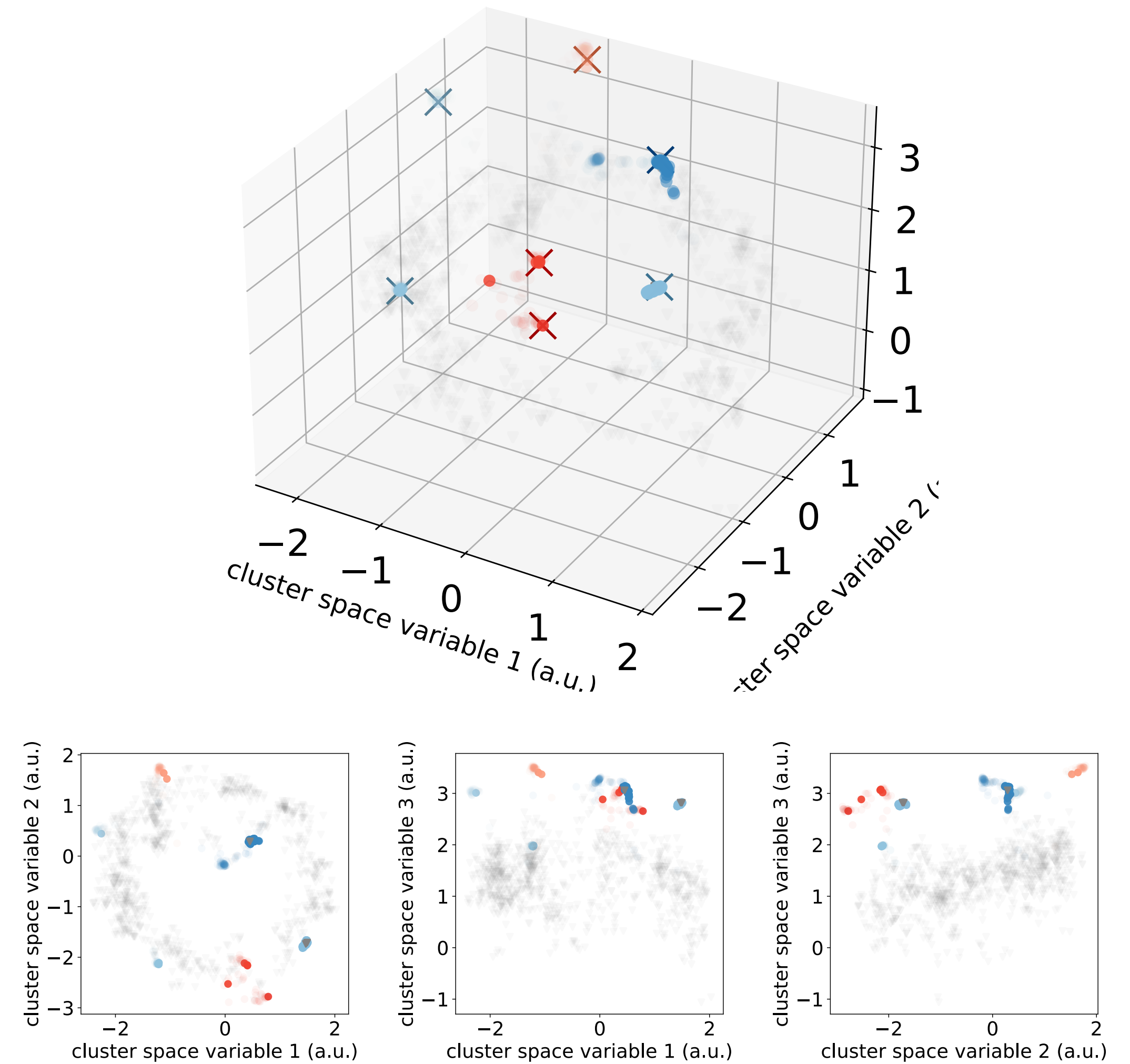
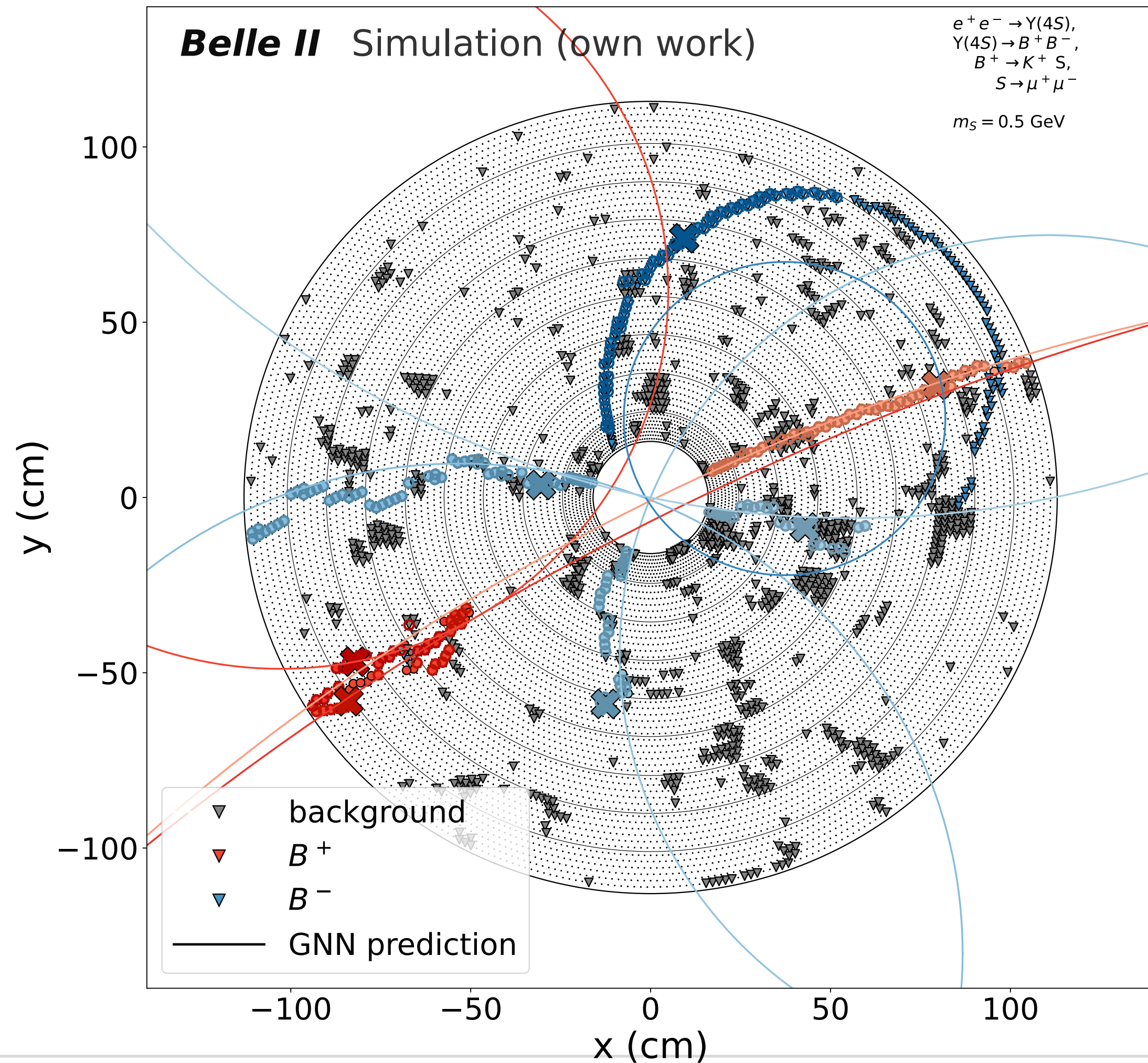


# Model Learning



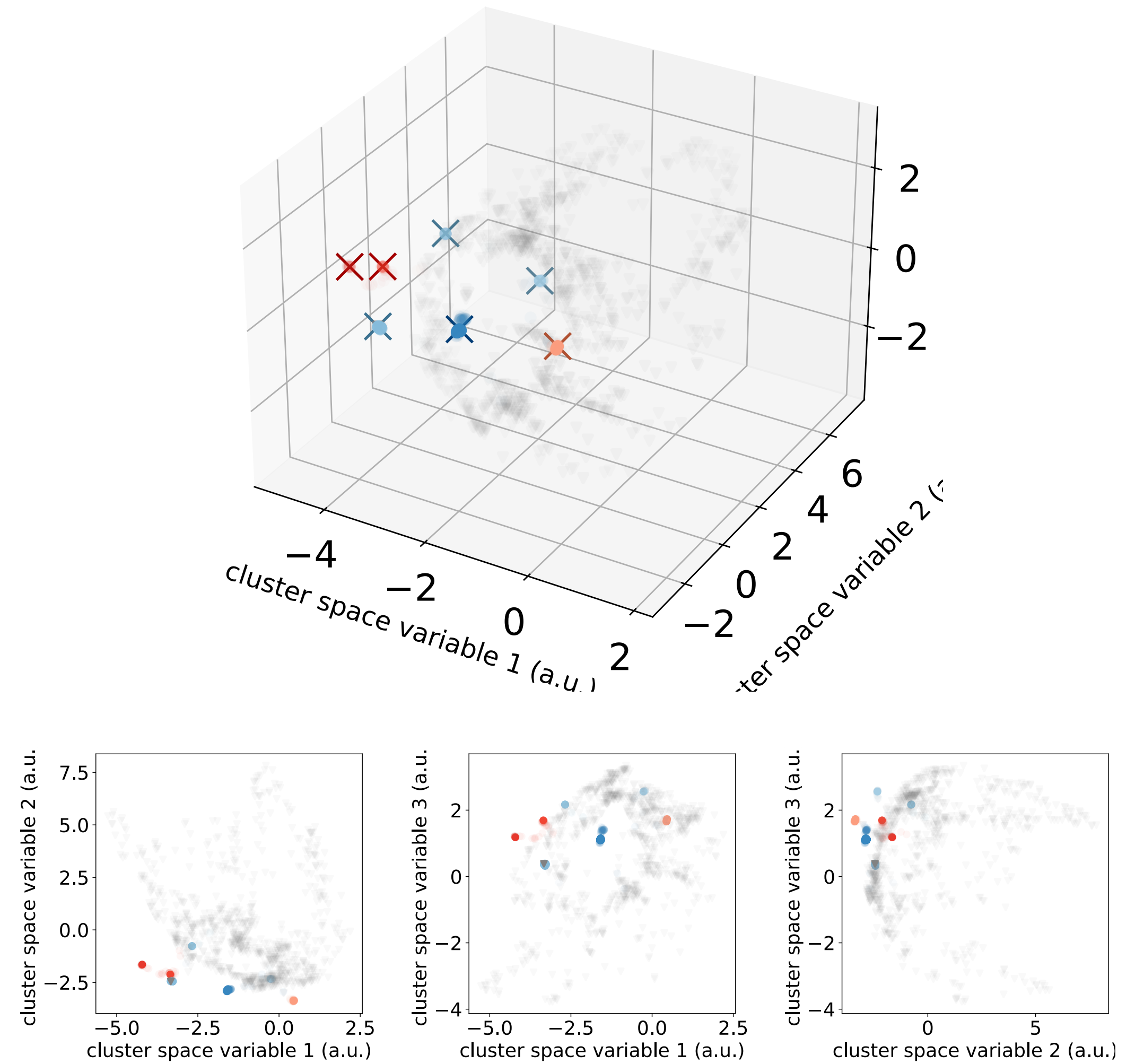
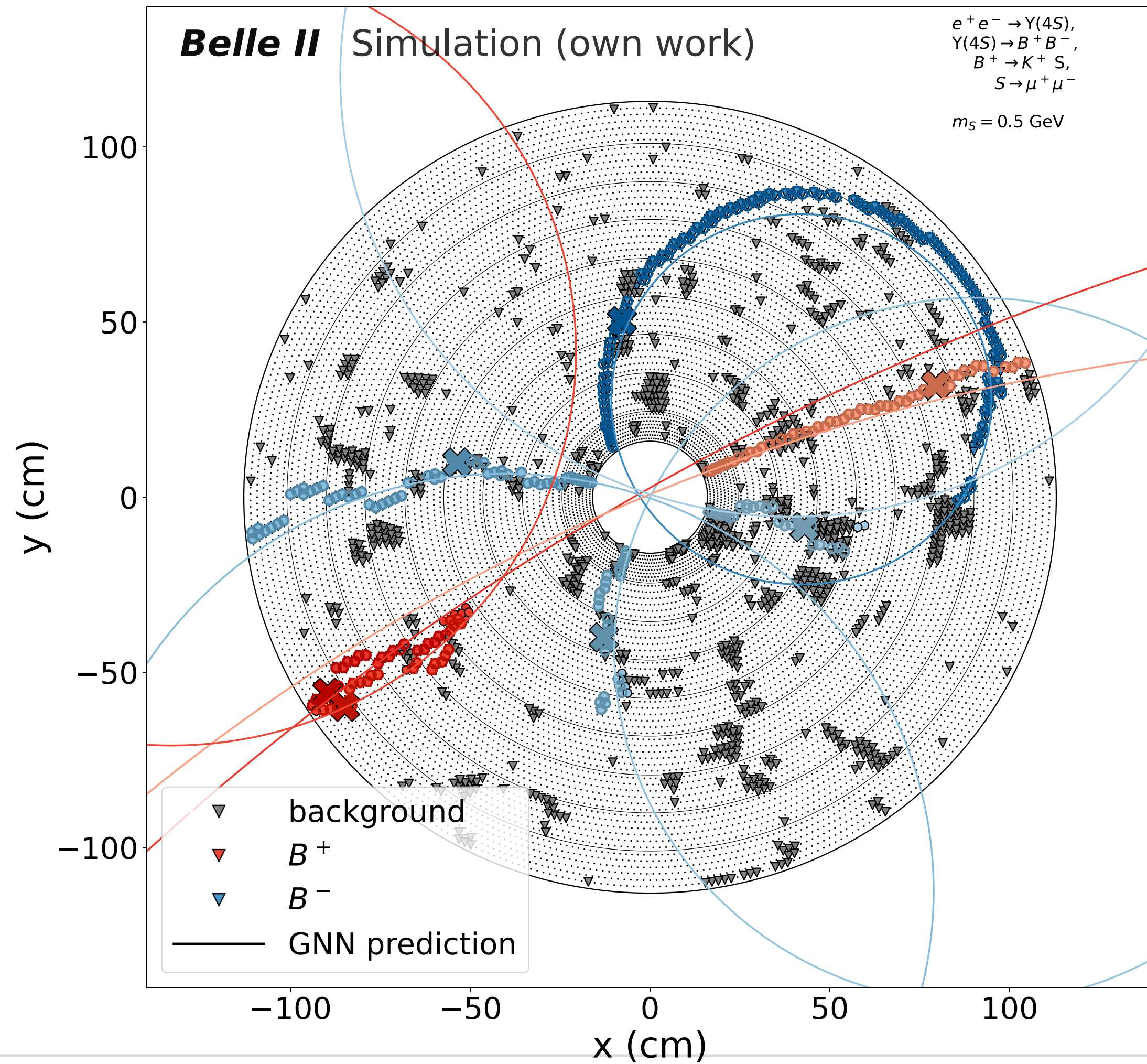


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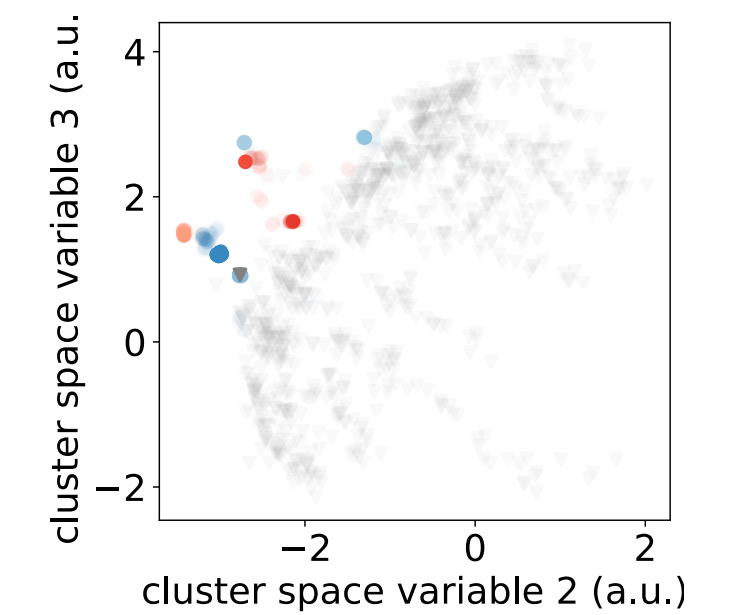
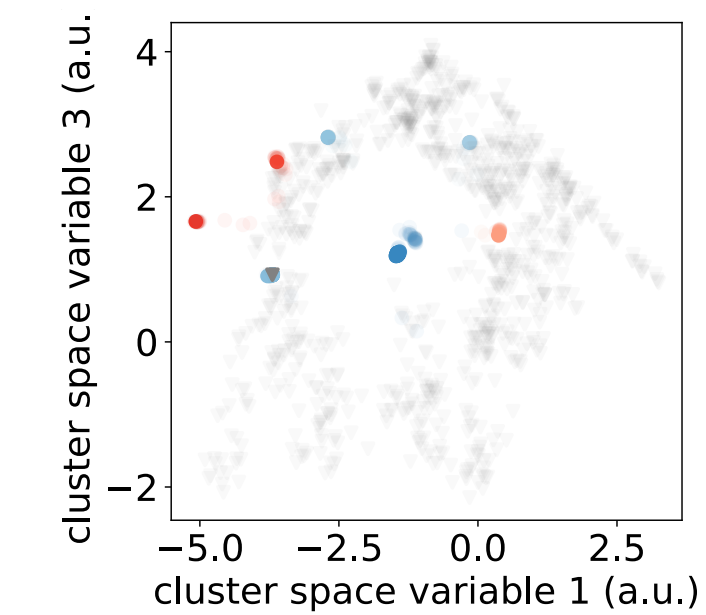
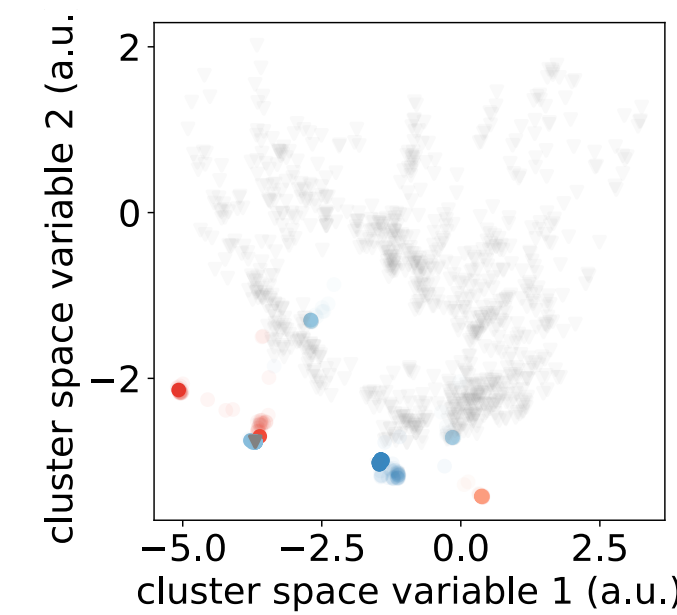
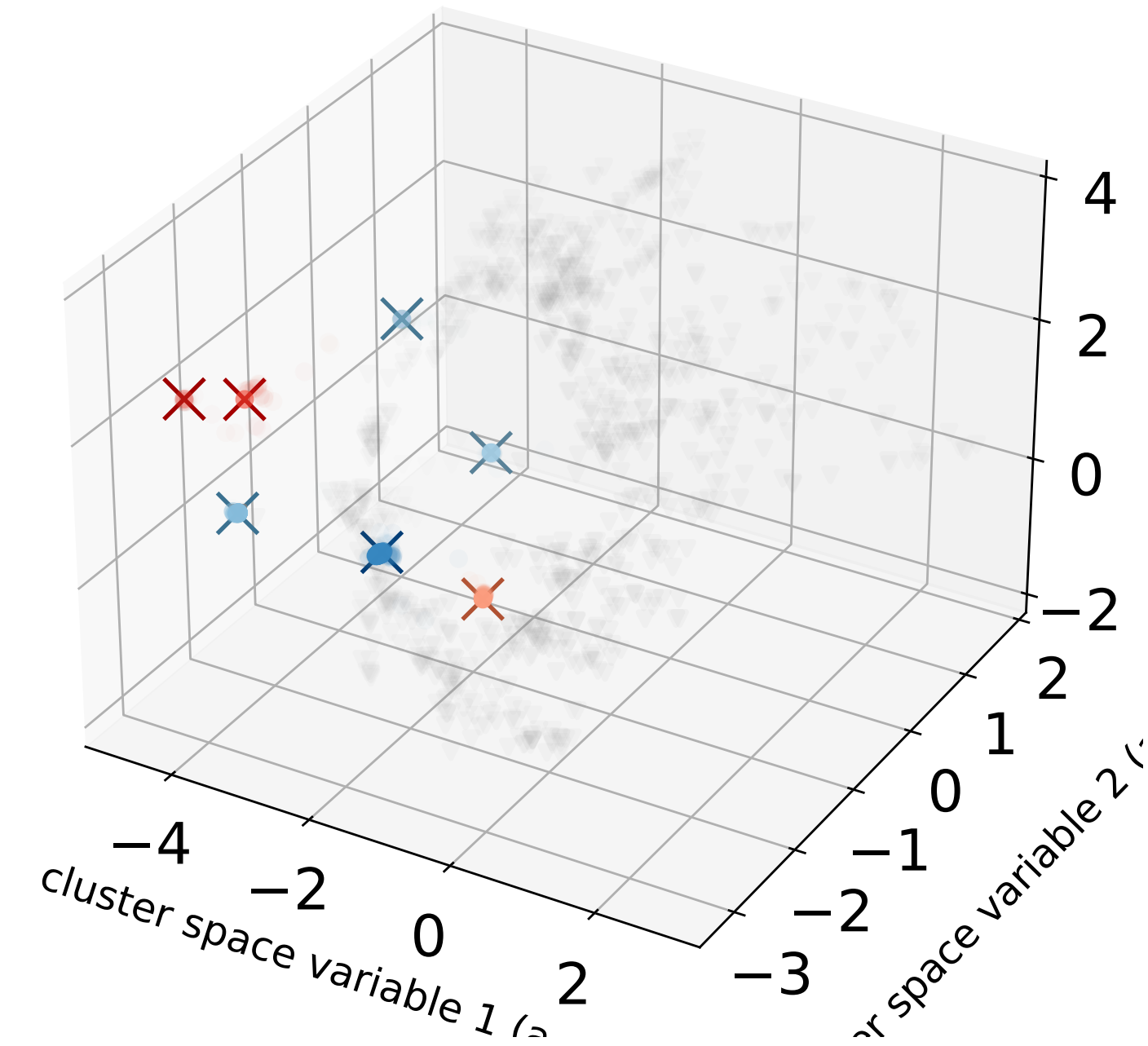
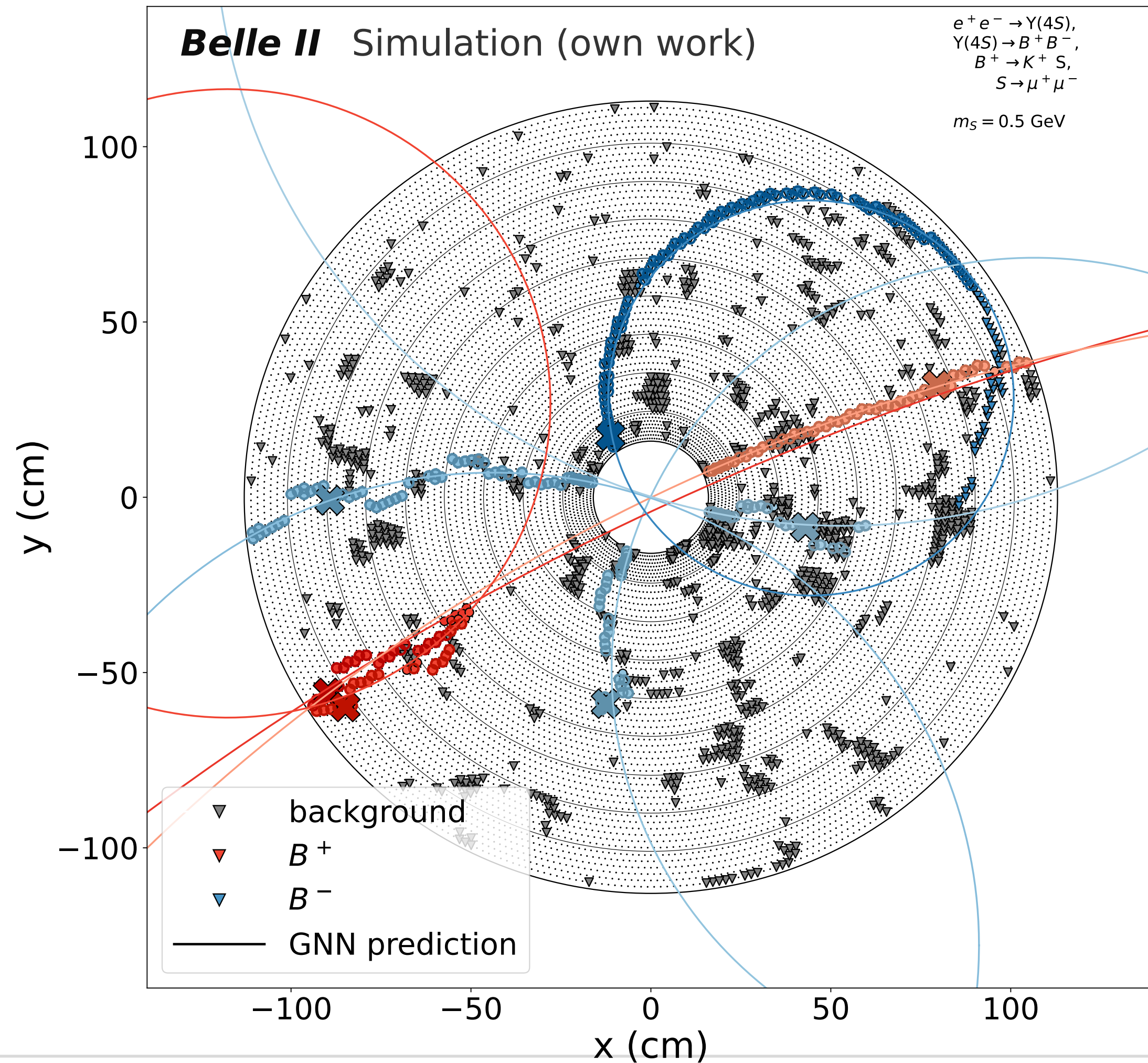


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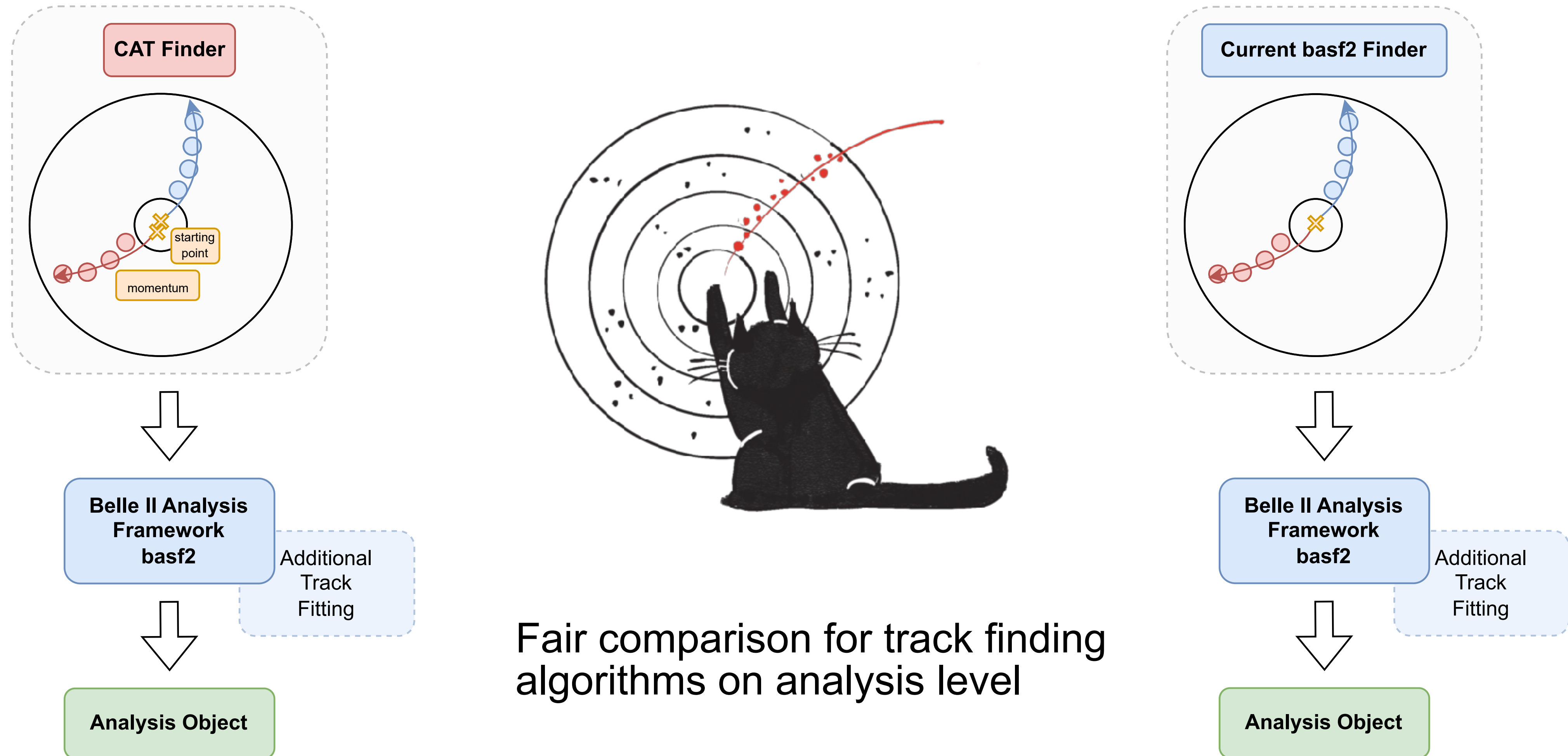


# Model Prediction



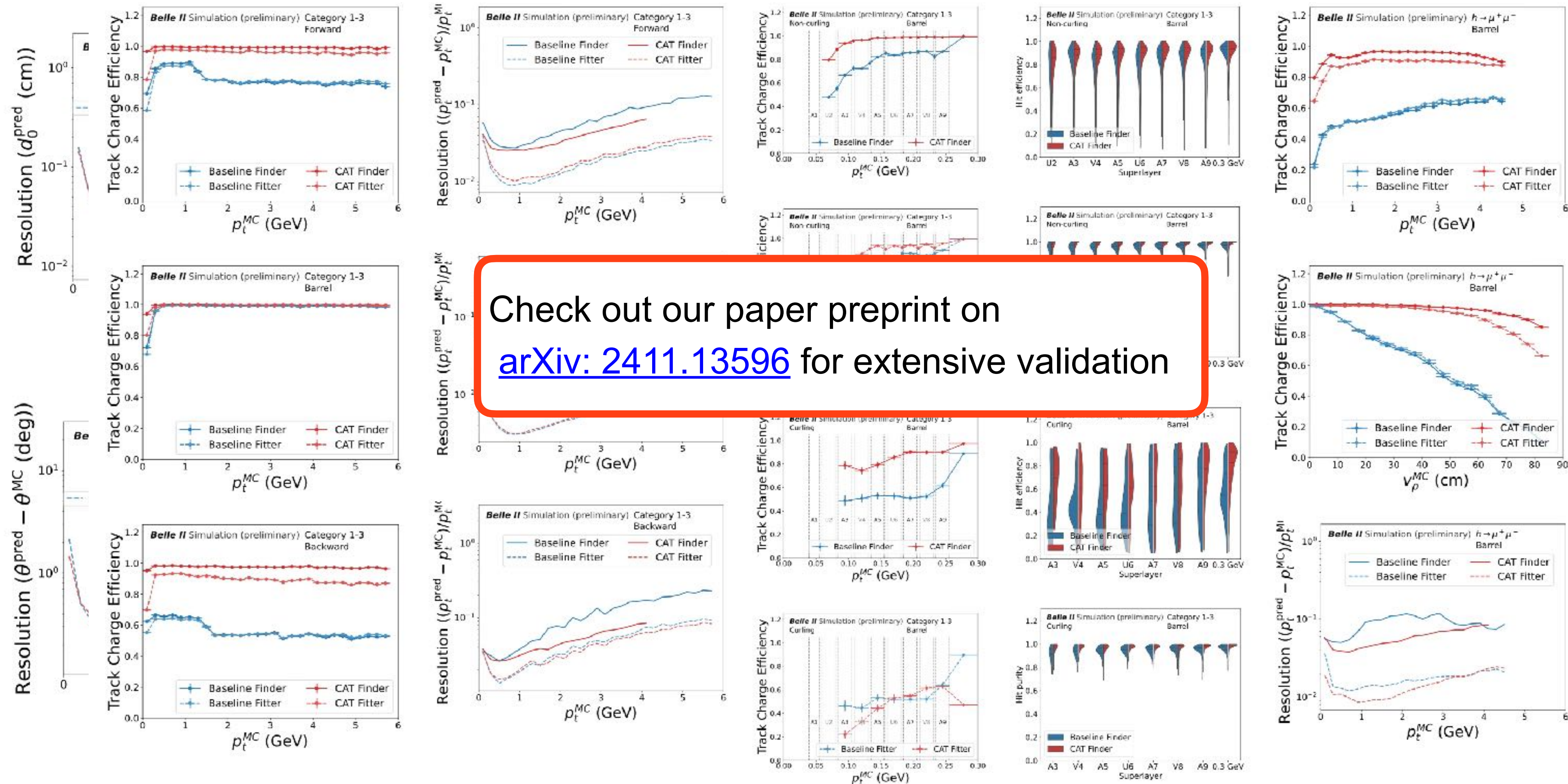


# Implementation in our analysis software for a comparison to Baseline Tracking





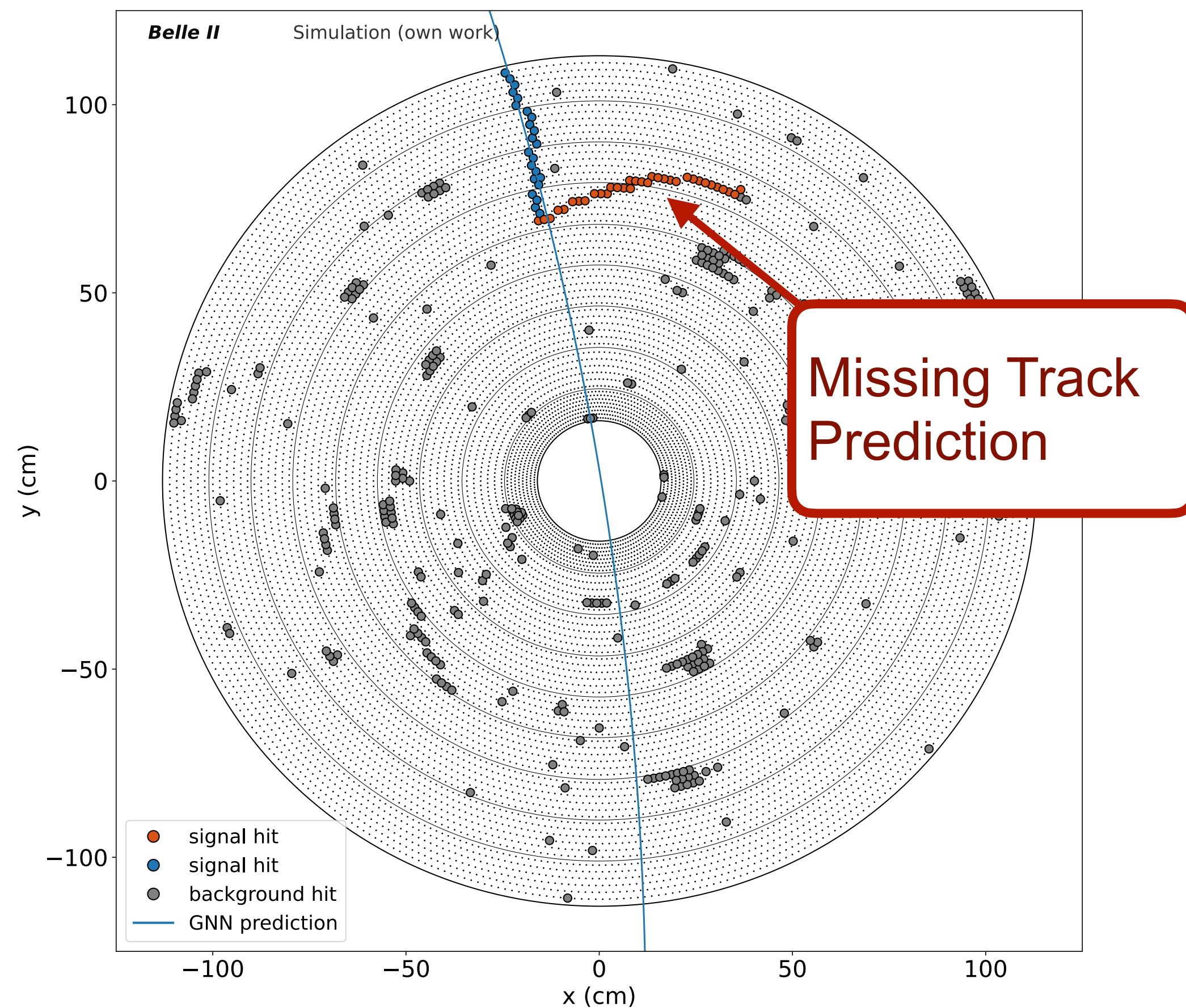
# Extensive studies and validation





# Important Metrics

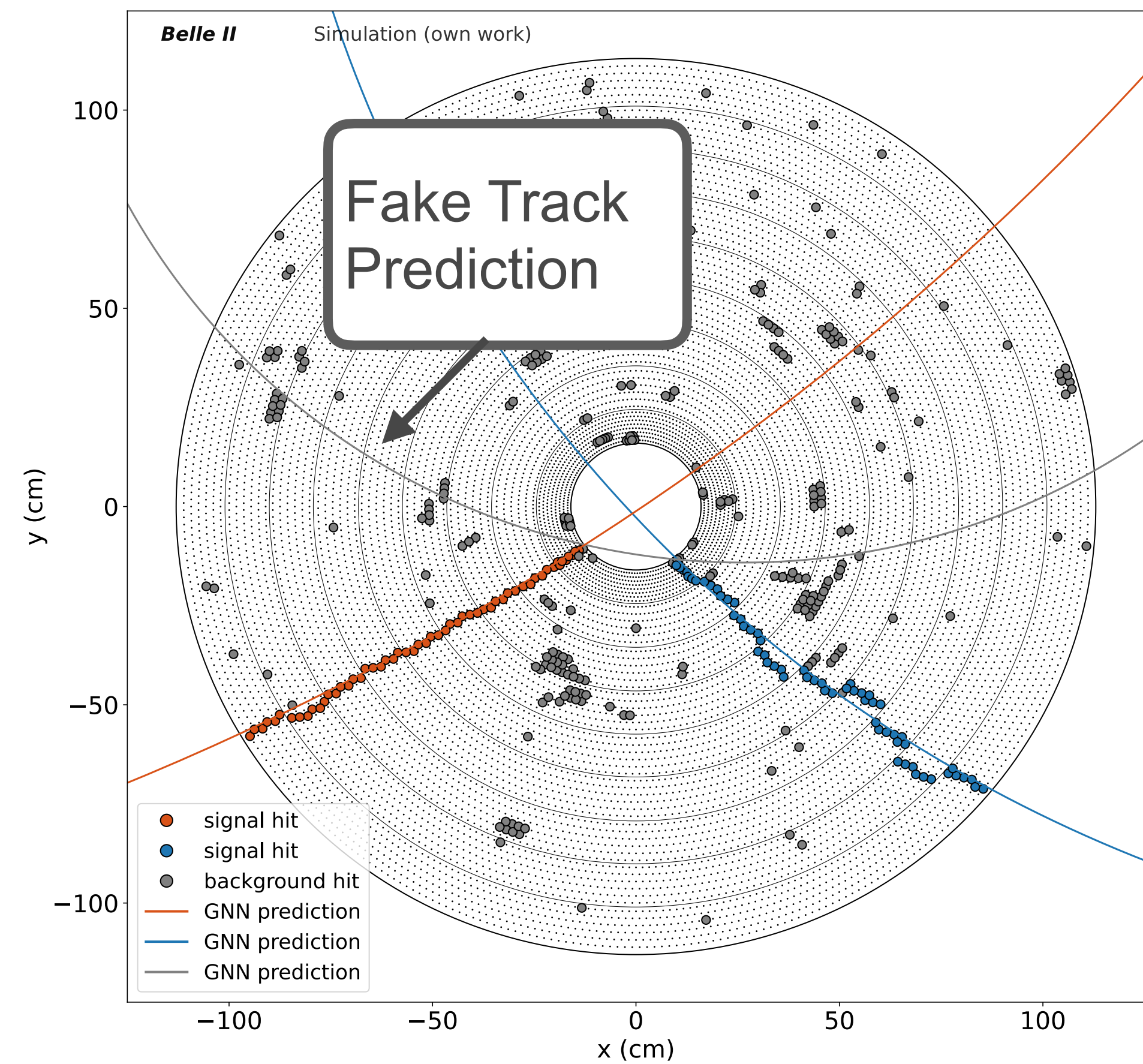
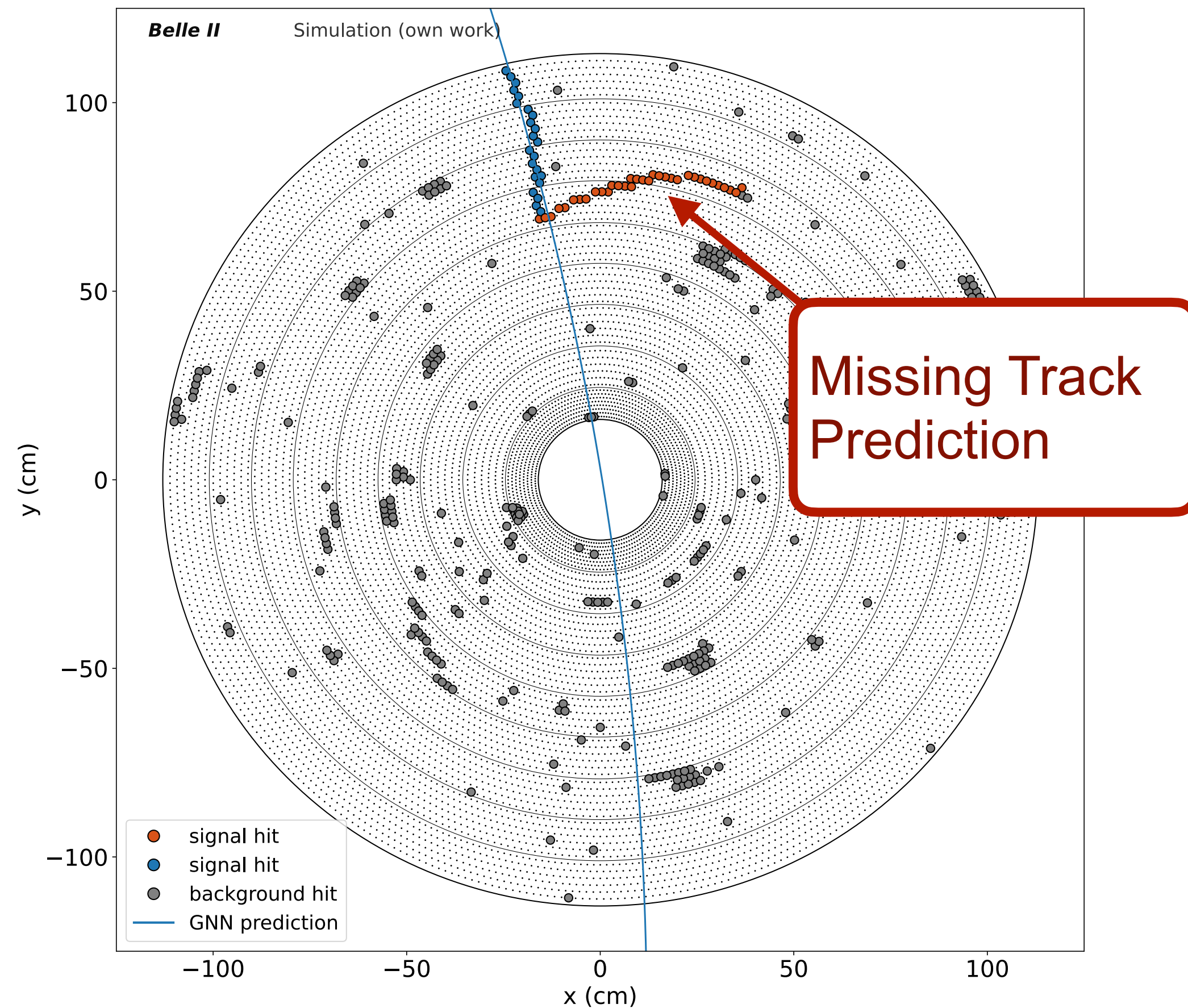
$$\text{Efficiency} = \frac{\text{true tracks}}{\text{all particles}}$$



# Important Metrics

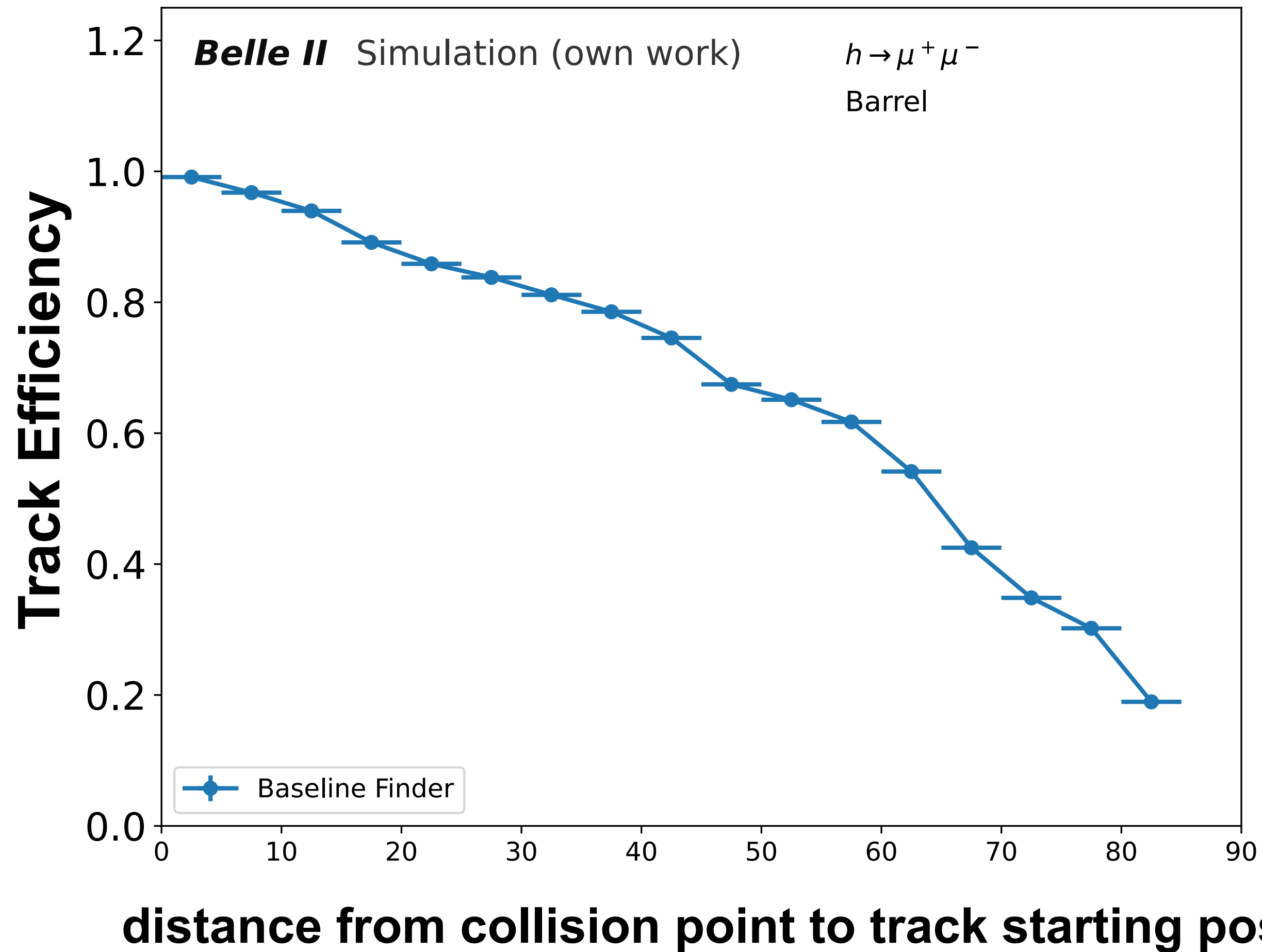
$$\text{Efficiency} = \frac{\text{true tracks}}{\text{all particles}}$$

$$\text{Purity} = \frac{\text{true tracks}}{\text{all tracks}}$$





# Performance on displaced particles



averaged

Efficiency

Purity

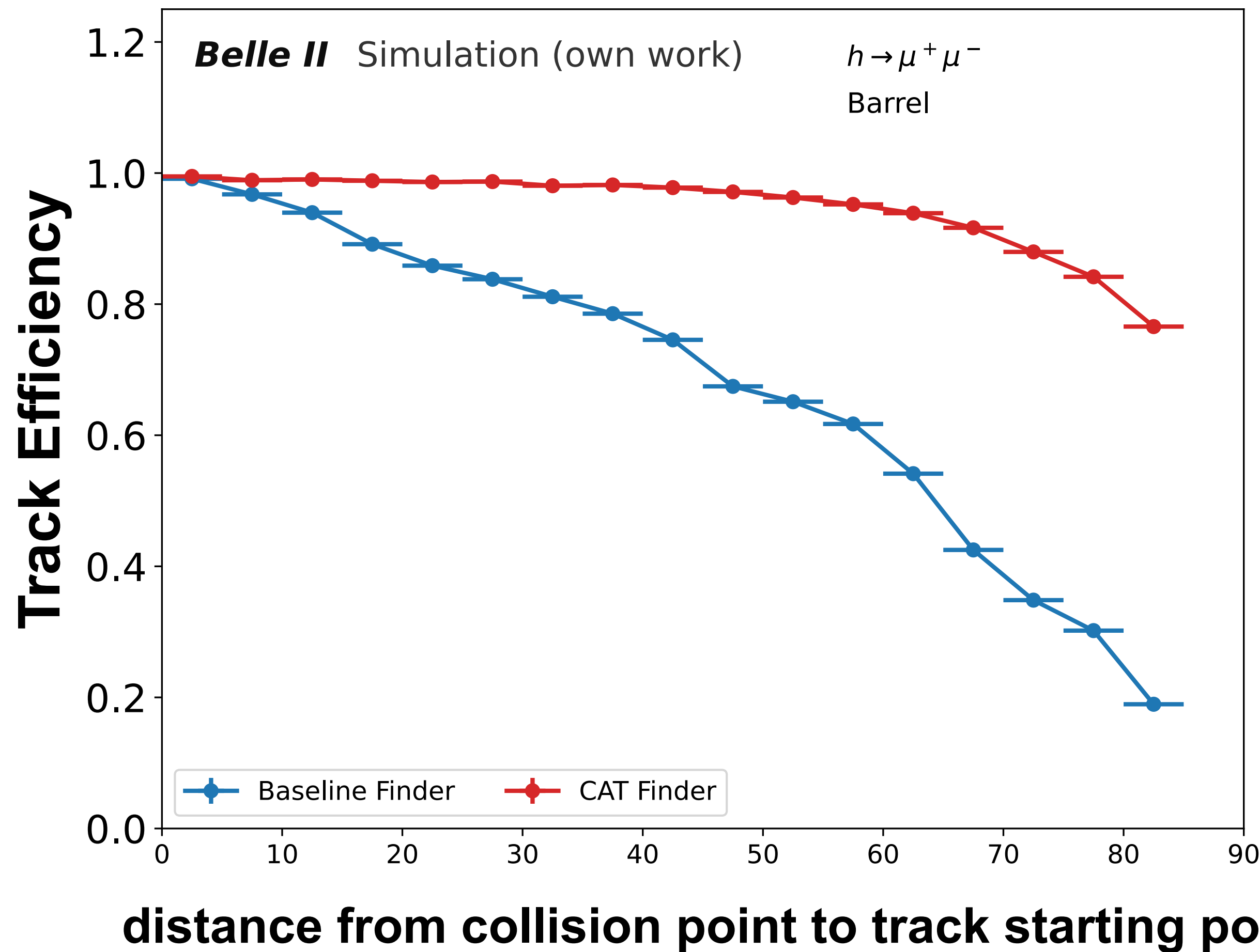
Baseline

**0.574 $\pm$ 0.001**

**0.964 $\pm$ 0.001**

Bertacchi et al., Track Finding at Belle II  
([arXiv:2003.12466](https://arxiv.org/abs/2003.12466))

# Performance on displaced particles



averaged

Efficiency

Purity

Baseline

**0.574 $\pm$ 0.001**

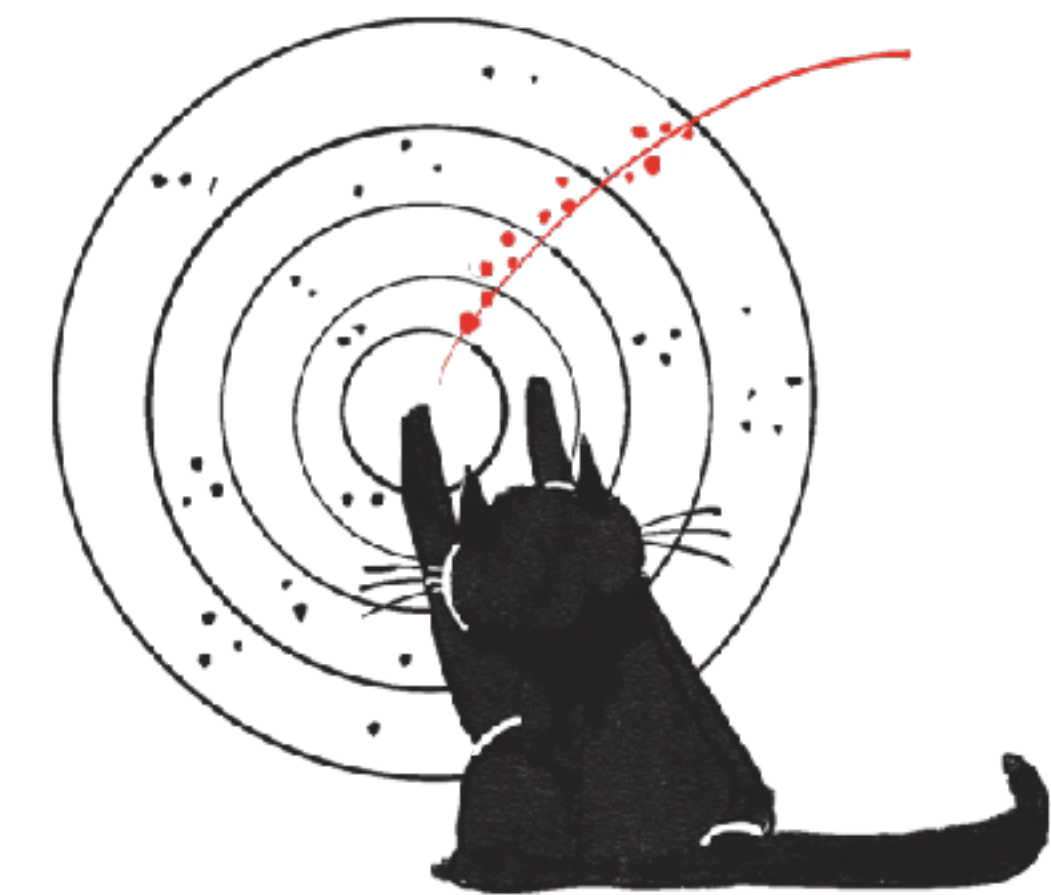
**0.964 $\pm$ 0.001**

Bertacchi et al., Track Finding at Belle II  
([arXiv:2003.12466](https://arxiv.org/abs/2003.12466))

CAT Finder

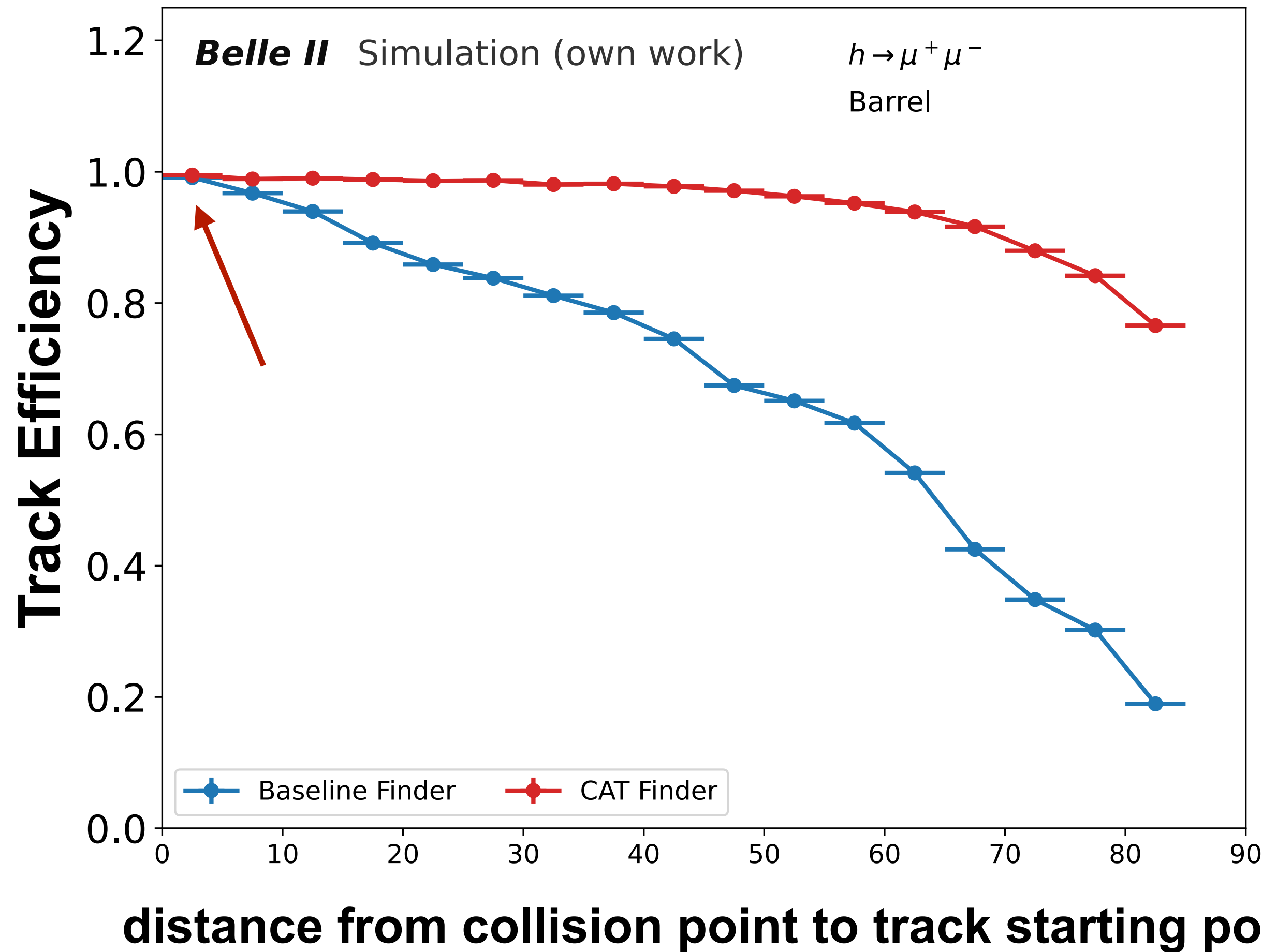
**0.892 $\pm$ 0.001**

**0.978 $\pm$ 0.001**





# Performance on displaced particles



averaged

Efficiency

Purity

Baseline

**0.574 $\pm$ 0.001**

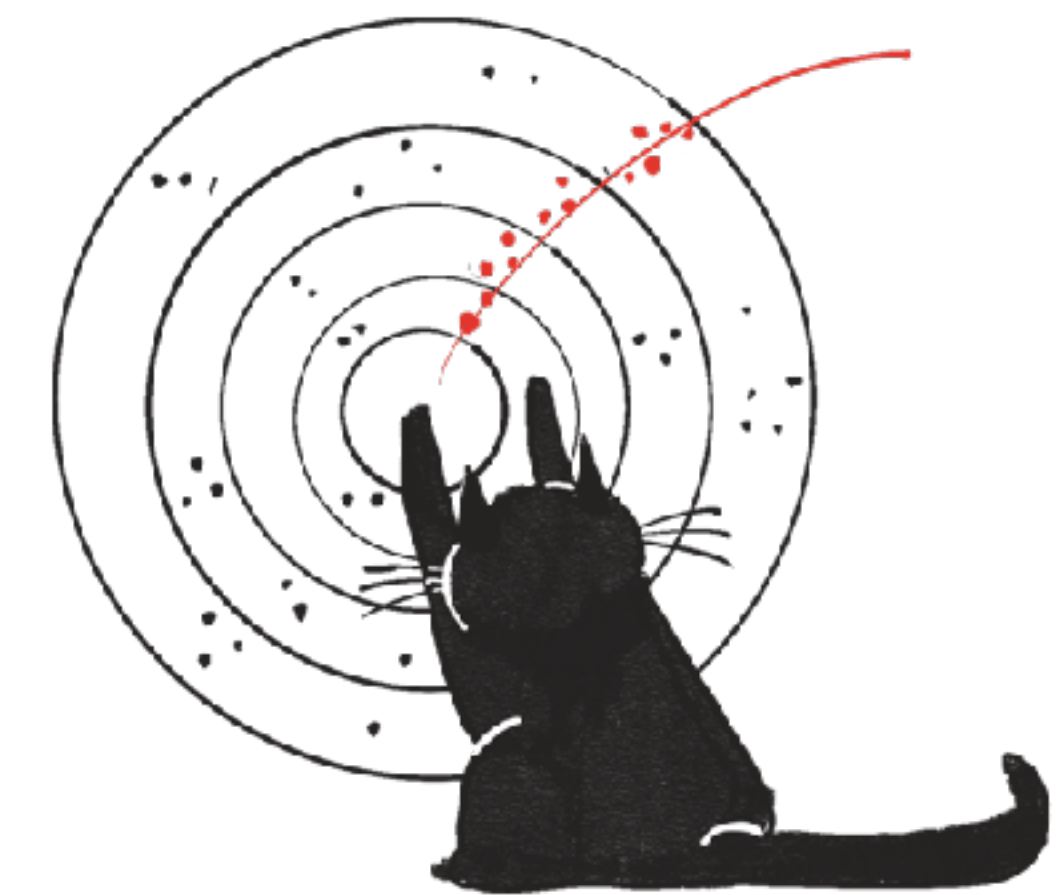
**0.964 $\pm$ 0.001**

Bertacchi et al., Track Finding at Belle II  
([arXiv:2003.12466](https://arxiv.org/abs/2003.12466))

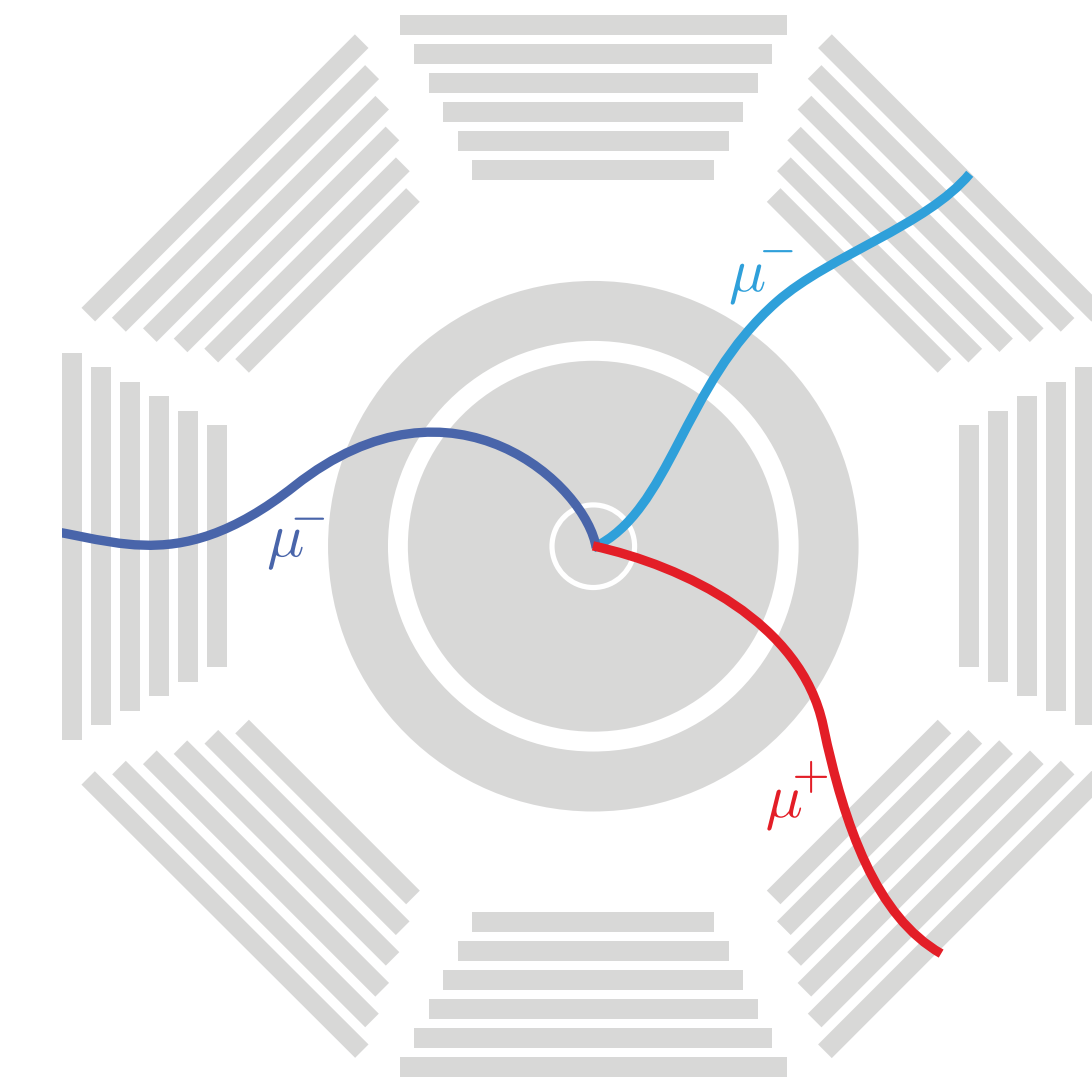
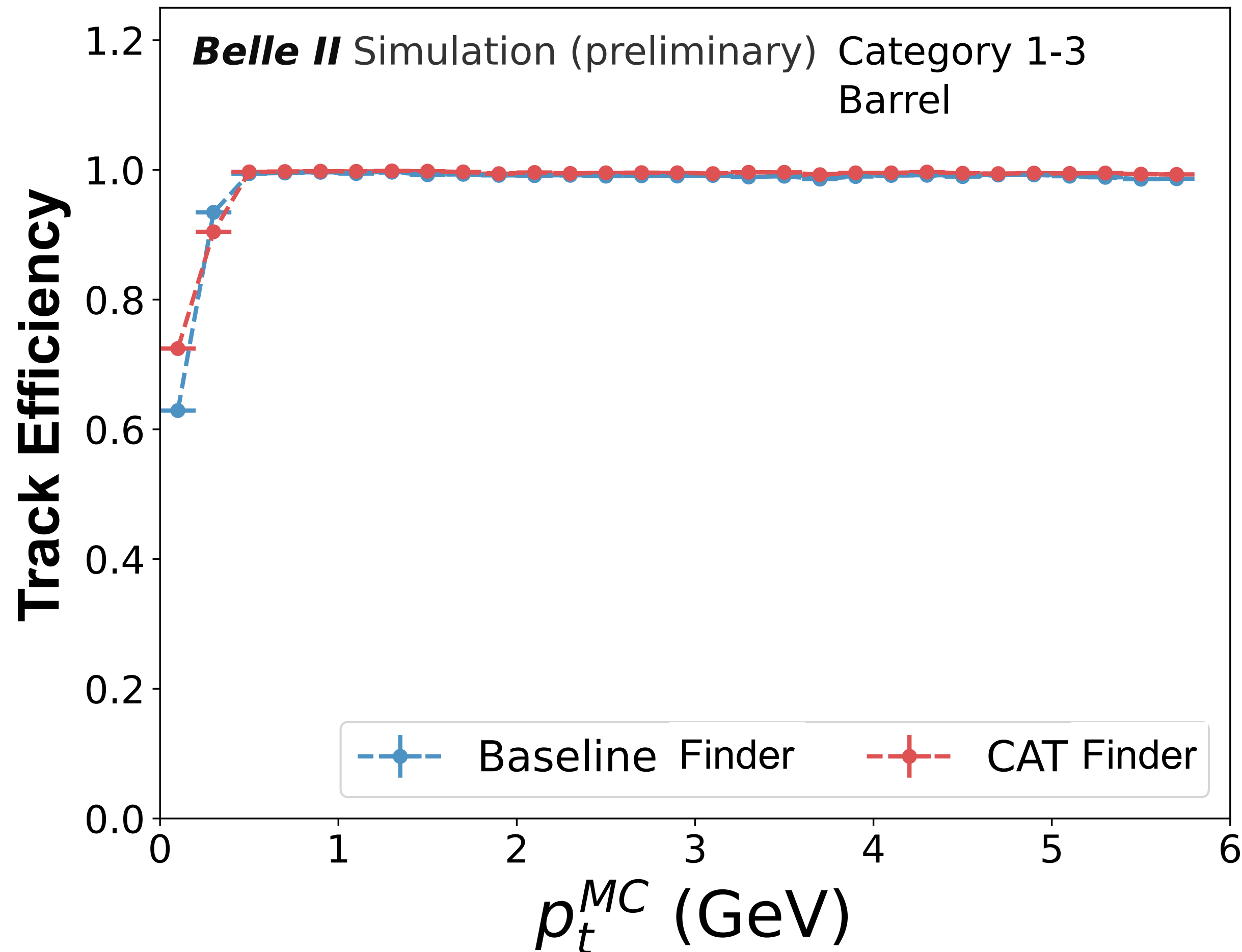
CAT Finder

**0.892 $\pm$ 0.001**

**0.978 $\pm$ 0.001**



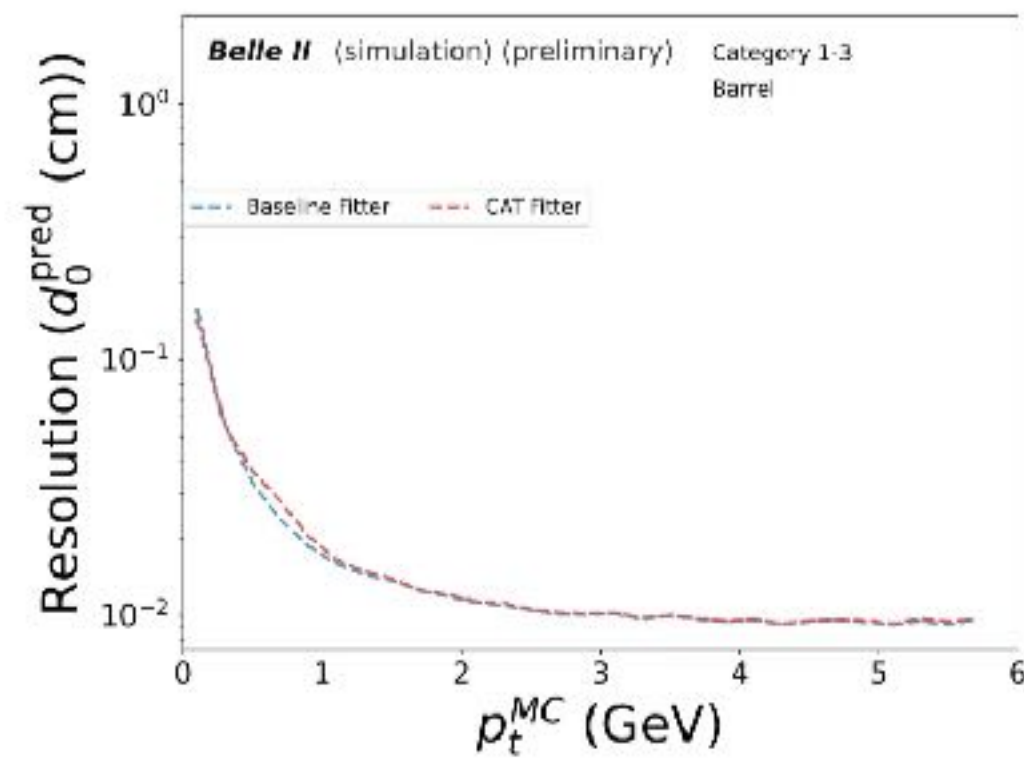
# Performance on prompt particles



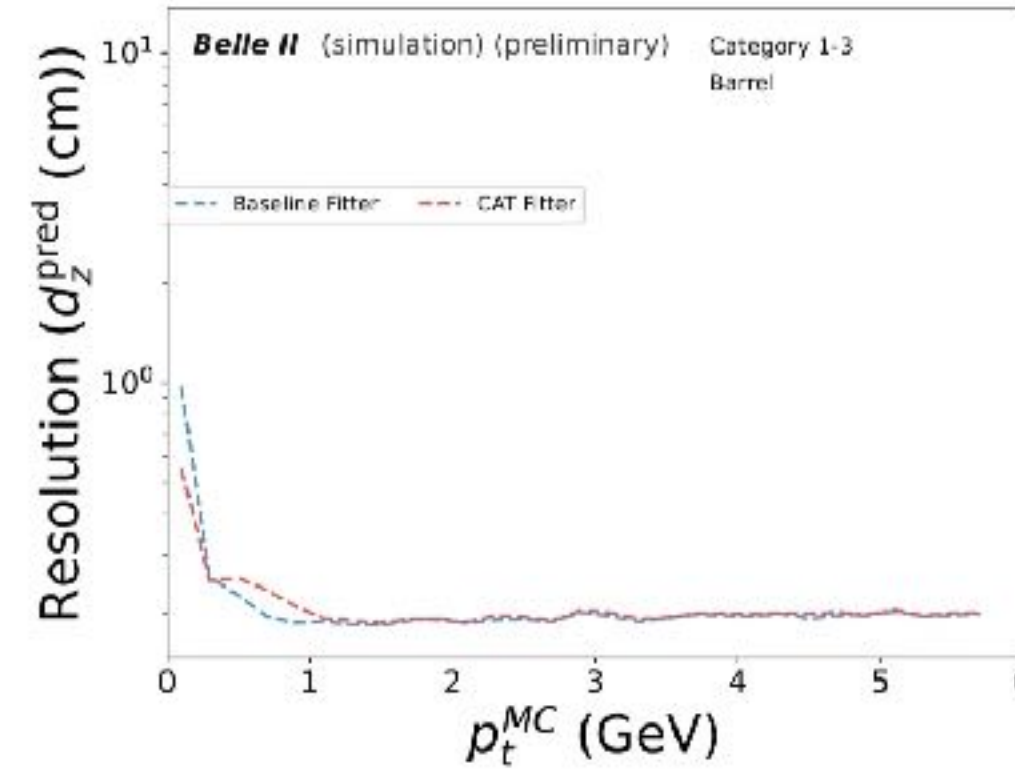
averaged	Efficiency	Purity
<b>Baseline</b>	<b>0.942<math>\pm</math>0.001</b>	<b>0.982<math>\pm</math>0.001</b>
<b>CAT Finder</b>	<b>0.951<math>\pm</math>0.001</b>	<b>0.987<math>\pm</math>0.001</b>



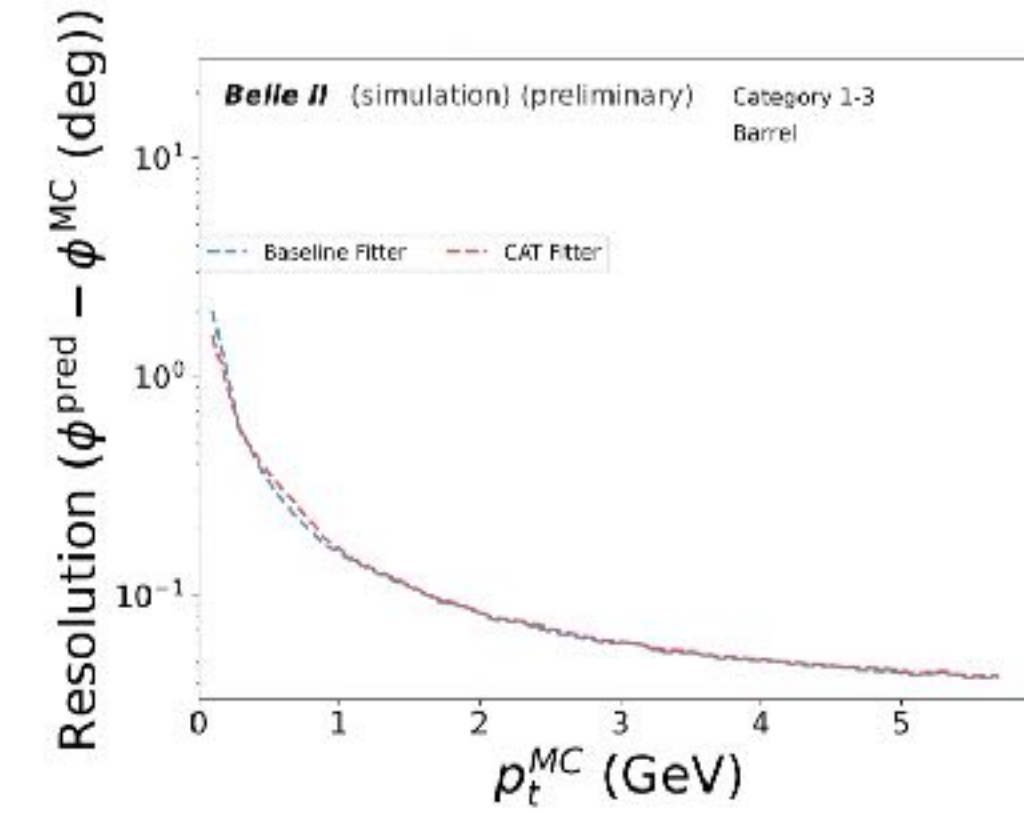
# Track parameter resolution



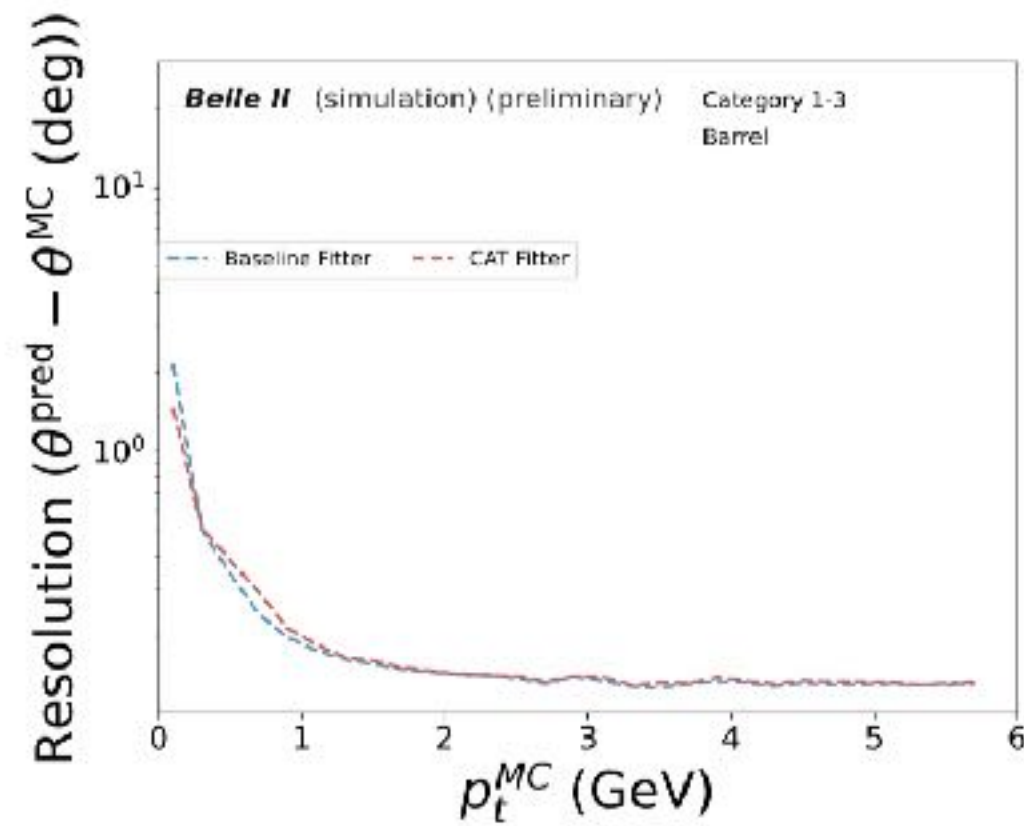
(a)  $d_0$  Resolution.



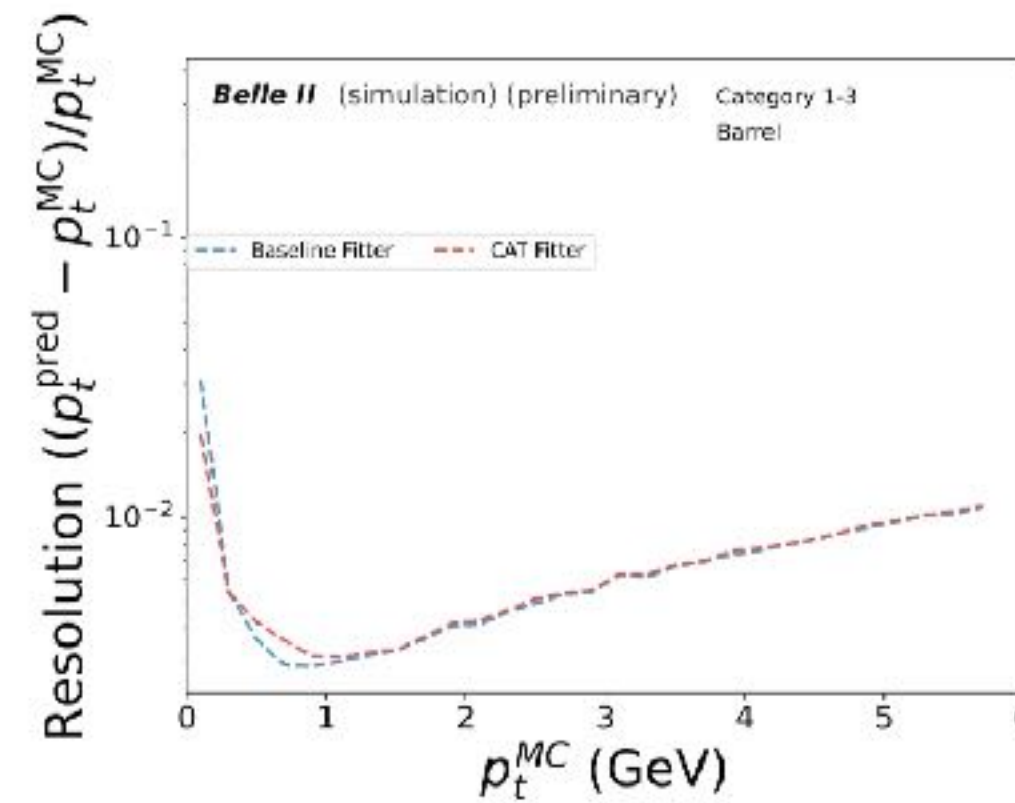
(b)  $d_z$  resolution.



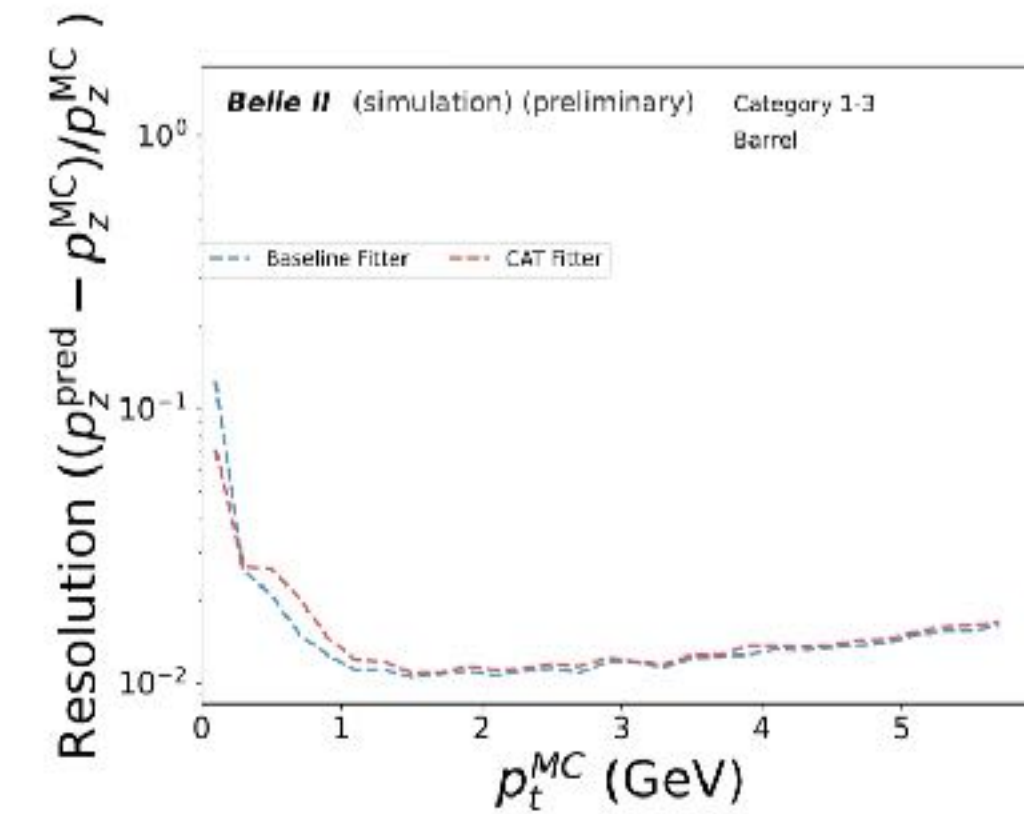
(c)  $\phi$  resolution.



(d)  $\theta$  resolution.



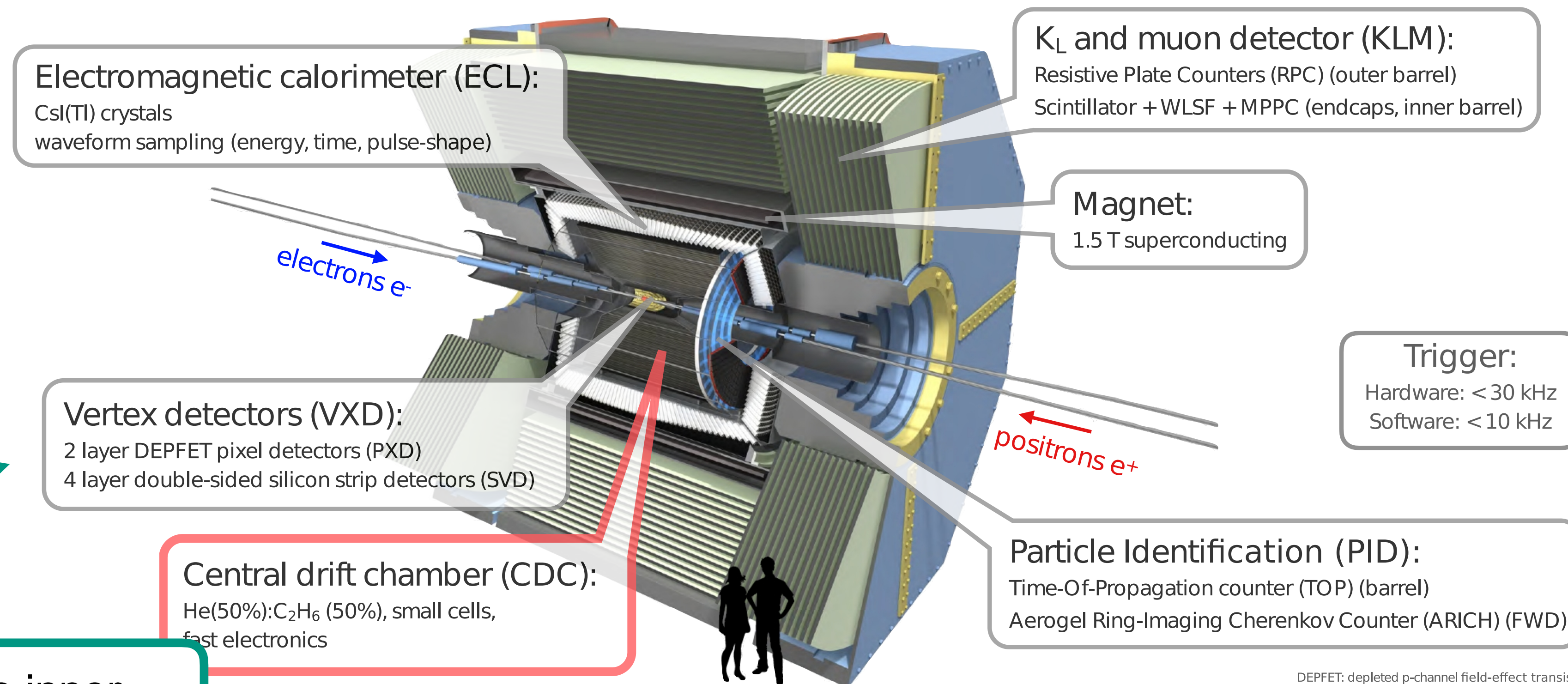
(e) relative  $p_t$  resolution.



(f) relative  $p_z$  resolution.



# Implemented the CAT Finder in the Full Reconstruction algorithm



Extrapolation to inner detectors is working (SVD, PXD)

Extrapolation to outer detectors (KLM, TOP) is working

DEPFET: depleted p-channel field-effect transistor  
WLSF: wavelength-shifting fiber  
MPPC: multi-pixel photon counter

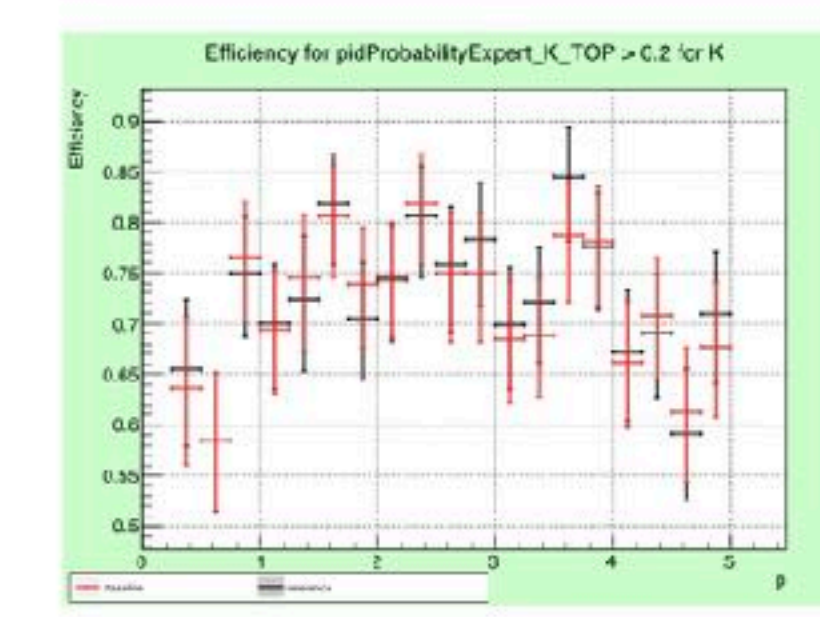
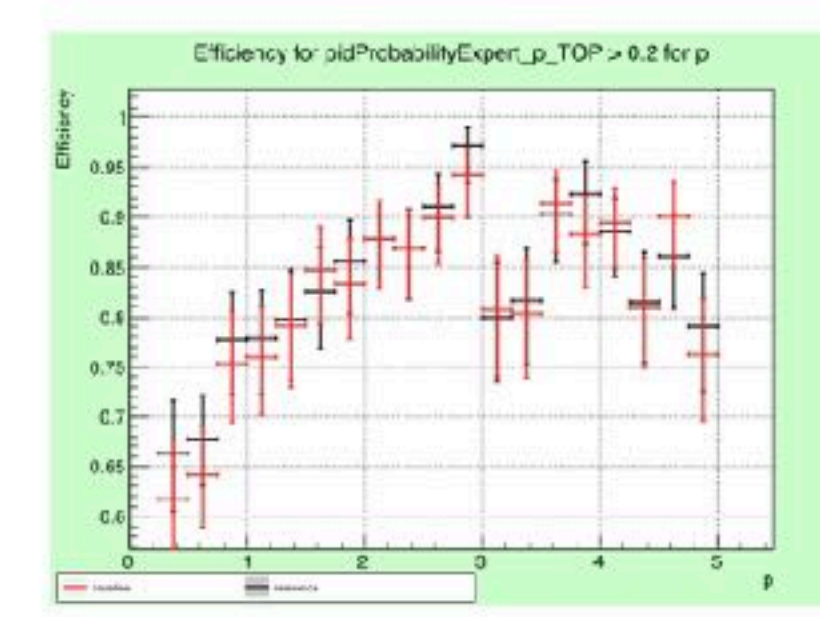
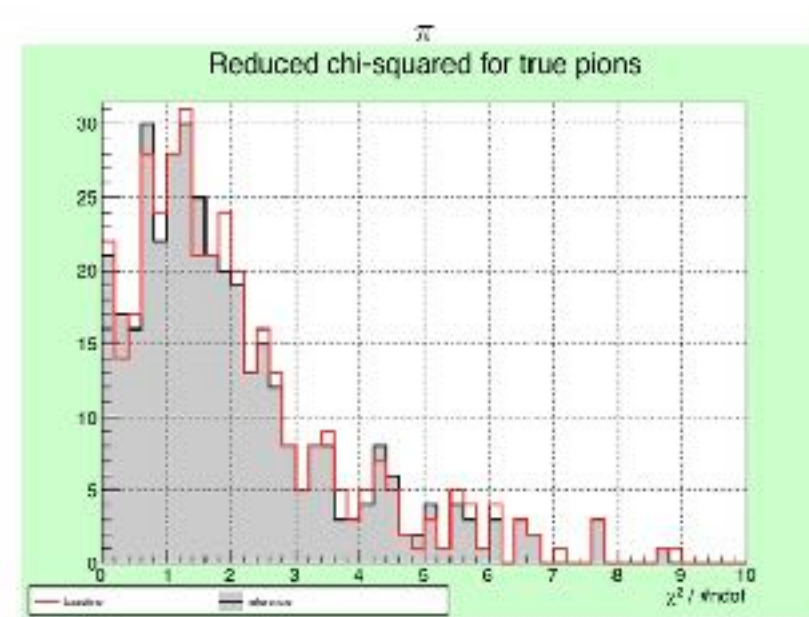
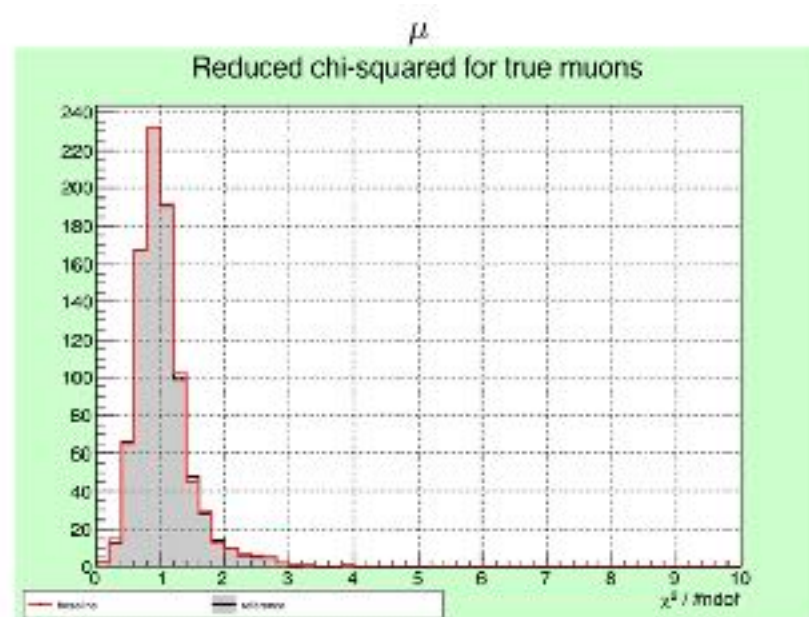


# Implemented the CAT Finder in the Full Reconstruction algorithm - Validation

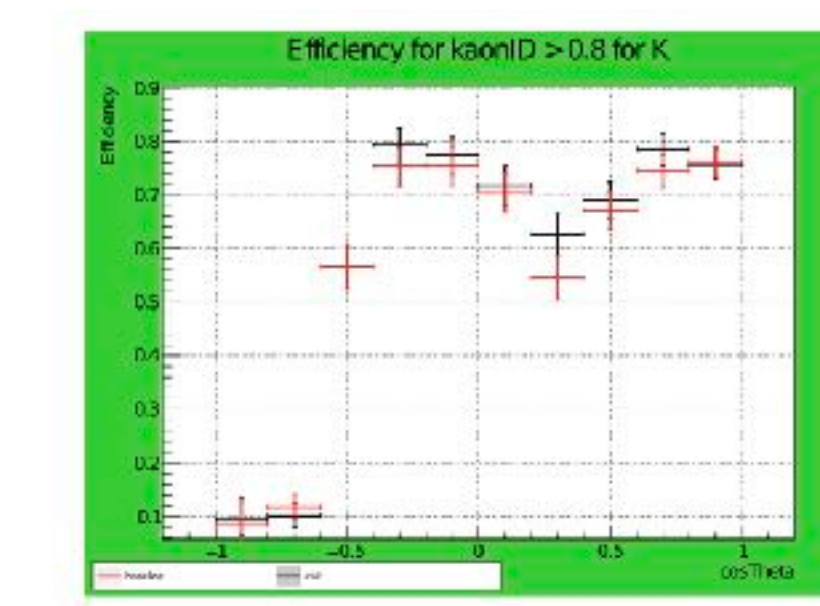
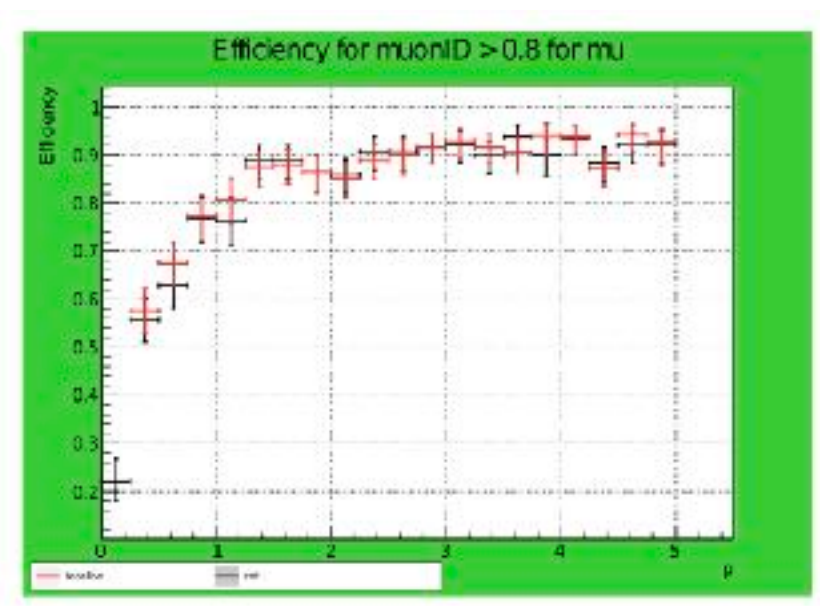
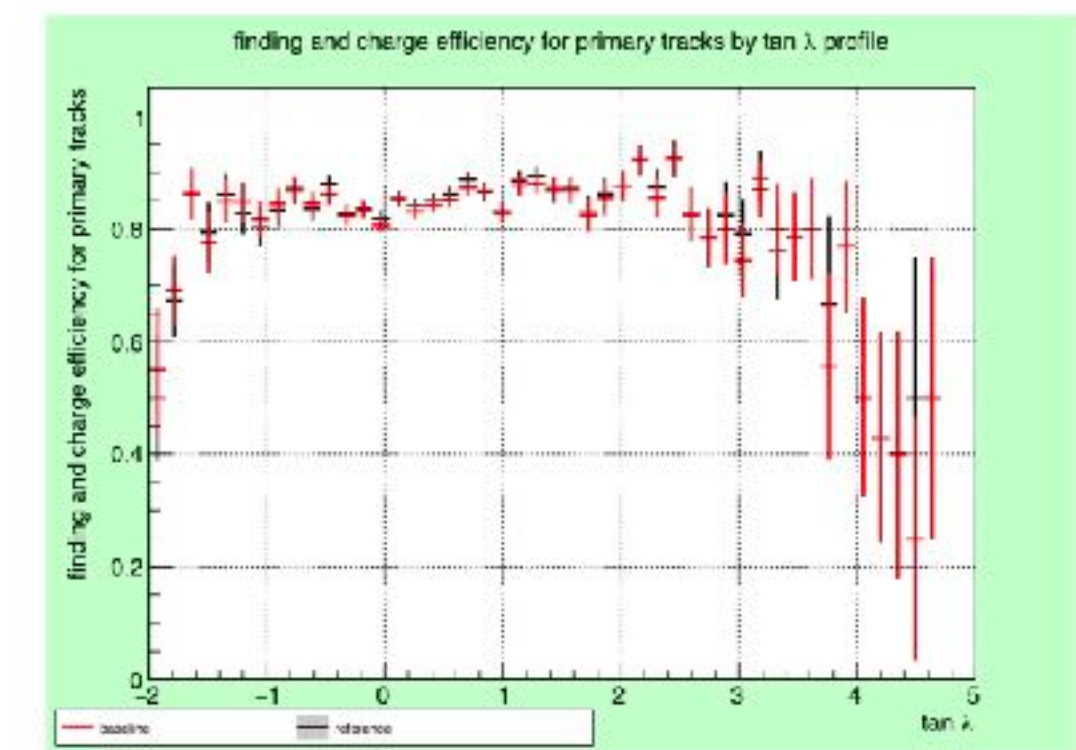
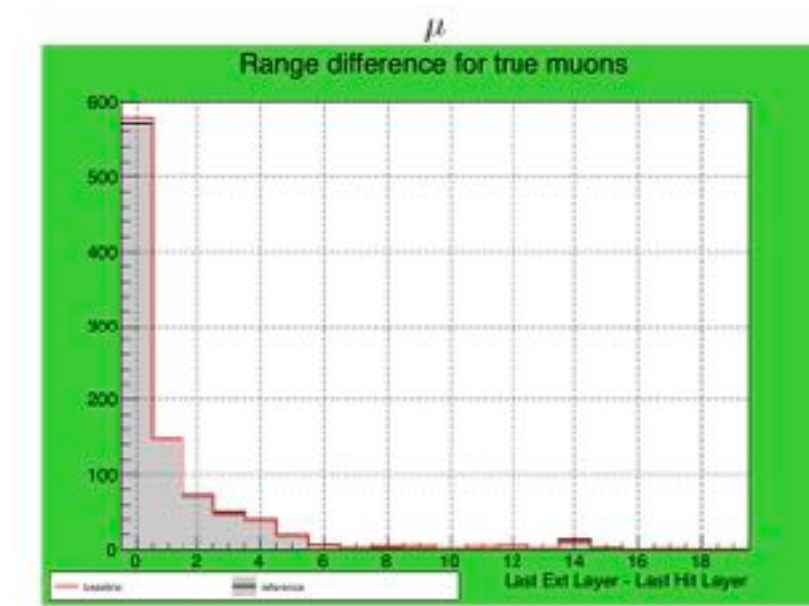
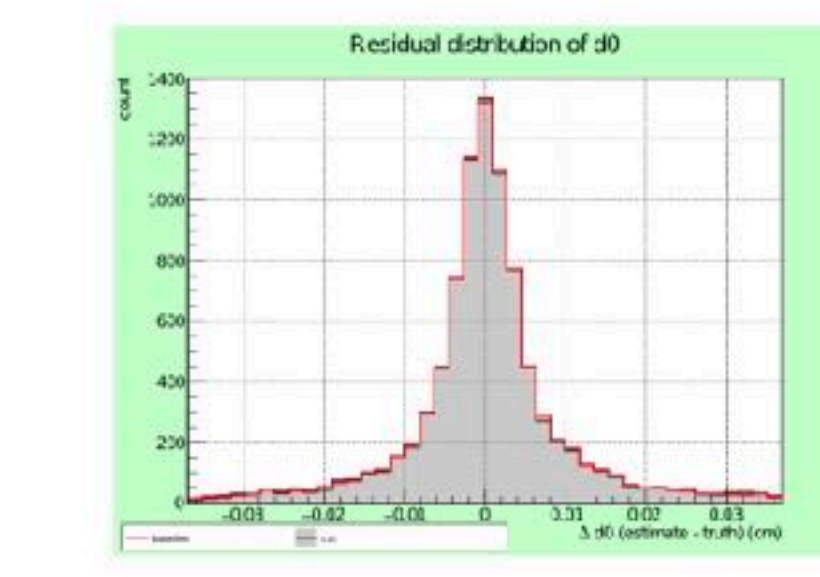
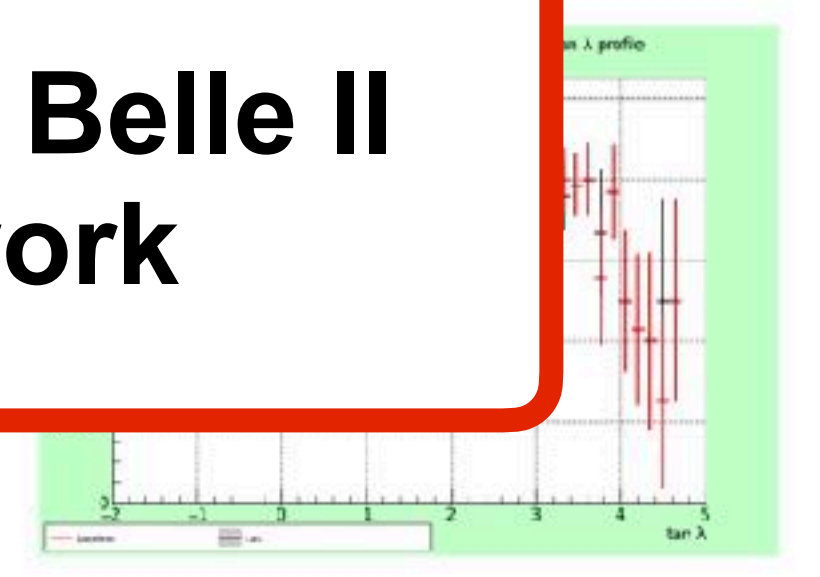
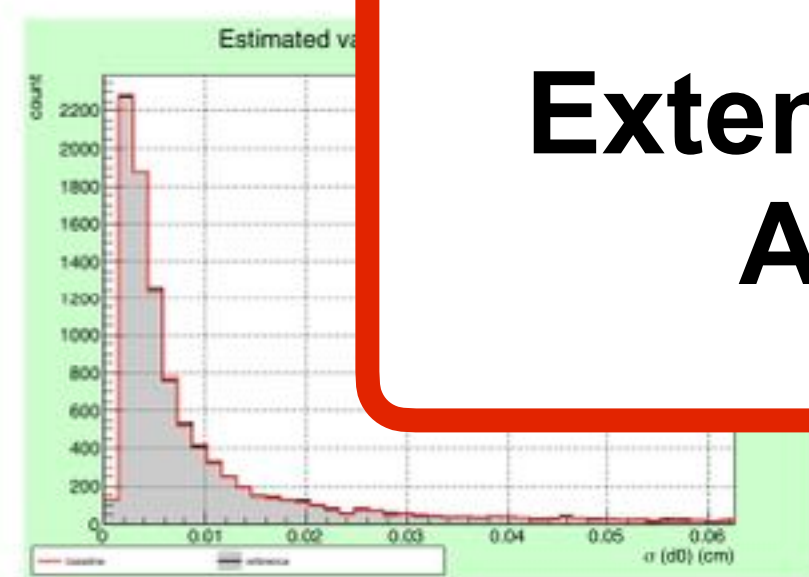
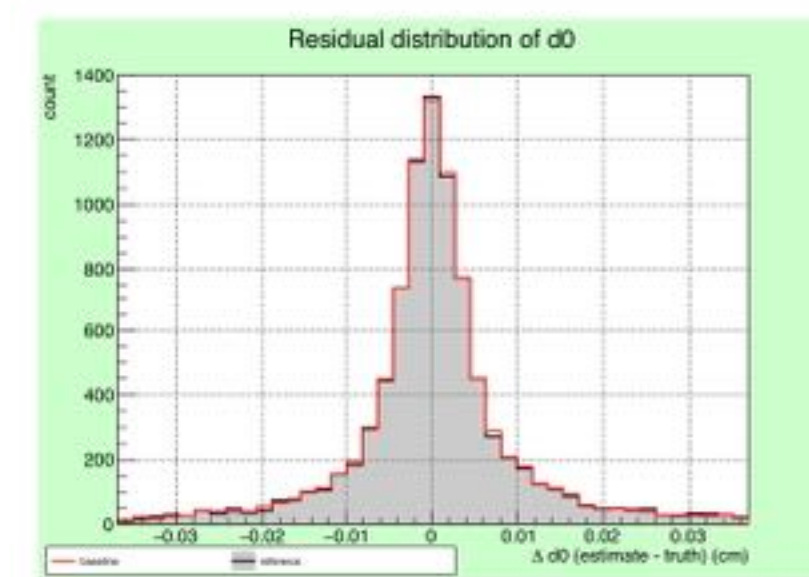
CAT Finder vs Baseline  
cat\_finder VS baseline

KSEff

	-0.7246 %	5.7445 %	9.5226 %	9.6065 %	-0.3171 %
	-3.2213 %	-0.0991 %	9.7743 %	4.2704 %	2.4130 %
	9.8739 %	1.6653 %	6.5992 %	4.3407 %	10.7041 %
	0.0000 %	4.1815 %	8.7288 %	7.4017 %	6.2256 %
	65.5844 %	7.6202 %	7.2699 %	6.2195 %	6.5570 %
	18.9780 %	10.1379 %	4.5910 %	3.5126 %	5.1140 %
	9.6974 %	8.5944 %	3.6692 %	6.2405 %	3.7280 %
	24.2460 %	9.7116 %	2.2043 %	4.7687 %	3.1972 %
	5.6072 %	0.2521 %	5.0162 %	4.1151 %	6.2550 %
	0.0000 %	0.0000 %	0.0000 %	0.0000 %	0.0000 %



**Extensive Validation within our Belle II Analyses Software Framework**





# Summary

- Implemented end-to-end multi-object track reconstruction algorithm using GNNs in official Belle II software
- Applicable to wide kinematic range and overlapping objects, efficiency largely independent from starting position
- Suppress the varying backgrounds and achieve higher purity than baseline algorithm
- Efficiency outperforms baseline algorithm for displaced tracks and has similar performance on tracks coming from the collision point
- Preprint on [arXiv: 2411.13596](https://arxiv.org/abs/2411.13596)



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# Outlook

- Implementation of CAT Finder as C++ module (Currently targeting SOFIE from ROOT)
- Tracking comparison on data ( $K_s^0$  peak in high-multiplicity events)
- Checking performance on analysis  $B \rightarrow KS$

