BESSY III orbit correction scheme layout and performance

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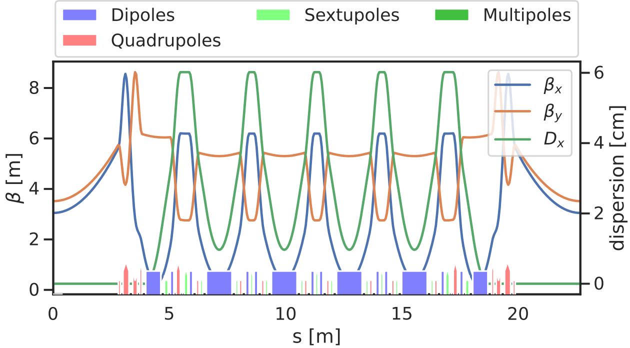
Fourth generation light source BESSY III

BESSY III is a 4th generation light source, located in Berlin, scheduled to start its operation by the mid-2030s.
Currently in its Conceptual Design Report (CDR) phase, BESSY III robustness against misaligments and errors is being carried out.

BESSY III lattice and key parameters



E [GeV]	C [m]	h	V_{RF} [MV]
2.5	362.9	607	1 - 2
ε_x [pm·rad]	σ_z^{RMS} [mm]	σ_δ^{RMS}	α_0
100	3.4	9.8×10^{-4}	$> 10^{-4}$

Table: BESSY III key parameters.

Lattice error propagation and correction

Once a lattice linear and non-linear behavior has been studied and optimized, the next step is to study its robustness including magnetic errors, misalignments, tilts, etc → **realistic lattice**.

To do so, SC[1] (and its python counterpart pySC[2]) can be used to include and propagate various error sources in an AT (or pyAT) lattice as well as providing trajectory/orbit correction algorithms.

SC and pySC are currently foreseen to be used to generate BESSY III error model and simulated commissioning.

[1] Simulated Commissioning website

[2] Python Simulated Commissioning Github

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Closed orbit errors

The presence of misalignments, tilts, and magnetic field errors leads to a distortion of the closed orbit.
Their effects can be expressed as dipolar kicks κ_i around the storage ring:

$$\Delta x(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi\nu)} \sum_{i=1}^{K} \kappa_i \sqrt{\beta(s_i)} \cos\left(|\phi(s) - \phi(s_i)| - \pi\nu\right) \quad (1)$$

Where s is the position along the ring, β is the beta function, ν is the tune, ϕ is the phase advance, and K dipolar kicks are assumed around the ring.

Global orbit correction (1/2)

In a real storage ring, the closed orbit can only be measured at the locations of Beam Position Monitors (BPM). The BPM closed orbit readings at their respective locations ($\overrightarrow{d}$) can now be expressed in a matrix form:

$$\overrightarrow{d} = \mathbf{A}\overrightarrow{\kappa} \tag{2}$$

$$A_{mk} = \frac{\sqrt{\beta_m \beta_k}}{2\sin(\pi\nu)} \cos(|\phi_m - \phi_k| - \pi\nu) \tag{3}$$

Where $\mathbf{A}$ is a matrix of dimensions M × K (# BPM × # dipolar kicks)

Global orbit correction (2/2)

In addition to BPM, a storage ring comprises corrector magnets (CM), which compensate for the closed orbit distortions along the ring. Keeping the same formalism as previously, $\overrightarrow{d}$ can be extended as:

$$\overrightarrow{d'} = \mathbf{A}\overrightarrow{\kappa} + \mathbf{B}\overrightarrow{\theta} \tag{4}$$

$$B_{mn} = \frac{\sqrt{\beta_m \beta_n}}{2\sin(\pi\nu)} \cos(|\phi_m - \phi_n| - \pi\nu) \tag{5}$$

Where $\mathbf{B}$ is a matrix of dimensions M $\times$ N (# BPM $\times$ # correctors)

The goal of orbit correction is to find the optimal corrector strengths $\overrightarrow{\theta}$ to minimize the impact of $\mathbf{A}\overrightarrow{\kappa}$.

Corrector strength calculations

To do so, a SVD decomposition coupled with some regularization technique can be employed. The SVD decomposition over $\mathbf{B}$ yields $\mathbf{B} = \mathbf{USV}^\top$. Here, $\mathbf{S}$ is the diagonal singular value matrix.

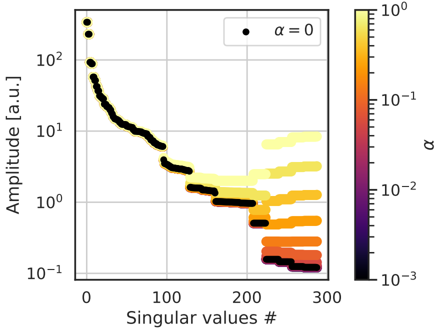
Then, the optimal CM strenghts $\overrightarrow{\theta}$ to minimize the closed-orbit displacements reads: $\overrightarrow{\theta} = \mathbf{VS^{-1}U}^\top \left(\overrightarrow{d} - \overrightarrow{d'} \right)$.

However, low singular values may lead to unwanted noise due to $\mathbf{S^{-1}}$. Two strategies can be applied to overcome this limitation:

- Truncating the matrix $\mathbf{B}$, keeping only the largest singular values
- Applying Tikhonov regularization

Tikhonov regularization

The Tikhonov regularization consists of modifying the $\mathbf{S}^{-1}$ elements to $\tilde{S}_{ii}^{-1} = \frac{\sigma_i}{\sigma_i^2+\alpha^2}$ where σ_i is the ith singular value of the matrix $\mathbf{B}$ and α the regularization parameter.



It effectively circumvents the problems linked to small singular values by artificially increasing the smallest ones and leaving almost untouched the largest ones.

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Critera for efficient orbit correction

$$B_{mn} = \frac{\sqrt{\beta_m \beta_n}}{2\sin(\pi\nu)} \cos(|\phi_m - \phi_n| - \pi\nu)$$

The matrix $\mathbf{B}$ is often referred to as an Orbit Response Matrix (ORM). It quantifies the sensibility of a storage ring to a change in the corrector strengths.

Finding an efficient correction scheme consists of maximizing the elements of $\mathbf{B}$.

Two criteria:

- Maximize both the BPM and corrector beta functions $\beta_m \beta_n$
- Find the optimal phase advance between a BPM and a corrector $\cos(|\phi_m - \phi_n| - \pi\nu)$

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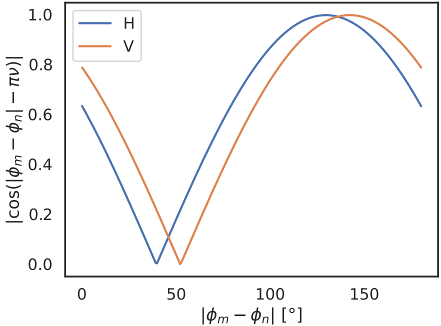
Phase advance criterion



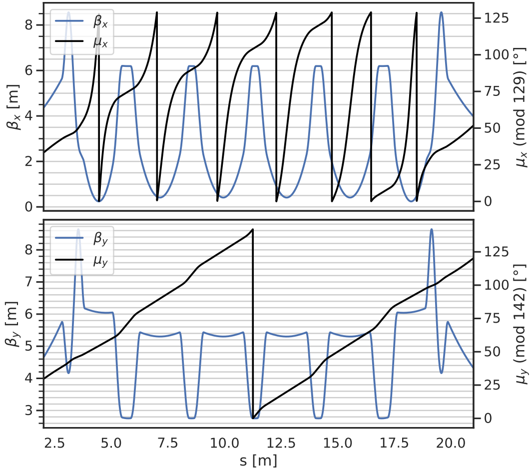
The optimal phase advance difference $|\phi_m - \phi_n|$ is given by the following condition:

$$\frac{|\phi_m - \phi_n| - \pi\nu}{2\pi} = integer \quad (6)$$

For the BESSY III reference lattice, we find 129° in the horizontal plane and 142° in the vertical plane.

⚠ The phase advance criterion depends on the tune itself; thus, changing it can impact the efficiency of an orbit correction scheme.

Beta function criterion



- Ideally, BPM and correctors should be placed where the beta function is maximum and along an isoline in phase advance
- In practice, both conditions are difficult to meet simultaneously

→ **Trade-off needed**

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Enforced lattice constraints

Technical constraints:

- CM are embedded in quadrupoles or sextupoles and BPM are placed next to them (**but** possibility of also having standalone CM → to be studied)
- Try to avoid both planes CM in a single element
- Sufficient space for CM (if standalone) and BPM

Beam dynamics constraints:

- BPM and CM are placed at large β_x, β_y
- Phase advance criterion is followed as closely as possible

Quality metrics

The efficiency of an orbit correction scheme is assessed through its:

- Number of CM and BPM
- Residual RMS orbit offset in:
  - BPM
  - Sextupoles (partly dictate final DA)
  - All elements
- CM strengths

The on and off-momentum dynamic apertures were neglected as they were comparable for most schemes. They are not relevant without complete LOCO corrections.

Error table

The misalignments used in SC/pySC are listed in the table below:

	Δx [μm]	Δy [μm]	Δz [μm]	Roll [μrad]
Small magnets	35	35	0	100
Dipoles	100	100	0	100
BPM	0	0	0	0

In addition, field errors of 1‰ were applied to all magnets.
The BPM and CM calibration errors were neglected in this study.
All errors are generated according to a 2σ truncated Gaussian process.

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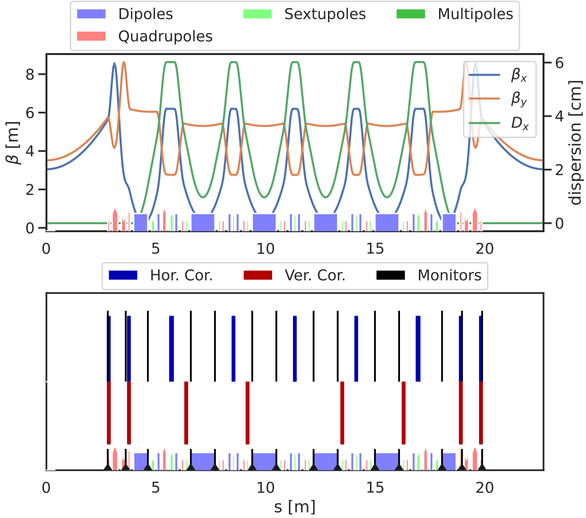
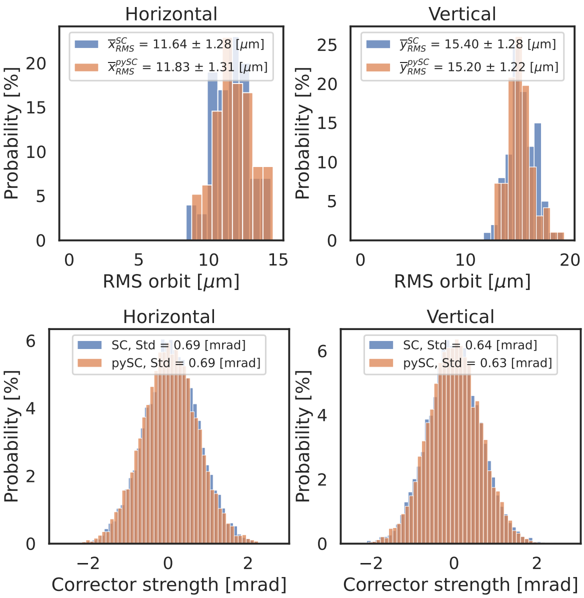
BESSY II-like scheme: BPM and CM layout



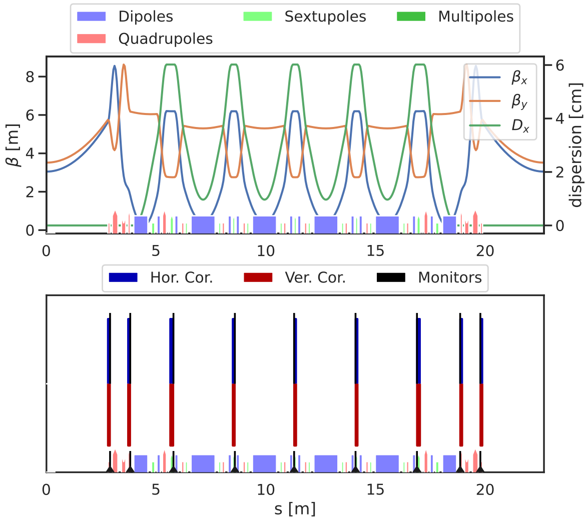
- BPM next to Q[1,4], O1, B2 (14 BPM/superperiod)
- H-corrector in Q[1,4], S[2,5,8] (9 HCM/superperiod)
- V-corrector in Q[1,4], S[3,6] (8 VCM/superperiod)

BESSY II-like scheme: RMS orbit & CM strengths



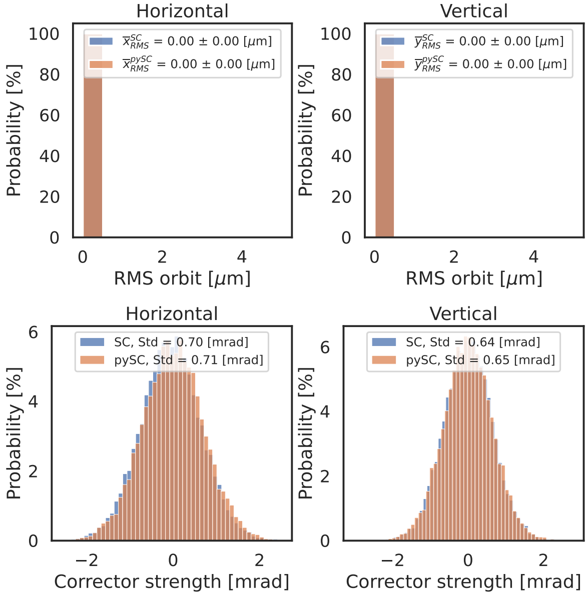
- Sub 15 μm RMS orbit in x and sub 20 μm in y
- Spread out RMS orbit distributions
- Large CM strengths in both planes

SLS-like scheme: BPM and CM layout



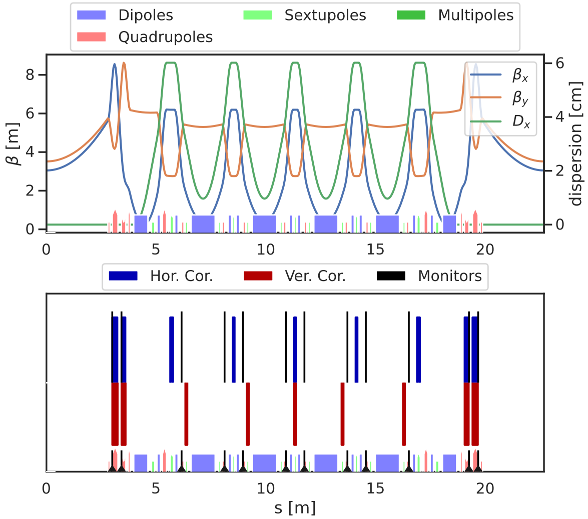
- BPM next to Q[1,4], S[2,5,8] (9 BPM/superperiod)
- H-corrector in Q[1,4], S[2,5,8] (9 HCM/superperiod)
- V-corrector in Q[1,4], S[2,5,8] (9 VCM/superperiod)

SLS-like scheme: RMS orbit & CM strengths



- 0 μm RMS orbit in both planes thanks to square scheme
- Non-zero orbit at sextupoles and all elements
- Large CM strengths in both planes

Current scheme: BPM and CM layout



- BPM next to Q[2,3], Q6 (12 BPM/superperiod)
- H-corrector in Q[2,3], S[2,5,8] (9 HCM/superperiod)
- V-corrector in Q[2,3], S[3,6,8] (9 VCM/superperiod)

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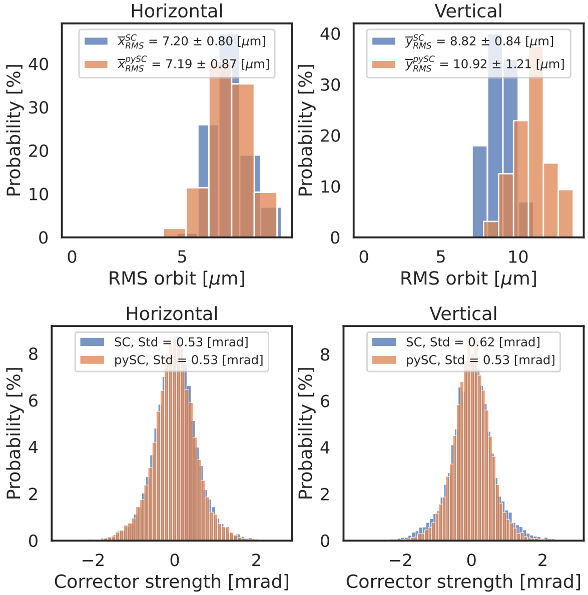
Current scheme: RMS orbit & CM strengths



- Sub 10 μm RMS orbit in x and sub 15 μm in y
- Reduced spread of the RMS orbits
- Small CM strengths in both planes

Summary

		BESSY II-like	SLS-like	Current
RMS orbit at BPM [μm]	x	11.8±1.3	0.0±0.0	7.2±0.9
	y	15.2±1.2	0.0±0.0	10.9±1.2
RMS orbit at sextupoles [μm]	x	30.5±12.2	16.0±1.2	16.1±1.1
	y	22.3±2.0	16.7±1.0	19.0±1.6
RMS orbit at all elements [μm]	x	27.8±10.5	14.4±1.0	15.4±1.1
	y	20.3±1.6	14.8±0.8	17.9±1.6
Std CM strength [mrad]	x	0.69	0.71	0.53
	y	0.63	0.65	0.53
Number of BPM/HCM/VCM		14/9/8	9/9/9	12/9/9

Observations:

- Adequate number and location of BPM → **similar RMS orbit at sextupoles and all elements**
- Square scheme ($n_{BPM} = n_{HCM} = n_{VCM}$) allows zero RMS orbit **at BPM only**
- With current scheme, smallest RMS orbit at sextupoles → **DA conservation** & smallest CM strength → **relaxed technical feasibility**

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Conclusion

- Presentation of physics-based criteria to guide the development of the orbit correction scheme:
  - Maximum $\beta_{x,y}$ at BPM and CM
  - Adequate $\phi_{x,y}$ between BPM and CM
- Capabilities of SVD and Tikhonov regularization approach
- Square orbit correction scheme allows for zero RMS orbit **at BPM but not at sextupoles** ⚠
- Current BESSY III scheme demonstrates good orbit correction at BPM/sextupoles/all elements in both planes while minimizing the CM strengths

Next steps

- Verify orbit correction schemes' performance with stand-alone CM
- Add CM calibration errors and BPM errors
- Perform LOCO/phase advance correction and assess lattice robustness
- Confirm orbit correction schemes' performance in the presence of linear coupling

Thank you for your attention!

Feel free to ask questions now or by email

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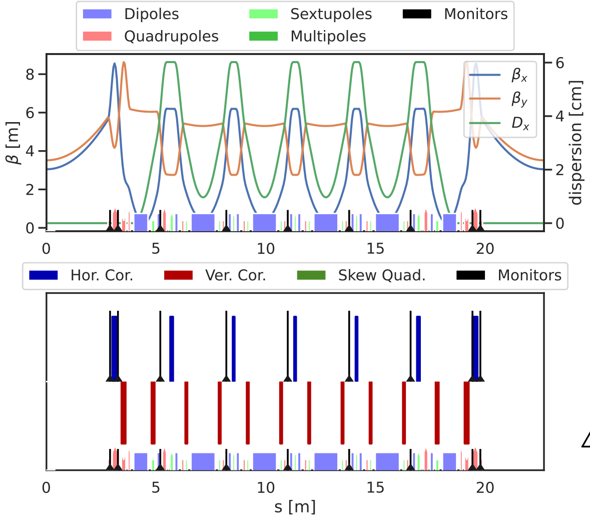
Backup slides

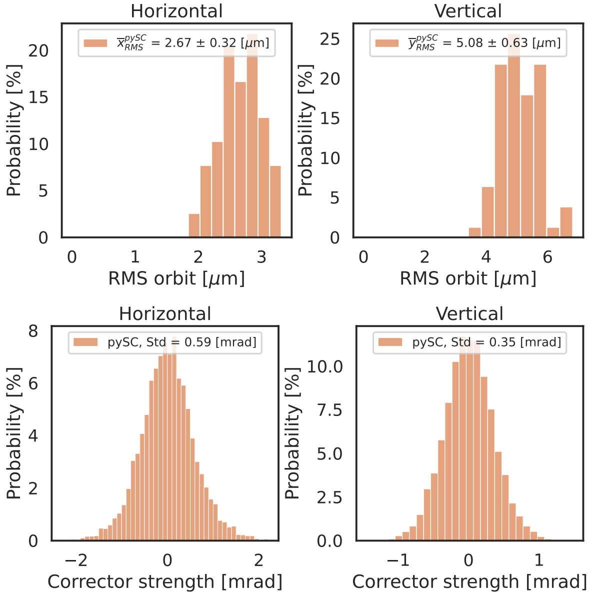
Alternative scheme: BPM and CM layout



- BPM next to Q[1,2], **1st** QB1, QB2 (9 BPM/superperiod)
- H-corrector in Q2, S[2,5,8] (7 HCM/superperiod)
- V-corrector in Q3, S[1,3,4,6,7] (12 VCM/superperiod)

⚠**Broken symmetry**

Alternative scheme: RMS orbit & CM strengths



- Sub 10 μm RMS orbit in both planes
- Slightly worse RMS orbit at sextupoles/all elements than current scheme
- Asymmetric CM strengths in both planes, extremely small in y thanks to the number of VCM

Condition number

In Linear Algebra, the condition number of a matrix M is defined as $C(M) = \frac{\sigma_{max}(M)}{\sigma_{min}(M)}$ where $\sigma(M)$ is a singular value (or eigenvalue for a square matrix) for the matrix M.
It quantifies how well-conditioned a matrix is for inversion → accurate inverse matrix and small numerical errors.

Truncating the number of kept singular values or applying a Tikhonov regularization changes the condition number of a matrix.