

I c e C u b e



EmCa

Electromagnetic Cascades

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KIT 13.Dec.18

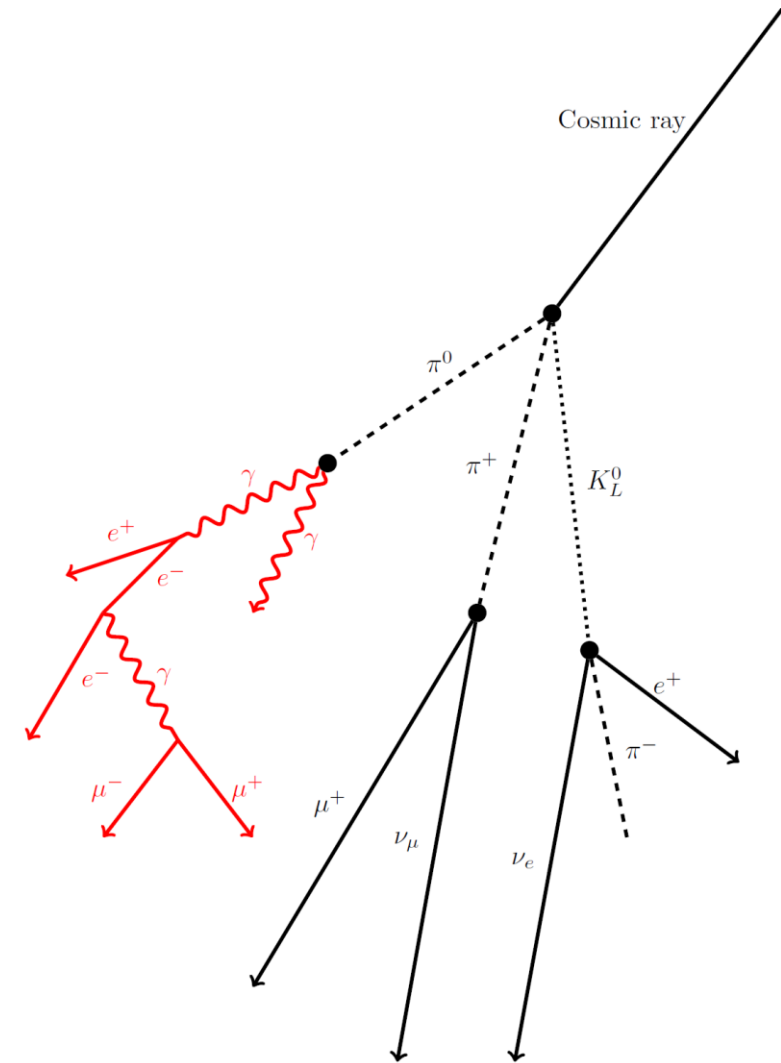
SFB 1258

Neutrinos
Dark Matter
Messengers



Contents

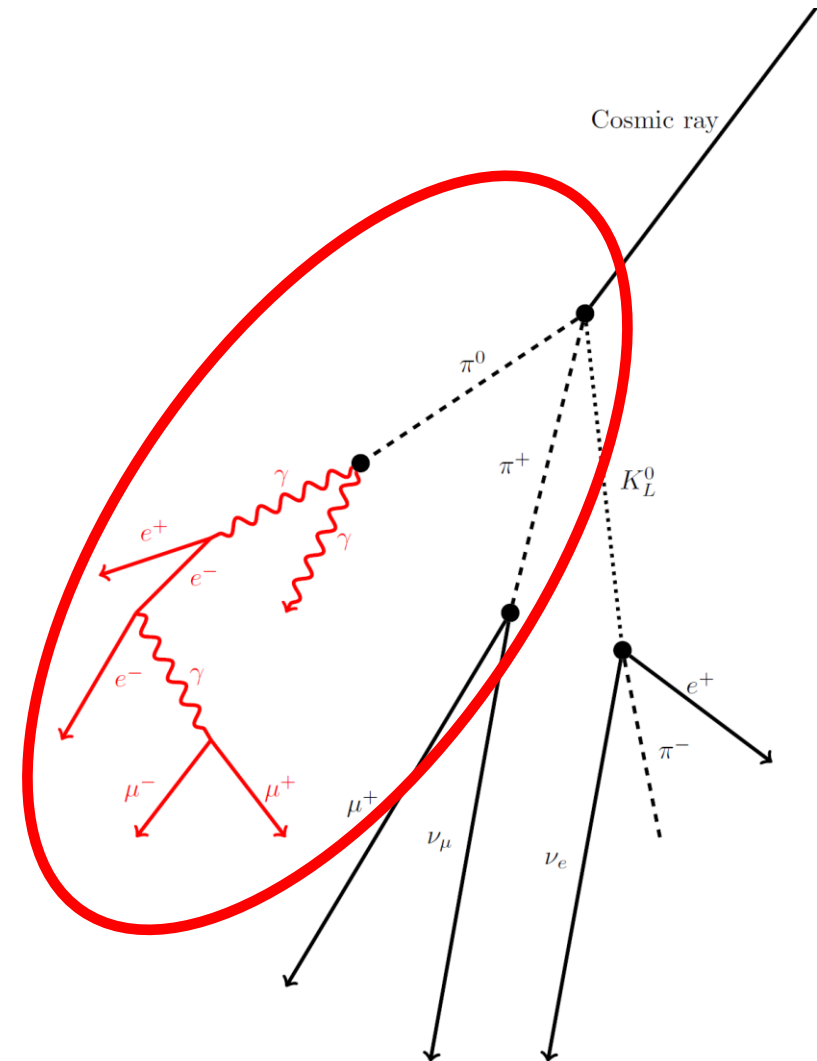
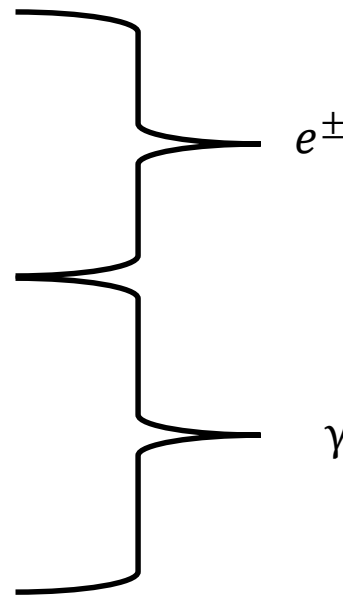
- EmCa Crash Course
 - Electromagnetic Showers
 - Cross-Checks
 - Current Status
 - Some technical stuff



Electromagnetic Cascade

- Relevant interactions

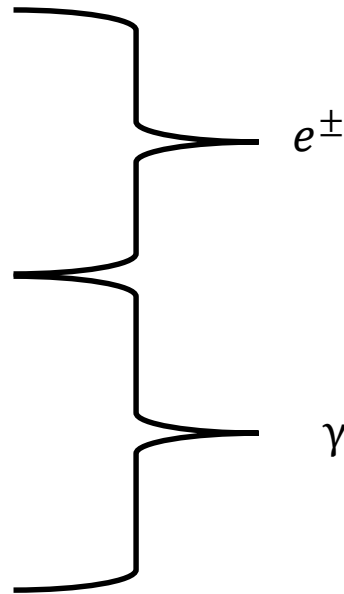
- Bremsstrahlung
- Ionization
- Moller
- Bhabha
- Pair production
- Photoelectric effect
- Rayleigh
- Compton



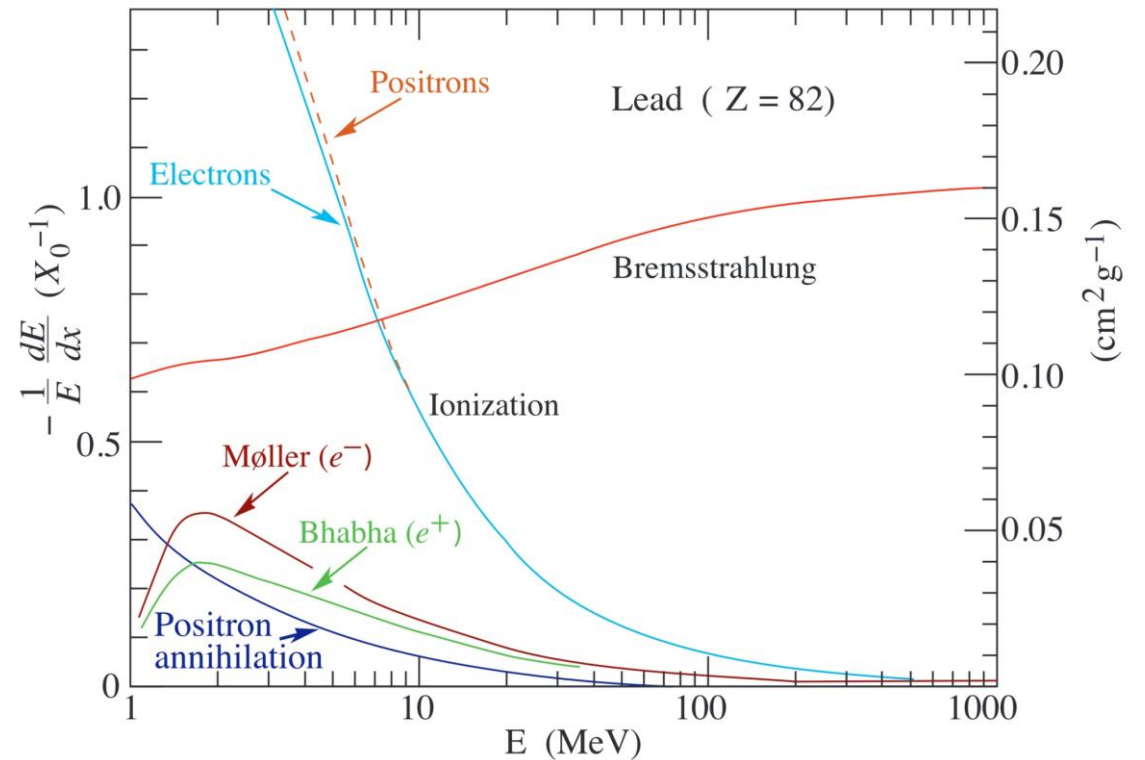
Electromagnetic Cascade

- Relevant interactions

- Bremsstrahlung
- Ionization
- ~~• Møller~~
- ~~• Bhabha~~
- Pair production
- Photoelectric effect
- Rayleigh
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Energy losses electron

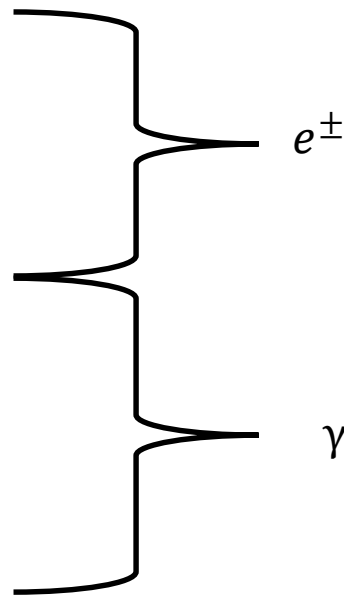


<http://pdg.lbl.gov/2014/reviews/rpp2014-rev-passage-particles-matter.pdf>

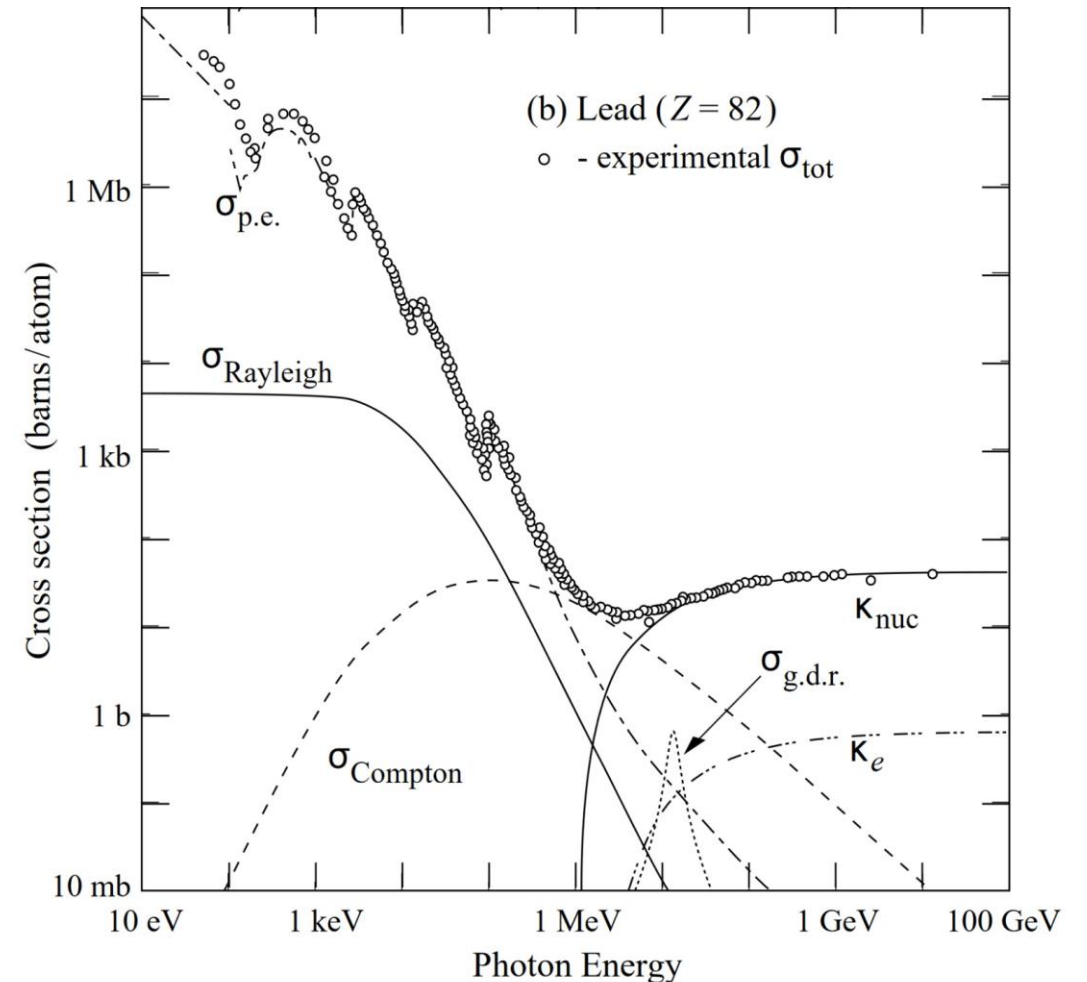
Electromagnetic Cascade

- Relevant interactions

- Bremsstrahlung
- Ionization
- ~~• Moller~~
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Energy losses photon



<http://pdg.lbl.gov/2014/reviews/rpp2014-rev-passage-particles-matter.pdf>

Available Cross-Sections: Bremsstrahlung (and Pair)



Data/Paper	Information	Energy range	Atomic Number Z	Remarks
Bethe and Heitler (1934,1944)	ω $\omega, \Theta_i, \Theta_f, \Phi$	1 keV - 1 GeV	7,8	energy range depends on Z
Koch and Motz (1959)	Total ω ω, Θ_i, Φ $\omega, \Theta_i, \Theta_f, \Phi$	different lower bounds, no upper bounds	depends on the used formulae	
Aiginger (1966)	ω, Θ_i	180, 380 keV	79, Al ₂ O ₃	experimental
Elwert and Haug (1969)	ω, Θ_i $\omega, \Theta_i, \Theta_f, \Phi$	keV range	13,79	
Penczynski and Wehner (1970)	ω, Θ_i	(300 ± 10) keV	82	experimental
Tseng and Pratt (1971)	$\omega, \Theta_i, \Theta_f, \Phi$	keV, MeV range	13,79	
Fink and Pratt (1973)	ω $\omega, \Theta_i, \Theta_f, \Phi$	keV, MeV range	6,13,79,92	also for pair production
Tsai (1974,1977)	ω, Θ_i	> few 10 MeV	all	
Seltzer and Berger (1985)	ω	1 keV - 10 GeV	Z=6,13,29,47,74,92	
EEDL (1991)	Total	5 eV - 1 TeV	all	see (Cullen et al., 1991)
Nackel (1994)	ω, Θ_i	keV	6,29,47,79	only twodimensional description
Schaffer et al. (1996)	ω, Θ_i, Φ	keV range	6,13,29,47,74,92	
Schaffer and Pratt (1997)	$\omega, \Theta_i, \Theta_f, \Phi$ ω, Θ_i	keV range	47,53,60,68,79	
Lehtinen (2000)	ω, Θ_i	1 keV - 1 GeV	7,8	Simple product ansatz for angular and frequency part
Geant 4 (2003)	Total ω Θ_i	5 eV - 1 TeV 1 keV - 10 GeV > few 10 MeV	all 6,13,29,47,74,92 all	based on EEDL based on Seltzer and Berger (1985) based on Tsai (1974, 1977)

+ Migdal

ω, θ_i

> 10MeV

All

<https://arxiv.org/pdf/1202.4879.pdf>

Cross - Sections

Bethe-Heitler:

- Used in EGS
- Corrections:
 - Coulomb
 - Screening
 - Empirical Corr.

Tsai:

- Used in Geant4
- Corrections:
 - Coulomb
 - Screening

Migdal:

- Full Screening
- LPM effect

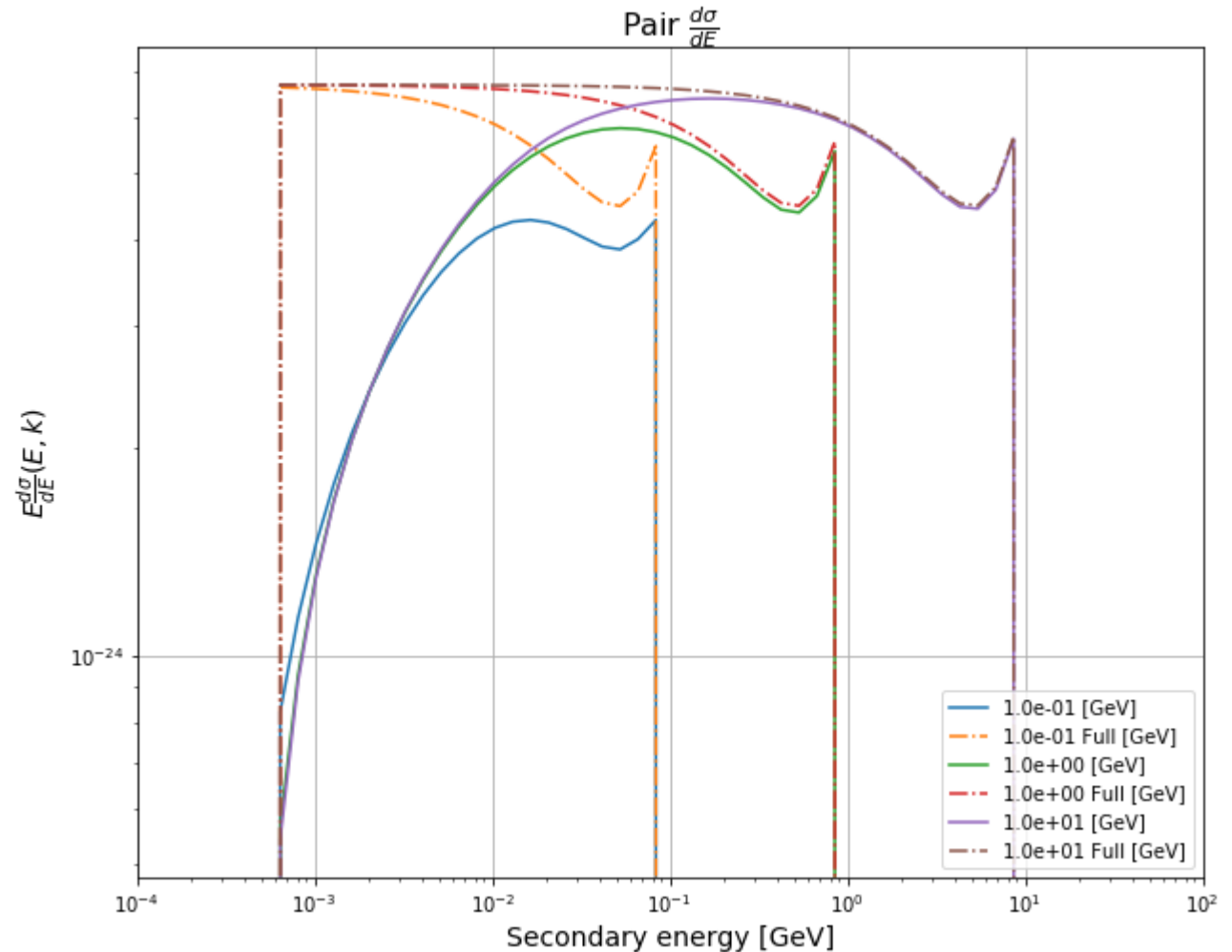
Need to include:

- Dielectric effect
- LPM effect

Atomic model: Thomas-Fermi-Moliere model

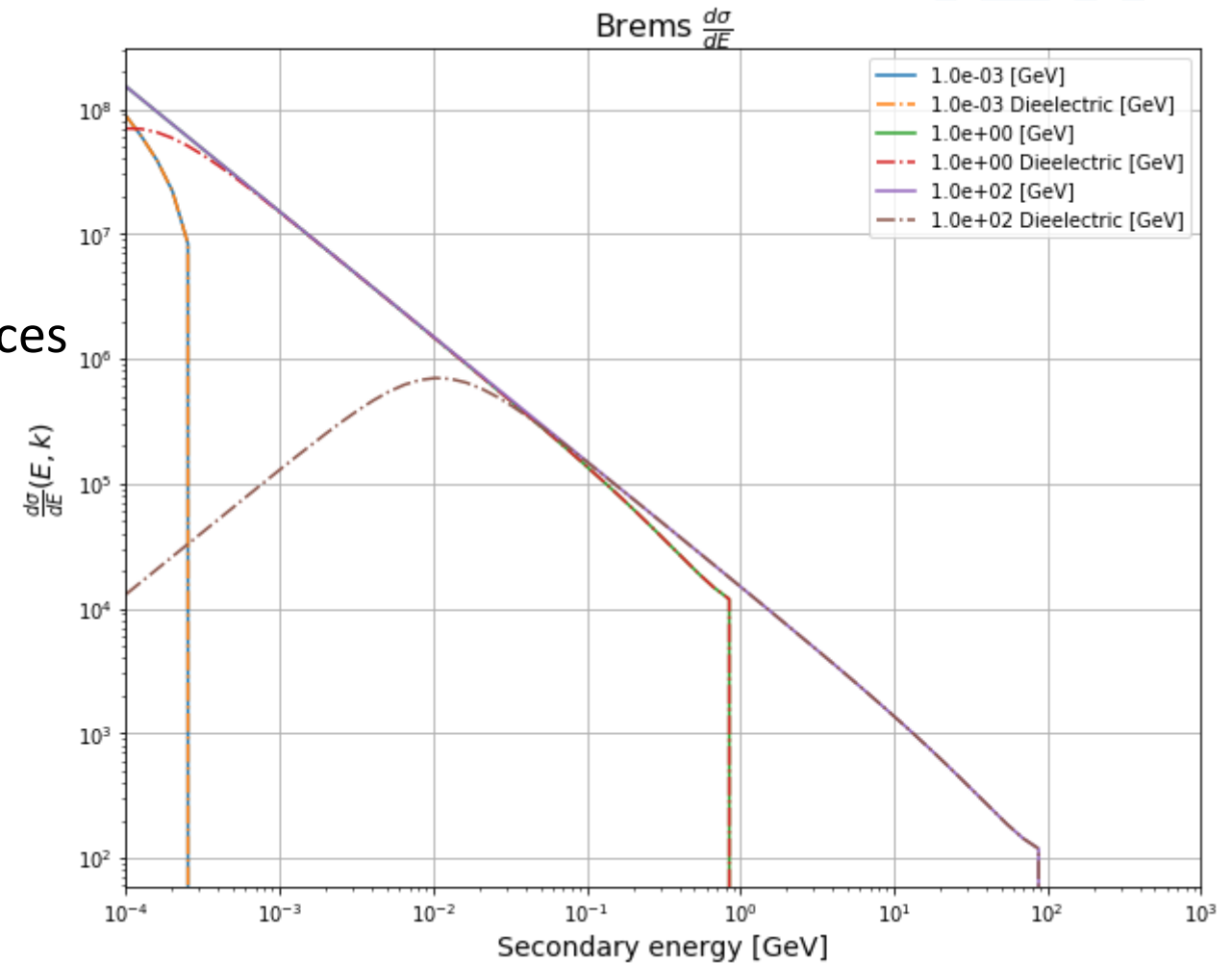
Screening

- Electron cloud screens nucleus
 - Bethe-Heitler: Correction term
 - Tsai: Included
 - Migdal: Full Screening Approx.
- Effect at $k < 10$ GeV
 - Predominantly pair production



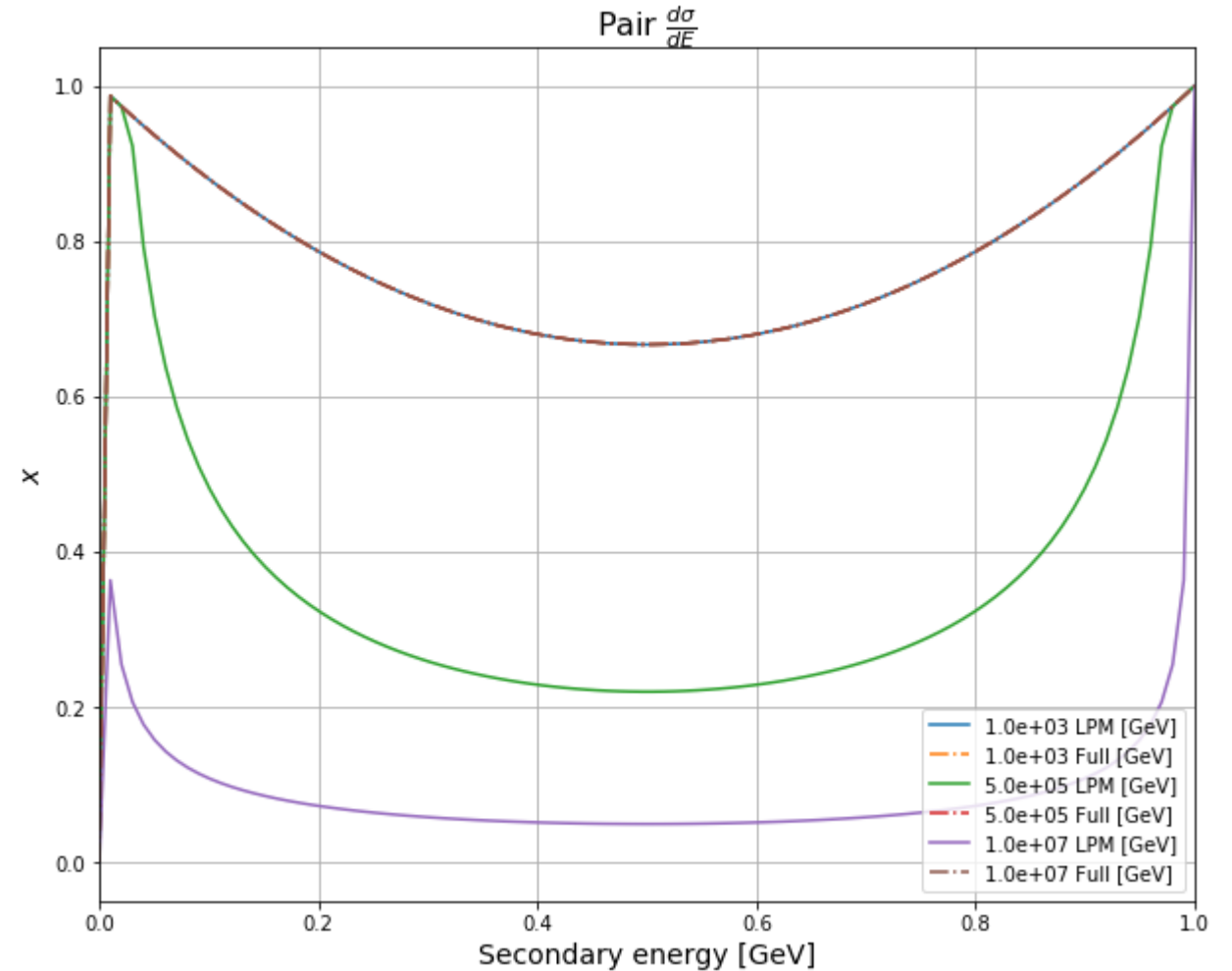
Dielectric Effect

- Material Effect
 - Emission takes place over long distances (High energy e to low energy γ)
 - Depends on the plasma frequency
 - In metals relevant at $\approx 10^{-4}$ *Prim*
- The cross-section no longer diverges!

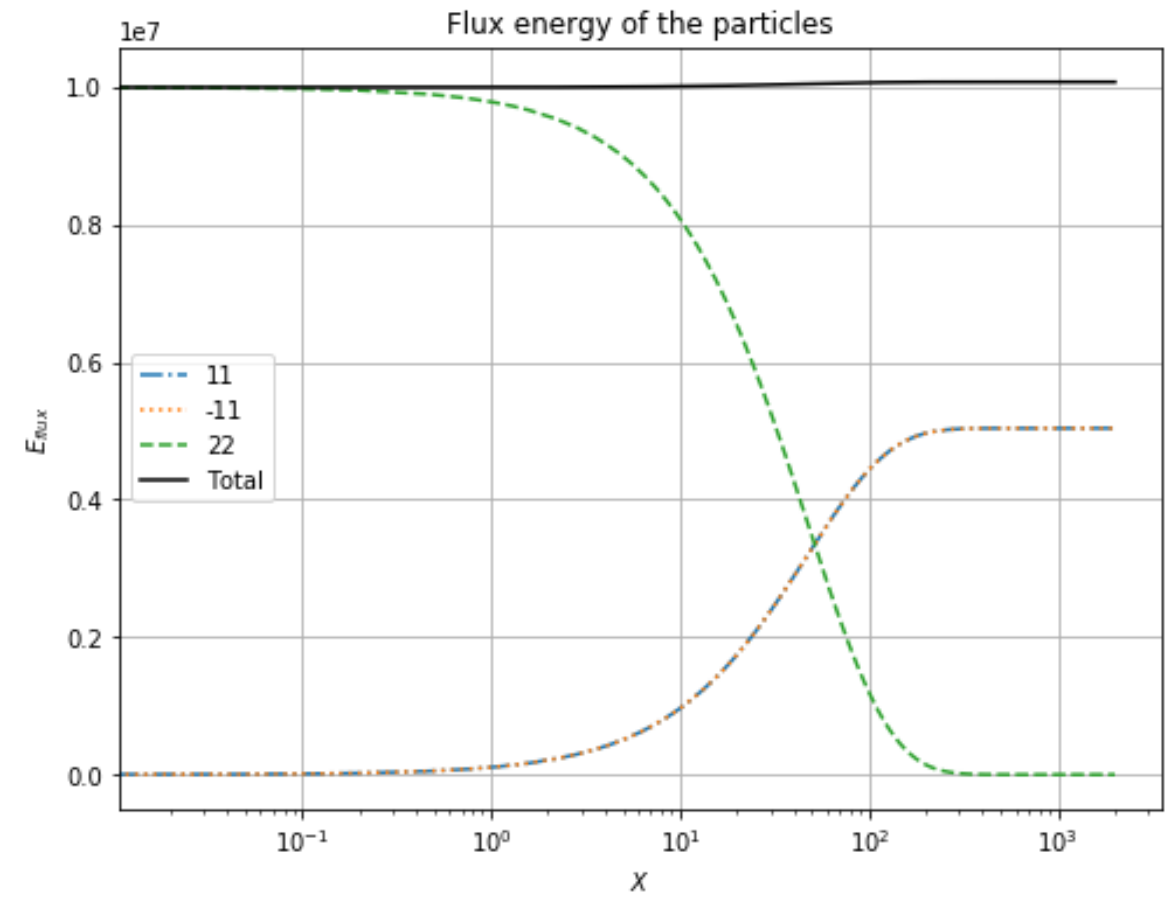
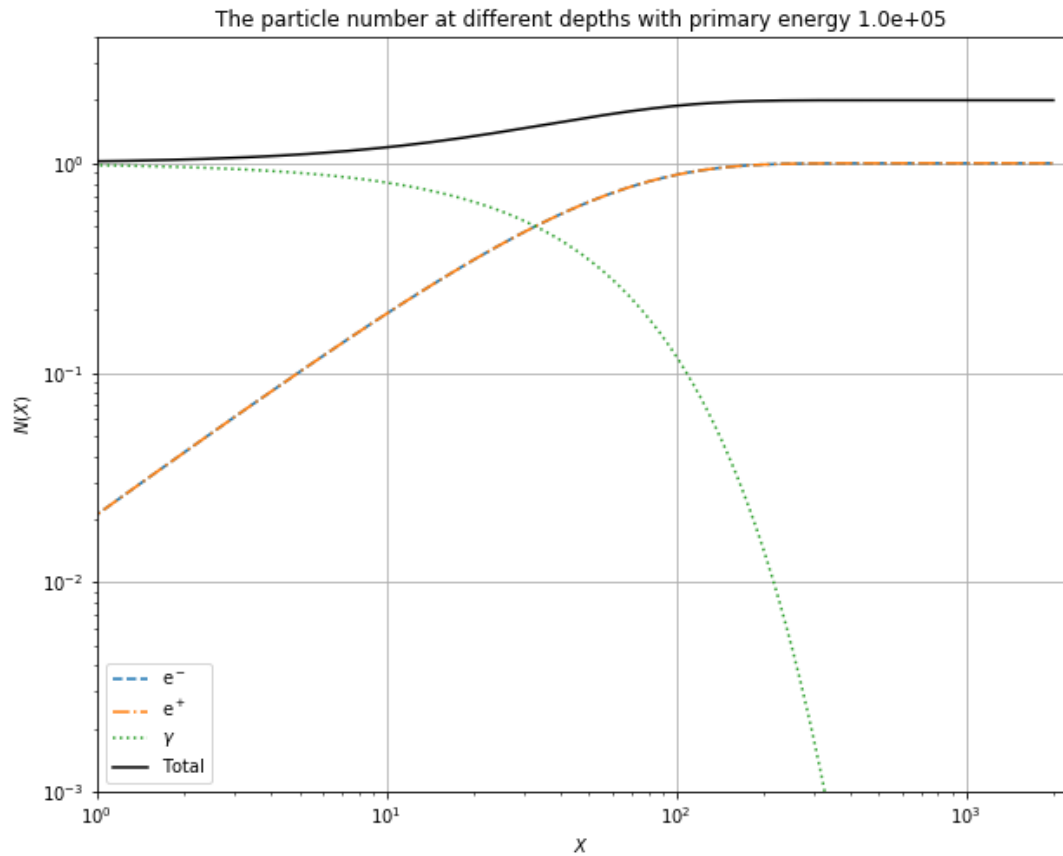


LPM Effect

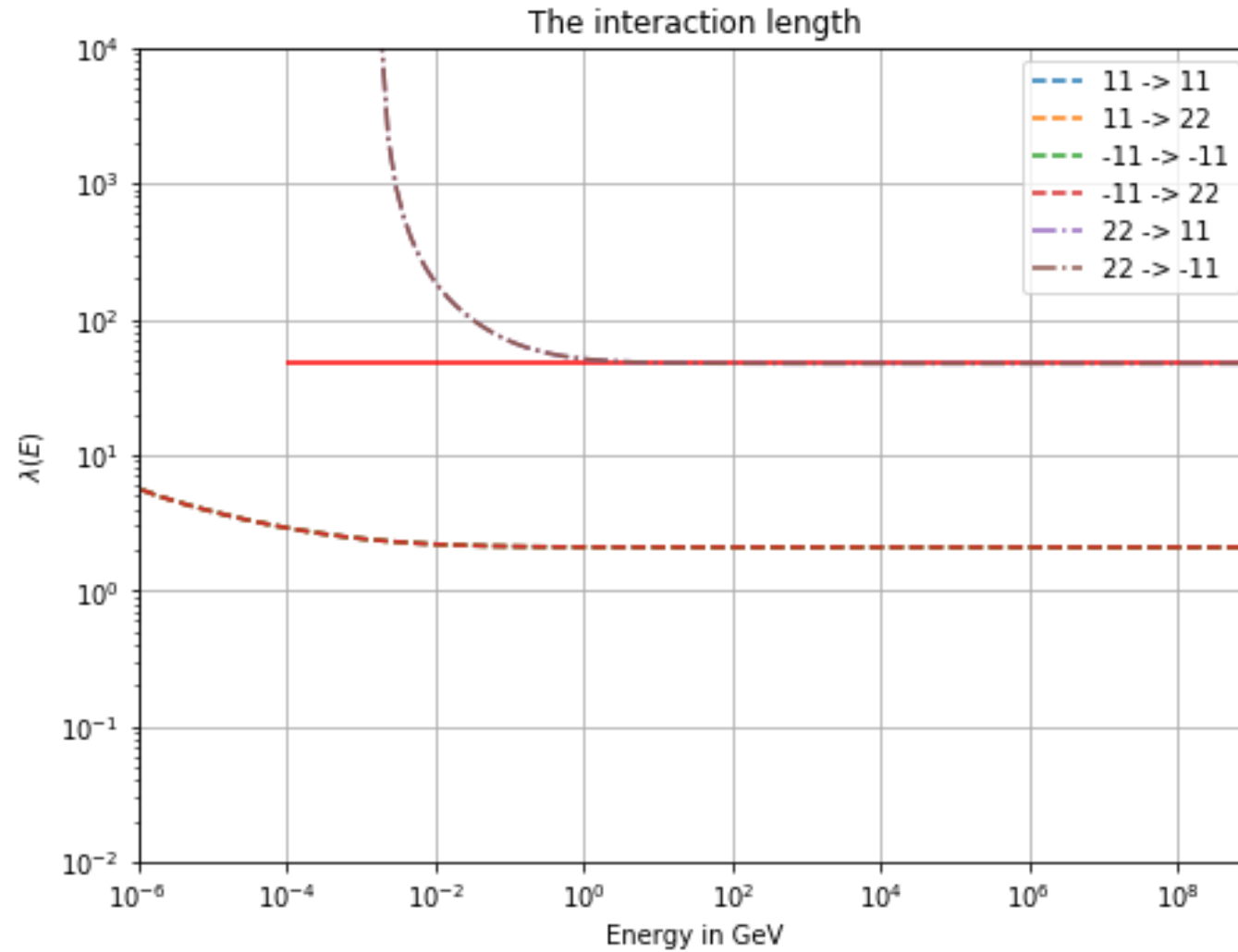
- Material and density dependent
 - Interaction vertices closer than reformation length
 - Scales with $\frac{1}{\sqrt{\rho}}$
- Relevant for high energies
 - Above E_{LPM}
- Here an approximation



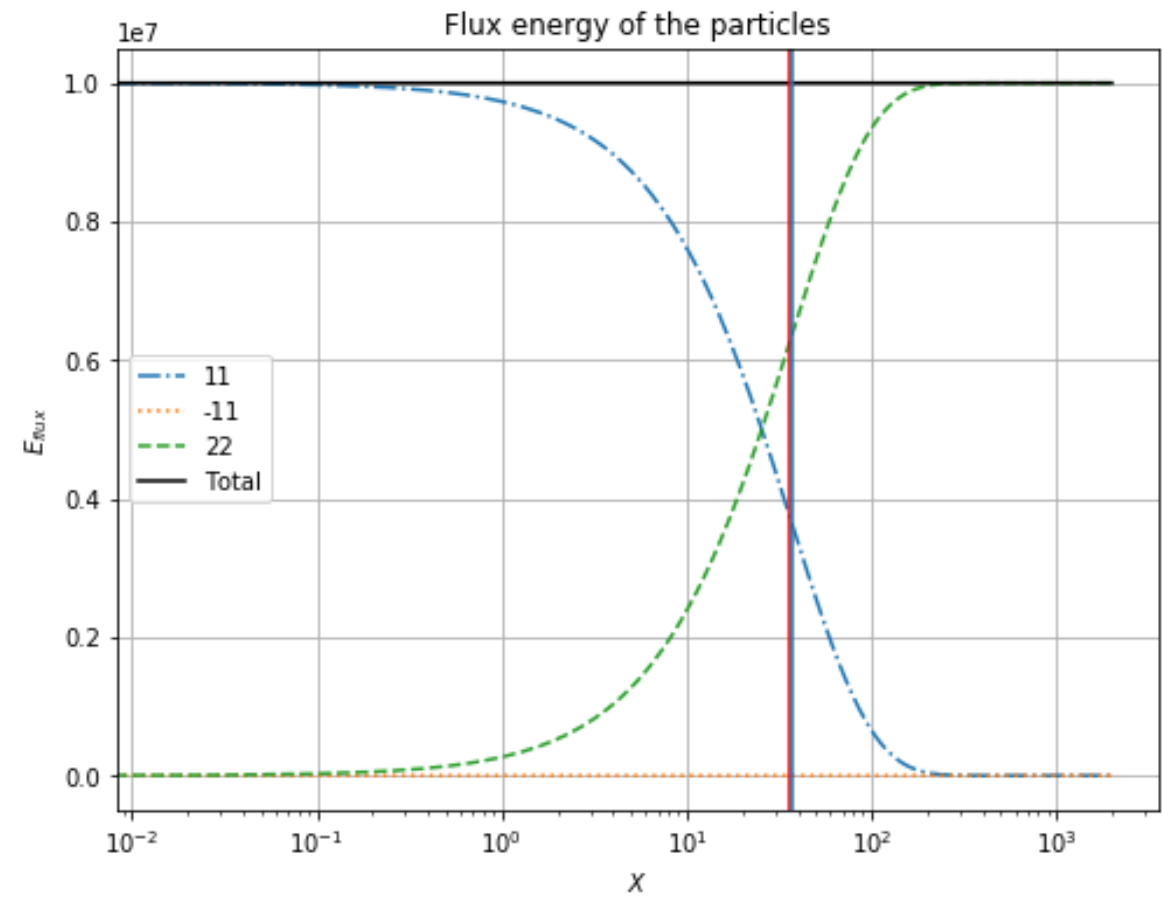
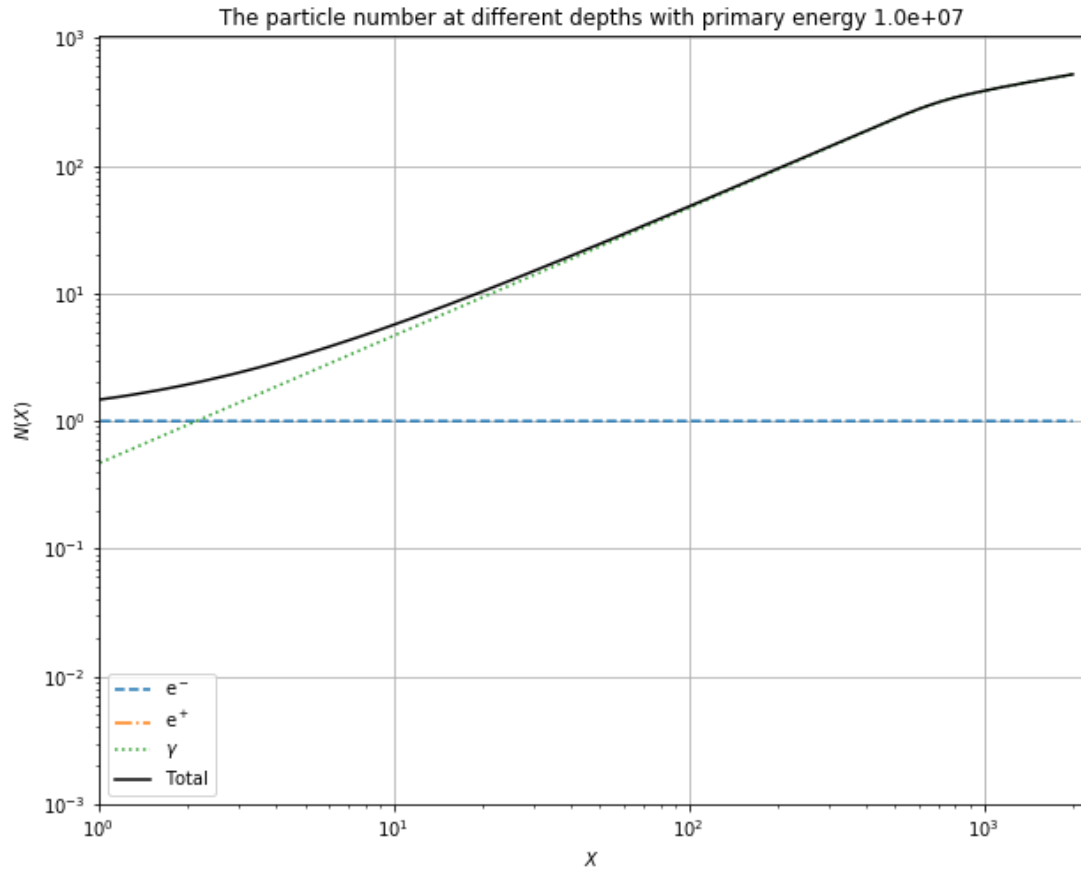
Cross-Checks - Pair



Cross-Checks – Pair II



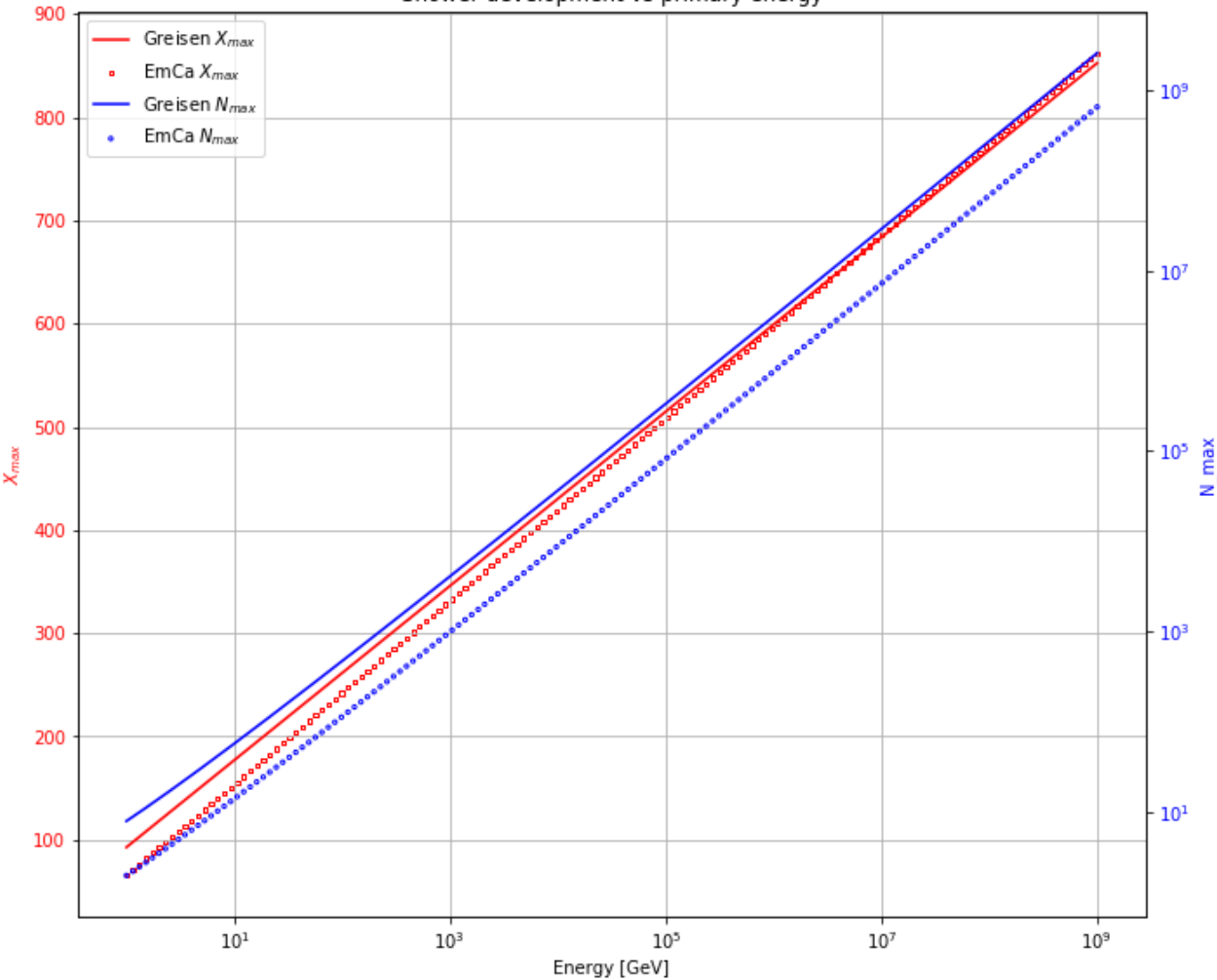
Cross-Checks - Bremsstrahlung



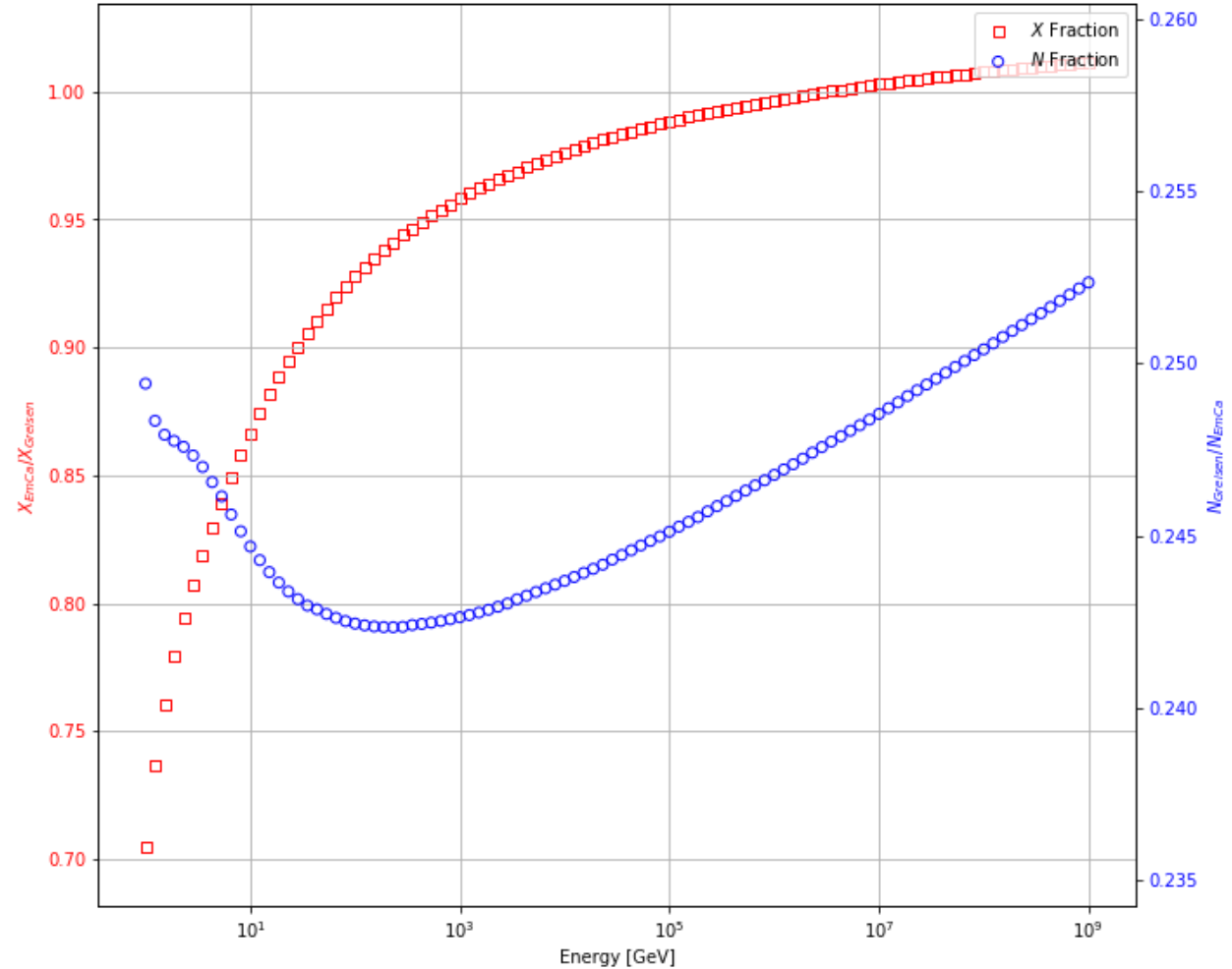
Cross-Checks – Shower Development



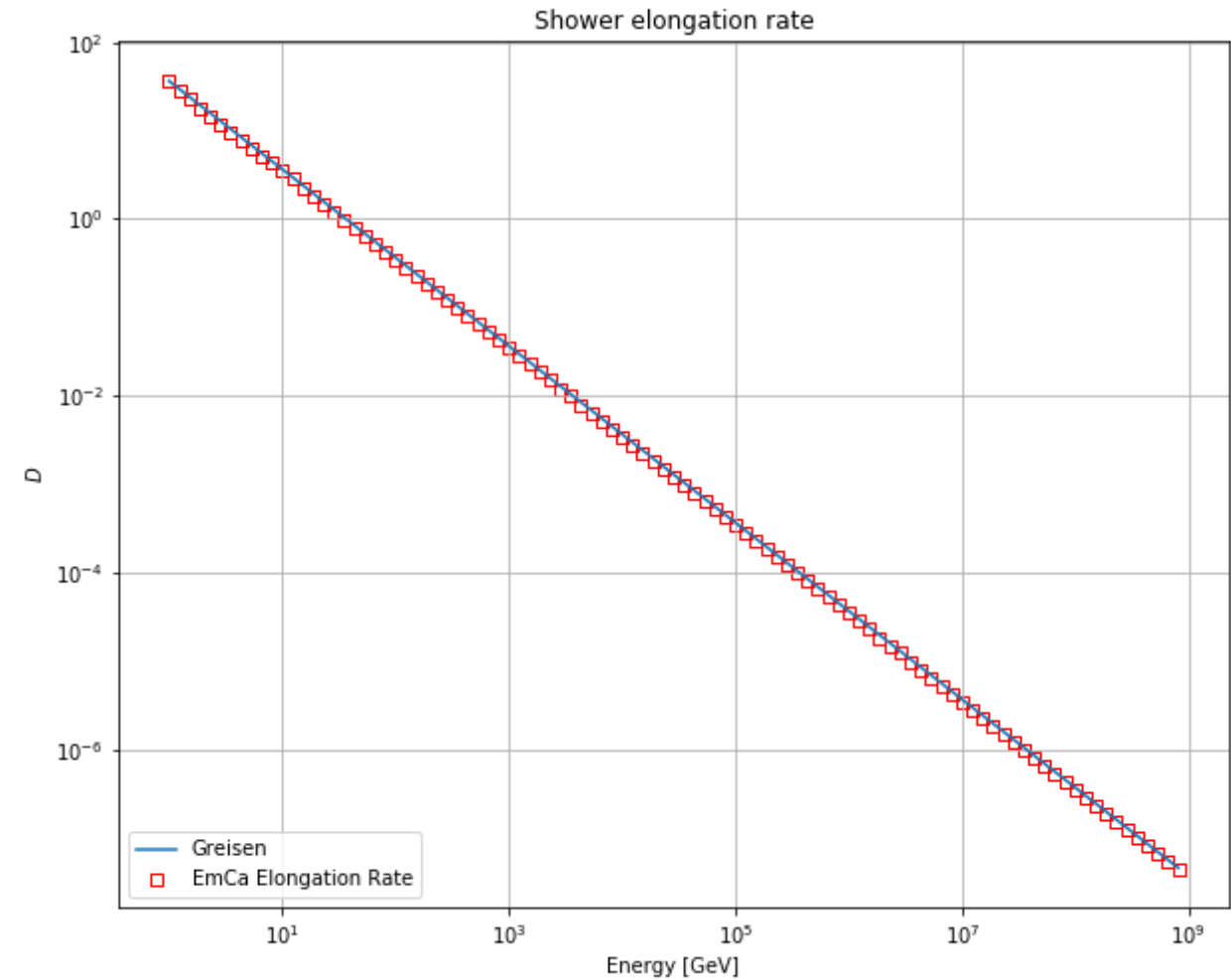
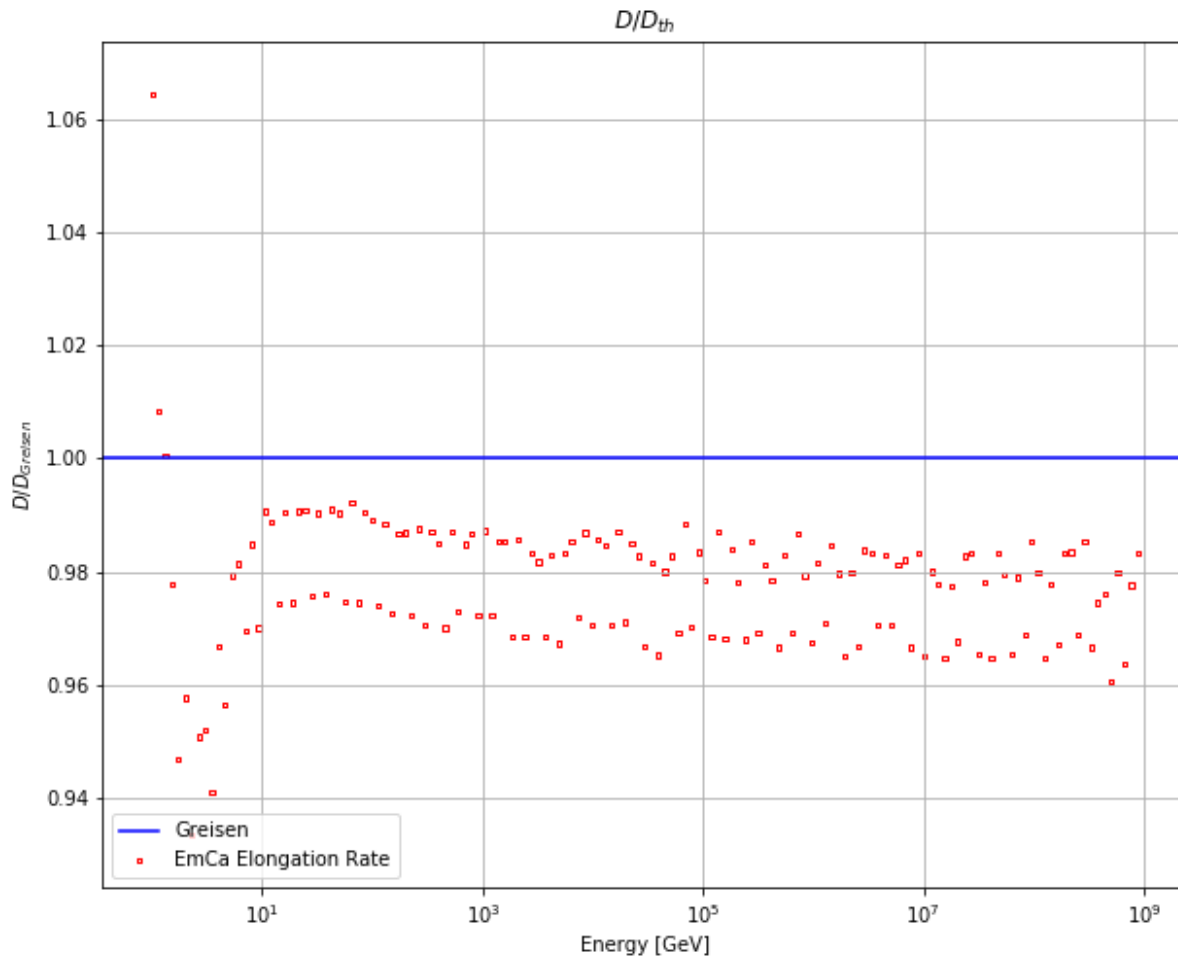
Shower development vs primary energy



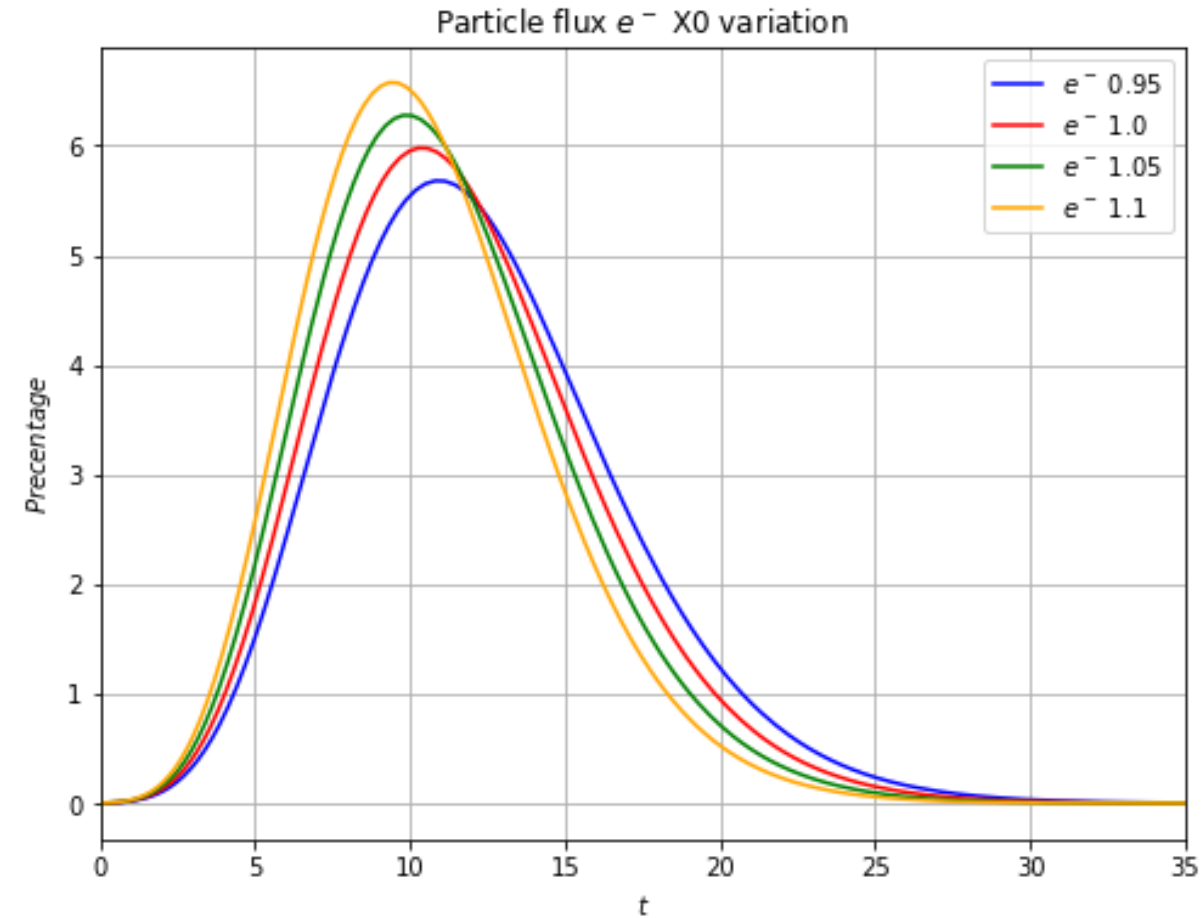
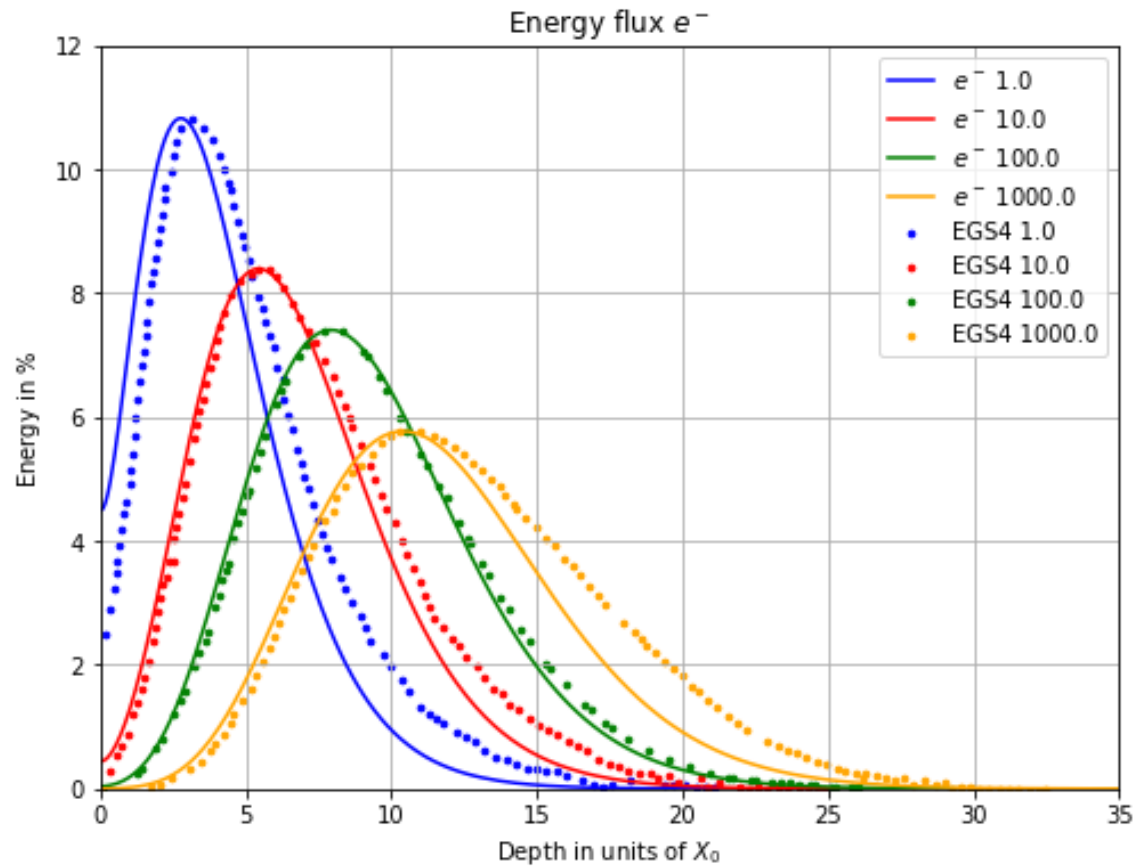
Shower development discrepancy



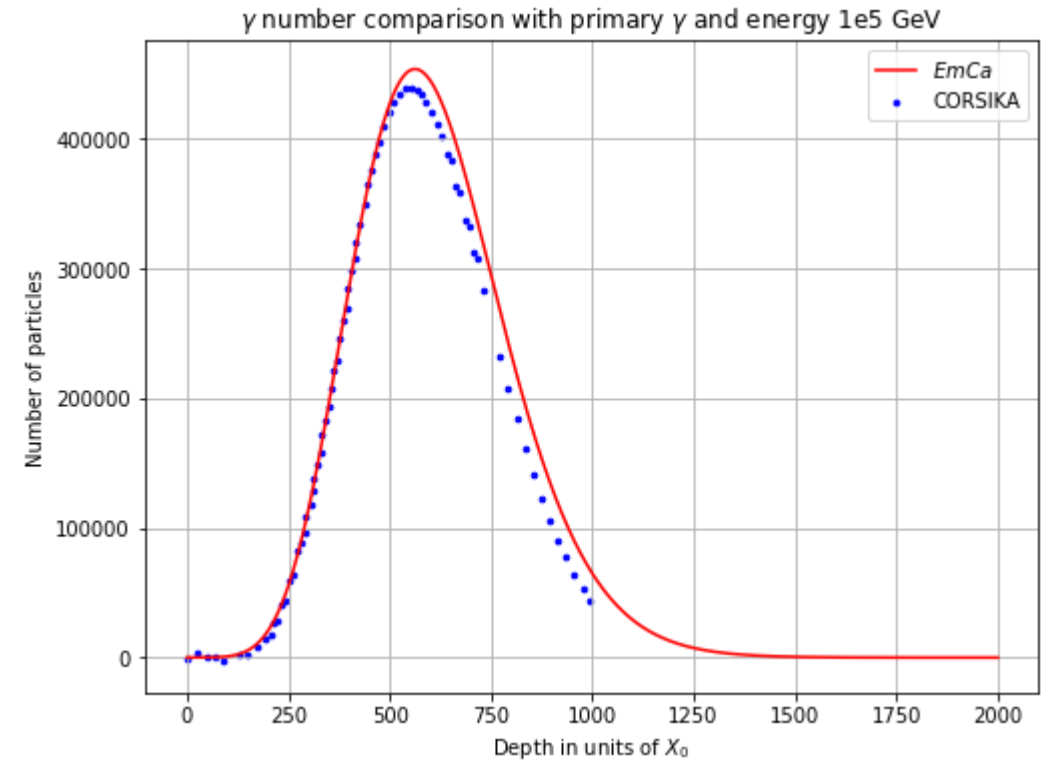
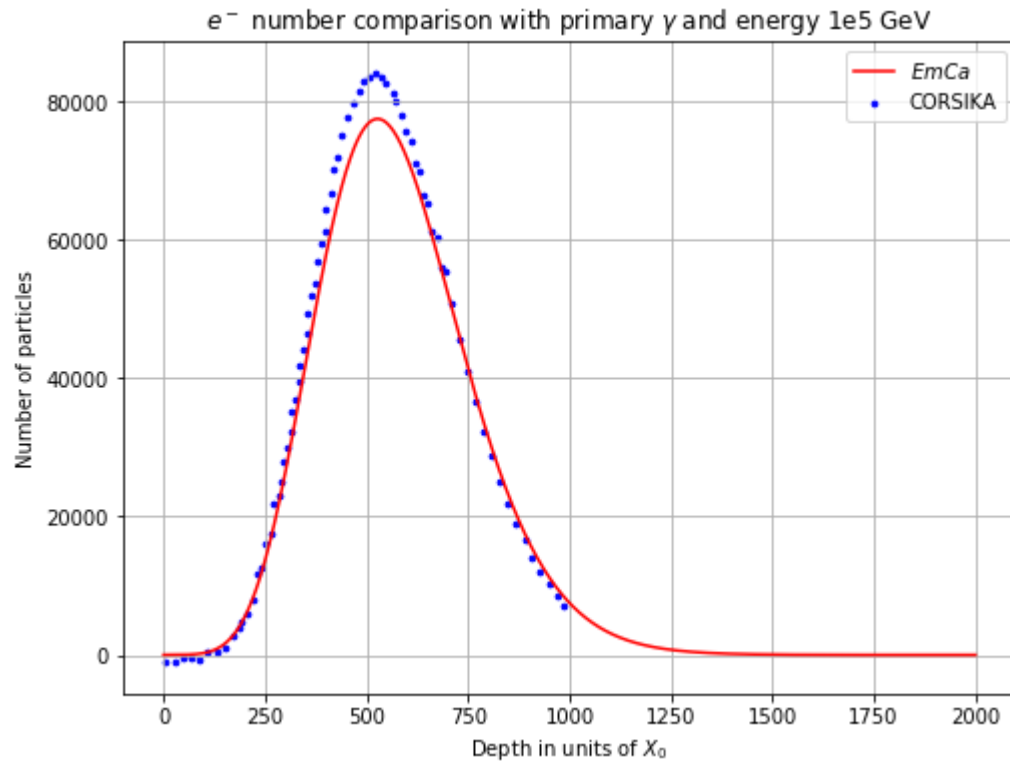
Cross-Checks – Shower Development II



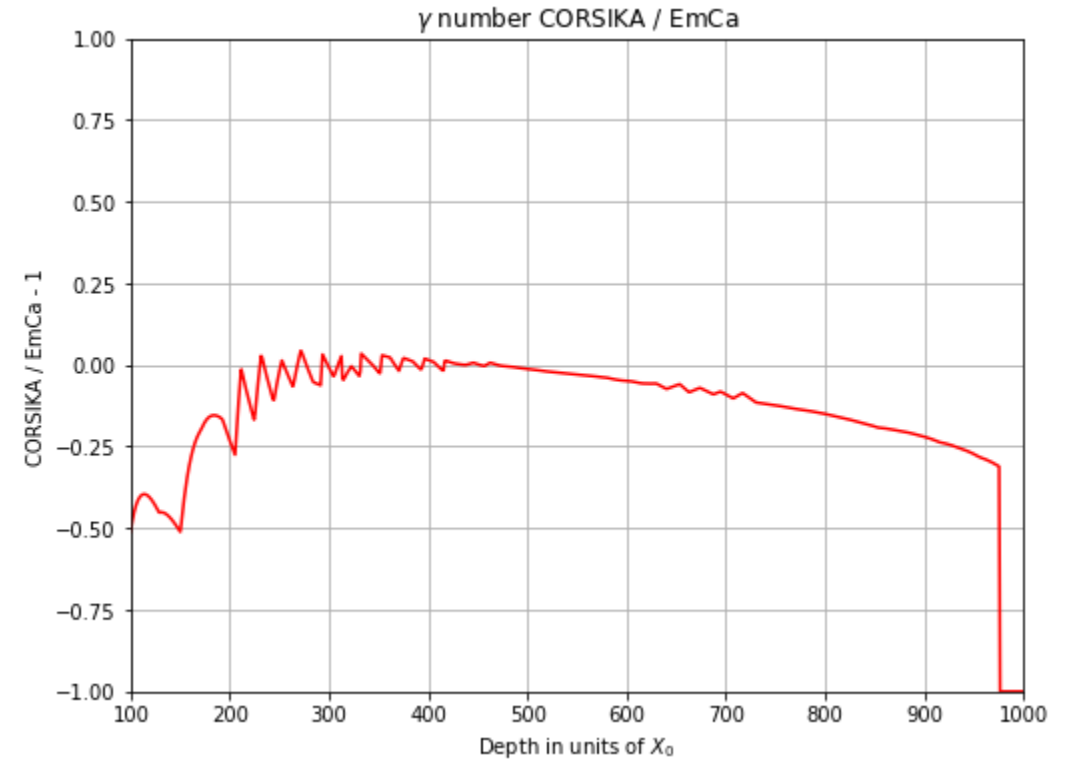
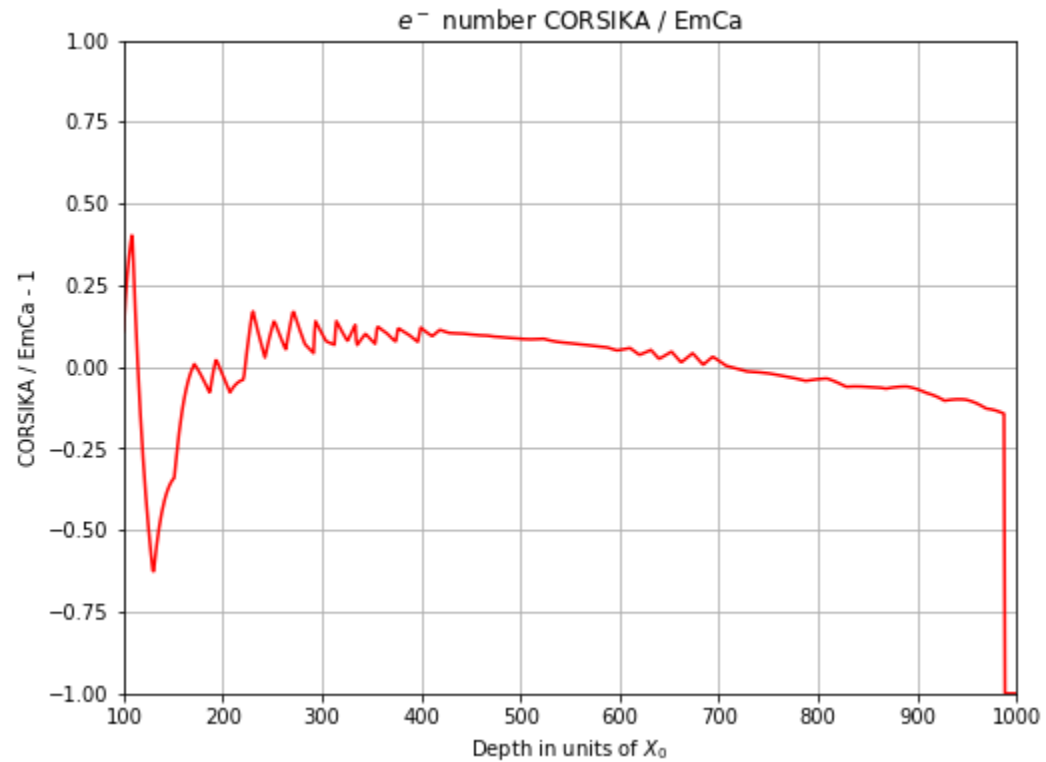
Cross-Checks – EGS4



Cross-Checks - CORSIKA



Cross-Checks - CORSIKA



Technicalities – Cross-Sections

- Pair

$$\frac{d\sigma}{dE} = \frac{\alpha r_0^2}{k} \left(\left[\frac{4}{3}x^2 - \frac{4}{3}x + 1 \right] \left[Z^2(\phi_1 - \frac{4}{3}\log Z - 4f) + Z(\psi_1 - \frac{8}{3}\log Z) \right] - \frac{2}{3}x [Z^2(\phi_1 - \phi_2) + Z(\psi_1 - \psi_2)] \right)$$

- Bremsstrahlung

$$\frac{d\sigma}{dk} = \frac{\alpha r_0^2}{k} \left(\left[y^2 - \frac{4}{3}y + \frac{4}{3} \right] \left[Z^2(\phi_1 - \frac{4}{3}\log Z - 4f) + Z(\psi_1 - \frac{8}{3}\log Z) \right] - \frac{2}{3}(1 - y) [Z^2(\phi_1 - \phi_2) + Z(\psi_1 - \psi_2)] \right)$$

Note there is an error in the Tsai paper concerning the Bremsstrahlung's cross-section

- Dielectric Suppression

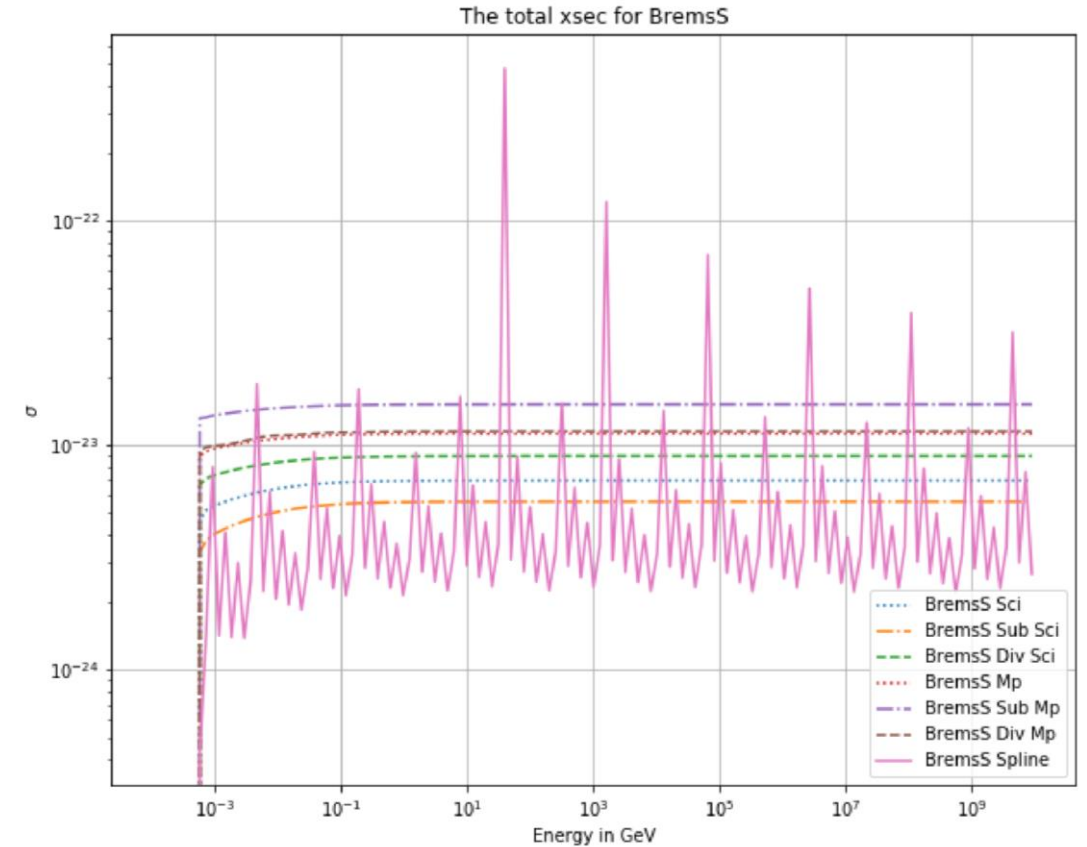
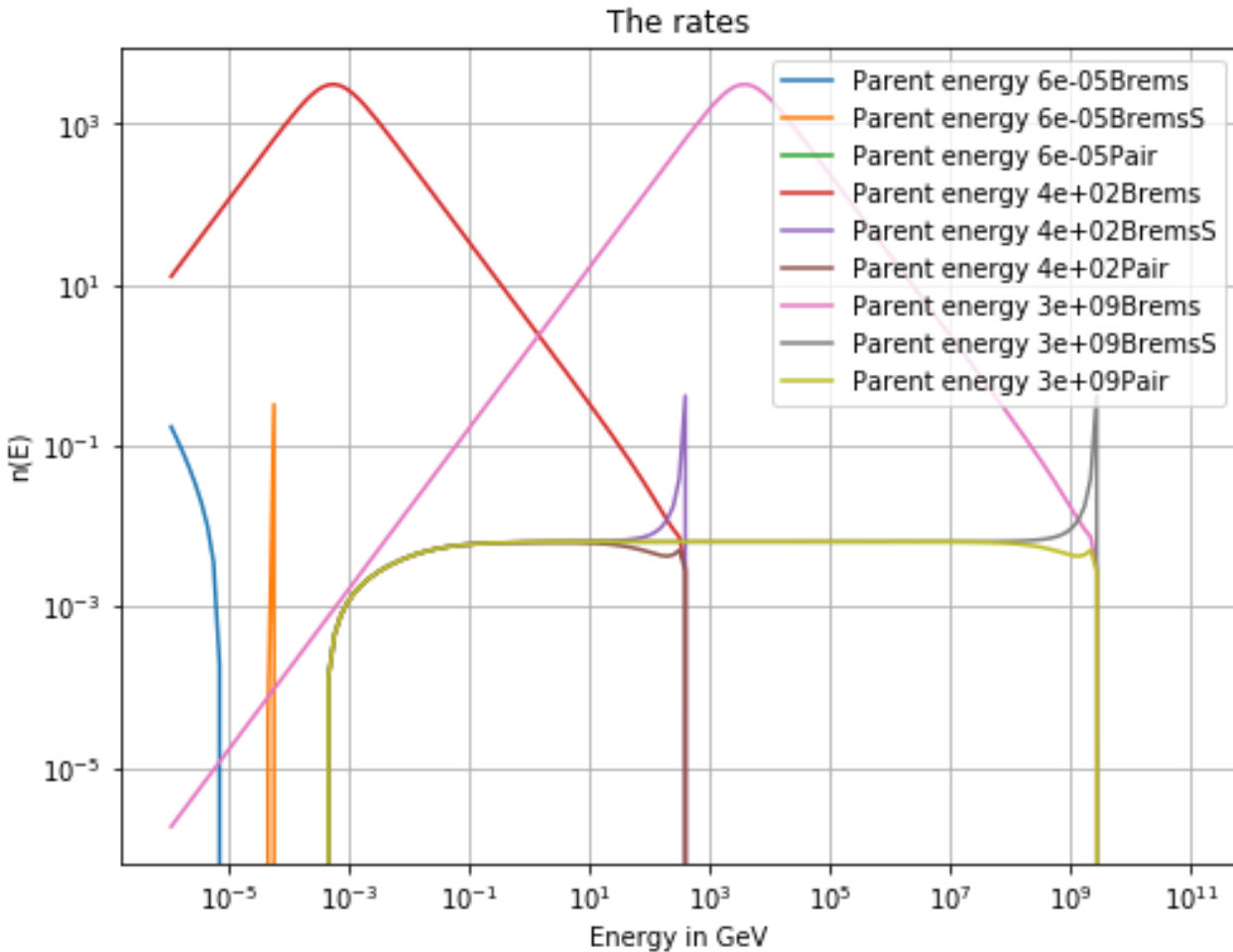
$$S_{die} = \frac{k^2}{k^2 + (\gamma \cdot \hbar\omega_p)^2}$$

- Bremsstrahlung

$$E_{LPM} = \frac{7.7 \text{TeV} X_0}{\rho} \quad S_{LPM} = \sqrt{\frac{k E_{LPM}}{E(E - k)}}$$

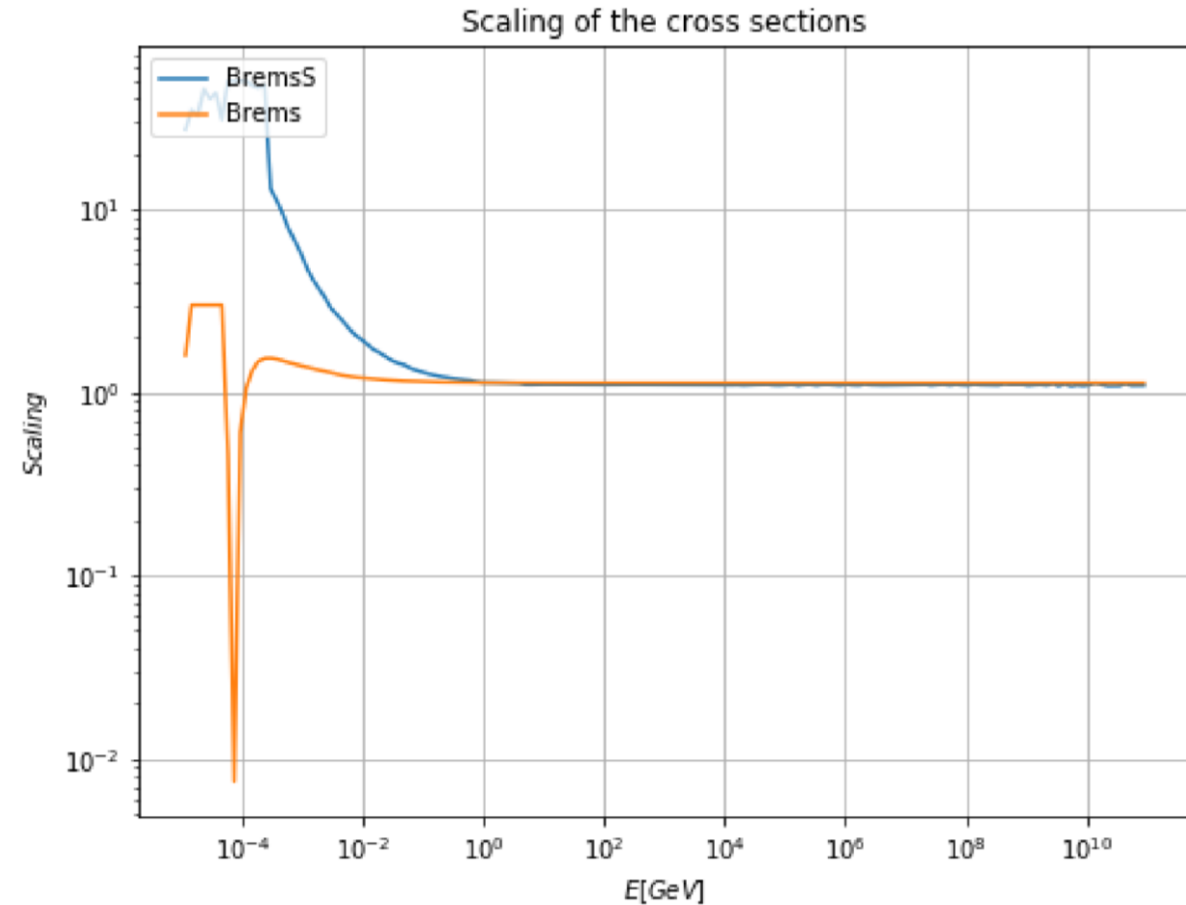
Note there is an error in the Tsai paper concerning the Bremsstrahlung's cross-section

Technicalities – Rates

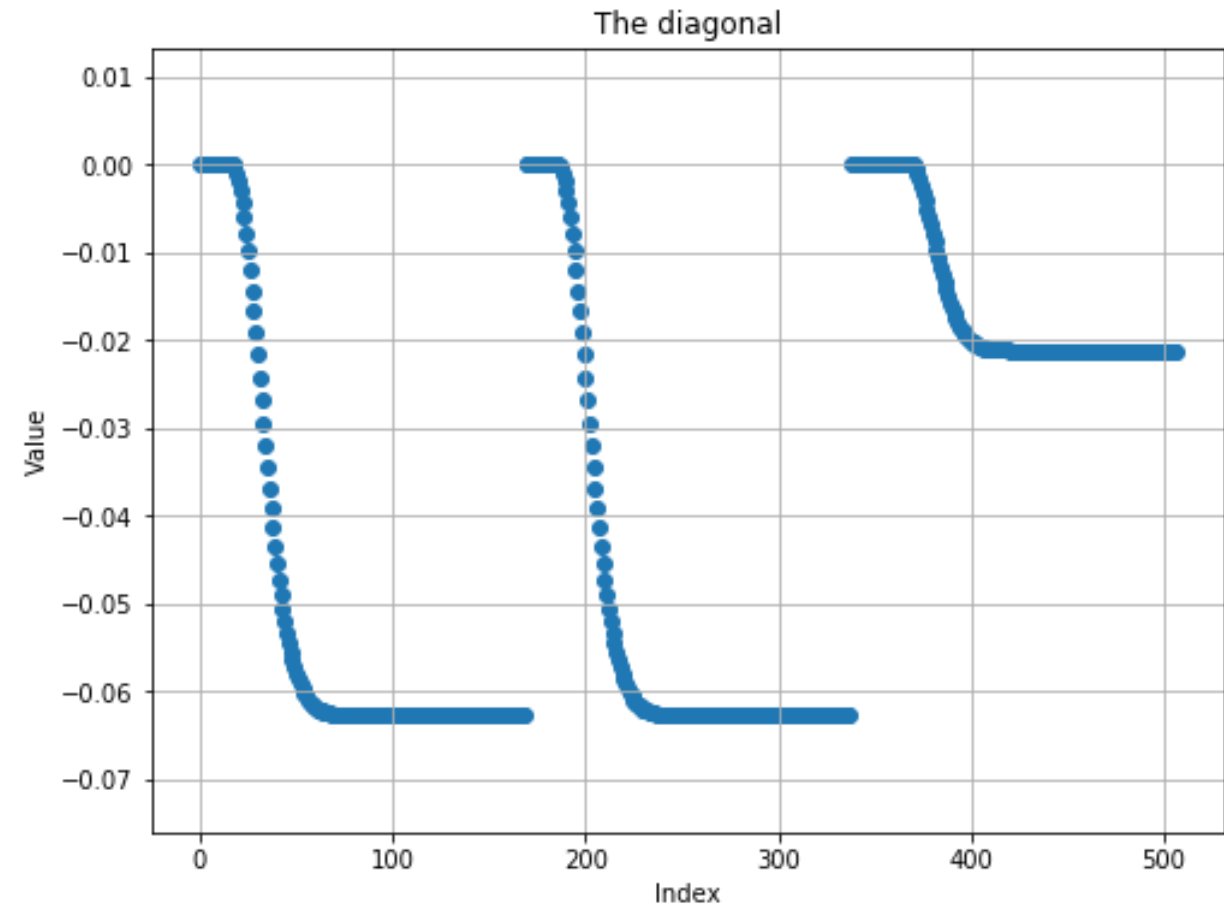
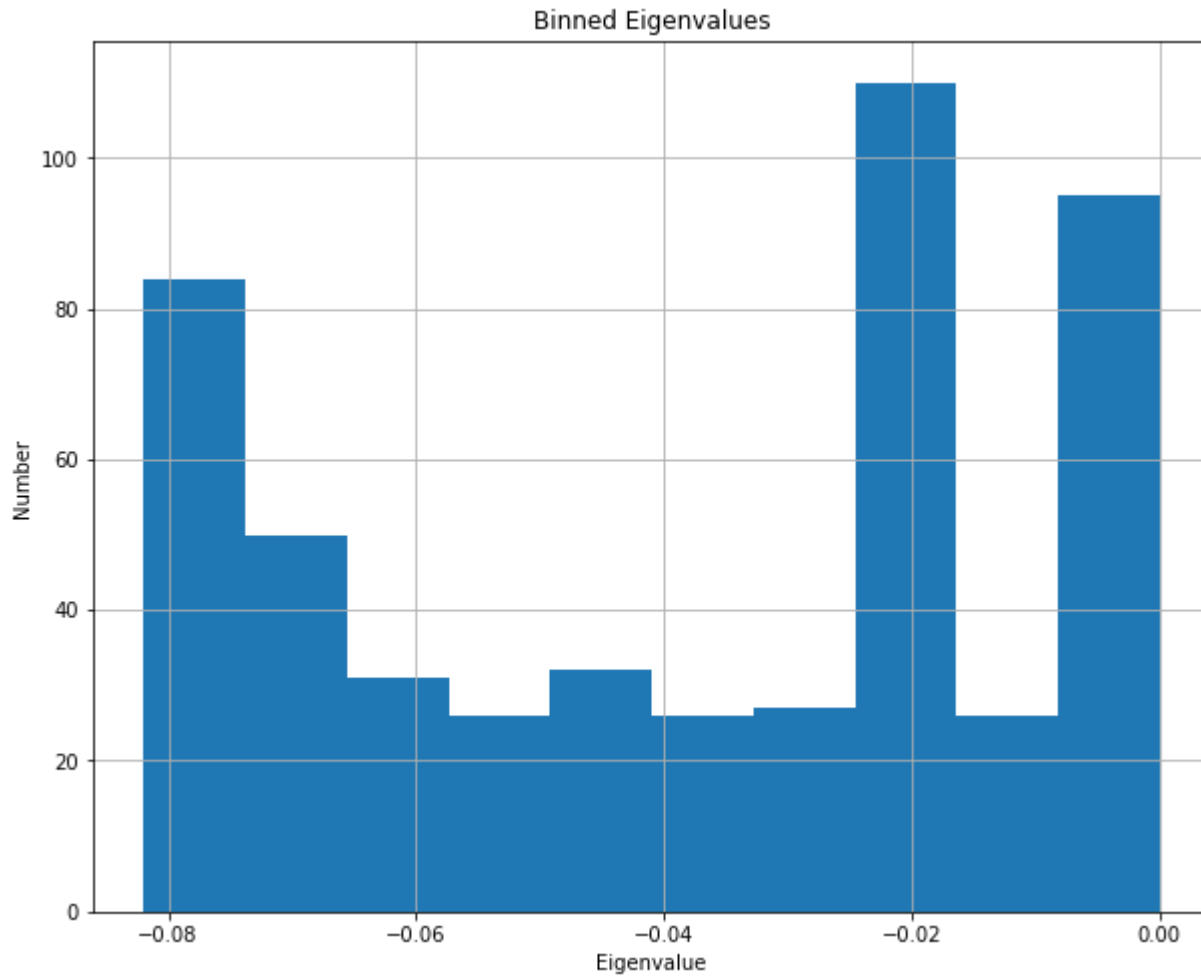


Technicalities - Scaling

- Observables as reference
 - Energy conservation
 - Radiation Length



Technicalities – Equation Matrix



Technicalities - Solver

- Ionization poses a problem
 - Major bottleneck
- Implemented
 - LSODA
 - => Fastest and least precise
 - BDF
 - => Slowest
 - RK45
 - => Intermediate and most precise
 - Euler
 - => Only usable without ionization

