

# Project C2a

## Hadronic Matrix Elements and Exclusive Semileptonic Decays

Thorsten Feldmann and Thomas Mannel

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## Many different decay modes, e.g.

- charged and neutral leptonic decays  
( $B_{u,c} \rightarrow \ell\nu$ ,  $B_{s,d} \rightarrow \mu^+\mu^-$ ,  $B \rightarrow \nu\nu$ , ...)
- exclusive and inclusive semileptonic  $b \rightarrow u(c)$  decays  
( $B \rightarrow X_{u,c}\ell\nu$ ,  $B \rightarrow D^{(*)}\ell\nu$ ,  $B \rightarrow \pi\ell\nu$ ,  $B \rightarrow \pi\pi\ell\nu$ ,  $B_s \rightarrow K\pi\ell\nu$ ,  $\Lambda_b \rightarrow p\ell\nu$ , ...)
- radiative and semileptonic FCNC decays  
( $B \rightarrow X_s\gamma$ ,  $B \rightarrow \rho\gamma$ ,  $B \rightarrow K^{(*)}\mu^+\mu^-$ ,  $B_s \rightarrow \phi\mu^+\mu^-$ ,  $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ ,  $B \rightarrow K\nu\nu$ , ...)
- non-leptonic decays with or without charm  
( $B \rightarrow J/\psi K$ ,  $B \rightarrow \pi\pi$ ,  $B \rightarrow K\pi$ ,  $B \rightarrow \pi\pi\pi$ , ...)

Many decay modes are suppressed by CKM- and/or loop factors

⇒ particular sensitivity to (indirect) NP effects !

## Many different observables, e.g.

- lifetimes and mixing parameters
- (partially) integrated decay rates
- (moments of) decay spectra, Dalitz distributions
- forward-backward asymmetries
- direct and mixing-induced CP asymmetries
- angular distributions and polarization fractions

Some observables are small or even vanish in the SM

⇒ particular sensitivity to (indirect) NP effects !

## Hadronic Uncertainties

- hadronic bound-state effects obscure underlying short-distance physics
- many process-dependent exclusive decay amplitudes that describe non-perturbative QCD dynamics

## Idea: Factorization / EFT

- decaying  $b$ -quark is heavy compared to  $\Lambda_{\text{QCD}}$
  - final states often involve (relatively) energetic particles (i.e.  $E \sim m_b/2$ )
- ⇒ different hadronic amplitudes can be expanded in terms of a handful of more fundamental (i.e. process-independent) quantities.
- also: control large logarithms in QCD perturbation theory

Particularly important:

- Large energy  $\leftrightarrow$  propagation close to the light-cone
- ⇒ hadronic matrix elements of non-local operators

- 1 Scope / Work Areas
- 2 Participating Scientists / Recruitment
- 3 Preliminary Work / Status of Subprojects
- 4 Recent Results – Example from WA1
- 5 Outlook

# 1. Scope / Work Areas

- reliable predictions for exclusive  $b \rightarrow c\ell\nu$  and  $b \rightarrow u\ell\nu$  transitions
- precision determinations of  $|V_{cb}|$  and  $|V_{ub}|$
- refined theoretical methods for hadronic input parameters

Work Area 1: Factorization and light-cone distribution amplitudes

Work Area 2: Theoretical development of QCD sum rules and related methods

Work Area 3: New channels and multi-hadron final states

Work Area 4: Inclusive rate from the sum over exclusive channels

- WA1 and WA3 will be coordinated by TF
- WA2 and WA4 will be coordinated by TM

## 2. Participating Scientists / Recruitment

Name	Position	Work Areas	Start – End	Affiliation
Yao Ji	Postdoc	WA1, WA4	Oct 19 –	SI
[N.N.]*	PhD Student	WA2, WA3	mid 19 –	SI
Björn O. Lange	Akad. Rat	WA1, WA4	permanent	SI

Additional personnel:

Alex Khodjamirian	Senior Prof.	WA2	– Dec 21	SI
Kevin Olschewsky	PhD Student	WA2, WA4	Feb 19 –	SI

\* (candidate comes for a presentation talk in the first week of april . . .)

# 3. Preliminary work / Status of Subprojects

## WA1: Factorization and LCDAs

### Preliminary Work

- QCD factorization in exclusive  $B$  decays:  
[Böer 18 [thesis]] [Böer/TF/van Dyk 17] [Beneke/TF 04] [BOL/Neubert 04] [Beneke/TF 00]
- Properties of  $B$ -meson LCDAs:  
[TF/BOL/Wang 14] [Bell/TF/Wang/Yip 13] [Bell/TF 08]  
[Khodjamirian/TM/Offen 07] [BOL/Neubert 03]

### Pending QFET Projects

- Resummation of rapidity logs in exclusive decays [Bell/TF/Böer/BOL]
- Renormalization of  $B$ -meson LCDA at 2-loop [Bell/Dehnadi/BOL/Piclum]

### Current P3H Activities

- improved parametrizations of  $B$ -meson LCDA [TF/van Dyk]  
[preliminary results](#) (see below)
- parametrizations for higher-twist  $B$ -meson LCDAs [TF/van Dyk]



# 3. Preliminary work / Status of Subprojects

## WA2: QCD sum rules and related methods

### Preliminary Work

- $B \rightarrow \gamma \ell \nu$  transition: [Braun/Khodjamirian 13]
- $B \rightarrow \pi \ell \nu$  transition: [Imson/Khodjamirian/TM/van Dyk 15] [Khodjamirian/TM/Offen/Wang 11] [De Fazio/TF/Hurth 07 ] [Khodjamirian/TM/Offen 06] [De Fazio/TF/Hurth 05]
- $B \rightarrow \pi \pi \ell \nu$  transition: [Cheng/Khodjamirian/Virto 17 (2x)] [Hambrock/Khodjamirian 16]
- semi-leptonic  $\Lambda_b$  decays: [Khodjamirian/Klein/TM/Wang 11] [TF/Yip 11]
- exclusive  $b \rightarrow c$  transitions: [TM/van Dyk 15]

### Pending QFET Projects

- isoscalar contribution to the  $B \rightarrow \pi \pi$  form factor [Khodjamirian/Virto/Vos]
- vector contribution to the  $B \rightarrow \pi K$  form factors [Khodjamirian/Virto]

### Current P3H Activities

- phenomenological re-analysis of  $B \rightarrow \gamma \ell \nu$  [TF/van Dyk; using results from WA1]
- a new QCD sum rule for  $B_c$  processes [Khodjamirian/TM]

# 3. Preliminary work / Status of Subprojects

## WA3: New channels and multi-hadron final states

### Preliminary Work

- phenomenology of  $B_{(s)} \rightarrow M_1 M_2 \ell \nu$  decays  
[TF/van Dyk/Vos 18] [TF/Müller/van Dyk 15] [Faller/TF/Khodjamirian/TM/van Dyk 13]
- phenomenology of  $\Lambda_b$  decays  
[TM/Wang 11]

### Current P3H Activities

- PhD project [K. Olschewsky](#): “Multibody  $B$ -decays”
- phenomenological analysis of  $B_c \rightarrow J/\psi \ell \bar{\nu}$  [Khodjamirian/TM/Kellermann]

# 3. Preliminary work / Status of Subprojects

## WA4: Inclusive rate from sum over exclusive rates

### Preliminary Work

- phenomenological analysis of  $B \rightarrow D, D^*, D^{**}$  effects:  
[TM/Rusov/Shahriaran 17] [Klein/TM/Shahriaran/van Dyk 15]
- differential decay spectra of charmless inclusive  $B$  decays:  
[BOL/Neubert/Paz 05]

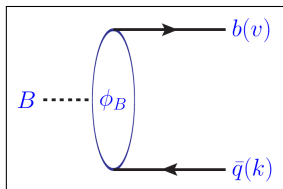
### Current P3H Activities

- ... waiting for postdoc to arrive in october ...

### **“Systematic parametrization of the $B$ -meson LCDAs”**

[TF, van Dyk (in preparation)]

## Parton picture: 2-particle Fock state



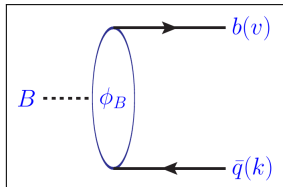
- Some external light-like momentum, e.g.  $p_\gamma^\mu = E_\gamma n^\mu$ ,  $n^2 = 0$
- with  $\omega \equiv n \cdot k$  light-cone projection of light antiquark momentum
- $\phi_B(\omega)$  as probability amplitude

- Field theoretical definition of  $\phi_B^+(\omega)$  from light-cone operators in HQET:

$$m_B f_B^{(\text{HQET})} \phi_B^+(\omega) = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0 | \bar{q}(\tau n) [\tau n, 0] \not{n} \gamma_5 h_V^{(b)}(0) | \bar{B}(m_B v) \rangle$$

[Grozin/Neubert]

## Parton picture: 2-particle Fock state



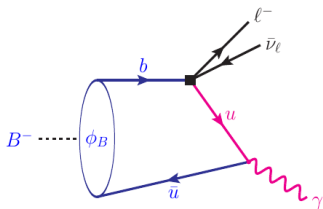
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- with  $\omega \equiv n \cdot k$  light-cone projection of light antiquark momentum
- $\phi_B(\omega)$  as probability amplitude

- in QCD factorization [Beneke et al.] one encounters **logarithmic moments**

$$L_n(\mu) = \int_0^\infty \frac{d\omega}{\omega} \ln^n\left(\frac{\mu}{\omega}\right) \phi_B^+(\omega) \quad (n=0,1,2, \dots)$$

- in QCD light-cone sum rules, one is sensitive to **low light-cone momenta**

$$\phi_B^{+\prime}(0), \quad \phi_B^{+\prime\prime}(0), \quad \text{etc.}$$



For large photon energy,  $E_\gamma \sim m_b/2$ :

$$(\mathbf{p}_\gamma - \mathbf{p}_{\bar{u}})^2 \simeq -2 \mathbf{p}_\gamma \cdot \mathbf{p}_{\bar{u}} \equiv -2 E_\gamma \omega$$

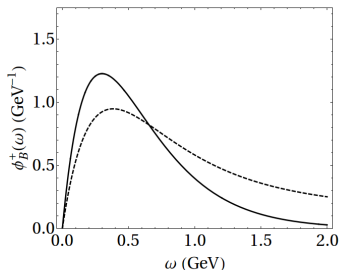
- Sensitive to light-cone projection  $\omega$  of light antiquark momentum in  $B$ -meson

$$F^{B \rightarrow \gamma}(E_\gamma) \simeq [\text{kinematic factor}] \times \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega)$$

Experimental bound on BR  $\rightarrow$  bound on  $\frac{1}{\lambda_B} \equiv \langle \omega^{-1} \rangle_B$

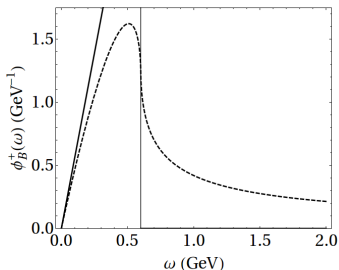
# Simple Models for $\phi_B^+(\omega)$ :

exponential model



$$\phi_B^+(\omega, \mu_0) = \frac{\omega}{\omega_0^2} \exp\left(-\frac{\omega}{\omega_0}\right)$$

free parton model



$$\phi_B^+(\omega, \mu_0) = \frac{\omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega)$$

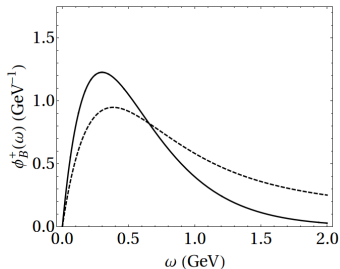
$$\bar{\Lambda} = M_B - m_b$$

- model LCDA at  $\mu_0$
- effect of RG evolution towards higher scales



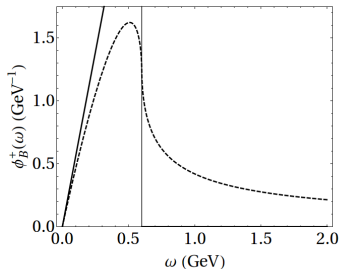
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$$\bar{\Lambda} = M_B - m_b$$

Can one do better?

# Complications:

Renormalization of light-cone operator in HQET non-trivial:

- local limit  $\lambda \rightarrow 0$  and renormalization do not commute
- $\phi_B^+(\omega, \mu)$  exhibits a “radiative tail”
- non-negative moments of  $\phi_B^+(\omega, \mu)$  become divergent

[Lange/Neubert, Lee/Neubert, Braun/Ivanov/Korchensky]

## What do we know:

- tree-level constraints from HQET parameters [Grozin/Neubert]
- dim-3 and dim-4 contributions to radiative tail [Lee/Neubert]
- dim-5 contributions to radiative tail (position space) [Kawamura et al.]
- simple hadronic models [Kawamura et al., Khodjamirian et al., Braun et al.]
- eigenfunctions of the 1-loop RG equations [Bell/TF/Wang ; Braun/Manashov]

## Step 1: Generic parametrization for low- $\omega$ region

$$\phi_B^+(\omega, \mu_m) \equiv \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^N c_k(\mu_m) L_k^{(1)}\left(\frac{\omega}{\omega_0}\right) + \text{“radiative tail” @ } \mu_m$$

- (truncated) series of **associated Laguerre polynomials**  $L_k^{(1)}$
- **exponential damping** for large momenta  $\omega$
- **hadronic information** encoded in
  - Laguerre coefficients  $c_k(\mu_m)$
  - hadronic reference momentum  $\omega_0 \sim \Lambda_{\text{had}}$
  - reference renormalization scale  $\mu_m \gg \Lambda_{\text{QCD}}$
- radiative tail will be treated separately

## Step 2: The generating function:

(basically Mellin transform)

$$\begin{aligned} F_{[+]}(t, \mu_m) &\equiv \frac{\Gamma(1-t)}{\Gamma(1+t)} \int_0^\infty \frac{d\omega}{\omega} \left( \frac{\mu e^{2\gamma E}}{\omega} \right)^{-t} \phi_B^+(\omega) \\ &= \frac{1}{\omega_0} \left( \frac{\mu_m e^{2\gamma E}}{\omega_0} \right)^{-t} \sum_{k=0}^N c_k(\mu_m) \frac{\Gamma(1+k-t)}{\Gamma(1+k)} \quad + \text{radiative tail} \end{aligned}$$

- Logarithmic moments can be obtained by expanding around  $t = 0$ , e.g.

$$L_0(\mu_m) = \frac{1}{\omega_0} \sum_{k=0}^N c_k(\mu_m) \quad + \text{radiative-tail contribution}$$

- Low-momentum behaviour from setting  $t = -1, -2, \dots$ , e.g.

$$\phi_B^{+'}(0, \mu_m) = \frac{1}{\omega_0^2} \sum_{k=0}^N k \cdot c_k(\mu_m) \quad + \text{radiative-tail contribution}$$

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- The generating function has a simple RG evolution:

$$F_{[+]}(t, \mu) = e^V \left( \frac{\mu}{\mu_m} \right)^{-t} F_{[+]}(t+g, \mu_m)$$

where  $V = V(\mu, \mu_m)$  and  $g = g(\mu, \mu_m)$  are standard RG functions in SCET.

# Step 3: Determining the Radiative Tail

from perturbative analysis in the parton picture

[Lee/Neubert] [Kawamura et al.]

$$\begin{aligned} \phi_B^+(\omega, \mu_m)|_{\text{radiative}} = & \theta(\omega - \omega_t) \frac{\alpha_s C_F}{\pi \omega} \left\{ \frac{1}{2} - \ln \frac{\omega}{\mu} + \frac{4\bar{\Lambda}}{3\omega} \left( 2 - \ln \frac{\omega}{\mu} \right) \right. \\ & \left. + \frac{\dots \bar{\Lambda}^2 + \dots \lambda_E^2(\mu) + \dots \lambda_H^2(\mu)}{\omega^2} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^3}{\omega^3}\right) + \dots \right\} \end{aligned}$$

- HQET parameter  $\bar{\Lambda} = m_B - m_b$  (pole-mass scheme)
- we also included the higher-order terms from dim-5 operators, with  $\lambda_{E,H}^2(\mu)$  (chromoelectric and chromomagnetic terms)
- forcing  $\phi_B^+$  to be continuous at  $\omega = \omega_t$ , one sets

$$\Rightarrow \quad \omega_t = \sqrt{e} \mu_m + 2\bar{\Lambda} + \dots$$

# Step 4: Improved Grozin-Neubert Relations

Constraints on Laguerre coefficients  $c_{0,1,2}$  to 1-loop accuracy, e.g.

$$c_0(\mu_m) = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{2} - \frac{\pi^2}{12} - \frac{8\bar{\Lambda}}{3\sqrt{e}\mu_m} + \dots \right\}$$
$$c_1(\mu_m) = \left( 1 - \frac{2\bar{\Lambda}}{3\omega_0} \right) \left( 1 + \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{2} - \frac{\pi^2}{12} + \dots \right\} \right) + \frac{\alpha_s C_F}{4\pi} \left\{ -\frac{2\sqrt{e}\mu_m}{\omega_0} + \dots \right\}$$

- naive tree-level normalization [← Grozin/Neubert]
  - radiative corrections [← Lee/Neubert]
  - power corrections, requiring  $\bar{\Lambda} \ll \mu_m$  [← Lee/Neubert] [← Kawamura et al.]
  - power-enhanced terms  $\leftrightarrow$  renormalon ambiguity in def. of  $\bar{\Lambda}$
- $\Rightarrow$  switching to renormalon-free definition  $\bar{\Lambda}_{\text{DA}}(\mu_m)$  [Lee/Neubert]

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$$c_1(\mu_m) = \left( 1 - \frac{2\bar{\Lambda}_{DA}(\mu_m)}{3\omega_0} \right) \left( 1 + \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{2} - \frac{\pi^2}{12} + \dots \right\} \right)$$

- naive tree-level normalization [← Grozin/Neubert]
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# Step 5: Analytic Form for Generating Function

$t < 1$

$$F_{[+]}(t, \mu_m)|_{\text{radiative}} = \frac{\alpha_s C_F}{\pi} \frac{\Gamma(1-t)}{\Gamma(1+t)} \frac{e^{-2\gamma_E t} e^{t/2}}{\sqrt{e\mu_m}} \left\{ -\frac{1}{(t-1)^2} + \frac{\bar{\Lambda}}{3\sqrt{e\mu_m}} \frac{8-6t}{(t-2)^2} + \dots \right\}$$

- Read off radiative contribution to logarithmic moments, e.g.

$$L_0(\mu_m)|_{\text{radiative}} = -\frac{\alpha_s C_F}{\sqrt{e\mu_m}\pi} \left\{ 1 - \frac{2\bar{\Lambda}}{3\sqrt{e\mu_m}} + \dots \right\}$$

(suppressed by  $\alpha_s/\pi$  and  $\omega_0/\mu_m$  compared to tree-level result)

- By construction, no contribution to low-momentum behaviour.

... work in progress ...

# 5. Outlook / ToDo List

## WA1:

- generic parametrizations for higher-twist  $B$ -meson LCDAs
- implications for phenomenology, in particular for  $B \rightarrow \gamma \ell \nu$  [→ C2b]
- applying the same ideas to shape functions for inclusive decays [→ WA4 + C1a]

## WA2:

- new sum rules for  $B_c$  form factors [→ WA3]
- sum rules for  $1/m_{b,c}$  corrections in  $B \rightarrow D \ell \bar{\nu}$

## WA3:

- semileptonic processes with bottom baryons
- $B_c$  phenomenology

## WA4:

- study of non-resonant contributions in  $b \rightarrow c$  and  $b \rightarrow u$
- decays into excited hadrons