Recursion relations in multiplet bases

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In collaboration with Yi-Jian Du and Malin Sjödahl

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Section 1

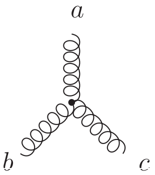
Introduction

Introduction

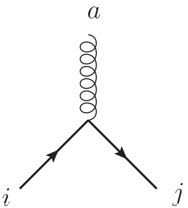
- The high energy of the LHC gives events with many colored partons in the perturbative regime.
- It is a challenge to deal with the color structure exactly for many partons, due to the non-Abelian structure of QCD.

Local symmetry of QCD

- The symmetry of QCD is $SU(3)$
- It enters the calculations from the vertices



$\propto i\, f^{abc}$



$\propto (T^a)^{ij}$

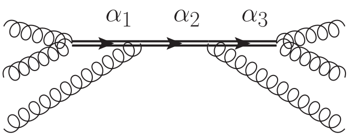
Section 2

Multiplet bases

Multiplet bases

- Orthogonal and minimal basis!

  Keppeler and Sjödahl, JHEP 1209, arXiv:1207.0609
- Number of vectors grow exponentially, not factorially.
- Orthogonality makes squaring easier!
- Downside is that decomposition and recursion are not as straightforward as with traditional bases.



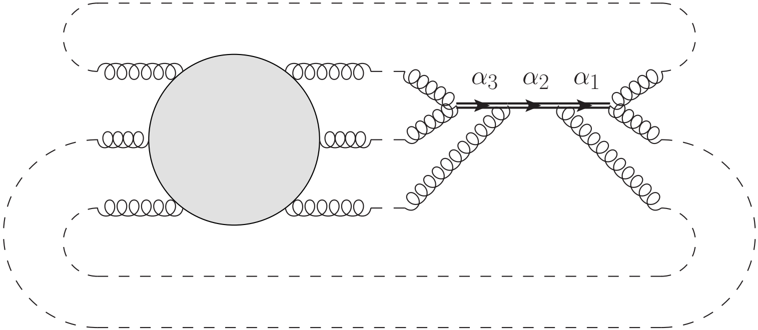
Quarks can be handled by combining the external quarks and antiquarks into pairs, which can then be in either a singlet or an octet.

Section 3

Decomposition into multiplet bases

Decomposition

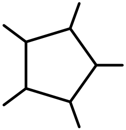
- The decomposition is just an evaluation of the scalar product between the basis vectors and the color structure



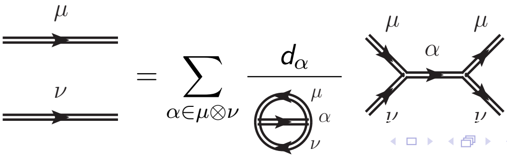
- This is just the evaluation of vacuum bubbles.

Vacuum bubbles

- Any vacuum bubble can rewritten as sums over factors of simpler vacuum bubbles.
- This is achieved by finding loops in the bubble

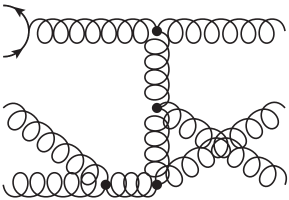


and repeatedly applying completeness relations, resulting in a reduction of the number of vertices in the bubble



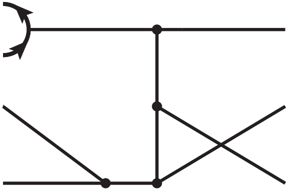
Example with a $q\bar{q}$ pair and 5 gluons

One of the color structures for $q\bar{q}2g \rightarrow 3g$ is



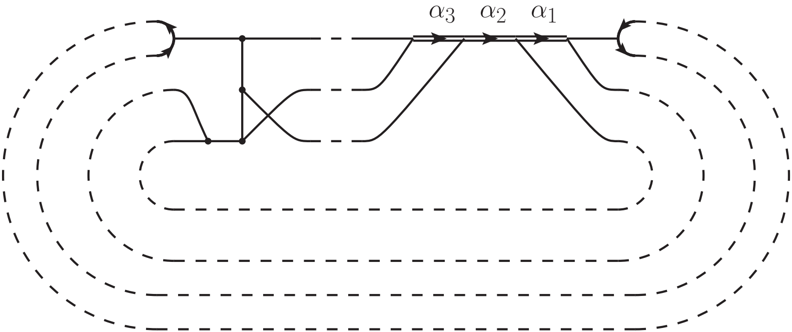
Example with a $q\bar{q}$ pair and 5 gluons

Using straight lines instead of curly lines for gluons gives



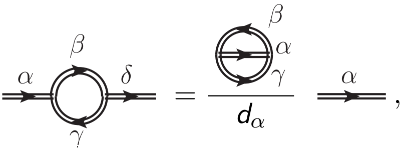
Example with a $q\bar{q}$ pair and 5 gluons

Decomposition into the basis vectors is equivalent to determining the scalar product between the basis vector and the color structure

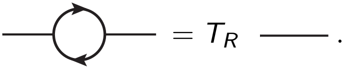


Example with a $q\bar{q}$ pair and 5 gluons

Two-vertex loops gives a factor

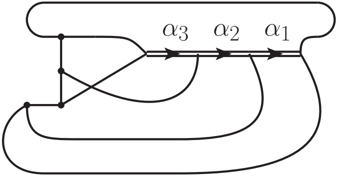


hence the quark loop gives



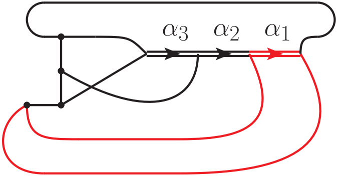
Example with a $q\bar{q}$ pair and 5 gluons

Removing the quark loop then gives

$A(\alpha_1, \alpha_2, \alpha_3) = T_R$ 

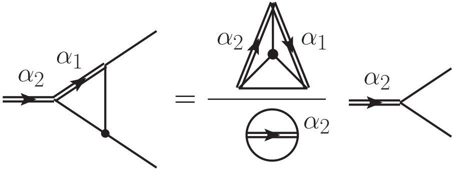
Example with a $q\bar{q}$ pair and 5 gluons

Highlighting the smallest loop

$A(\alpha_1, \alpha_2, \alpha_3) = T_R$ 

Reducing the loop

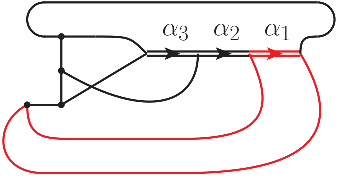
A vertex correction only gives a factor (given by small bubbles)



with a possible sum over vertices.

Example with a $q\bar{q}$ pair and 5 gluons

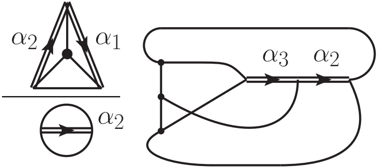
Using the vertex correction result on

$A(\alpha_1, \alpha_2, \alpha_3) \; = \; T_R$ 

Example with a $q\bar{q}$ pair and 5 gluons

Gives

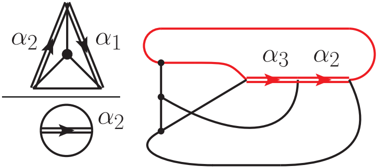
$$A(\alpha_1, \alpha_2, \alpha_3) = T_R$$



Example with a $q\bar{q}$ pair and 5 gluons

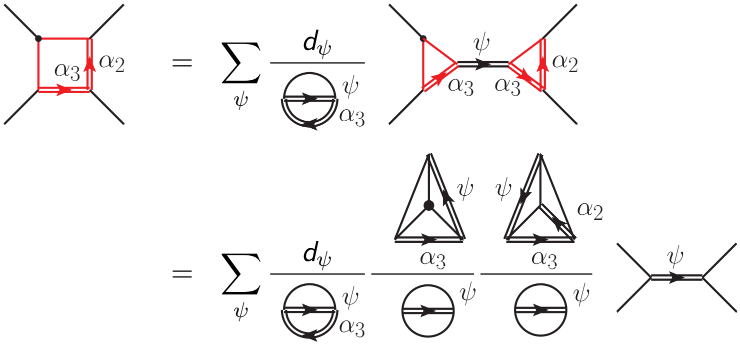
Now we must pick a 4-vertex loop

$$A(\alpha_1, \alpha_2, \alpha_3) = T_R$$



Reducing the second loop

A 4-vertex loop is less trivial than a vertex correction



Reducing the second loop

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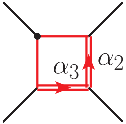
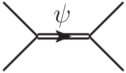


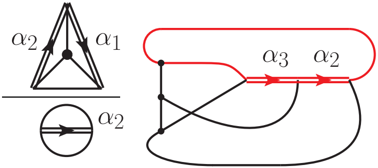
$$= \sum_{\psi} C(\psi, \alpha_2, \alpha_3)$$



Example with a $q\bar{q}$ pair and 5 gluons

Now we are to remove the 4-vertex loop

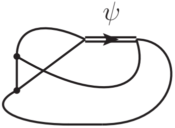
$$A(\alpha_1, \alpha_2, \alpha_3) \quad = \quad T_R$$



Example with a $q\bar{q}$ pair and 5 gluons

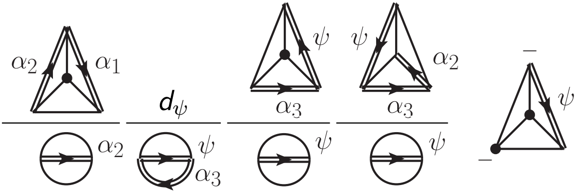
Giving us the final expression

$$A(\alpha_1, \alpha_2, \alpha_3) = \sum_{\psi} C'(\psi, \alpha_1, \alpha_2, \alpha_3)$$



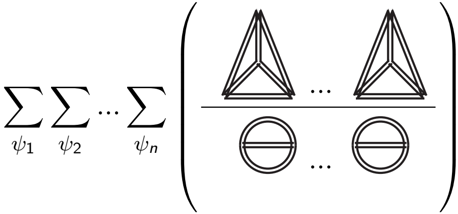
$=$

$= T_R \sum_{\psi}$



Decomposing Feynman diagrams

- Any vacuum bubble can be rewritten as a sum of factors of smaller vacuum bubbles, called Wigner 3j and 6j coefficients.



- Rewriting into smaller bubbles can be done without specifying for which basis vector it is.
- These smaller bubbles can be calculated once and then be stored.

Wigner coefficients

An upper limit of the number of required Wigner coefficients are

$n = N_g + N_{q\bar{q}}$	4	5	6	7	8	9	10	11	12
LO $N_c = 3$	21	39	106	152	254	318	452	536	705
NLO $N_c = 3$	29	55	120	176	272	350	476	576	733
LO $N_c \to \infty$	28	68	313	636	1 777	3 095	7 289	12 009	25 487
NLO $N_c \to \infty$	44	108	389	808	2 023	3 693	8 077	13 783	27 613

I have calculated the coefficients required for gluons for LO up to $N_g = 6$.

M. Sjödahl and J. Thorén, Decomposing color structure into multiplet bases, to be submitted very soon.

Section 4

Recursion relations

Recursion

- Avoid Feynman diagrams, the number grows factorially!
- Recursion relation in external particles.
- Has been explored for traditional bases (trace, DDM [1], color flow [2]).
- The orthogonality of the multiplet bases is promising.

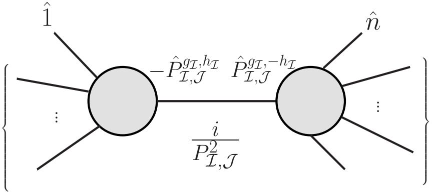
[1] V. Del Duca, L. Dixon and F. Maltoni, hep-ph/9910563.

[2] F. Maltoni, K. Paul, T. Stelzer and S. Willenbrock, hep-ph/0209271.

BCFW

A recursion relation for on-shell amplitudes is the BCFW recursion relation,

$$\mathcal{M}(g_1, \ldots, g_n) = \sum_{\mathcal{I},\mathcal{J}} \sum_{g_\mathcal{I}, h_\mathcal{I}} \mathcal{I}$$

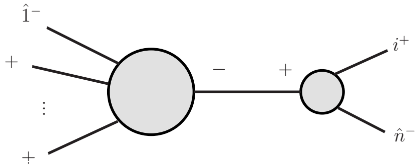


$\mathcal{J}$

R. Britto, F. Cachazo, B. Feng and E. Witten, hep-th/0501052

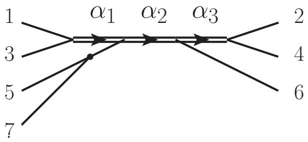
BCFW for MHV amplitudes

If the amplitude is maximally helicity violating, the relation simplifies.
It is possible to choose the momentum shift such that only one factorization channel contributes



Color structures for MHV

The color structures of interest are then multiplet basis vectors emitting a gluon,



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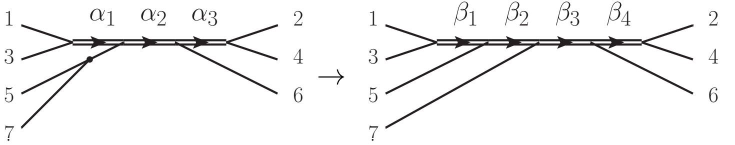
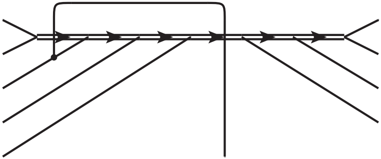
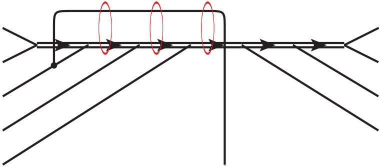


Basis vectors radiating gluons



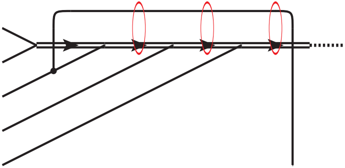
Basis vectors radiating gluons

Applying completeness relations



Basis vectors radiating gluons

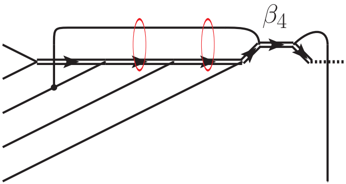
Applying completeness relations



Basis vectors radiating gluons

Applying completeness relations

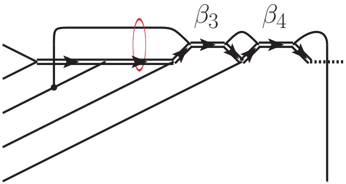
$$\sum_{\beta_4} d_{\beta_4}$$



Basis vectors radiating gluons

Applying completeness relations

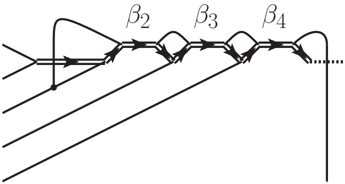
$$\sum_{\beta_3,\beta_4} d_{\beta_3} d_{\beta_4}$$



Basis vectors radiating gluons

Applying completeness relations

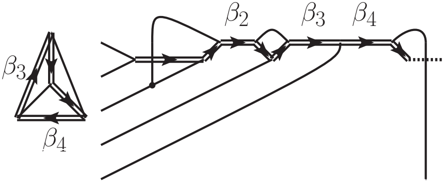
$$\sum_{\beta_2,\beta_3,\beta_4} d_{\beta_2} d_{\beta_3} d_{\beta_4}$$



Basis vectors radiating gluons

Removing vertex corrections

$$\sum_{\beta_2,\beta_3,\beta_4} d_{\beta_2} d_{\beta_3} d_{\beta_4}$$



Basis vectors radiating gluons

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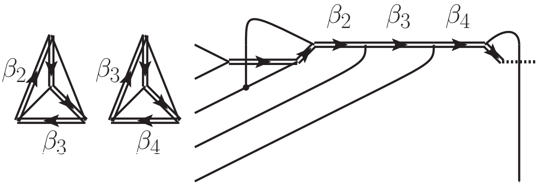
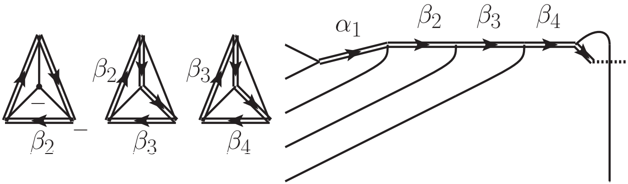


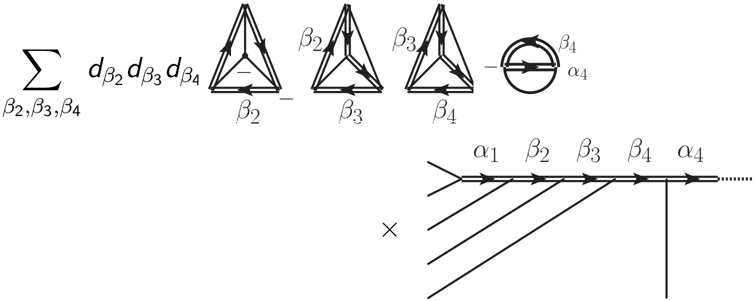
Basis vectors radiating gluons

Removing vertex corrections

$$\sum_{\beta_2,\beta_3,\beta_4} d_{\beta_2} d_{\beta_3} d_{\beta_4}$$



Basis vectors radiating gluons



a specific vertex normalization has been chosen to simplify the expression.

Radiation matrices

Preliminary numbers for multiplet basis vectors radiating a gluon.

	n	5	6	7	8	9	10	11	12
Avg	$N_c \geq n$	6.8	14.4	24.6	57.9	109	299	593	1 775
over $2, \ldots, n-1$	QCD	6.0	10.8	17.5	32.5	54.6	106	185	268
Max	$N_c \geq n$	9	44	44	400	400	4 006	4 006	41 256
	QCD	8	33	33	178	178	962	962	5 220
Vectors	$N_c \geq n$	44	265	1 854	14 833	133 496	1 334 961	14 684 570	176 214 841
(all orders)	QCD	32	145	702	3 598	19 280	107 160	614 000	3 609 760
Trace (LO)	any N_c	24	120	720	5 040	40 320	362 880	3 628 800	39 916 800
DDM (LO)	any N_c	6	24	120	720	5 040	40 320	362 880	3 628 800

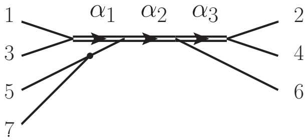
For 10 gluons, $(9!)^2 \sim 10^{11}$ for the trace basis, $(8!)^2 \sim 10^9$ for DDM, and $19\,280 \cdot 8 \cdot 106 \sim 10^7$ for multiplet bases.

Y-J. Du, M. Sjödahl and J. T., Recursion in multiplet bases for tree-level MHV gluon amplitudes, to be submitted soon.

Representative also for parton showers.

Radiation matrices

Color structures of interest for parton showers are of the same form,



Section 5

Conclusions and outlook

Conclusions

- The multiplet bases are minimal and orthogonal.
- Decomposing color structures into them is non-trivial.
- A manageable number of vacuum bubbles has to be calculated and stored (even for general N_c).
- Recursion in multiplet bases requires fewer terms when squaring than traditional bases.
- The number of terms required for recursion is also representative for parton showers.

Outlook

- Calculate $6j$ coefficients for more than 6 external gluons.
- Implement recursion relations efficiently.
- Handle recursion with quarks.
- Go beyond MHV amplitudes in recursion.
- Handle NLO amplitudes.

Backup slide: momentum shift

In the spinor helicity formalism the chosen momentum shift is

$$\lambda_1 \to \lambda_1, \quad \widetilde{\lambda}_1 \to \widetilde{\lambda}_1 - z\widetilde{\lambda}_n,$$
$$\widetilde{\lambda}_n \to \widetilde{\lambda}_n, \quad \lambda_n \to \lambda_n + z\lambda_1,$$

this causes the $(3, n-1)$ factorization channel to vanish.