# Gauge Dependence in the NLO corrections to the process $H^{\pm} \to W^{\pm} h_i$ in the real NMSSM

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# Outline i

# Introduction

# The MSSM

Minimal Supersymmetric extension of the Standard Model (MSSM)

- Every degree of freedom of the SM gets a superpartner
- We have two complex Higgs doublets for anomaly cancellation and supersymmetry
  After EWSB:

$$H_d = \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}\left(h_d + i a_d\right) \\ h_d^- \end{pmatrix} \quad H_u = \begin{pmatrix} h_u^+ \\ v_u + \frac{1}{\sqrt{2}}\left(h_u + i a_u\right) \end{pmatrix}$$

- $\rightsquigarrow$ Physical Higgs bosons $h, H, A$ and $H^{\pm}$

# The MSSM

SUSY-conserving interactions are determined by the superpotential

$$\mathcal{W}_{\text{MSSM}} = \underbrace{\hat{\bar{u}} Y_u \hat{Q} \cdot \hat{H}_u - \hat{\bar{d}} Y_d \hat{Q} \cdot \hat{H}_d - \hat{\bar{e}} Y_e \hat{L} \cdot \hat{H}_d}_{\text{Yukawa-couplings}} + \underbrace{\mu \hat{H}_d \cdot \hat{H}_u}_{\mu\text{-term}}$$

- $\mu$ has to be set to EWSB scale ad-hoc for proper phenomenology
- At tree level we have an upper bound on the light Higgs mass

$$m_h^2 \leq m_Z^2 \cos^2(2\beta)$$

Large loop corrections needed to get a $125$ GeV Higgs boson

# The NMSSM

The superpotential gets extended to

$$\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{MSSM}} + \frac{\kappa}{3}\hat{S}^3 + \lambda \hat{S}\hat{H}_u \cdot \hat{H}_d$$

There are 7 physical Higgs states (ordered by ascending mass):

- 3 CP-even Higgs bosons $H_1, H_2, H_3$
- 2 CP-odd Higgs bosons $A_1, A_2$
- 2 charged Higgs bosons $H^\pm$

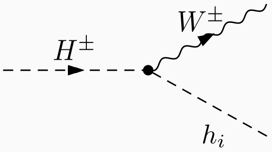
# The decay $H^{\pm} \to W^{\pm} h_i$

High precision calculations for the charged Higgs decay channels are important to

- properly interpret exclusion limits
- properly derive particle properties in case of discovery

# Calculation

# $H^\pm \to W^\pm h_i$ at tree level

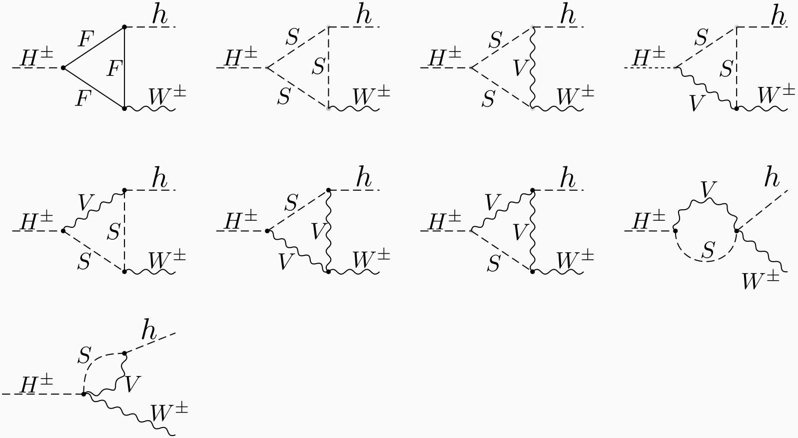



$$= \pm \frac{i}{2} g_2 \left( R^h_{i1} R^{H^+}_{11} - R^h_{i2} R^{H^+}_{12} \right) \left( p_{H^\pm} + p_h \right)^\mu$$

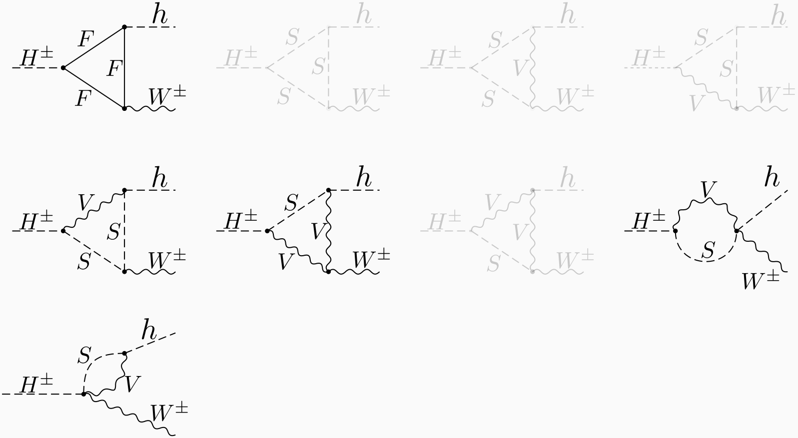
$$\Rightarrow M^{\text{tree}} = \pm i g_2 \left( R^h_{i1} R^{H^+}_{11} - R^h_{i2} R^{H^+}_{12} \right)$$

- $R^h$ and $R^{H^\pm}$ are the rotation matrices at tree-level from gauge to mass eigenstates
- $g_2$ is the $\mathcal{SU}(2)$ gauge coupling

# Vertex corrections at one loop



# UV divergent diagrams



# Renormalization Conditions (Parameters)

- Tadpoles are renormalized on-shell, i.e.

$$\delta t_\phi = T_\phi$$

- $m_Z^2$, $m_W^2$ and $m_{H^\pm}^2$ are renormalized on-shell, i.e.

$$\delta m_\phi^2 = \mathrm{Re}\left(\Sigma_{\phi\phi}(m_\phi^2)\right)$$

- $\tan\beta$ is renormalized $\overline{DR}$

$$\delta \tan\beta = \frac{1}{2}\tan\beta \left(\delta Z_{H_u} - \delta Z_{H_d}\right)\big|_{\text{div.}}$$

- $\lambda, \kappa, A_\kappa$ and $v_S$ are renormalized $\overline{DR}$, such that the renormalized neutral Higgs two-point functions are finite

$$\hat{\Sigma}_{h_i h_j}\Big|_{\text{div}} = 0$$

# Renormalization Conditions (Wavefunction Renormalization)

- $\delta Z_W$ is renormalized on-shell, i.e.

$$\delta Z_W = -\mathrm{Re}\left( \left.\frac{\partial \Sigma_{WW}(p^2)}{\partial p^2}\right|_{p^2=m_W^2} \right)$$

- The Higgs sector is renormalized $\overline{DR}$

$$-\mathrm{Re}\left.\frac{\partial \Sigma_{h_i h_i}}{\partial p^2}\right|_{p^2=m_{h_i}^2}^{\text{div}} = |R_{i1}|^2 \delta Z_{H_d} + |R_{i2}|^2 \delta Z_{H_u} + |R_{i3}|^2 \delta Z_S$$

# On-Shell Properties

External Particles have to fulfill on-shell properties

- They do not mix with other particles
- $p^2 = m^2_{\text{pole}}$
- Residue of the propagator has to be one

# Z-matrix

On-shell properties can be summarized:

$$\lim_{p^2 \to M_{h_i}^2} \frac{-i}{p^2 - M_{h_i}^2} \left( \mathbf{Z} \hat{\Gamma}^h \mathbf{Z}^T \right)_{ij} = \delta_{ij} \quad \text{for} \quad i, j = 1, 2, 3$$

$\mathbf{Z}$ transforms from tree-level states $h_i$ to loop-corrected states $H_i$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \mathbf{Z} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

$\hat{\Gamma}^h$ are the two-point functions for the CP-even Higgs bosons

# Higgs boson in the final state

For neutral Higgs bosons:

tree-level state $\neq$ loop-corrected state

$\Rightarrow$ For processes with (loop-corrected) external Higgs boson:

$$\mathcal{M}_{H^\pm \to W^\pm H_i} = \sum_{j=1}^{3} \mathbf{Z}_{ij} \mathcal{M}_{H^\pm \to W^\pm h_j}$$

# $\mathbf{Z}$ calculated

$$\mathbf{Z}_{ij} = \sqrt{Z_{h_i}} \cdot \left. \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)} \right|_{p^2 = M^2_{h_i}},$$

where $\quad \Delta = -\left(\hat{\Gamma}^h\right)^{-1} \quad$ and $\quad Z_{h_i} = \frac{1}{\partial_{p^2}\left(\frac{i}{\Delta_{ii}(p^2)}\right)\Big|_{p^2 = M^2_{h_i}}}.$

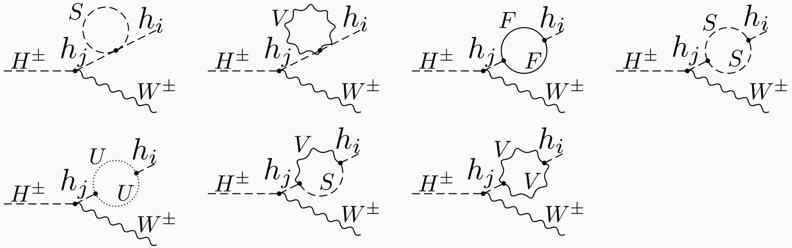
[Fuchs, Weiglein '17]

Expanding to one-loop order

$$\delta\mathbf{Z}_{ij} \approx \frac{\hat{\Sigma}_{h_i h_j}}{p^2 - m^2_{h_j}}$$

$\Rightarrow$ At one loop the mixing can be taken into account by including 1 particle reducible diagrams

# External Leg Contributions

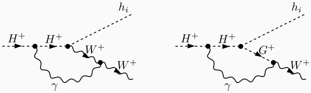


# Matrix element at strict one-loop order

$$
\begin{aligned}
\mathcal{M}_{H^\pm\to W^\pm H_i} =& \mathcal{M}^{\text{tree}}_{H^\pm\to W^\pm h_i} + \mathcal{M}^{\text{loop}}_{H^\pm\to W^\pm h_i} \\
\mathcal{M}^{\text{loop}}_{H^\pm\to W^\pm h_i} =& \mathcal{M}^{\text{vertex}}_{H^\pm\to W^\pm h_i} + \mathcal{M}^{\text{ext}\ h}_{H^\pm\to W^\pm h_i} + \mathcal{M}^{\text{ext}\ H^\pm}_{H^\pm\to W^\pm h_i}
\end{aligned}
$$

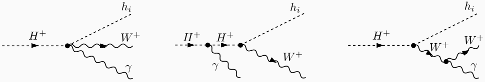
# IR Divergence

Charged external legs $\rightarrow$ IR divergent diagrams




Also $\sqrt{Z_{H^\pm}}$ and $\delta Z_W$ are IR divergent

According to the Kinoshita-Lee-Nauenberg theorem IR divergence cancels at every order with real photon emission



At this point our decay width is

- UV finite ✓
- IR finite ✓
- Gauge independent ✓
- calculated with tree-level Higgs mass $\rightsquigarrow$ not phenomenologically desirable
- not fully accounting for Higgs mixing at higher orders

$\rightarrow$ Use loop-corrected masses and Higgs mixing from NMSSMCALC[1] for the Higgs-boson

[1] [Baglio et al. '14, http://www.itp.kit.edu/maggie/NMSSMCALC/]

The mixing of the Higgs bosons $\mathbf{Z}_{ij}$ is calculated by `NMSSMCALC` to be consistent with the Higgs masses
$\rightarrow$ Do not expand $\mathbf{Z}_{ij}$ to one loop

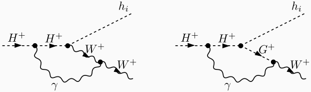
$$\mathcal{M}^{\text{impr.}}_{H^\pm\to W^\pm H_i} = \underbrace{\mathbf{Z}_{ij}\mathcal{M}^{\text{tree}}_{H^\pm\to W^\pm h_j}}_{\text{impr. LO}} + \mathbf{Z}_{ij}\left(\mathcal{M}^{\text{vertex}}_{H^\pm\to W^\pm h_j} + \mathcal{M}^{\text{ext}\ H^\pm}_{H^\pm\to W^\pm h_j}\right)$$

$\mathcal{M}^{\text{loop}}$ now does not contain external leg contributions on the Higgs boson, as these are contained in $\mathbf{Z}_{ij}$

The cancelation of IR divergences depends on the relation

$$g_{H^\pm G^\mp h_i} = \frac{p^2_{H^\pm} - p^2_{h_i}}{m_W} g_{H^\pm W^\mp h_i}$$

in the following diagrams




which is fulfilled at $p^2_{H^\pm} = m^2_{H^\pm}$ and $p^2_{h_i} = m^2_{h_i}$
$\rightarrow$ enforce this relation for loop-corrected masses

# Gauge Dependence

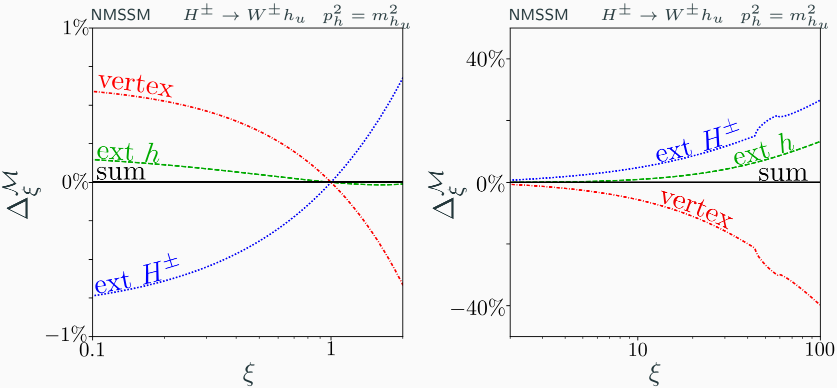
# Example Parameter Point

$M_{H^\pm} = 624\,\text{GeV} \quad \tan\beta = 3.1 \quad \lambda = 0.367 \quad \kappa = 0.584 \quad \mu_{\text{eff}} = 227\,\text{GeV}$

Higgs Bosons:

| | | |
|---|---|---|
| $m_{h_1} = 9.8\,\text{GeV}$ | $M_{h_1} = 94\,\text{GeV}$ | $h_s$-like |
| $m_{h_2} = 91\,\text{GeV}$ | $M_{h_2} = 125\,\text{GeV}$ | $h_u$-like |
| $m_{h_3} = 627\,\text{GeV}$ | $M_{h_3} = 628\,\text{GeV}$ | $h_d$-like |

# $\xi$ Dependence at strict one-loop order

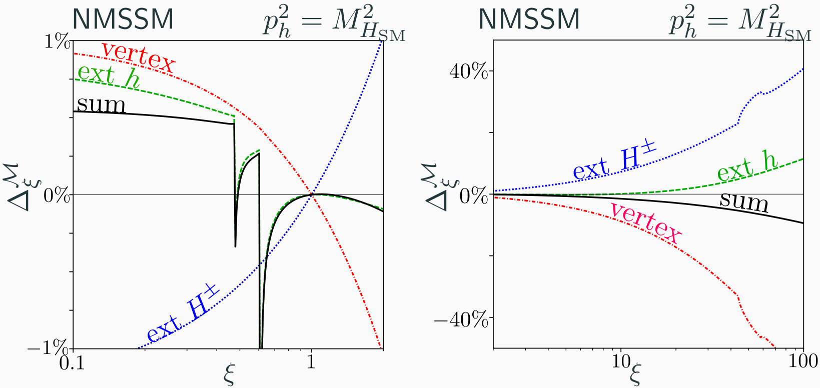



$$\Delta_\xi^\mathcal{M} = \frac{\mathcal{M} - \mathcal{M}|_{\xi=1}}{\mathcal{M}|_{\xi=1}}$$

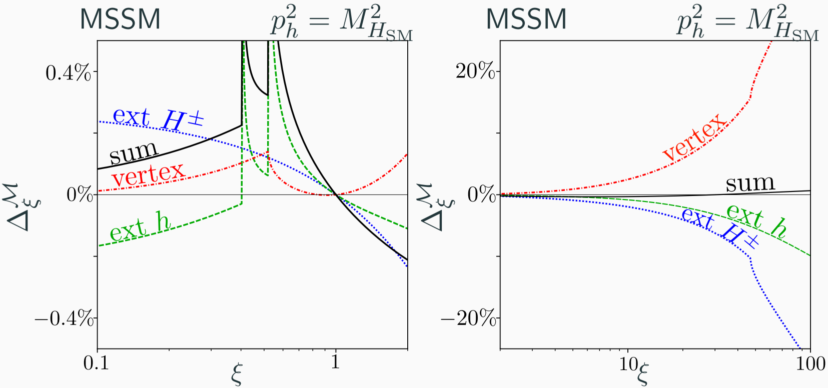
# Gauge Invariance

When using loop-corrected masses for $p_h^2$ we break gauge invariance

# $\xi$ Dependence

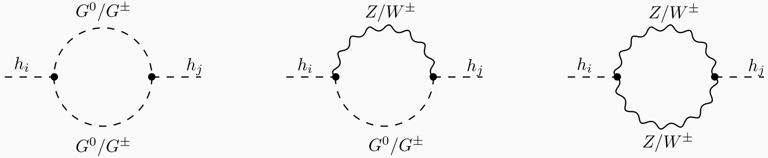


# $\xi$ Dependence MSSM



# Singularities

Mass singularities that occur in the diagrams




do not cancel anymore because

$$g_{h_i G^+ G^-} = \frac{-p_h^2}{2m_W^2} g_{h_i W^+ W^-}$$

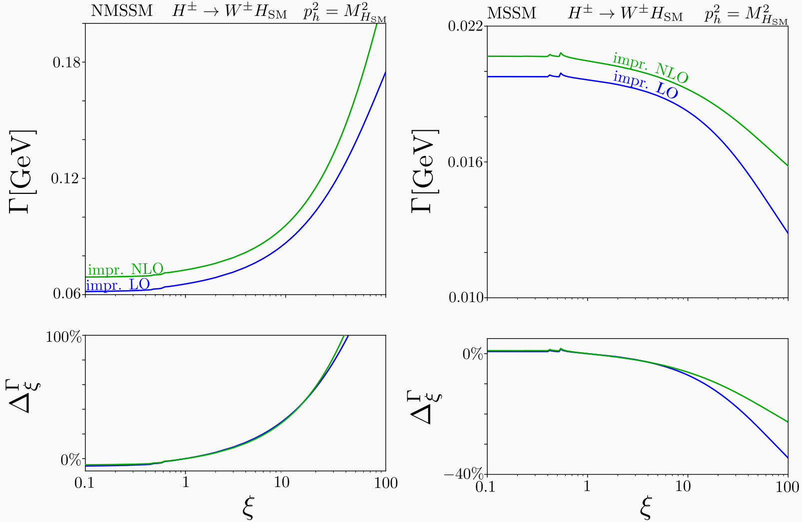
$$g_{h_i G_0 G_0} = \frac{-p_h^2}{2m_Z^2} g_{h_i Z Z}$$

only hold for $p_h^2 = m_{h_i}^2$

# UV divergence

Enforcing these relations by changing the Goldstone couplings leads to UV divergence

# $\xi$ Dependence Resummed

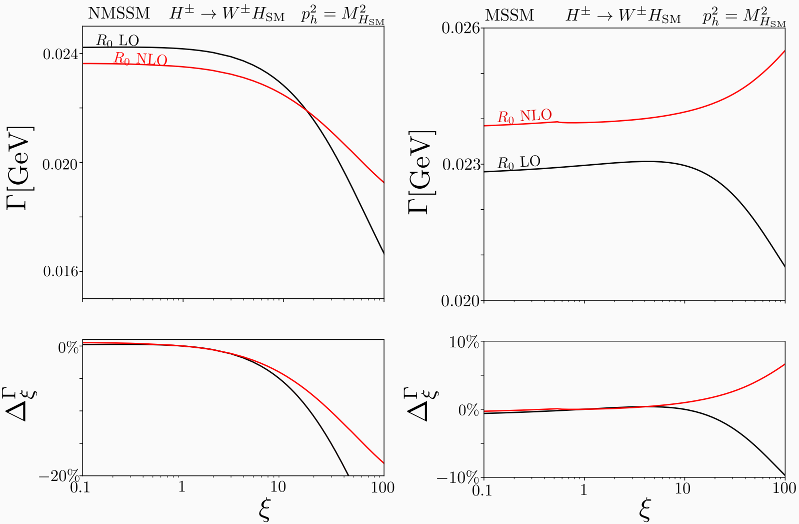



$$\Delta_\xi^\Gamma = \frac{\Gamma - \Gamma|_{\xi=1}}{\Gamma|_{\xi=1}}$$

# $R_0$ Matrix

Instead of using $\mathbf{Z}$, use the matrix $R_0$ that diagonalizes the loop-corrected mass matrix at $p^2 = 0$

# $\xi$ Dependence Resummed




$$\Delta_\xi^\Gamma = \frac{\Gamma - \Gamma|_{\xi=1}}{\Gamma|_{\xi=1}}$$

# Conclusion

# Conclusions

- At one-loop order we encounter both UV and IR divergences that are removed through renormalization and adding real photon emission
- Gauge dependence arising from mixing orders has a big impact on the result in the NMSSM

Thank you for your attention

# Backup: SUSY breaking parameters

SUSY-breaking parameters:

$$
\begin{aligned}
M_1 &= 423\,\text{GeV} & M_2 &= 669\,\text{GeV} & M_3 &= 1850\,\text{GeV} \\
A_t &= 2178\,\text{GeV} & A_b &= -358\,\text{GeV} & A_\tau &= 1401\,\text{GeV} \\
A_\kappa &= -1423\,\text{GeV} & M_{\text{SUSY}} &= 3\,\text{TeV} & & \\
m_{\tilde{l}_L} &= 1170\,\text{GeV} & m_{\tilde{\tau}_R} &= 2872\,\text{GeV} & & \\
m_{\tilde{Q}_L} &= 2365\,\text{GeV} & m_{\tilde{t}_R} &= 1036\,\text{GeV} & m_{\tilde{b}_R} &= 2360\,\text{GeV}
\end{aligned}
$$