Impact of Vacuum Stability on the Phenomenology of SUSY Models

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The Standard Model (In)Stability

$$V_{\text{SM}} = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$$

- large field values: $V \sim \lambda(H^\dagger H)^2$
- RGE: $\lambda \to \lambda(Q)$, where $Q \sim H$
- $\lambda \to 0$ around $Q \sim 10^{10}$ GeV, new minimum beyond M_{Planck}

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The MSSM: less simple

$$V_{\rm MSSM} = V_F + V_{\rm soft} + V_D$$

with (only 3rd generation squarks and Higgses)

$$\begin{aligned} V_{\rm soft} = {} & m_{H_{\rm d}}^2 |h_{\rm d}|^2 + m_{H_{\rm u}}^2 |h_{\rm u}|^2 - \left(B_\mu h_{\rm d} \cdot h_{\rm u} + {\rm h.\,c.}\right) \\ & + \tilde{t}_{\rm L}^* \tilde{m}_Q^2 \tilde{t}_{\rm L} + \tilde{t}_{\rm R}^* \tilde{m}_t^2 \tilde{t}_{\rm R} + \tilde{b}_{\rm L}^* \tilde{m}_Q^2 \tilde{b}_{\rm L} + \tilde{b}_{\rm R}^* \tilde{m}_b \tilde{b}_{\rm R} \\ & + \left(A_{\rm t} h_{\rm u} \tilde{t}_{\rm L}^* \tilde{t}_{\rm R} + A_{\rm b} h_{\rm d} \tilde{b}_{\rm L}^* \tilde{b}_{\rm R} + {\rm h.\,c.}\right) \end{aligned}$$

A multi-scalar theory

- 2 Higgs doublets
- 2×6 scalar quarks, $6 + 3$ scalar leptons
- 12 colored and 18 + 2 charged directions
- charged Higgs directions "safe" [Casas et al. 1996]
- SM Higgs potential: SO(4) symmetry

- large couplings to Higgs doublets (y_t and y_b comparably large)
- large stop contribution (X_t, A_t) to light Higgs mass needed
- SUSY threshold corrections for m_b influence y_b

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An analytic solution?

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impossible!

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An analytic solution?

impossible! *only approximative*

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}_L{}^* \left(\tilde{m}_L^2 + |y_t h_2|^2\right) \tilde{t}_L + \tilde{t}_R{}^* \left(\tilde{m}_t^2 + |y_t h_2|^2\right) \tilde{t}_R \\
& + \tilde{b}_L{}^* \left(\tilde{m}_L^2 + |y_b h_1|^2\right) \tilde{b}_L + \tilde{b}_R{}^* \left(\tilde{m}_b^2 + |y_b h_1|^2\right) \tilde{b}_R \\
& - \left[\tilde{t}_L{}^* (\mu^* y_t \, h_1{}^* - A_t h_2) \tilde{t}_R + \text{h.c.}\right] \\
& - \left[\tilde{b}_L{}^* (\mu^* y_b \, h_2{}^* - A_b h_1) \tilde{b}_R + \text{h.c.}\right] \\
& + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
& + \frac{g_1^2}{8} \left(|h_2|^2 - |h_1|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2 + \frac{1}{3}|\tilde{t}_L|^2 - \frac{4}{3}|\tilde{t}_R|^2\right)^2 \\
& + \frac{g_2^2}{8} \left(|h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2\right)^2 \\
& + \frac{g_3^2}{8} \left(|\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2 \\
& + (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\text{Re}(B_\mu \, h_1 h_2).
\end{aligned}
$$

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}_L{}^* \left(\tilde{m}_L^2 + |y_t h_2|^2\right) \tilde{t}_L + \tilde{t}_R{}^* \left(\tilde{m}_t^2 + |y_t h_2|^2\right) \tilde{t}_R \\
& + \tilde{b}_L{}^* \left(\tilde{m}_L^2 + |y_b h_1|^2\right) \tilde{b}_L + \tilde{b}_R{}^* \left(\tilde{m}_b^2 + |y_b h_1|^2\right) \tilde{b}_R \\
& - \left[\tilde{t}_L{}^* (\mu^* y_t\, h_1{}^* - A_t h_2)\, \tilde{t}_R + \text{h.c.}\right] \\
& - \left[\tilde{b}_L{}^* (\mu^* y_b\, h_2{}^* - A_b h_1)\, \tilde{b}_R + \text{h.c.}\right] \\
& + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
& + \frac{g_1^2}{8} \left(|h_2|^2 - |h_1|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2 + \frac{1}{3}|\tilde{t}_L|^2 - \frac{4}{3}|\tilde{t}_R|^2\right)^2 \\
& + \frac{g_2^2}{8} \left(|h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2\right)^2 \\
& + \frac{g_3^2}{8} \left(|\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2 \\
& + (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\text{Re}(B_\mu\, h_1 h_2).
\end{aligned}
$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|,\ |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$

$$
\begin{aligned}
V_{\tilde q,h} = &\tilde t^{\,*}\left(\tilde m_L^2 + |y_t h_2|^2\right)\tilde t + \tilde t^{\,*}\left(\tilde m_t^2 + |y_t h_2|^2\right)\tilde t \\
&+\tilde b^{\,*}\left(\tilde m_L^2 + |y_b h_1|^2\right)\tilde b + \tilde b^{\,*}\left(\tilde m_b^2 + |y_b h_1|^2\right)\tilde b \\
&-\left[\tilde t^{\,*}(\mu^* y_t\, h_1{}^* - A_t h_2)\tilde t + \text{h.c.}\right] \\
&-\left[\tilde b^{\,*}(\mu^* y_b\, h_2{}^* - A_b h_1)\tilde b + \text{h.c.}\right] \\
&+|y_t|^2|\tilde t|^2|\tilde t|^2 + |y_b|^2|\tilde b|^2|\tilde b|^2 \\
&+\frac{g_1^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde b|^2 - |\tilde t|^2\right)^2 \\
&+\frac{g_2^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde b|^2 - |\tilde t|^2\right)^2 \\
&+(m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\text{Re}(B_\mu\, h_1 h_2).
\end{aligned}
$$

$|\tilde t_L| = |\tilde t_R| = |\tilde t|$, $|\tilde b_L| = |\tilde b_R| = |\tilde b|$

The tree-level scalar potential

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}^{\,*}\left(\tilde{m}_L^2 + |y_t h_2|^2\right)\tilde{t} + \tilde{t}^{\,*}\left(\tilde{m}_t^2 + |y_t h_2|^2\right)\tilde{t} \\
& + \tilde{b}^{\,*}\left(\tilde{m}_L^2 + |y_b h_1|^2\right)\tilde{b} + \tilde{b}^{\,*}\left(\tilde{m}_b^2 + |y_b h_1|^2\right)\tilde{b} \\
& - \left[\tilde{t}^{\,*}(\mu^* y_t\, h_1{}^* - A_t h_2)\tilde{t} + \text{h.c.}\right] \\
& - \left[\tilde{b}^{\,*}(\mu^* y_b\, h_2{}^* - A_b h_1)\tilde{b} + \text{h.c.}\right] \\
& + |y_t|^2|\tilde{t}|^2|\tilde{t}|^2 + |y_b|^2|\tilde{b}|^2|\tilde{b}|^2 \\
& + \frac{g_1^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2\right)^2 \\
& + \frac{g_2^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2\right)^2 \\
& + (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\text{Re}(B_\mu\, h_1 h_2).
\end{aligned}
$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; $|\tilde{b}| = |h_1| = |\phi_1|$, $|\tilde{t}| = |h_2| = |\phi_2|$

The tree-level scalar potential

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \phi_2{}^* \left(\tilde{m}_L^2 + |y_t \phi_2|^2\right) \phi_2 + \phi_2{}^* \left(\tilde{m}_t^2 + |y_t \phi_2|^2\right) \phi_2 \\
& + \phi_1{}^* \left(\tilde{m}_L^2 + |y_b \phi_1|^2\right) \phi_1 + \phi_1{}^* \left(\tilde{m}_b^2 + |y_b \phi_1|^2\right) \phi_1 \\
& - \left[\phi_2{}^* (\mu^* y_t \, \phi_1{}^* - A_t \phi_2) \, \phi_2 + \text{h.c.} \right] \\
& - \left[\phi_1{}^* (\mu^* y_b \, \phi_2{}^* - A_b \phi_1) \, \phi_1 + \text{h.c.} \right] \\
& + |y_t|^2 |\phi_2|^2 |\phi_2|^2 + |y_b|^2 |\phi_1|^2 |\phi_1|^2
\end{aligned}
$$

$$
+ (m_{h_2}^2 + |\mu|^2) |\phi_2|^2 + (m_{h_1}^2 + |\mu|^2) |\phi_1|^2 - 2 \, \text{Re}(B_\mu \, \phi_1 \phi_2).
$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; $|\tilde{b}| = |h_1| = |\phi_1|$, $|\tilde{t}| = |h_2| = |\phi_2|$

$$V_{\tilde{q},h} = \phi_2{}^* \left(\tilde{m}_L^2 + |y_t \phi_2|^2\right) \phi_2 + \phi_2{}^* \left(\tilde{m}_t^2 + |y_t \phi_2|^2\right) \phi_2$$

$$-\left[\phi_2{}^* (\qquad\qquad -A_t \phi_2)\, \phi_2 + \text{h.c.}\right]$$

$$+ |y_t|^2 |\phi_2|^2 |\phi_2|^2$$

$$+ (m_{h_2}^2 + |\mu|^2)|\phi_2|^2 \qquad .$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|,\ |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; ~~$|\tilde{b}| = |h_1| = |\phi_1|$~~, $|\tilde{t}| = |h_2| = |\phi_2|$

Minimize the potential

$$V(\phi) = m^2\phi^2 - A\phi^3 + \lambda\phi^4,$$

with $m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2$, $A = -A_t$ and $\lambda = 3y_t^2$.

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with $m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2$, $A = -A_t$ and $\lambda = 3y_t^2$.

Answer:

$$\phi_0 = 0, \qquad \phi_\pm = \frac{3A \pm \sqrt{9A^2 32\lambda m^2}}{8\lambda}.$$

Condition to be safe from non-standard (i.e. non-trivial) minima:

$$V(\phi_\pm) > 0 \quad \hookrightarrow \quad m^2 > \frac{A^2}{4\lambda}$$

Mathematics for the kindergarden

Minimize the potential

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$$V(\phi_\pm) > 0 \quad \hookrightarrow \quad m^2 > \frac{A^2}{4\lambda}$$

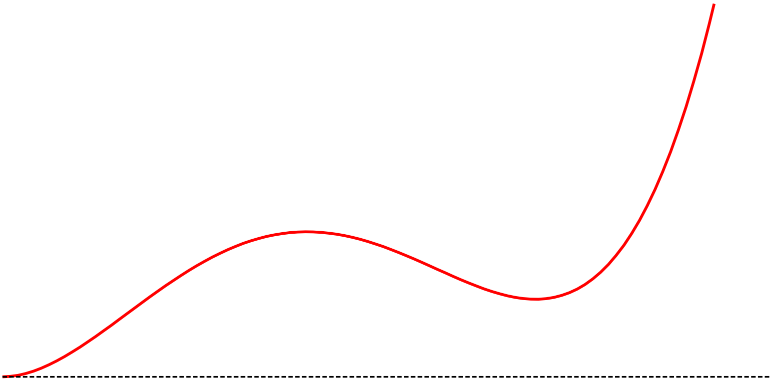
Well-known constraints [Gunion, Haber, Sher '88]

$$|A_t|^2 < 3y_t^2\left(m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2\right)$$

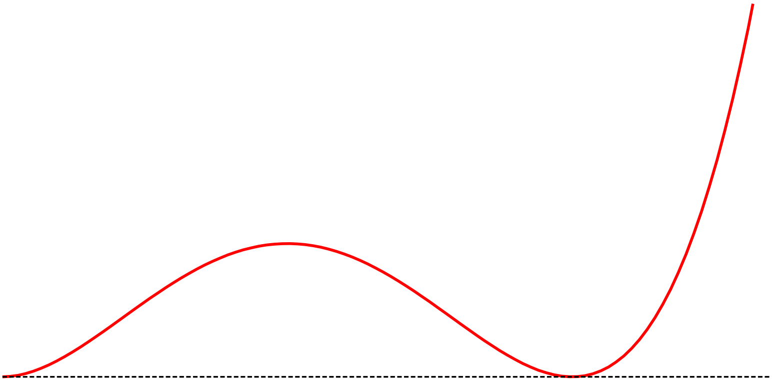
$$|A_b|^2 < 3y_b^2\left(m_{h_1}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_b^2\right)$$

for the limiting cases $|\tilde{t}_L| = |\tilde{t}_R| = |h_2|$ and $|\tilde{b}_L| = |\tilde{b}_R| = |h_1|$!

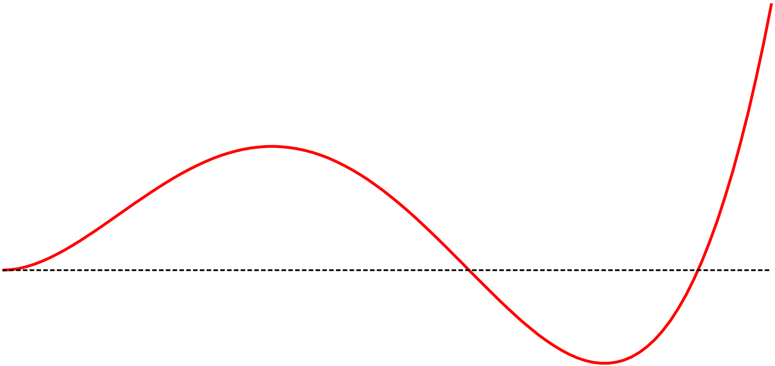
$A^2 < 4\lambda m^2$



$$A^2 = 4\lambda m^2$$



$$A^2 > 4\lambda m^2$$



$$h_2 = \phi, \qquad |\tilde{t}| = \alpha|\phi|, \qquad h_1 = \eta\phi, \qquad |\tilde{b}| = \beta|\phi|$$

$$\begin{aligned} V_\phi = & \Big(m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1+\eta^2)\mu^2 - 2B_\mu \eta \\ & \qquad + (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2 \tilde{m}_t^2 + \beta^2 \tilde{m}_b^2 \Big) \phi^2 \\ & - 2\left(\alpha^2(\mu y_t \eta - A_t) + \beta^2(\mu y_t - \eta A_b) \right) \phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2)\phi^4 \\ & + \left(\frac{g_1^2 + g_2^2}{8}(1 - \eta^2 + \beta^2 - \alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2 \right) \phi^4 \\ \equiv & \, \boldsymbol{M^2(\eta, \alpha, \beta)\phi^2 - \mathcal{A}(\eta, \alpha, \beta)\phi^3 + \lambda(\eta, \alpha, \beta)\phi^4}, \end{aligned}$$

$$h_2 = \phi, \qquad |\tilde{t}| = \alpha|\phi|, \qquad h_1 = \eta\phi, \qquad |\tilde{b}| = \beta|\phi|$$

$$\begin{aligned} V_\phi =& \Big(m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1+\eta^2)\mu^2 - 2B_\mu \eta \\ & + (\alpha^2+\beta^2)\tilde{m}_L^2 + \alpha^2 \tilde{m}_t^2 + \beta^2 \tilde{m}_b^2\Big)\phi^2 \\ & - 2\left(\alpha^2(\mu y_t \eta - A_t) + \beta^2(\mu y_t - \eta A_b)\right)\phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2)\phi^4 \\ & + \left(\frac{g_1^2+g_2^2}{8}(1-\eta^2+\beta^2-\alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2\right)\phi^4 \\ \equiv & \boldsymbol{M^2(\eta,\alpha,\beta)\phi^2 - \mathcal{A}(\eta,\alpha,\beta)\phi^3 + \lambda(\eta,\alpha,\beta)\phi^4}, \end{aligned}$$

with

$$\begin{aligned} M^2 =& \; m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1+\eta^2)\mu^2 - 2B_\mu\eta \\ & + (\alpha^2+\beta^2)\tilde{m}_L^2 + \alpha^2\tilde{m}_t^2 + \beta^2\tilde{m}_b^2, \\ \mathcal{A} =& \; 2\alpha^2\eta\mu y_t - 2\alpha^2 A_t + 2\beta^2\mu y_b - 2\eta\beta^2 A_b, \\ \lambda =& \; \frac{g_1^2+g_2^2}{8}(1-\eta^2+\beta^2-\alpha^2)^2 \\ & + (2+\alpha^2)\alpha^2 y_t^2 + (2\eta^2+\beta^2)\beta^2 y_b^2. \end{aligned}$$

[Gunion, Haber, Sher '88; Casas, Lleyda, Muñoz '96]

The same but different ("A-parameter bounds")

$$\mathcal{A}^2 < 4\lambda M^2$$

$$\downarrow$$

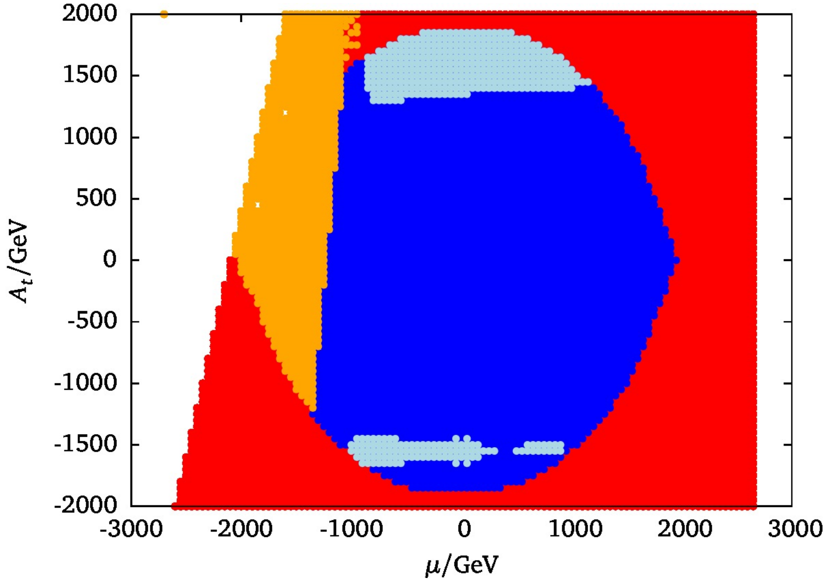
$$4 \min_{\{\eta,\alpha,\beta\}} \lambda(\eta,\alpha,\beta) M^2(\eta,\alpha,\beta) > \max_{\{\eta,\alpha,\beta\}} (\mathcal{A}(\eta,\alpha,\beta))^2$$

$h_u = \tilde{b}$, $h_d^0 = 0$ [WGH'15]

$$m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2}$$

$|h_d|^2 = |h_u|^2 + |\tilde{b}|^2$, $\tilde{b} = \alpha h_u$ [WGH'15]

$$m_{11}^2(1+\alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1+\alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{4\mu^2\alpha^2}{2+3\alpha^2}$$


2000
1500
1000
500
0
-500
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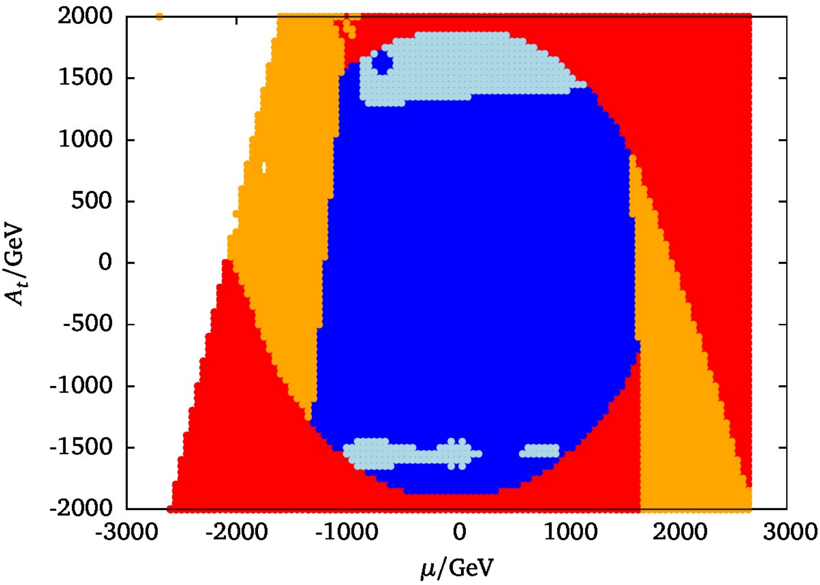
-2000
A_t/GeV
-3000
-2000
-1000
0
1000
2000
3000
μ/GeV

[WGH 2016]



2000
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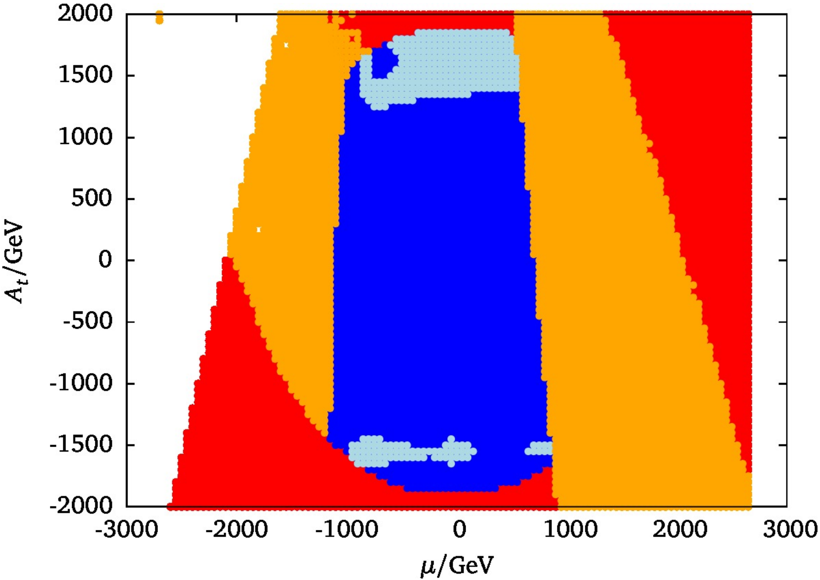
A_t/GeV
μ/GeV

[WGH 2016]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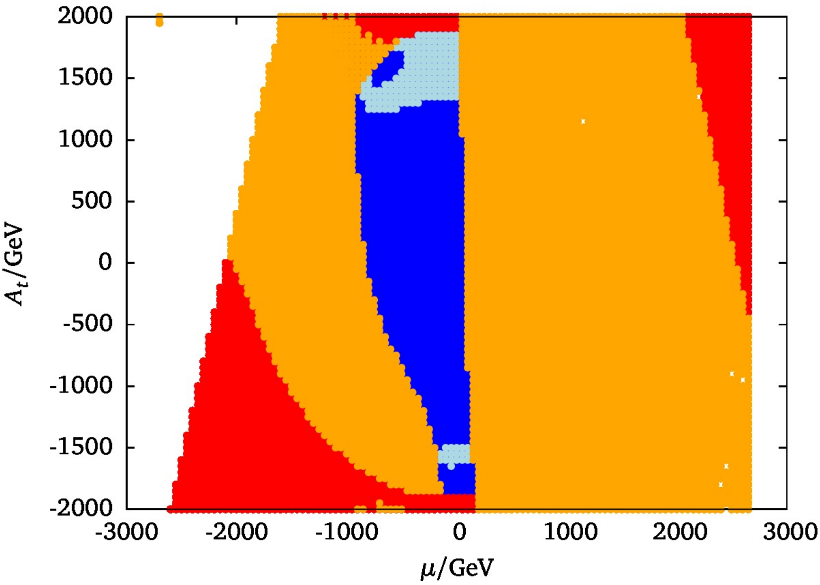
μ/GeV

[WGH 2016]



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A_t/GeV
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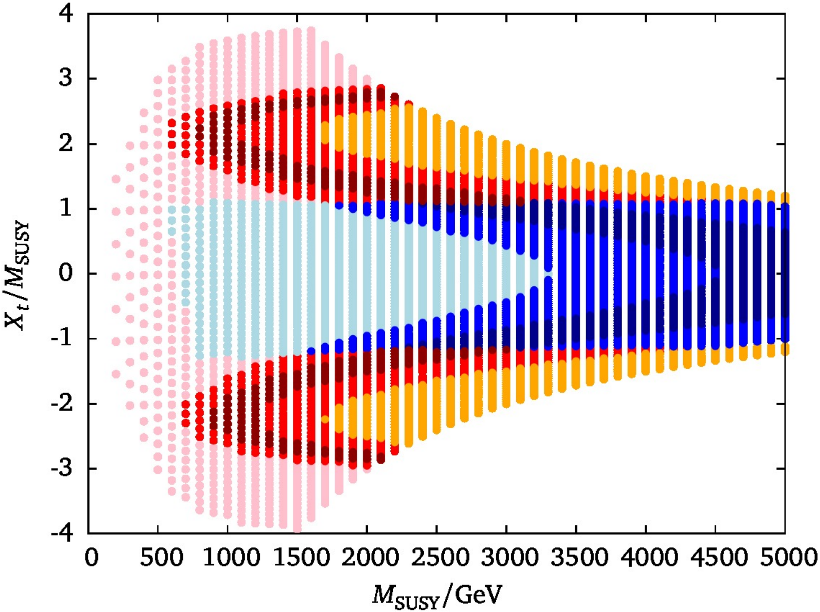
μ/GeV

[WGH 2016]


4
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-3
-4
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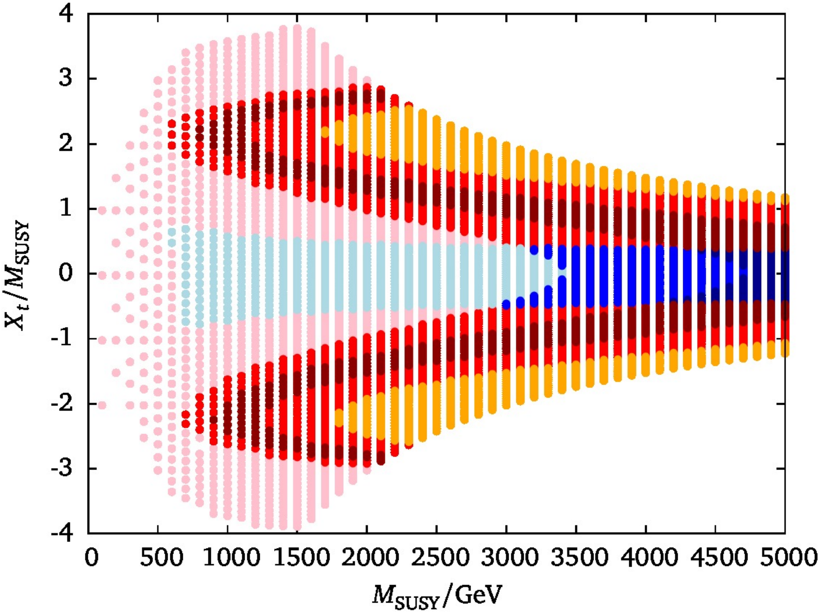
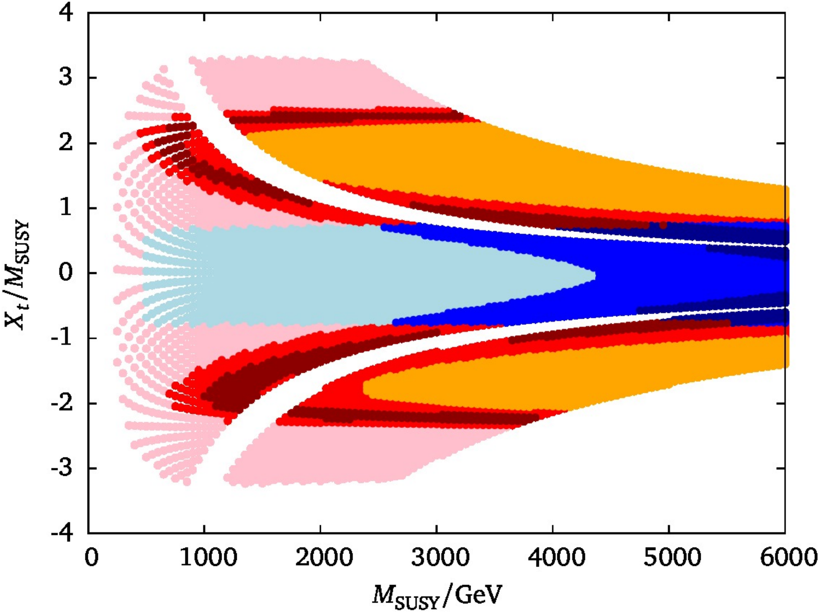
4500
5000
X_t / M_{SUSY}
$M_{\mathrm{SUSY}}/\mathrm{GeV}$



4
3
2
1
0
-1
-2
-3
-4
X_t / M_{SUSY}
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
M_{SUSY}/GeV



4
3
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0
-1
-2
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-4
0
1000
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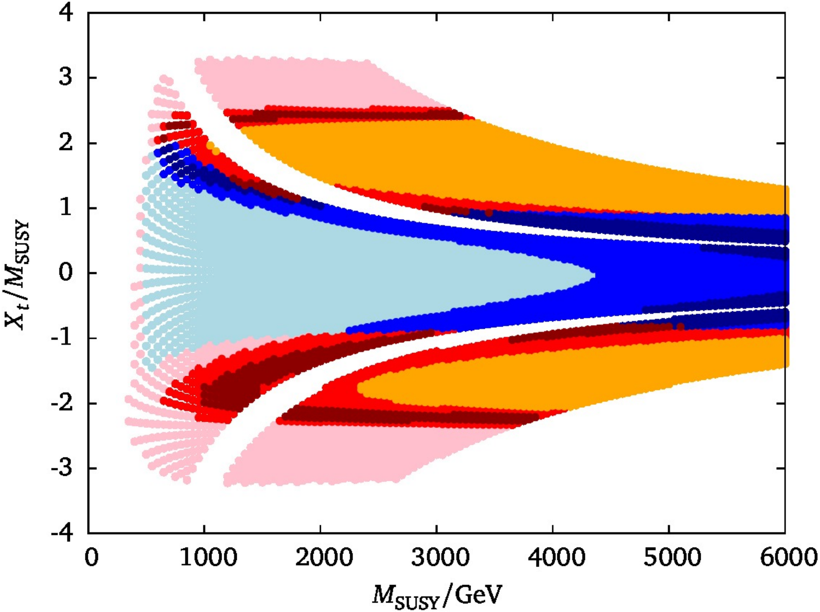
4000
5000
6000
X_t/M_{SUSY}
$M_{\text{SUSY}}/\text{GeV}$


4
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0
-1
-2
-3
-4
X_t/M_{SUSY}
0
1000
2000
3000
4000
5000
6000
$M_{\text{SUSY}}/\text{GeV}$

A comment on metastability and quantum tunneling

Cosmological stability

bounce action

$$B \gtrsim 400$$

$\hookrightarrow$ life-time longer than age of the universe

Decay probability (per unit volume)

$$\frac{\Gamma}{V} = A e^{-B/\hbar}$$

[Coleman '77]

Death and doom

- value of B crucially depends on field space path
- multifield spaces: reduction to single field space (!)
- *independent* of SUSY parameter choice

A general n scalar potential

$$V(\vec{\phi}) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

- includes up to 3^n stationary points
- initial vacuum at $\vec{\phi} = \vec{\phi}_v$

$$\left.\frac{\partial V}{\partial \phi_a}\right|_{\vec{\phi}=\vec{\phi}_v} = 0$$

Expanding around the vacuum

- $\vec{\phi} = \vec{\phi}_v + \vec{\varphi}$, with $\vec{\varphi} = (\varphi_1, \ldots, \varphi_n)^{\mathrm{T}}$

$$V(\vec{\varphi}) = \lambda'_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d + A'_{abc}\varphi_a\varphi_b\varphi_c + m'^2_{ab}\varphi_a\varphi_b$$

- rewrite $\vec{\varphi} \to \varphi\hat{\varphi}$ with unit vector $\hat{\varphi}$, $\varphi = \sqrt{\varphi_1^2 + \ldots + \varphi_n^2}$

$$V(\varphi, \hat{\varphi}) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2$$

A general n scalar potential

$$V(\vec{\phi}) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

- includes up to 3^n stationary points
- initial vacuum at $\vec{\phi} = \vec{\phi}_v$

$$\left.\frac{\partial V}{\partial \phi_a}\right|_{\vec{\phi}=\vec{\phi}_v} = 0$$

A semi-analytic approximation

A quartic potential: $V(\phi) = \lambda\,\phi^4 - A^2\,\phi^3 + m^2\,\phi^2$

$$B = \frac{\pi^2}{3\lambda}(2-\delta)^{-3}\left(13.832\,\delta - 10.819\,\delta^2 + 2.0765\,\delta^3\right)$$

with

$$\delta = \frac{8\lambda m^2}{A^2}$$

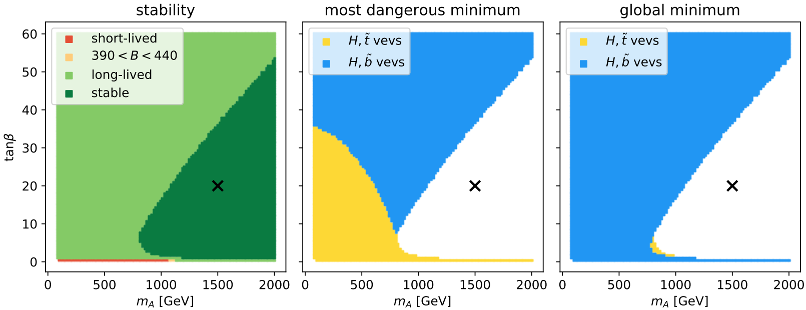
[Adams 1993]

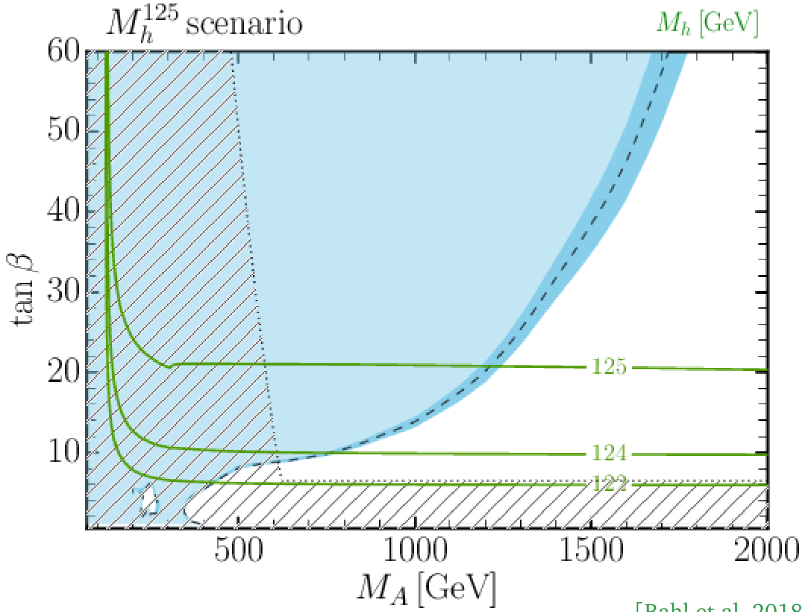
Benchmark scenario M_h^{125} [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,, \quad \mu = 1\,\text{TeV}\,,$$

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_\tau = A_t\,,$$

$$M_1 = M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



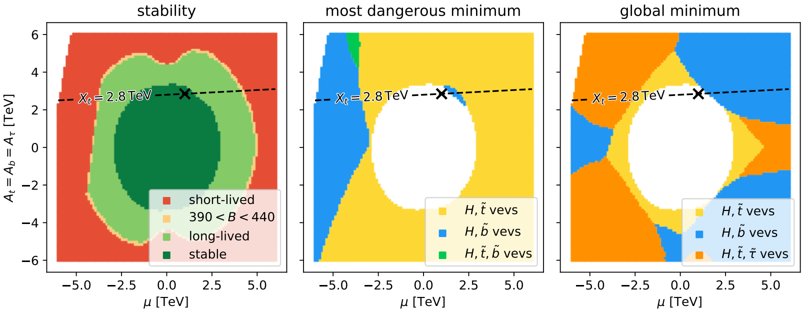


[Bahl et al. 2018]

An absolutetely stable and experimentally allowed point

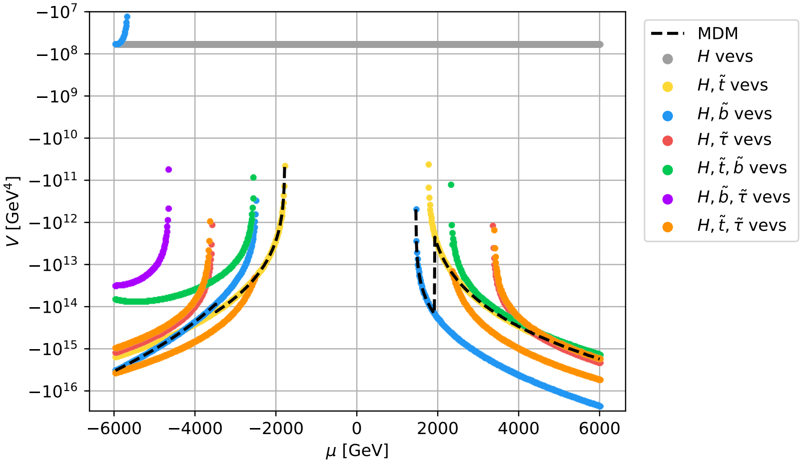
$$\tan\beta = 20 \qquad m_A = 1500\,\text{GeV}$$

$A \equiv A_t = A_b = A_\tau$



- tachyonic sbottom masses upper left corner
- caveat: still limited numbers of fields included!

How is the “most dangerous minimum” (MDM) defined?
Go along the dashed line with $X_t = 2.8\,\mathrm{TeV}$



A vast set of constraints

$$A_t^2 + 3\mu^2 < (m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2) \cdot \begin{cases} 3 & \text{stable,} \\ 7.5 & \text{long-lived.} \end{cases}$$

[Casas, Lleyda, Muñoz 1996, Kusenko, Langacker, Segre 1996]

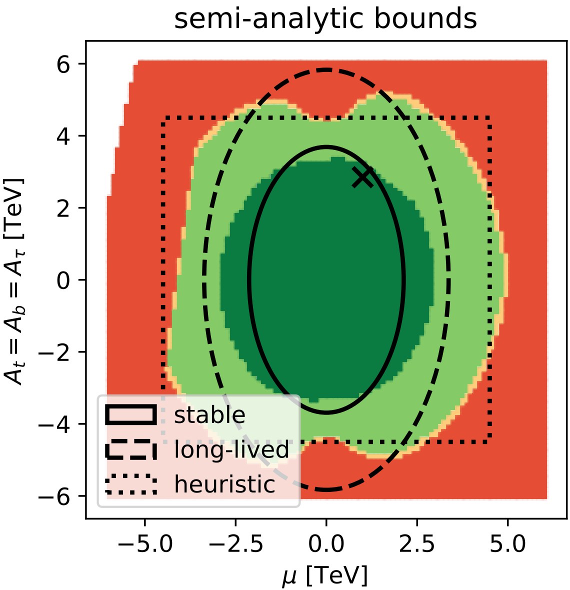
Furthermore, a “heuristic” bound of

$$\frac{\max(A_{\tilde{t},\tilde{b}}, \mu)}{\min(m_{Q_3,U_3})} \lesssim 3$$

exists. [Bechtle, Haber, Heinemeyer, Stefaniak, Stål, Weiglein, Zeune 2016]

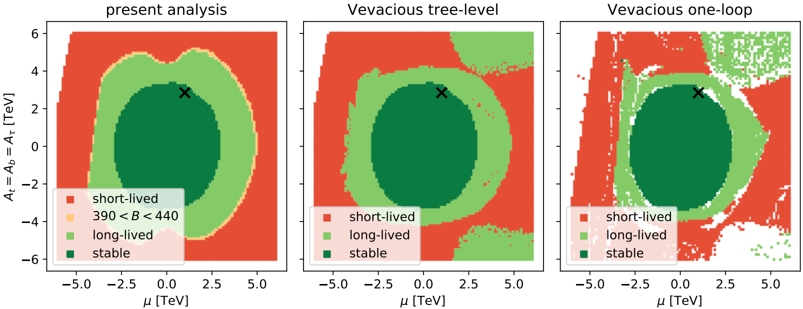
- vacuum tunneling weakens the “traditional” constraint
- metastable vacuum vs. absolute minimum
- quick and dirty versus sophisticated and precise (i. e. slow)

$\hookrightarrow$ needs numerical evaluation!


semi-analytic bounds
$A_t = A_b = A_\tau$ [TeV]
6
4
2
0
−2
−4
−6
−5.0
−2.5
0.0
2.5
5.0
μ [TeV]
stable
long-lived
heuristic

- inclusion of one-loop effective potential
- thermal corrections
- quantum tunneling by `CosmoTransitions` [Wainwright 2011]

`Vevacious` [Camargo-Molina, O'Leary, Porod, Staub 2013]



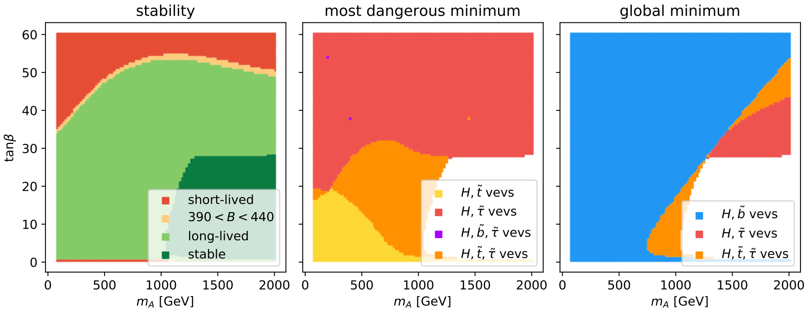
[WGH, Weiglein, Wittbrodt 2019]

Benchmark scenario $M_h^{125}(\tilde{\tau})$: light stau [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 350\,\text{GeV}\,, \quad \mu = 1\,\text{TeV}\,,$$

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_t\,, \quad A_\tau = 800\,\text{GeV}\,,$$

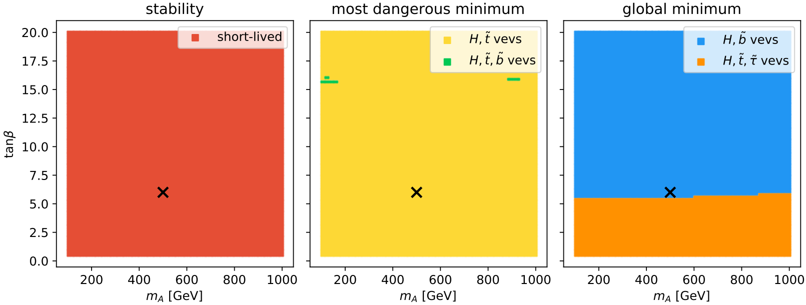
$$M_1 = M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



large $\tan\beta$ values: short-lived

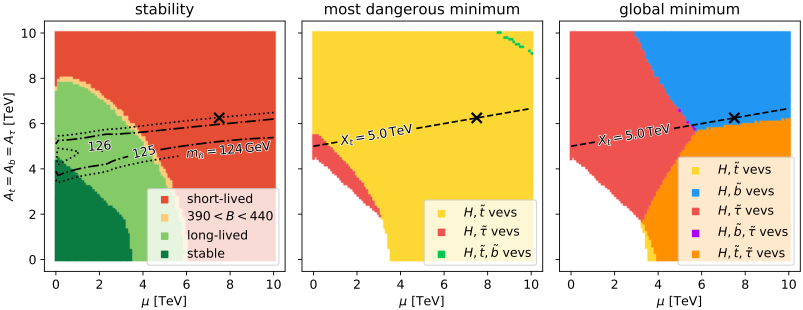
Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



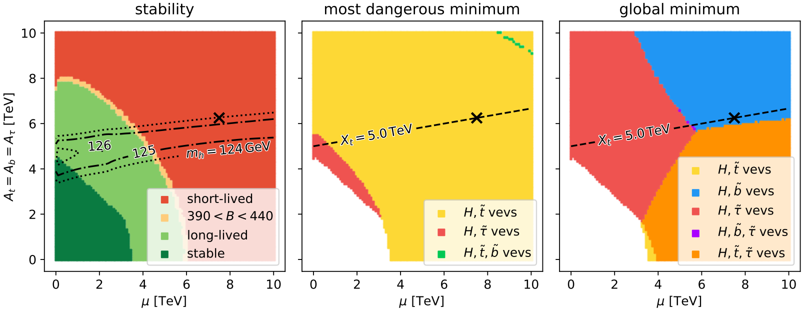
Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$

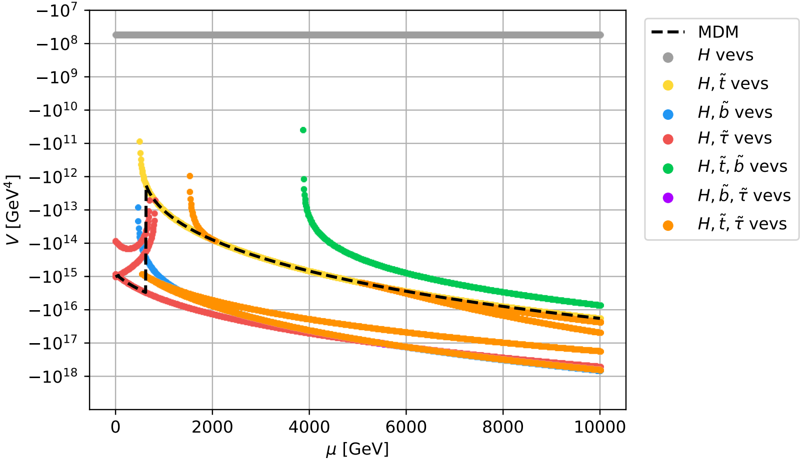


Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$

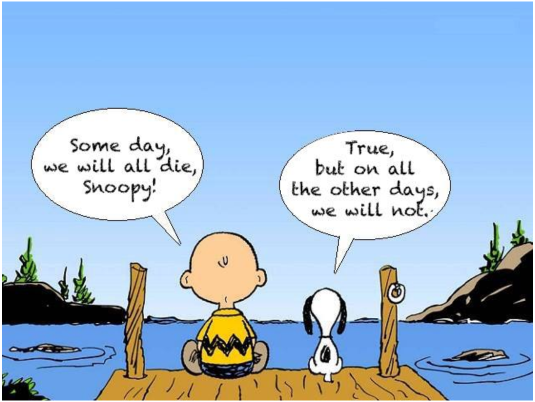


$$\mu = 7.5\,\text{TeV} \rightarrow 4\,\text{TeV} \quad \text{and} \quad A = 6.25\,\text{TeV} \rightarrow 5\,\text{TeV}$$



-10^{7}
-10^{8}
-10^{9}
-10^{10}
-10^{11}
-10^{12}
-10^{13}
-10^{14}
-10^{15}
-10^{16}
-10^{17}
-10^{18}
V [GeV^4]
0
2000
4000
6000
8000
10000
μ [GeV]
MDM
H vevs
$H, \tilde{t}$ vevs
$H, \tilde{b}$ vevs
$H, \tilde{\tau}$ vevs
$H, \tilde{t}, \tilde{b}$ vevs
$H, \tilde{b}, \tilde{\tau}$ vevs
$H, \tilde{t}, \tilde{\tau}$ vevs

- severe constraints from vacuum (meta)stability in SUSY
- fast and numerically stable approach
- global minimum not the “most dangerous” one
- tree-level analysis sufficient (in comparison with 1-loop)



Backup

Slides

False vacuum decay

The bounce solution [Coleman 1977]

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\phi}\frac{d\phi}{d\rho} = \frac{\partial U}{\partial \phi}$$

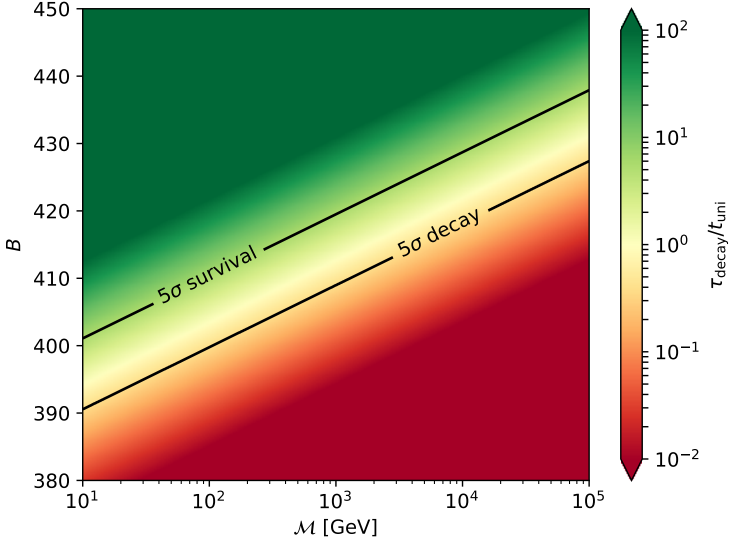
with boundary conditions

$$\phi(\infty) = \phi_v, \quad \left.\frac{d\phi}{d\rho}\right|_{\rho=0} = 0$$

U is the euclidean scalar potential, ρ is a spacetime variable and ϕ_v is the location of the metastable minimum.

The bounce action B is the stationary point of the euclidean action given by the integral

$$B = 2\pi^2 \int_0^\infty \rho^3 \, d\rho \left[\frac{1}{2}\left(\frac{d}{d\rho}\phi_B(\rho)\right)^2 + U(\phi_B(\rho))\right]$$


450
440
430
420
410
400
390
380
B
10^1
10^2
10^3
10^4
10^5
$\mathcal{M}$ [GeV]
5σ survival
5σ decay
10^2
10^1
10^0
10^{-1}
10^{-2}
$\tau_{\text{decay}}/t_{\text{uni}}$

Reduction to a single real scalar field

$$\phi \to \frac{1}{\sqrt{2}} \mathrm{Re}(\phi) + \frac{i}{\sqrt{2}} \mathrm{Im}(\phi)$$

- φ is canonically normalised after expanding $\vec{\varphi} = \varphi \hat{\varphi}$
- EW vacuum is given by

$$\mathrm{Re}(h_u^0) = v \sin\beta \,, \quad \mathrm{Re}(h_d^0) = v \cos\beta$$

where $v = \sqrt{v_u^2 + v_d^2} \approx 246\,\mathrm{GeV}$ is the SM Higgs vev

Unfeasible to vary all real scalar degrees of freedom simultaneously $\hookrightarrow$ selection of fields

$$\left\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{t}_L), \mathrm{Re}(\tilde{t}_R), \mathrm{Re}(\tilde{b}_L), \mathrm{Re}(\tilde{b}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{t}_L), \mathrm{Re}(\tilde{t}_R), \mathrm{Re}(\tilde{\tau}_L), \mathrm{Re}(\tilde{\tau}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{b}_L), \mathrm{Re}(\tilde{b}_R), \mathrm{Re}(\tilde{\tau}_L), \mathrm{Re}(\tilde{\tau}_R)\right\}$$