Three-loop Quark Jet Function

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Jet function: applications

Quark jet function is an universal ingredient in SCET factorization

→ needed for resummation of many observables:

- thrust
- C-parameter
- heavy jet mass
- DIS
- $b \to s\gamma$

History

Jet function introduced in [Bauer, Pirjol, Stewart, '01]

quark jet function

- one-loop: [Bauer, Manohar, '03; Bosch, Lange, Neubert, Paz, '04]
- two-loop: [Becher, Neubert, '06]
- three-loop: [RB, Liu, Stahlhofen, '18]

gluon jet function

- one-loop: [Becher, Schwartz, '09]
- two-loop: [Becher, Bell, '10]
- three-loop: [Banerjee, Dhani, Ravindran '18]

Overview

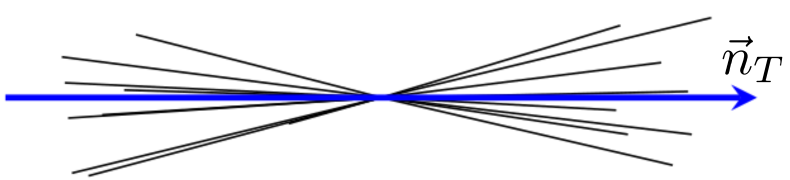
1. Thrust
2. Definition of the quark jet function
3. Three-loop calculation
4. Summary

Thrust: definition

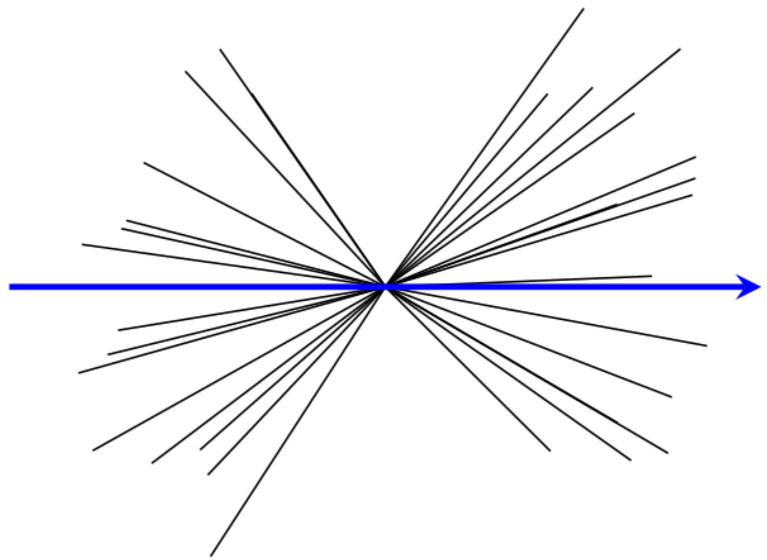
Thrust is an event shape observable: $e^+e^- \to$ hadrons

[Farhi '77]

$$T = \frac{1}{Q} \max_{\vec{n}_T} \sum_j |\vec{n}_T \cdot \vec{p}_j|$$



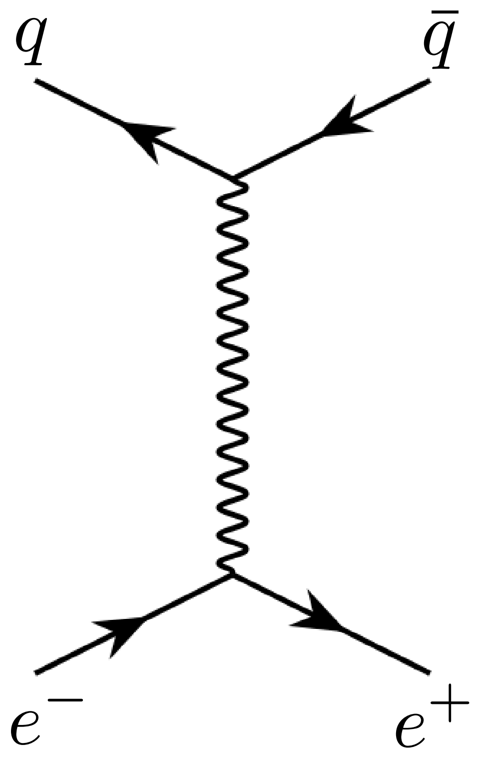
$\tau = 1 - T = 0.00039$



$\tau = 1 - T = 0.0073$

Thrust: tree level

At leading order we have: $e^+e^- \to \gamma^* \to \bar{q}q$



$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \delta(\tau) + \mathcal{O}(\alpha_s)$$

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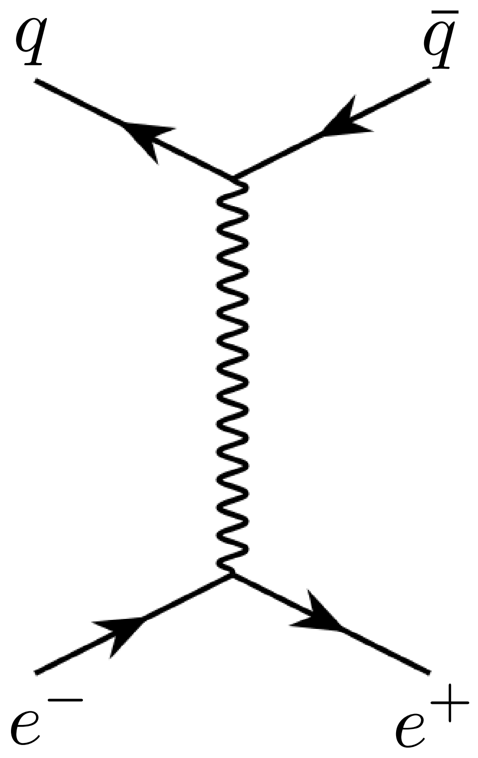


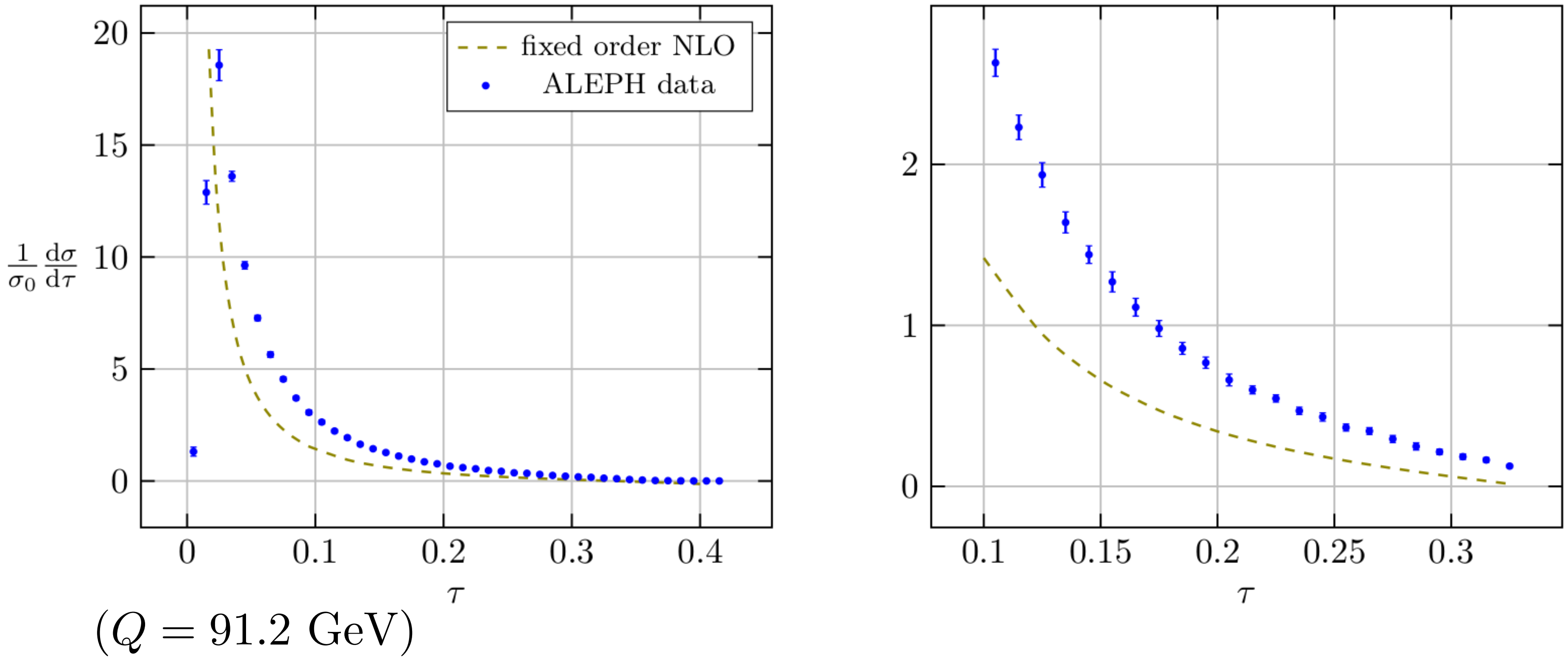
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \delta(\tau) + \mathcal{O}(\alpha_s)$$

allows for a precise determination of α_s

[Becher, Schwartz, '08; Abbate, Fickinger, Hoang, Mateu, Stewart, '10, '12]

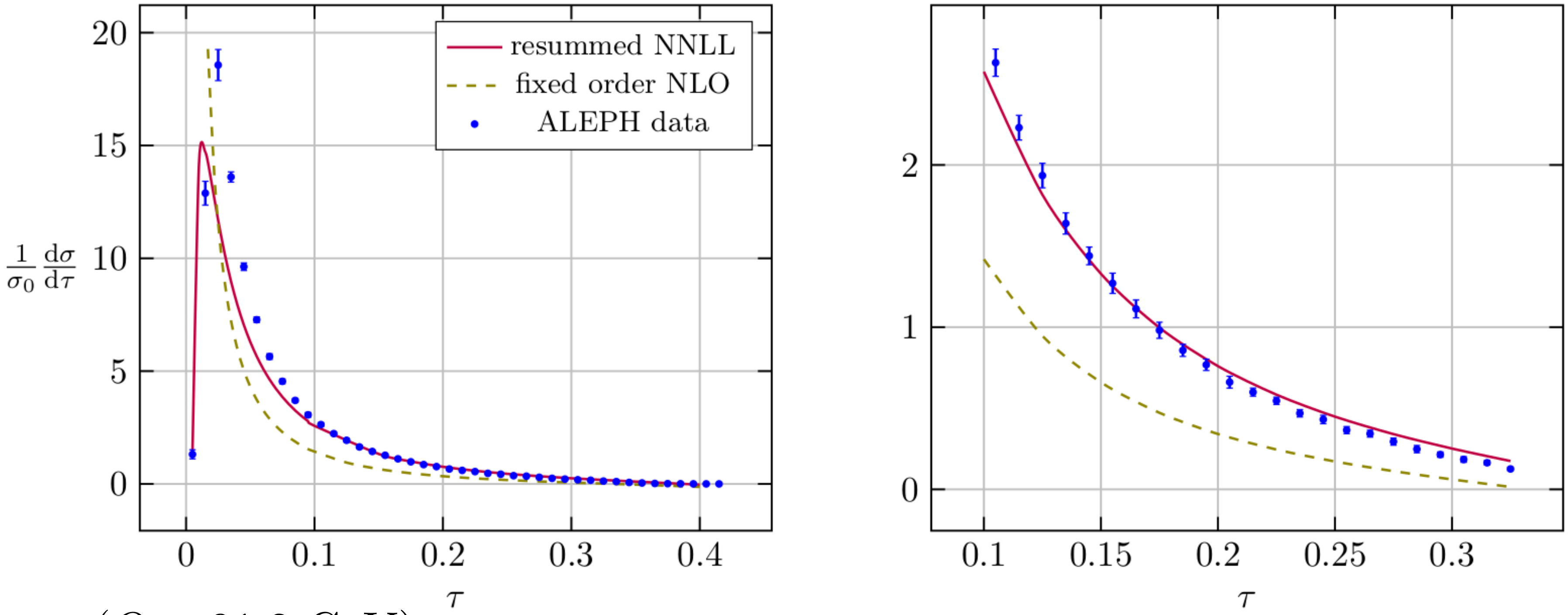
Thrust: theory vs experiment

- For small τ perturbation series is spoiled by large logarithms $\log^n(\tau)$



Thrust: theory vs experiment

- For small τ perturbation series is spoiled by large logarithms $\log^n(\tau)$
- Resum large logs within SCET framework



$(Q = 91.2 \text{ GeV})$

Thrust: factorisation formula

For small τ the differential cross section factorizes:

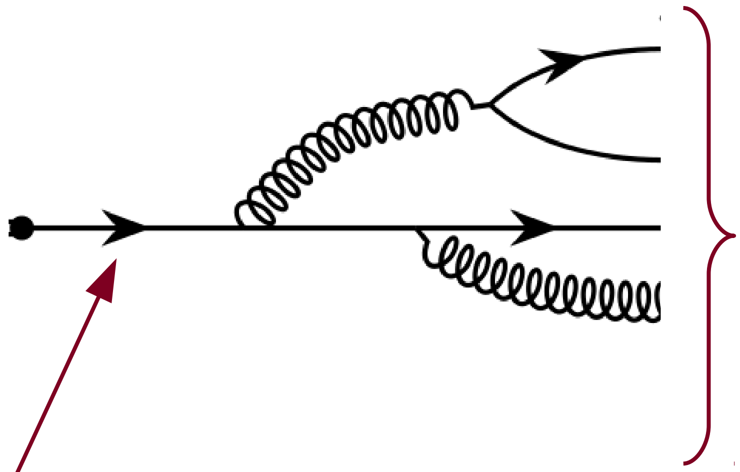
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = H(Q^2) \times J_c \otimes J_{\bar{c}} \otimes S(\tau) + \mathcal{O}(\tau^0)$$

hard function | jet function | soft function

$$Q^2 \gg \tau Q^2 \gg \tau^2 Q^2$$

Jet function

Jet function $J_c(p^2)$ is related to probability to find jet with invariant mass p^2

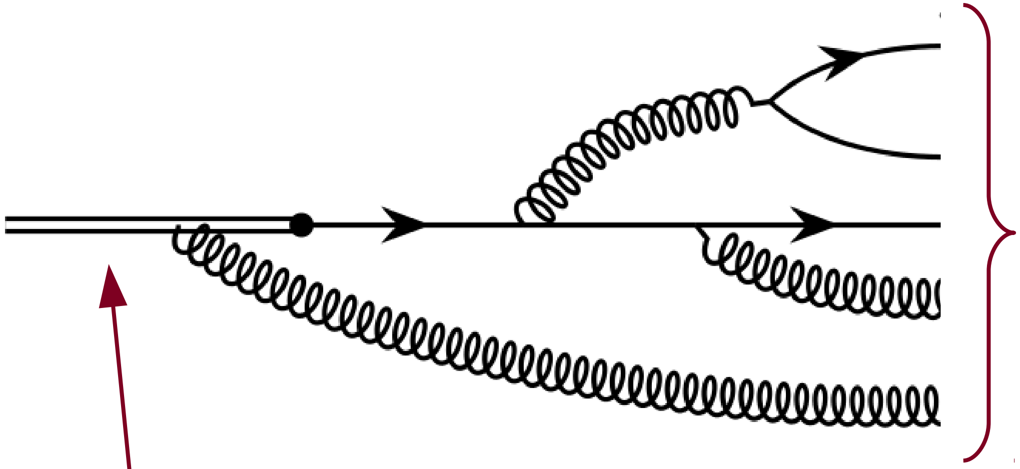


$\langle X_c | \bar{\xi}_c(0) \qquad |0\rangle$

collinear quark, propagats (mostly) in direction $n^\mu = (1, \vec{n}_T)$

Jet function

Jet function $J_c(p^2)$ is related to probability to find jet with invariant mass p^2



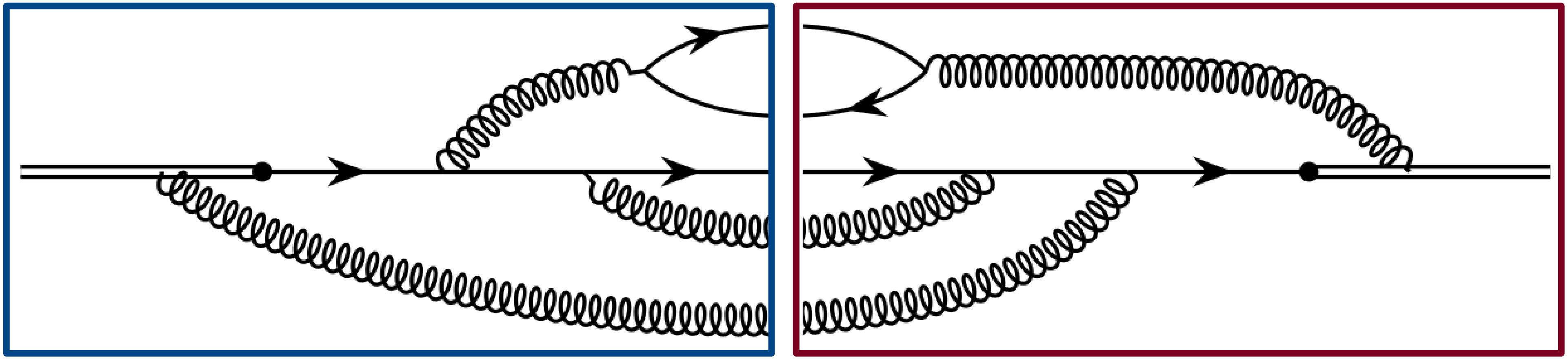
final state X_c

$$\langle X_c | \bar{\xi}_c(0) W_c(0) | 0 \rangle$$

collinear Wilson line $\quad W_c(x) = \mathrm{P}\exp\left[i g_{\mathrm{YM}} \int_{-\infty}^{0} \mathrm{d}s\, \bar{n} \cdot A_c(x + s\bar{n}) \right]$

Jet function - continued

Jet function $J_c(p^2)$ is related to probability to find jet with invariant mass p^2

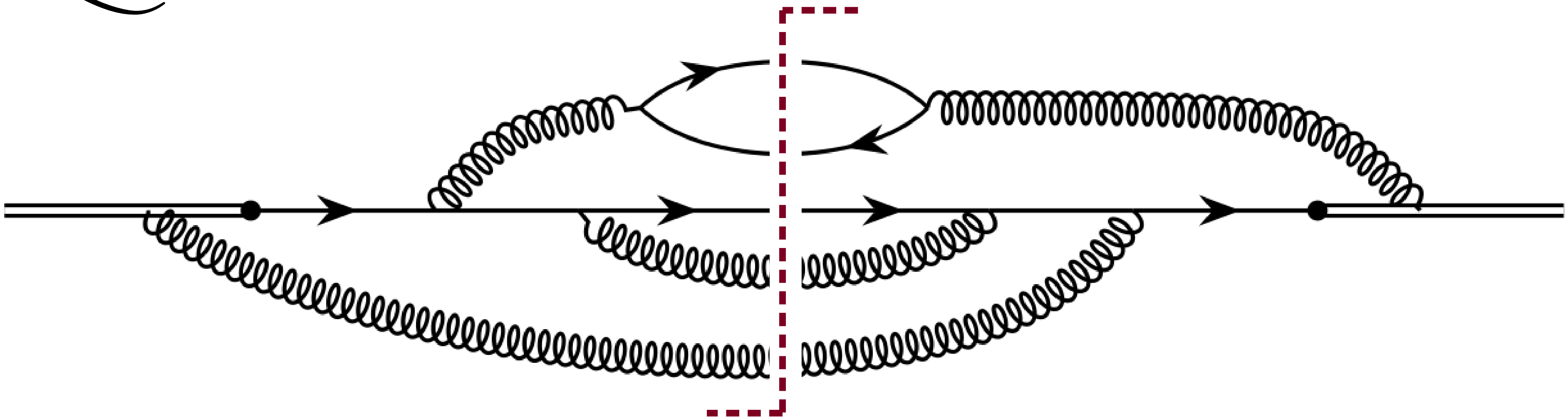


$$J(p^2) = \frac{1}{2N_c} \sum\!\!\!\!\!\!\int \mathrm{d}\Pi_{X_c} \, \mathrm{Tr}\left[\bar{n}\!\!\!/ \langle 0 | W_c^\dagger(0) \xi_c(0) | X_c \rangle \langle X_c | \bar{\xi}_c(0) W_c(0) | 0 \rangle \right]$$

$$\times (2\pi)^{d-2} \delta^{(d-2)}(p_{X_c}^\perp)(2\pi)\delta(Q/2 - p_{X_c}^0)\delta(p^2 - p_{X_c}^2)$$

Jet function

- Use optical theorem to rewrite J as a two point function
- At leading power in τ: collinear sector equivalent to QCD



$$J(p^2) = \frac{1}{\pi N_c} \operatorname{Im}\left[\frac{i}{\bar{n}\cdot p}\int \mathrm{d}^d x\, e^{-ip\cdot x}\, \langle 0|\, \mathrm{Tr}\left[\frac{\bar{\not{n}}}{4} W^\dagger(0)\psi(0)\overline{\psi}(x)W(x)\right]|0\rangle\right]$$

Three-loop calculation

1. Generate diagrams (QGRAF) [Nogueira, '93]
2. Map diagrams to integral families
3. Lorentz, Dirac and color algebra

$$\text{Diagram} = \sum_i c_i \times (\text{scalar integral})_i$$

Three-loop calculation

1. Generate diagrams (QGRAF) [Nogueira, '93]
2. Map diagrams to integral families
3. Lorentz, Dirac and color algebra
4. Partial fractioning of linear dependent propagators
5. IBP reduction (FIRE5 + LiteRed) [Smirnov, '14] [Lee, '12]

$$J^{(3)} = \sum_{i=1}^{30} \tilde{c}_i \times (\text{master integral})_i$$

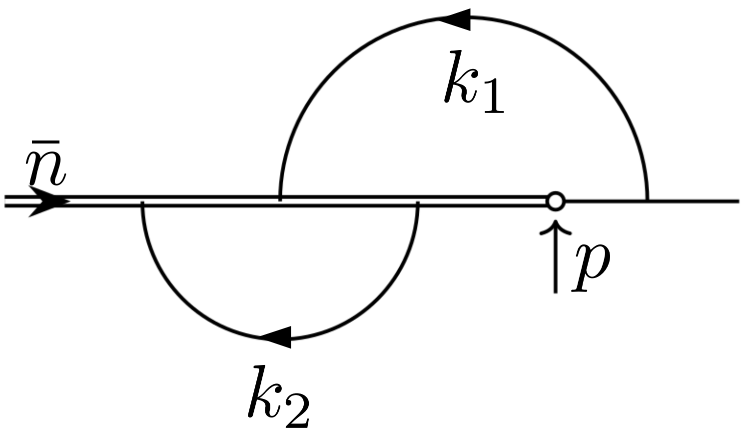
Three-loop calculation

1. Generate diagrams (QGRAF) [Nogueira, '93]
2. Map diagrams to integral families
3. Lorentz, Dirac and color algebra
4. Partial fractioning of linear dependent propagators
5. IBP reduction (FIRE5 + LiteRed) [Smirnov, '14] [Lee, '12]
6. Compute master integrals (analytically)
7. Renormalization

→ DONE

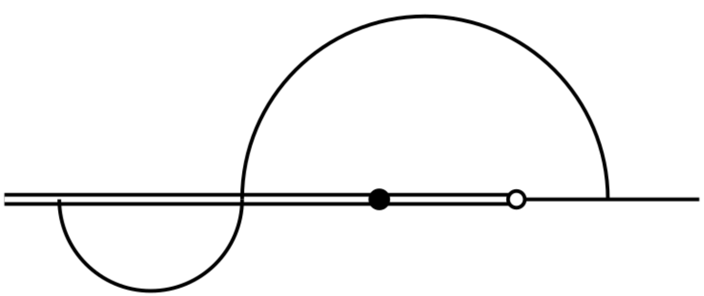
Partial fractioning

- For straightforward IPB reduction need linear independent propagators
- Consider two-loop integral:



$$\longrightarrow \frac{1}{[\bar{n}\cdot k_1][\bar{n}\cdot k_2][\bar{n}\cdot(k_1+k_2)]}$$

$=$



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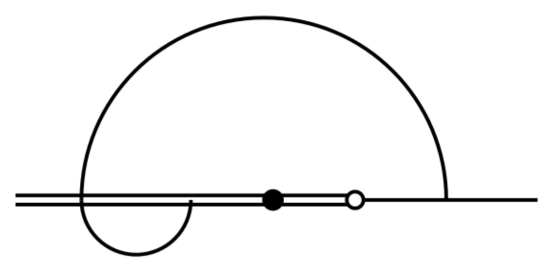

Partial fractioning

Use multivariate partial fraction decomposition algorithm

- Output for two loop example: [Pak, '12]

$$\frac{1}{[\bar{n}\cdot k_2][\bar{n}\cdot(k_1+k_2)]} \to \frac{1}{[\bar{n}\cdot k_1][\bar{n}\cdot(k_2)]} - \frac{1}{[\bar{n}\cdot k_1][\bar{n}\cdot(k_1+k_2)]}$$

$$[\bar{n}\cdot(k_1+k_2)] \to [\bar{n}\cdot k_1] + [\bar{n}\cdot k_2]$$

$$\frac{[\bar{n}\cdot k_2]}{[\bar{n}\cdot(k_1+k_2)]} \to 1 - \frac{[\bar{n}\cdot k_1]}{[\bar{n}\cdot(k_1+k_2)]}$$

- Apply rules recursively
- Use of Gröbner basis ensures termination

Master integrals

- Integrals depend on p^2 and $\bar{n}\cdot p$

fixed by dimensional analysis

fixed by rescaling $\bar{n} \to \lambda\bar{n}$

Master integrals

- Integrals depend on p^2 and $\bar{n}\cdot p$
- Can set $p^2 = -1$ and $\bar{n}\cdot p = 1$
- Compute master integrals with HyperInt:

[Panzer, '14]

1. start with Feynman parameter

$$I = \frac{\Gamma(a - Ld/2)}{\prod_{k=1}^{n} \Gamma(a_k)} \int_0^\infty \mathrm{d}x_1 \cdots \int_0^\infty \mathrm{d}x_n \, \delta\left(1 - \sum_{k\in I} x_k\right) \prod_{k=1}^{n} x_k^{a_k - 1} \frac{\mathcal{U}^{a-(L+1)d/2}}{\mathcal{F}^{a-Ld/2}}$$

Master integrals

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3. perform parameter integrals with HyperInt

2. expand in ϵ

Note: parameter integrals have to be finite!

Master integrals

Strategy: [Manteufel, Panzer, Schabinger, '14 '15]

1. Find quasi-finite integral in even dimension, e.g. $d = 6 - 2\epsilon$

- integrate out trivial bubbles
- integration of Feynman parameters associated to eikonal propagators straightforward
- $\Gamma(a - Ld/2)$

} allowed divergences

Master integrals

Strategy:

1. Find quasi-finite integral in even dimension, e.g. $d = 6 - 2\epsilon$
2. Compute it using HyperInt
3. Use dimensional recurrence relations: [Tarasov, '96; Lee, '10]

$$I^{6-2\epsilon} = \sum_i c_i I_i^{4-2\epsilon} \qquad \text{(LiteRed)}$$

Master integrals

Strategy:

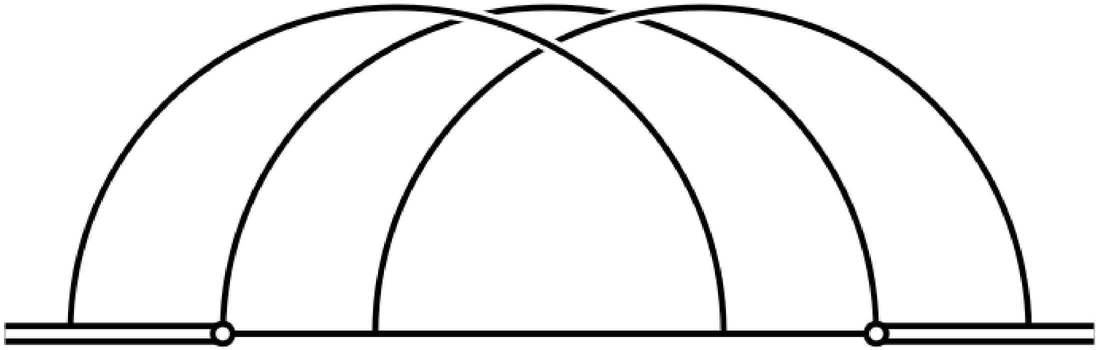
1. Find quasi-finite integral in even dimension, e.g. $d = 6 - 2\epsilon$
2. Compute it using HyperInt
3. Use dimensional recurrence relations [Tarasov, '96; Lee, '10]
4. IBP reduction:

$$I^{6-2\epsilon} = \tilde{c}_0 G_0^{4-2\epsilon} + \sum_i \tilde{c}_i G_i^{4-2\epsilon}$$

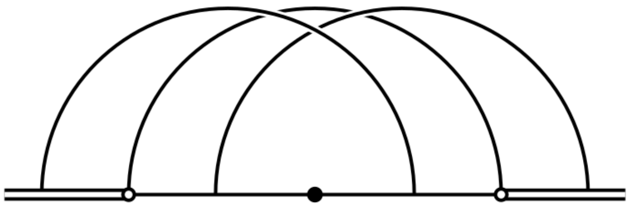
interested in,
same propagators as $I^{6-2\epsilon}$

fewer propagators,
known

Want to compute the following integral in $d = 4 - 2\epsilon$



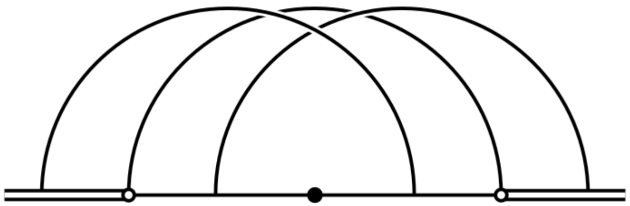
Master integrals: example



in $d = 6 - 2\epsilon$ using HyperInt

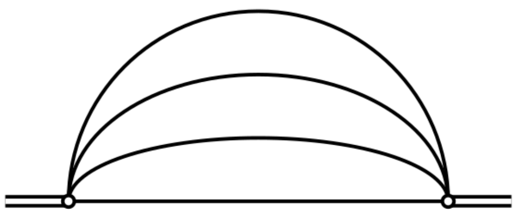
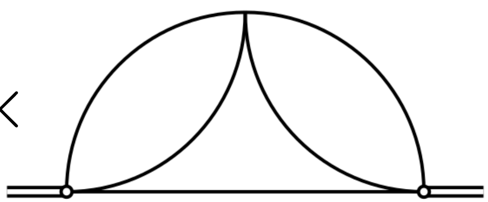
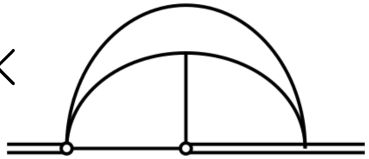
$$= \frac{\zeta_3}{4\epsilon} + \left(\frac{13\zeta_3}{8} + \frac{89\pi^4}{4320}\right) + \mathcal{O}(\epsilon)$$

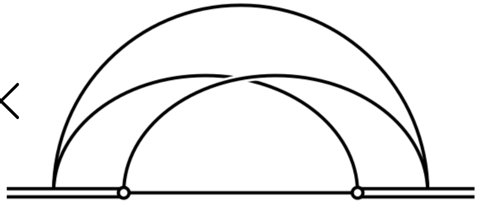
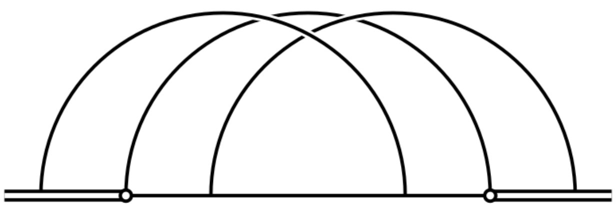
Master integrals: example



in $d = 6 - 2\epsilon$ using HyperInt

$$= \frac{\zeta_3}{4\epsilon} + \left(\frac{13\zeta_3}{8} + \frac{89\pi^4}{4320}\right) + \mathcal{O}(\epsilon)$$

$= \tilde{c}_1 \times$  $+ \tilde{c}_2 \times$  $+ \tilde{c}_3 \times$ 

$+ \tilde{c}_4 \times$  $+ \tilde{c}_0 \times$ 

$$= -\frac{4\pi^2}{27\epsilon^3} + \frac{4(7\zeta_3 + \pi^2)}{9\epsilon^2} + \mathcal{O}(1/\epsilon)$$

in $d = 4 - 2\epsilon$

dimensional recurrence
+ IBP reduction

Crosschecks

- Gauge invarinace: compute in R_ξ gauge, after IBP reduction dependence on ξ drops out

- Numerical check of master integrals with FIESTA4

  [Smirnov, '16]

- Renormalization group equation provide consistency relations

Summary

- Motivated the quark jet function on the example of thrust

- Three-loop calculation

    partial fractioning

    quasi-finite master integrals

- Quark jet function needed for resummation of many observables: thrust, $b \to s\gamma$, ...