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The Kinetic Heavy Quark Mass to Three Loops

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based on Fael et al. 2020

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Outline

1. Motivation
2. Calculation
3. Results

Motivation

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The Heavy Quark Expansion (HQE):

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

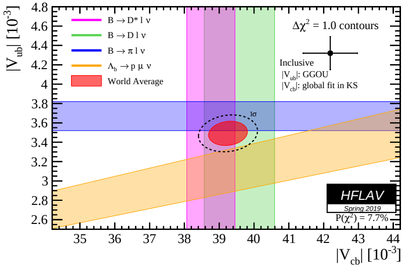
- $\mathrm{d}\Gamma_i$ are computed in perturbative QCD
- dependece on non-perturbative HQE parameters: $\mu_\pi, \mu_G, \rho_D, \rho_{LS}, \ldots$
- Perturbative corrections to $b \to c\ell\nu$ exhibit a bad convergence in the on-shell and the $\overline{\mathrm{MS}}$ scheme for the heavy quark masses.

Motivation

- Non-perturbative parameters of the HQE are extracted from data together with m_c, m_b and $|V_{cb}|$.
- Error from scheme conversion between $\overline{\mathrm{MS}}$ to kinetic scheme large:

$$n_l = 4: \qquad m_b^{\mathrm{kin}}(1\,\mathrm{GeV}) = (4163 + 259 + 77 + ??)\,\mathrm{MeV}$$

- Better knowledge of scheme conversion can further constrain the global fit.



The kinetic heavy quark mass

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- The kinetic mass scheme is tailored for $b \to c$ transitions.
- It is based on the heavy quark – hadron mass relation in HQET:

Definition

$$m^{\text{kin}} = m^{\text{OS}} - \overline{\Lambda}(\mu)\Big|_{\text{pert}} - \frac{\mu_\pi^2(\mu)\Big|_{\text{pert}}}{2m^{\text{kin}}} + \dots$$

[Bigi, Shifman, N. Uraltsev, et al. 1997; Czarnecki et al. 1998]

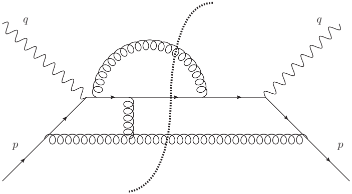
- Key mass scale is m_b, not M_B:

$$\Gamma_{\rm sl} \simeq \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (M_B - \overline{\Lambda})^5$$

Calculation

- Definitions of $\overline{\Lambda}(\mu)$ and $\mu_\pi^2(\mu)$ are given by small velocity sum rules.
- $\overline{\mathrm{MS}}$ – kinetic mass relation at $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ are known
  [Bigi, Shifman, N.G. Uraltsev, et al. 1995; Czarnecki et al. 1998]
- Goal: calculate the relation to $\mathcal{O}(\alpha_s^3)$
- Need to compute inclusive scattering

$$b + j \to b + X\,(g, q)$$



- $\overline{\Lambda}(\mu)$ and $\mu_\pi^2(\mu)$ do not depend on the current j
- They are given by the leading term of the expansions around the threshold $y = s - m_b^2$ and the momentum of the external current q.

Implemenation of expansions:

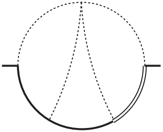
1. threshold expansion $y = s - m_b^2 \ll m_b^2$: expansion by regions [Beneke and Smirnov 1998]
   - loop momenta can either scale:
     hard (h) $k_i \sim m_b$ or ultrasoft (u) $k_i \sim y/m_b$
   - region where all momenta scale hard does not contribute
2. afterwards naive expansion in external momentum

Calculational details:

- (uhh)-region: 3 master integrals
- (uuh)-region: 3 master integrals
- (uuu)-region: 20 new master integrals with linear propagators

Calculation

Example:



$$= \prod_{i=1}^{3} \int \frac{\mathrm{d}k_i}{(2\pi)^d} \frac{1}{[k_1^2][k_2^2][(k_1-k_3)^2][(k_2-k_3)^2][2k_1.p-y][2k_3.p-y][2k_2.p]}$$

$$= \mathcal{N}\left\{ \frac{\zeta_2}{6\epsilon^2} + \frac{1}{\epsilon}\left(\frac{4\zeta_2}{3} + \frac{\zeta_3}{3}\right) + \frac{26\zeta_2}{3} + \frac{8\zeta_3}{3} + \frac{187\zeta_2^2}{60} + \ldots \right\}$$

Analytic results for all master integrals up to weight 5 have been obtained through:

- PSLQ
- analytic summation
- differential equations in auxillary variable

Results

$$
\begin{aligned}
\frac{m^{\text{kin}}}{m^{\text{OS}}} = {} & 1 - \frac{\alpha_s^{(n_l)}}{\pi} C_F \left(\frac{4}{3} \frac{\mu}{m^{\text{OS}}} + \frac{1}{2} \frac{\mu^2}{(m^{\text{OS}})^2} \right) + \left(\frac{\alpha_s^{(n_l)}}{\pi} \right)^2 C_F \left\{ \frac{\mu}{m^{\text{OS}}} \left[C_A \left(-\frac{215}{27} + \frac{2\pi^2}{9} + \frac{22}{9} l_\mu \right) + n_l T_F \left(\frac{64}{27} - \frac{8}{9} l_\mu \right) \right] \right. \\
& \left. + \frac{\mu^2}{(m^{\text{OS}})^2} \left[C_A \left(-\frac{91}{36} + \frac{\pi^2}{12} + \frac{11}{12} l_\mu \right) + n_l T_F \left(\frac{13}{18} - \frac{1}{3} l_\mu \right) \right] \right\} + \left(\frac{\alpha_s^{(n_l)}}{\pi} \right)^3 C_F \left\{ \frac{\mu}{m^{\text{OS}}} \left[C_A^2 \left(-\frac{130867}{1944} \right. \right. \right. \\
& + \frac{511\pi^2}{162} + \frac{19\zeta_3}{2} - \frac{\pi^4}{18} + \left(\frac{2518}{81} - \frac{22\pi^2}{27} \right) l_\mu - \frac{121}{27} l_\mu^2 \right) + C_A n_l T_F \left(\frac{19453}{486} - \frac{104\pi^2}{81} - 2\zeta_3 \right. \\
& \left. + \left(-\frac{1654}{81} + \frac{8\pi^2}{27} \right) l_\mu + \frac{88}{27} l_\mu^2 \right) + C_F n_l T_F \left(\frac{11}{4} - \frac{4\zeta_3}{3} - \frac{2}{3} l_\mu \right) + n_l^2 T_F^2 \left(-\frac{1292}{243} + \frac{8\pi^2}{81} + \frac{256}{81} l_\mu - \frac{16}{27} l_\mu^2 \right) \right] \\
& + \frac{\mu^2}{(m^{\text{OS}})^2} \left[C_A^2 \left(-\frac{96295}{5184} + \frac{445\pi^2}{432} + \frac{57\zeta_3}{16} - \frac{\pi^4}{48} + \left(\frac{2155}{216} - \frac{11\pi^2}{36} \right) l_\mu - \frac{121}{72} l_\mu^2 \right) + C_A n_l T_F \left(\frac{13699}{1296} - \frac{23\pi^2}{54} \right. \right. \\
& \left. \left. \left. - \frac{3\zeta_3}{4} + \left(-\frac{695}{108} + \frac{\pi^2}{9} \right) l_\mu + \frac{11}{9} l_\mu^2 \right) + C_F n_l T_F \left(\frac{29}{32} - \frac{\zeta_3}{2} - \frac{1}{4} l_\mu \right) + n_l^2 T_F^2 \left(-\frac{209}{162} + \frac{\pi^2}{27} + \frac{26}{27} l_\mu - \frac{2}{9} l_\mu^2 \right) \right] \right\},
\end{aligned}
$$

Results

Using as inputs $\overline{m}_b(\overline{m}_b) = 4163\,\text{MeV}$, $\alpha_s^{(5)}(M_Z) = 0.1179$:

$$n_l = 3: \quad m_b^{\rm kin}(1\,\text{GeV}) = (4163 + 248 + 81 + 30)\,\text{MeV} = 4521(15)\,\text{MeV}$$

$$n_l = 4: \quad m_b^{\rm kin}(1\,\text{GeV}) = (4163 + 259 + 77 + 25)\,\text{MeV} = 4523(12)\,\text{MeV}$$

To be compared with:

- scheme conversion uncertainty at two loops: $\delta m_b^{\rm kin} = 30\,\text{MeV}$
  [Gambino 2011]
- m_b from $b \to c\ell\nu$ global fit: $m_b^{\rm kin}(1\,\text{GeV}) = 4554 \pm 18\,\text{MeV}$
  [Amhis et al. 2019]

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$m_c \neq 0$:

$$n_l = 3: \quad m_b^{\text{kin}}(1\,\text{GeV}) = (4163 + 248 + 80 + 30)\,\text{MeV} = 4520(15)\,\text{MeV}$$

$$n_l = 4: \quad m_b^{\text{kin}}(1\,\text{GeV}) = (4163 + 259 + 78 + 26)\,\text{MeV} = 4526(12)\,\text{MeV}$$

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Summary:

- We've calculated the relation between the kinetic heavy quark mass and the on-shell mass up to $\mathcal{O}(\alpha_s^3)$.
- This result can be used to extract $|V_{cb}|$ more precisely from global fits.

Outlook:

- Include finite m_c effects into the relation. ✓
- How to extend the calculation to higher orders in the $1/m_b$ expansion?

For further discussions on heavy quark masses:
CRC Mini Workshop on Quark Masses
`https://indico.scc.kit.edu/event/899`

References I

- Yasmine Sara Amhis et al. "Averages of *b*-hadron, *c*-hadron, and τ-lepton properties as of 2018". In: (Sept. 2019). arXiv: 1909.12524 [hep-ex].
- M. Beneke and Vladimir A. Smirnov. "Asymptotic expansion of Feynman integrals near threshold". In: *Nucl. Phys. B* 522 (1998), pp. 321–344. DOI: 10.1016/S0550-3213(98)00138-2. arXiv: hep-ph/9711391.
- Ikaros I.Y. Bigi, Mikhail A. Shifman, N.G. Uraltsev, et al. "Sum rules for heavy flavor transitions in the SV limit". In: *Phys. Rev. D* 52 (1995), pp. 196–235. DOI: 10.1103/PhysRevD.52.196. arXiv: hep-ph/9405410.

References II

- Ikaros I.Y. Bigi, Mikhail A. Shifman, N. Uraltsev, et al. “High power n of m(b) in beauty widths and n=5 —> infinity limit”. In: *Phys. Rev. D* 56 (1997). Ed. by Ikaros I Bigi, Paolo Gambino, and Thomas Mannel, pp. 4017–4030. DOI: 10.1103/PhysRevD.56.4017. arXiv: hep-ph/9704245.
- A. Czarnecki, K. Melnikov, and N. Uraltsev. “NonAbelian dipole radiation and the heavy quark expansion”. In: *Phys. Rev. Lett.* 80 (1998), pp. 3189–3192. DOI: 10.1103/PhysRevLett.80.3189. arXiv: hep-ph/9708372.
- Matteo Fael, Kay Schönwald, and Matthias Steinhauser. “Kinetic Heavy Quark Mass to Three Loops”. In: *Phys. Rev. Lett.* 125.5 (2020), p. 052003. DOI: 10.1103/PhysRevLett.125.052003. arXiv: 2005.06487 [hep-ph].

References III

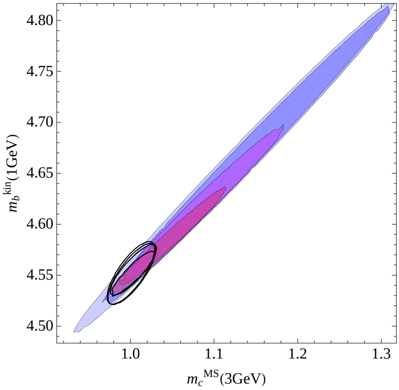
- Paolo Gambino. “B semileptonic moments at NNLO”. In: *JHEP* 09 (2011), p. 055. DOI: 10.1007/JHEP09(2011)055. arXiv: 1107.3100 [hep-ph].
- Paolo Gambino and Christoph Schwanda. “Inclusive semileptonic fits, heavy quark masses, and V_{cb}”. In: *Phys. Rev. D* 89.1 (2014), p. 014022. DOI: 10.1103/PhysRevD.89.014022. arXiv: 1307.4551 [hep-ph].

Motivation

- Error from scheme conversion between $\overline{\text{MS}}$ to kinetic scheme large:

$$n_l = 4: \qquad m_b^{\text{kin}}(1\,\text{GeV}) = (4163 + 259 + 77 + 25)\,\text{MeV}$$

- Better knowledge of scheme conversion can further constrain the global fit. [Gambino and Schwanda 2014]



References

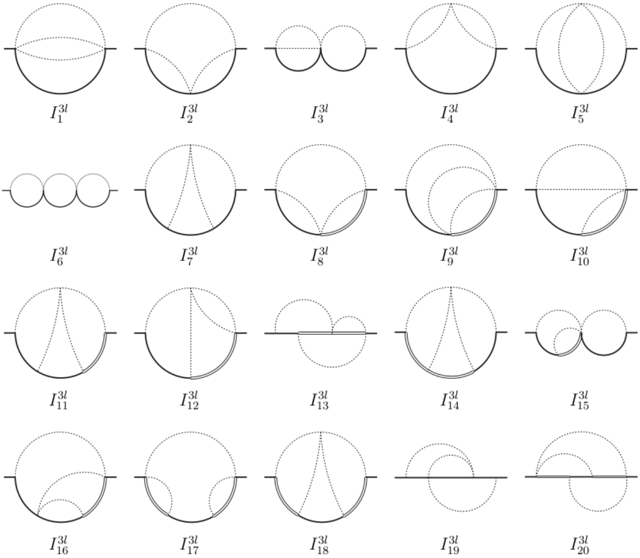
Calculation

Small velocity sum rules:

$$\overline{\Lambda}(\mu)\Big|_{\text{pert}} = \lim_{\vec{v}\to 0}\lim_{m\to\infty}\frac{2}{\vec{v}^2}\frac{\int_0^\mu \mathrm{d}\omega\,\omega\,W(\omega,\vec{v})}{\int_0^\mu \mathrm{d}\omega\,W(\omega,\vec{v})}, \quad \mu_\pi^2(\mu)\Big|_{\text{pert}} = \lim_{\vec{v}\to 0}\lim_{m\to\infty}\frac{3}{\vec{v}^2}\frac{\int_0^\mu \mathrm{d}\omega\,\omega^2\,W(\omega,\vec{v})}{\int_0^\mu \mathrm{d}\omega\,W(\omega,\vec{v})}$$

- $W(\omega,\vec{v}) = 2\,\mathrm{Im}\left[\frac{i}{2m}\int \mathrm{d}^4x\, e^{-iqx}\langle Q|\, TJ(x)J(0)\,|Q\rangle\right]$
  (J is an arbitrary current)
- $\vec{v}$: velocity of the heavy quark
- ω: excitation energy of the heavy quark

Masters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