Contribution of the Darwin operator to non-leptonic decays of heavy quarks

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In collaboration with A. Lenz and A. Rusov

Motivation

The lifetime ratio $\tau(B_s)/\tau(B_d)$

◇ Use Heavy Quark Expansion (HQE) [Shifman, Voloshin '85]

$$\frac{\tau(B_s)}{\tau(B_d)} = \frac{\Gamma_b + \delta\Gamma_{B_d}}{\Gamma_b + \delta\Gamma_{B_s}} = 1 + \underbrace{\tau(B_s)\left(\delta\Gamma_{B_d} - \delta\Gamma_{B_s}\right)}_{0.0007 \pm 0.0025}$$

[Kirk, Lenz, Rauh '17]

∗ Γ_b - leading contribution, free b-quark decay ∗ $\delta\Gamma_{B_q}$ - subleading terms

◇ Multiple cancellations arise

◇ Unique possibility

∗ to compete with increasing experimental precision

∗ to validate HQE

∗ to test for BSM scenarios and search for invisible decays

The theoretical framework

◇ From the optical theorem:

$$\Gamma_{B_q} = \frac{1}{2m_{B_q}} \mathrm{Im} \langle B_q | \, i \int d^4x \, \mathcal{T}\{\mathcal{L}_{\rm eff}(x), \mathcal{L}_{\rm eff}(0)\} \, | B_q \rangle$$

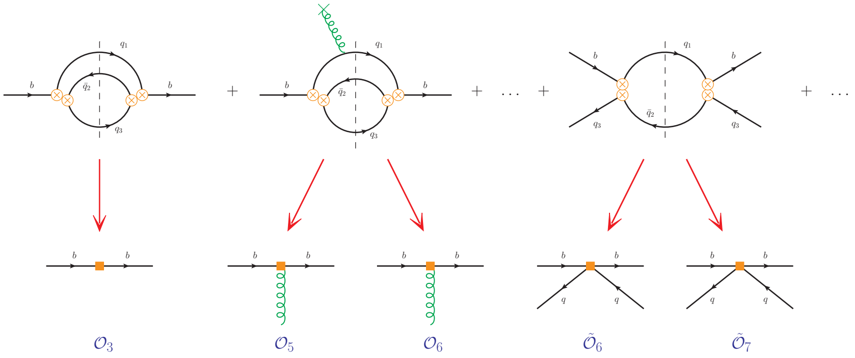
◇ OPE in inverse power of m_b: $\quad p^\mu = m_b v^\mu + k^\mu$

$$\Gamma_{B_q} = \underbrace{\Gamma_0 \langle \mathcal{O}_3 \rangle}_{\Gamma_b} + \underbrace{\Gamma_2 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_3 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_3 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_4 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]}_{\delta\Gamma_{B_q}}$$

* $\Gamma_i, \tilde{\Gamma}_i$ - short distance coefficients
* $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local quark operator of dimension d
* $\dfrac{\delta\Gamma^{(d)}_{B_q}}{\Gamma_b} \sim \left(\dfrac{k}{m_b} \right)^{d-3} \sim \left(\dfrac{1\,\mathrm{GeV}}{4.5\,\mathrm{GeV}} \right)^{d-3}$ - small parameter

The theoretical framework

$$\Gamma_{B_q} = \Gamma_0 \langle \mathcal{O}_3 \rangle + \Gamma_2 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_3 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + ... + 16\pi^2 \Big[\tilde{\Gamma}_3 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_4 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + ... \Big]$$



The theoretical framework

◇ What has been included so far …

	"Two – loop" contributions		"One – loop" contributions	
	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$
$\mathcal{O}(1)$	✓	✓	–	–
$\mathcal{O}\left(\frac{1}{m_b^2}\right)$	✓	✗	–	–
$\mathcal{O}\left(\frac{1}{m_b^3}\right)$	✗	✗	✓	✓
$\mathcal{O}\left(\frac{1}{m_b^4}\right)$	✗	✗	✓	✗

Some peculiarities

◇ Suppression of $1/m_b^2$ contributions

◇ "One-loop" $1/m_b^3$ corrections expected to be dominant, but

$q = d, s$

$$\delta\tilde{\Gamma}^{(6)}_{B_q} \sim \Bigg\{ \underbrace{\left(\frac{C_1^2}{3} + 2\,C_1 C_2 + 3\,C_2^2\right)}_{\approx 10^{-2}} \left((\underbrace{B_2^q}_{\approx 1} - \underbrace{B_1^q}_{\approx 1}) + \mathcal{O}\underbrace{\left(\frac{m_c^2}{m_b^2}\right)}_{\approx 0.05} \right) + \underbrace{2C_1^2}_{\approx 2}\ \underbrace{f(\epsilon_2, \epsilon_1)}_{\text{color suppr.}} \Bigg\}$$

◇ Strong suppression despite loop enhancement

◇ "Two-loop" $1/m_b^3$ corrections found sizeable in the SL case

[Gremm, Kapustin, '96]

* What about the NL case?

Computation of the Darwin term for NL decays

Contribution of two-quark operators

◇ Restart from optical theorem

$$\Gamma_{\rm NL}(B) = \frac{1}{2m_B}\, {\rm Im}\, \langle B(p_B)|\, i \int d^4x\, T\left\{\mathcal{L}_{\rm eff}(x), \mathcal{L}_{\rm eff}(0)\right\} |B(p_B)\rangle$$

◇ The effective Lagrangian [Buchalla, Buras, Lautenbacher '96]

$$\mathcal{L}_{\rm eff}(x) = -\frac{4G_F}{\sqrt{2}} V^*_{q_1 b} V_{q_2 q_3}\left[C_1\, Q_1(x) + C_2\, Q_2(x)\right] + {\rm h.c.}$$

◇ $\Delta B = 1$ operator basis

$$Q_1 = (\bar{q}_1^i \Gamma_\mu b^i)(\bar{q}_3^j \Gamma^\mu q_2^j) \qquad Q_2 = (\bar{q}_1^i \Gamma_\mu b^j)(\bar{q}_3^j \Gamma^\mu q_2^i)$$

$q_1, q_2 = u, c \quad q_3 = d, s$

Contribution of two-quark operators

- Three contributions

$Q_1 \otimes Q_1$ $\qquad$ $Q_1 \otimes Q_2$ $\qquad$ $Q_2 \otimes Q_2$

$\left[t^a_{ij} t^a_{lm} = \frac{1}{2}\left(\delta_{im}\delta_{jl} - \frac{1}{N_c}\delta_{ij}\delta_{lm}\right)\right]$ $\downarrow$

Use Fierz-transformation

$$\frac{1}{N_c}\,(Q_1 \otimes Q_1) + 2\,(Q_1 \otimes T)$$

$$T = (\bar{q}_1^{\,i}\Gamma_\mu\, t^a_{ij}\, b^j)(\bar{q}_3^{\,l}\,\Gamma^\mu\, t^a_{lm}\, q_2^m)$$

- Consider two-quark operators only

$$\Gamma^{(2q)}_{\rm NL}(B) = \left[C_1^2\,\Gamma^{(2q)}_{11} + 2\,C_1 C_2\,\left(\frac{1}{N_c}\Gamma^{(2q)}_{11} + 2\,\Gamma^{(2q)}_{1T}\right) + C_2^2\,\Gamma^{(2q)}_{22}\right]$$

Contribution of two-quark operators

$$\Gamma^{(2q)}_{11(1T)} = -\frac{4G_F^2|V_{q_1b}|^2|V_{q_2q_3}|^2}{m_B}\,\mathrm{Im}\,\langle B(p_B)|\, i\int d^4x\,\bar{b}(0)\,\Gamma_\mu\,(t^a)\,iS^{(q_1)}(0,x)\Gamma_\nu\,b(x)$$
$$\times\,\mathrm{Tr}\left[\Gamma^\mu\,(t^a)\,iS^{(q_3)}(0,x)\,\Gamma^\nu\,iS^{(q_2)}(x,0)\right]|B(p_B)\rangle + (x\leftrightarrow 0)$$

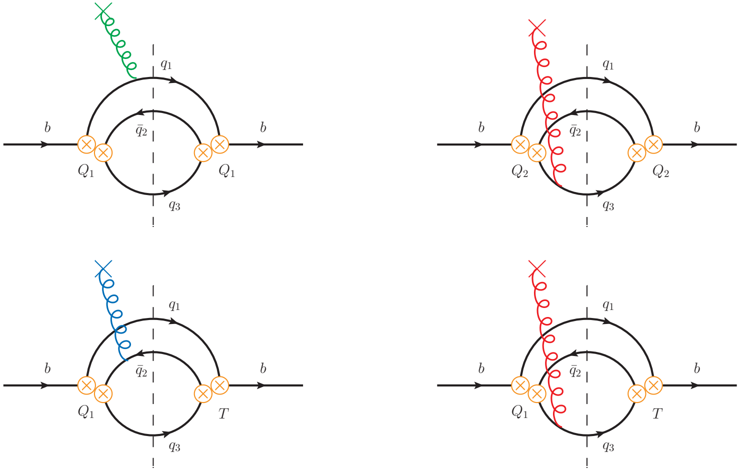
- Need to expand up to $\mathcal{O}(D^3)$ $\quad D_\mu = \partial_\mu - i\,A_\mu(x)$ $\quad G_{\mu\nu} = i\,[D_\mu, D_\nu]$
- Use background field method [Novikov et al. '84]

$$S_{ij}(k) = S^{(0)}(k)\,\delta_{ij} + S^{(1)\,a}(k)\,t^a_{ij}$$

$$S^{(0)} = \frac{\not{k}+m}{k^2-m^2} \qquad S^{(1)\,a} \sim G^a_{\mu\nu}\,,D_\rho G^a_{\mu\nu}$$

- Straightforward treatment of colour

Contribution of two-quark operators



Contribution of two-quark operators

- Two-loop tensor integrals of rank $r = 1, \ldots, 4$

- Build all possible r-tensors with $g^{\mu\nu}$ and p^{μ}

- Coefficients given by scalar integrals of the type

$$\int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{f(p, k_1, k_2)}{[k_1^2 - m_1^2]^{n_1} \, [k_2^2 - m_2^2]^{n_2} \, [(p - k_1 - k_2)^2 - m_3^2]^{n_3}}$$

$n_i = 1, 2, 3$

- IBP reduction implemented with *LiteRed* [Lee '12]

Contribution of two-quark operators

◇ Master integrals from sunset diagram

$s = p^2$

λ Källen function

$$\mathrm{Im}\, S(s; m_1, m_2, m_3) = \frac{1}{256\pi^3} \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} dt\, \frac{\sqrt{\lambda(t, m_2^2, m_3^2)\, \lambda(s, t, m_1^2)}}{t\, s}$$

[Remiddi, Tancredi '16]

◇ No UV divergences at LO-QCD

◇ IR divergencies from expansion of light-quark propagator

- ∗ Keep m_q as IR regulator in four dimensions

$q = u, d, s$

Alternatively use dimensional regularisation with $m_q = 0$

[Mannel, Moreno, Pivovarov '20]

Contribution of two-quark operators

- Obtain following matrix elements

$b(x) = e^{-im_b v\cdot x} b_v(x)$

$$\langle B(p_B)|\bar{b}_v(0)\,\mathcal{F}(p)\,b_v(0)|B(p_B)\rangle$$

$$\langle B(p_B)|\bar{b}_v(0)\,\mathcal{F}_{\mu\nu}(p)\,(iD^\mu)(iD^\nu)b_v(0)|B(p_B)\rangle$$

$$\langle B(p_B)|\bar{b}_v(0)\,\mathcal{F}_{\mu\nu\rho}(p)\,(iD^\mu)(iD^\nu)(iD^\rho)b_v(0)|B(p_B)\rangle$$

At $1/m_b^2$ only expansion of anti-quark is not vanishing

- Expand up to $\mathcal{O}(D^3)$

$$p^\mu = m_b v^\mu + iD^\mu$$

- Keep track of order of D^μ 's

$$p^{\mu_1}\dots p^{\mu_n} = \frac{1}{n!}\sum_{\sigma\in\mathcal{S}_n} p^{\sigma(\mu_1)}\dots p^{\sigma(\mu_n)}$$

Contribution of two-quark operators

- Systematic expansion

$$a\langle \bar{b}_v b_v \rangle + b_\mu \langle \bar{b}_v (iD^\mu) b_v \rangle + c_{\mu\nu} \langle \bar{b}_v (iD^\mu)(iD^\nu) b_v \rangle + d_{\mu\nu\rho} \langle \bar{b}_v (iD^\mu)(iD^\nu)(iD^\rho) b_v \rangle + \ldots$$

- $a, b_\mu, c_{\mu\nu}, d_{\mu\nu\rho}$ depend only on quark masses and v^μ

- Decompose matrix elements in terms of basic parameters

[Dassinger, Mannel, Turczyk, '07]

$$\langle B(p_B)|\bar{b}_v b_v|B(p_B)\rangle = 2m_B \left(1 - \frac{\mu_\pi^2(B) - \mu_G^2(B)}{2m_b^2}\right)$$

$$\vdots \qquad\qquad \vdots$$

Contribution of two-quark operators

◇ Finally

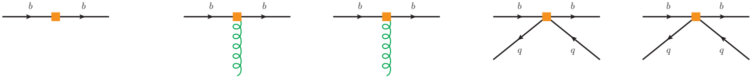
$$\Gamma_{\rm NL}^{(2q)}(B) = \bar{\Gamma}_0 \Big[\left(3\, C_1^2 + 2\, C_1 C_2 + 3\, C_2^2\right) \mathcal{C}_0^{(q_1 \bar{q}_2 q_3)} \left(1 - \frac{\mu_\pi^2(B)}{2 m_b^2}\right) + \left(3\, C_1^2\, \mathcal{C}_{G,11}^{(q_1 \bar{q}_2 q_3)} + 2\, C_1 C_2\, \mathcal{C}_{G,12}^{(q_1 \bar{q}_2 q_3)} + 3\, C_2^2\, \mathcal{C}_{G,22}^{(q_1 \bar{q}_2 q_3)}\right) \frac{\mu_G^2(B)}{m_b^2} + \left(3\, C_1^2\, \mathcal{C}_{D,11}^{(q_1 \bar{q}_2 q_3)} + 2\, C_1 C_2\, \mathcal{C}_{D,12}^{(q_1 \bar{q}_2 q_3)} + 3\, C_2^2\, \mathcal{C}_{D,22}^{(q_1 \bar{q}_2 q_3)}\right) \frac{\rho_D^3(B)}{m_b^3} \Big]$$

Or almost ...

$$\mathcal{C}_D \underset{m_q \to 0}{\sim} \log\left(m_q^2 / m_b^2\right)$$

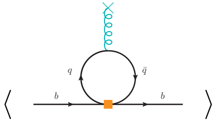
Role of four-quark operators

◇ Let's go back



$\mathcal{O}_3$ $\mathcal{O}_5$ $\mathcal{O}_6$ $\tilde{\mathcal{O}}_6$ $\tilde{\mathcal{O}}_7$

◇ $\tilde{\mathcal{O}}_6$ and $\mathcal{O}_6$ mix under renormalisation



$$\sim \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_q^2}\right) + c\right] \langle \mathcal{O}_{\rho_D} \rangle + \mathcal{O}\left(\frac{1}{m_b}\right)$$

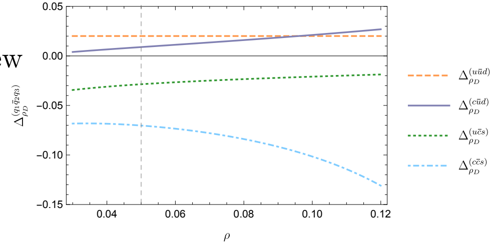
◇ All $\log(m_q^2)$ vanish!

◇ Constant c depends on choice of operator basis

Result

$$\Gamma_{\rm NL}^{(\rho_D)}(B) = \bar{\Gamma}_0 \left(3\, C_1^2\, \mathcal{C}_{\rho_D,\,11}^{(q_1 \bar{q}_2 q_3)} + 2\, C_1 C_2\, \mathcal{C}_{\rho_D,\,12}^{(q_1 \bar{q}_2 q_3)} + 3\, C_2^2\, \mathcal{C}_{\rho_D,\,22}^{(q_1 \bar{q}_2 q_3)}\right) \frac{\rho_D^3}{m_b^3}$$

- We confirmed the SL case $\mathcal{C}_{\rho_D,\,11}^{(q_1 \bar{q}_2 q_3)}$
- Results for $\mathcal{C}_{\rho_D,\,12}^{(q_1 \bar{q}_2 q_3)}$, $\mathcal{C}_{\rho_D,\,22}^{(q_1 \bar{q}_2 q_3)}$ new
- Computed for all NL modes
- Relative size of order $2 - 7\%$



$$\rho = \frac{m_c^2}{m_b^2}$$

Conclusion

◇ Large coefficient of the Darwin operator, but

$$\frac{\tau(B_s)}{\tau(B_d)} = 1 + \Bigg\{ \underbrace{\Gamma_5 \Big(\langle \mathcal{O}_5 \rangle_{B_d} - \langle \mathcal{O}_5 \rangle_{B_s} \Big)}_{\checkmark} \frac{1}{m_b^2} + \underbrace{\Gamma_6}_{\checkmark} \underbrace{\Big(\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B_s} \Big)}_{\text{✗}} \frac{1}{m_b^3}$$

$$+ \Bigg[\underbrace{\Big(\tilde{\Gamma}_6^{B_d} - \tilde{\Gamma}_6^{B_s} \Big) \langle \tilde{\mathcal{O}}_6 \rangle_{B_d}}_{\checkmark} - \tilde{\Gamma}_6^{B_s} \underbrace{\Big(\langle \tilde{\mathcal{O}}_6 \rangle_{B_s} - \langle \tilde{\mathcal{O}}_6 \rangle_{B_d} \Big)}_{\text{✗}} \Bigg] \frac{1}{m_b^3} + \ldots \Bigg\} \tau(B_s)$$

◇ $SU(3)_f$ violation effects crucial

Further steps:

- ∗ Determination of $\Big(\langle \mathcal{O}_6 \rangle_{B_d} - \langle \mathcal{O}_6 \rangle_{B_s} \Big)$ [Lenz, Piscopo, Rusov]
- ∗ Computation of $\langle \tilde{\mathcal{O}}_6 \rangle_{B_s}$ $\underbrace{\text{[King, Lenz, Rauh}}_{\text{HQET SumRules}}, \underbrace{\text{Witzel]}}_{\text{Lattice}}$

Thanks for the attention

Backup slides

Dimension-six operator basis

Two-quark operators

$$\mathcal{O}_{\rho_D} = \bar{b}_v (iD_\mu)(iv\cdot D)(iD^\mu) b_v$$

$$\mathcal{O}_{\rho_{LS}} = \bar{b}_v (iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})\, b_v$$

Four-quark operators

$$\tilde{\mathcal{O}}_{6,1}^{(q)} = (\bar{b}_v^i \gamma_\mu (1-\gamma_5)\, q^i)(\bar{q}^j \gamma^\mu (1-\gamma_5)\, b_v^j) \quad \tilde{\mathcal{O}}_{6,2}^{(q)} = (\bar{b}_v^i \not{v} (1-\gamma_5)\, q^i)(\bar{q}^j \not{v} (1-\gamma_5)\, b_v^j)$$

$$\tilde{\mathcal{O}}_{6,3}^{(q)} = (\bar{b}_v^i \gamma_\mu (1-\gamma_5)\, q^j)(\bar{q}^j \gamma^\mu (1-\gamma_5)\, b_v^i) \quad \tilde{\mathcal{O}}_{6,4}^{(q)} = (\bar{b}_v^i \not{v} (1-\gamma_5)\, q^j)(\bar{q}^j \not{v} (1-\gamma_5)\, b_v^i)$$

Parametrisation of four-quark operators matrix elements

$$\langle B_{q'} |\, \tilde{\mathcal{O}}_{6,i}^{(q)}\, | B_{q'} \rangle = A_i\, m_B^2\, f_B^2 \left(\mathcal{B}_i^{(q)}(B)\, \delta_{qq'} + \tau_i^{(q)}(B) \right)$$

with $A_1 = A_3 = 1,\ A_2 = A_4 = (m_B/(m_b + m_q))^2,\qquad q = u, d, s$

Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

$$\text{QHD violation} \equiv \begin{cases} 1/m_Q \text{ corrections in } \Gamma \\ \text{oscillatory terms in } \Gamma \end{cases}$$

⋆ In the '90s appears discrepancy:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} \sim 0.96 & \text{[Shifman, Voloshin '86]} \\ 0.798 \pm 0.034 & \text{[HFAG '03]} \end{cases}$$

⋆ 2019 status:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} 0.935 \pm 0.054 & \text{[Lenz '14]} \\ 0.969 \pm 0.006 & \text{[HFLAV '19]} \end{cases}$$

◇ Shift of $4.9\,\sigma$

Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

- ⋆ Compare HQE with experiments:
  - ◇ No sign of any significant deviation
  - ◇ $\Delta\Gamma_s$ highly sensitive (fewer states, smaller phase space)
    - ∗ Good agreement
- ⋆ Simplified models of QCD:
  - ◇ SV limit: no duality violation for SL and NL decays

    [Boyd, Grinstein, Manohar '95; Grinstein, Savrov '03]
  - ◇ 'tHooft model: no $1/m_Q$ corrections, tiny oscillatory terms

    [Grinstein, Lebed '97, '98, '01]

    [Bigi, Shifman, Uraltsev, Vainshtein '98, '99, '00]