Study of the $b \to u$ contamination in the inclusive V_{cb} determination

Thomas Mannel[1], Muslem Rahimi[1], K. Keri Vos[2]

[1]University of Siegen, [2]Technical University of Munich

June 17, 2020

UNIVERSITÄT SIEGEN

Motivation

- Entering era of high precision: Determine the CKM element V_{cb} via inclusive decays.

$$\text{Belle-II} \quad B \to X\ell\nu = B \to \sum_{i=u,c} X_i\ell\nu$$

- In Belle experiment the inclusive background $B \to X_u\ell\nu$ is subtracted with PYTHIA (Monte-Carlo generator).
- Recent results: $|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3}$ [Gambino et al., arXiv:1606.06174v2]

Motivation

- Problem: A high uncertainty is assumed for the subtraction.
- Goal: Compute differential decay width with lepton energy cut for $B \to X_u \ell \nu$ and compare it with PYTHIA. Two possible scenarios:
  - Theory and Experiment are in good agreement $\Rightarrow$ Possibly reduce the uncertainty of V_{cb}
  - Theory and Experiment are not in good agreement $\Rightarrow$ Joint effort between experimental and theoretical physicists in order to simulate $B \to X_u \ell \nu$ background correctly.

Inclusive $b \to c\ell\nu$ decay

- Effective Hamiltonian:

$$\mathcal{H} = \frac{G_F V_{cb}}{\sqrt{2}} J_L^\alpha J_{H,\alpha} + \text{h.c.},$$

where $J_L^\alpha = \ell\gamma^\alpha(1-\gamma^5)\nu$ and $J_H^\alpha = \bar{c}\gamma^\alpha(1-\gamma^5)b$ are the leptonic and hadronic currents.

- Differential Decay Width:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell \mathrm{d}q^2 \mathrm{d}E_\nu} = \frac{G_F^2 |V_{cb}|^2}{16\pi^3} L_{\mu\nu} W^{\mu\nu}$$

- Lepton Tensor:

$$L^{\mu\nu} = \sum_{\text{lepton spin}} \langle 0 | J_L^{\dagger\,\mu} | \ell\bar{\nu} \rangle \langle \ell\bar{\nu} | J_L^\nu | 0 \rangle$$

Inclusive $b \to c\ell\nu$ decay

- The hadronic tensor is the imaginary part of the forward matrix element of a time-ordered product of weak currents:

$$T^{\mu\nu} = \langle B(v)| \bar{b}_V \Gamma^\mu i S_{\text{BGF}} \Gamma^{\dagger\nu} b_V |B(v)\rangle$$
$$W^{\mu\nu} = -\frac{1}{\pi} \text{Im}\, T^{\mu\nu}$$

Charm quark propagates in the background field.

$$S_{\text{BGF}} = \frac{1}{\not{Q} - m_c + i\epsilon}$$

Expand S_{BGF} with momentum $Q^\mu = m_b v^\mu - k^\mu - q^\mu$ in powers of k^μ / m_b
$\Rightarrow$ Heavy Quark Expansion = Operator Product Expansion (OPE).

Inclusive $b \to c\ell\nu$ decay

- Decay rate of $B \to X_c \ell\nu$:

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + \cdots .$$

  Γ_i are power series in $\alpha_s(m_b)$.

- Leading order starts at $\mathcal{O}(1/m_b^2)$:

$$2m_B\mu_\pi^2 = -\langle B(v)|\,\bar{b}_v(iD)^2 b_v\,|B(v)\rangle$$
$$2m_B\mu_G^2 = -\langle B(v)|\,\bar{b}_v(iD_\mu)(iD_\nu)(-\sigma^{\mu\nu})b_v\,|B(v)\rangle$$

- Number of non-pertubative parameters are increasing at higher order of $1/m_b$:
  - 2 parameters at order $\mathcal{O}(1/m_b^3)$
  - 9 parameters at order $\mathcal{O}(1/m_b^4)$
  - 18 parameters at order $\mathcal{O}(1/m_b^5)$

Decay width: Contributions

- PYTHIA:
  - Tree level and α_s-corrections [de Fazio, Neubert, arXiv:hep-ph/9905351].
- OPE:
  - Tree level, α_s-corrections and power-corrections up to order $1/m_b^2$.
  - Obtain inclusive decay $b \to u\ell\nu$ by taking the limit $m_c \to 0$
- Remember: Our goal is to compute observables to compare PYTHIA with OPE.

Observable: Moments

Differential decay width such as $\mathrm{d}\Gamma/\mathrm{d}E_\ell, \mathrm{d}\Gamma/\mathrm{d}M_x^2$ and $\mathrm{d}\Gamma/\mathrm{d}E_H$ are not good observable since there are corners in the phase space where the OPE breaks down.

- Moments:

$$\langle E_\ell^n \rangle_{E_\ell > E_{\mathrm{cut}}} = \frac{1}{\Gamma_{E_\ell > E_{\mathrm{cut}}}} \int_{E_\ell > E_{\mathrm{cut}}} \mathrm{d}E_\ell \, E_\ell^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell} .$$

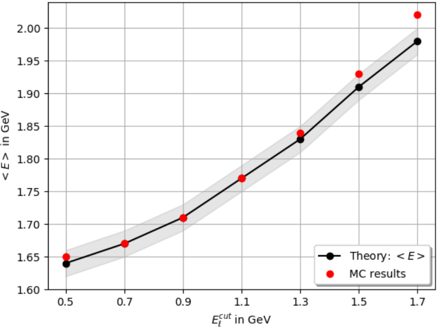
- Central Moments (less correlated):

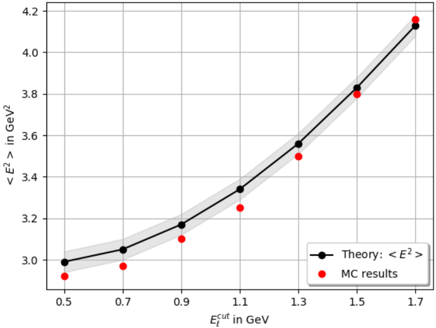
$$\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle = \langle E_\ell^2 \rangle - \langle E_\ell \rangle^2 .$$

Choice of Mass Scheme

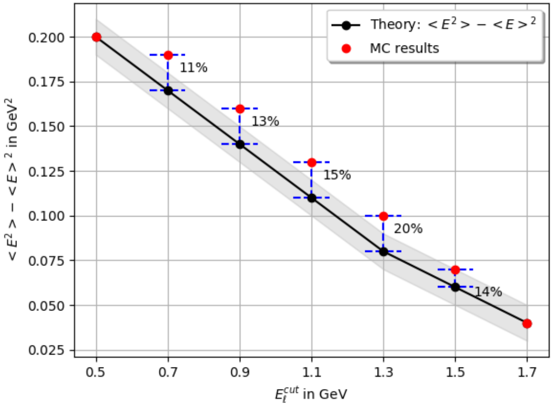
- Mass scheme determines size of radiative corrections
- Absorb radiative corrections into the definition of the mass
- Pole scheme: Intrinsic uncertainty of order Λ_{QCD} due to infrared renormalon
- Better choice: Short-distance masses, which do not have this problem $\Rightarrow$ Kinetic Scheme [Bigi et al., arXiv:hep-ph/9704245]

Results:Charged Lepton Energy Moments

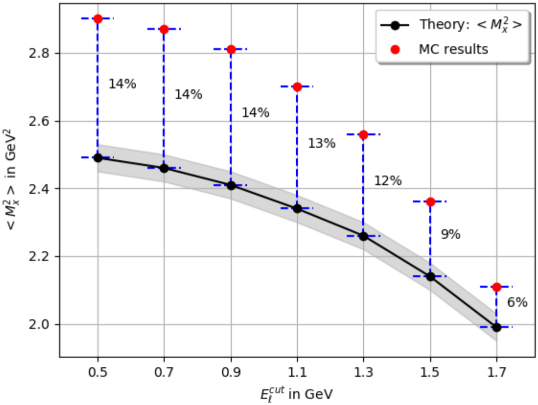




Results:Charged Lepton Energy Moments



Results:: Hadronic Invariant Mass Moments



Conclusion

- Preliminary results:
  - First and second charged lepton moments are in agreement with the MC-results. However, second central moment deviates up to 20% from the central value.
  - Hadronic invariant mass moment deviates up to 14% from the central value.
- Present status: Internal discussion with experimental physicists on how to go to the ultimate precision.
- Work is still on going $\Rightarrow$ Stay tuned!

Backup: Pole Scheme to Kinetic Scheme

$$m_b^{\text{pole}} = m_b^{\text{kin}}(\mu) + \left[\bar{\Lambda}(\mu)\right]_{\text{pert}} + \frac{\left[\mu_\pi^2(\mu)\right]_{\text{pert}}}{2m_b^{\text{kin}}(\mu)} + \cdots ,$$

$$\mu_\pi^2(0) = \mu_\pi^2(\mu) - \left[\mu_\pi^2(\mu)\right]_{\text{pert}} ,$$

$$\mu_G^2(0) = \mu_G^2(\mu) - \left[\mu_G^2(\mu)\right]_{\text{pert}} ,$$

$$\left[\bar{\Lambda}(\mu)\right]_{\text{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_b)}{\pi}\mu \left[1 + \frac{\alpha_s(m_b)\beta_0}{2\pi}\left(\log\left(\frac{m_b^{\text{kin}}}{2\mu}\right)\right) + \frac{8}{3}\right],$$

$$\left[\mu_\pi^2(\mu)\right]_{\text{pert}} = C_F \frac{\alpha_s(m_b)}{\pi}\mu^2 \left[1 + \frac{\alpha_s(m_b)\beta_0}{2\pi}\left(\log\left(\frac{m_b^{\text{kin}}}{2\mu}\right) + \frac{13}{6}\right)\right.$$
$$\left. - \frac{\alpha_s}{\pi} C_A \left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right] + \mathcal{O}\left(\frac{\mu^3}{m_b^{\text{kin}}}\right),$$

$$\left[\mu_G^2(\mu)\right]_{\text{pert}} = \mathcal{O}\left(\frac{\mu^3}{m_b^{\text{kin}}}\right).$$

Backup: Inclusive $b \to u\ell\nu$ decay

Observable quantities: Hadronic invariant mass, charged lepton energy and hadronic energy.

$$M_X^2 = m_b^2\,\hat{p}^2 + \bar{\Lambda}\,m_b\,z + \bar{\Lambda}^2, \quad E_H = v\cdot p + \bar{\Lambda},$$

where $\bar{\Lambda} = m_B - m_b$.

$$\begin{aligned}\mathcal{M}_{(i,j)} &= \frac{1}{\Gamma_0}\int \mathrm{d}z\,\mathrm{d}\hat{p}^2\,\mathrm{d}x(\hat{p}^2)^i z^j\,\frac{\mathrm{d}^3\Gamma}{\mathrm{d}z\mathrm{d}\hat{p}^2\mathrm{d}x} \\ &= M_{(i,j)} + \frac{\alpha_s}{\pi}A^{(1)}_{(i,j)} + \cdots\,,\end{aligned}$$

[Gambino et al., arXiv:hep-ph/0505091]

Introduce dimensionless variables:

$$x = \frac{2E_\ell}{m_b}, \quad \hat{p}^2 = \frac{p^2}{m_b^2}, \quad z = \frac{2v\cdot p}{m_b}$$

Backup: Numerical Inputs

m_b^{kin}	(4.546 ± 0.021) GeV	[1]
m_b^{pole}	(4.78 ± 0.06) GeV	[2]
m_B	$(5.279 \pm 0.26\ )$ GeV	[2]
$\mu_\pi^2(\mu)$	(0.432 ± 0.068) GeV^2	[1]
$\mu_G^2(\mu)$	(0.355 ± 0.060) GeV^2	[1]
$\alpha_s(m_b)$	0.223	

Table: The scale of the kinetic scheme here is $\mu = 1$ GeV.

[1] = [Gambino et al. arXiv:1606.06174]
[2] = [PDG 2020]