Towards completion of the four-body contributions to $\bar{B} \to X_s\gamma$ at NLO

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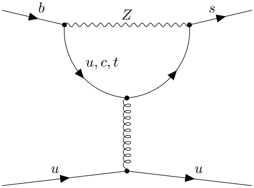
$\bar{B} \to X_s\gamma$

$\bar{B} \to X_s\gamma$ is one of the most suitable processes for the search for new physics in the quark flavor sector

$b \to s\gamma$ forbidden at tree-level, dominant contributions loop induced by weak decays
$\to$ small Standard Model rate
$\to$ sensitive to new particles running in the loop



$\Rightarrow$ Use framework of the effective weak theory to calculate the CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$ (with $E_\gamma > 1.6$ GeV) for this process. Current predictions, including calculations up to NNLO:

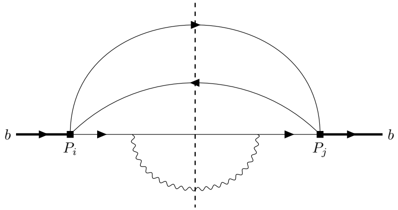
$$\mathcal{B}_{s\gamma}^{SM} = (3.36 \pm 0.23) \cdot 10^{-4}$$

[Misiak et al., arXiv:1503.01789]

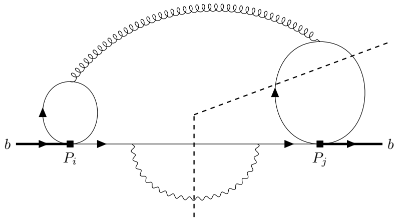
$$\mathcal{B}_{s\gamma}^{exp} = (3.32 \pm 0.15) \cdot 10^{-4}$$

[HFLAV, arXiv:1612.07233]

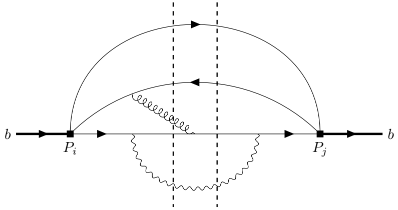
Four-body contributions to $\bar{B} \to X_s + \gamma$



[Kamiński et al., arXiv:1209.0965]



[Huber et al., arXiv:1411.7677]



this talk

- diagrams as above only contain 4-particle cuts
- uncalculated until now: subleading contributions on the left, where additional 5-particle cuts have to be taken into account
- formally completing NLO QCD

Effective Operators

The relevant processes in our case are described by a subset of dimension-6 operators from the effective weak theory:

$$\mathcal{L}_{eff} = \mathcal{L}_{QED+QCD} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \Big[\sum_{i=1}^{2} \mathcal{C}_i^u P_i^u + \sum_{i=3}^{6} \mathcal{C}_i P_i \Big]$$

$$P_1^u = (\bar{s}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \qquad P_2^u = (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

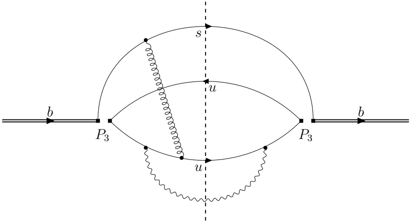
$$P_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q) \qquad P_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q)$$

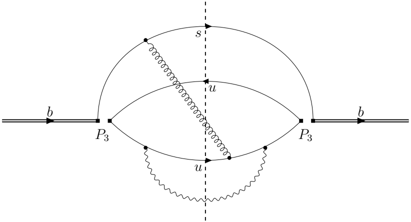
$$P_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho q) \qquad P_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho T^a q)$$

For this calculation, the sum over the quarks includes up-, down- and strange-quark (bottom is kinematically forbidden and final states including charm are excluded from $B \to X_s \gamma$ per definition)

Example Processes: Real & virtual contributions

11968 virtual & 14400 real contributions (including all six operators)
$\rightarrow$ reduced by a factor of four through color identities





$\Rightarrow$ Virtual: 4-particle-cuts with up to three massive propagators
(up to two of them contained in the loop)
$\Rightarrow$ Real: 5-particle-cuts of with up to four massive propagators

Steps and status of the calculation

this talk:

a) Evaluation and processing of the cut-diagrams

b) Integration over the four- and five-particle (massless) phase space

future steps:

c) Renormalization of UV divergences

d) Treatment of IR divergences

a) Evaluation and processing of the diagrams

Program Setup:

i) Generation of the Diagrams: QGRAF
[P. Nogueira, J. Comput. Phys. 105 (1993)]

ii) Algebraic simplification of trace structures and kinematics: FORM
[B. Ruijl et al., arXiv:1707.06453]

iii) Integration-By-Parts Reduction: FIRE6
[A. V. Smirnov, F. S. Chukharev, arXiv:1901.07808]

$$\Rightarrow |\mathcal{M}(s_{ij})|^2$$

Aside: Subtleties in the evaluation of the traces

If the operators $P^u_{1/2} \sim (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$ are used, the evaluation of the diagrams leads to expressions with either four γ_5 in one trace or products of two traces with up to two γ_5 each.
$\Rightarrow$ Not straightforward to treat these in d dimensions

Our Method: Use the relation

$$P^u_1 = -\frac{4}{27} P^u_3 + \frac{1}{9} P^u_4 + \frac{1}{27} P^u_5 - \frac{1}{36} P^u_6 + \mathcal{O}(\epsilon)$$

But: This leads to extra evanescent operators that have to be considered

in the end, when renormalizing.

b) Phase space

Need to integrate the Kernels $\mathcal{K}(s_{ij})$ $(= |\mathcal{M}(s_{ij})|^2)$ from a) over the four- and five-particle massless phase space in $d = 4 - 2\epsilon$ dimensions

For the four-particle-cuts this looks the following:

$$\int [ds_{ij}]\ \delta(1 - \sum s_{ij}) \mathcal{K}(s_{ij}) (-\Delta_4)^{\frac{d-5}{2}} \Theta(-\Delta_4)$$

$\Rightarrow$ Express cut on energy as $E_\gamma > \frac{m_b}{2}(1-\delta)$, translating (in the restframe of the bottom-quark) to

$$s_{14} + s_{24} + s_{34} > 1 - \delta.$$

This condition can then be incorporated into the integral:

$$\int_0^\delta dz \int_0^1 [ds_{ij}] \delta(1 - z - s_{14} - s_{24} - s_{34}) \delta(z - s_{12} - s_{23} - s_{13}) \times$$

$$\times\ \mathcal{K}(s_{ij}) (-\Delta_4)^{\frac{d-5}{2}} \Theta(-\Delta_4)$$

IBP Reduction

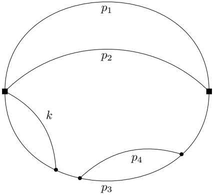
One subtlety: IBP relations do not know about the cut on the photon energy
$\rightarrow$ have to put it in by hand, using reversed unitarity:

$$\delta(p^2) \rightarrow \frac{1}{p^2 + i\varepsilon} - \frac{1}{p^2 - i\varepsilon}$$

for the delta-function $\delta(z - s_{12} - s_{13} - s_{23})$ introduced by the cut.

[Anastasiou, Melnikov, arXiv:hep-ph/0207004]

This relation is also used for the final state particles with $p_i^2 = 0$, leading to four-loop topologies that are getting reduced:

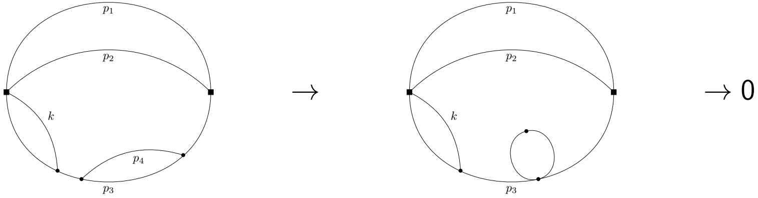


Master integrals

Occuring topologies can be classified and reduced together in three families. These families are determined by the number of massive propagators in the loop being either *zero*, *one* or *two*.
$\Rightarrow$ resulting in 1/16/16 (non-vanishing) master integrals.

If during reduction, the power of one of the 5 propagators from the reversed unitarity becomes non-positive, throw away that contribution:

$$\frac{1}{(p^2)^0} = 1 = \frac{p^2}{p^2} \to p^2\delta(p^2) = 0$$



Evaluation of the master integrals

- A lot of the master integrals (typically those with the least amount of propagators) can be written down in closed form
- For some MIs, the last phase space integration can not be carried out straightforwardly:

$$\int_0^1 dw \;\, z^{-2\epsilon}(1-z)^{1-2\epsilon} \;{}_3F_2(a1, a2, a3, a4, a5, 1-wz)$$

  $\Rightarrow$ expansion in ϵ and order-by-order integration over w
- Differential equations approach is also very helpful:
  Write master integrals as

$$\partial_z \vec{f}(z,\epsilon) = \left[\sum_k \frac{a_k(\epsilon)}{z - z_k}\right] \vec{f}(z,\epsilon)$$

  $\rightarrow$ to solve the equations, only need solution in one point (e.g. z=0) for the boundary conditions, where the integrals are solvable in closed form

Summary

Status of the calculation:

- creation and reduction of the expression is done
- subtleties, such as the treatment of γ_5 and the implementation of the cut are under control
- master integrals under evaluation

Outlook:

- finish solving the master integrals
- treatment of the UV and IR divergences, including the evanescent contributions
- investigate phenomenological aspects of the result

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Thanks for your attention!

Backup

State of the art

- NLO QCD completed in 2002

  [Buras et al., hep-ph/0203135]

- Estimate of corrections $\mathcal{O}(\alpha_s^2)$

  [Misiak et al., hep-ph/0609232]

- Multi-parton contributions

  - Completion of BLM corrections

    [Misiak, Poradziński, arXiv:1009.5685]

  - Tree level contributions

    [Kamiński et al., arXiv:1209.0965]

  - Most NLO four-body corrections $B \to s\gamma + q\bar{q}$

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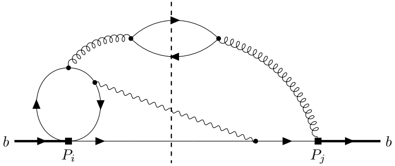

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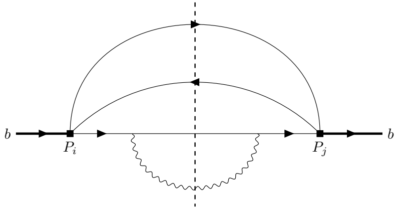

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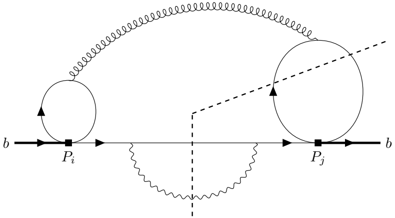

Sample Kernel

$$\mathcal{I} = \int dPS_4 \int \frac{d^4\ell}{(4\pi)^d} \frac{s_{13}s_{24}}{\ell^2(\ell+k_1+k_2+k_3)^2 s_{34}} (-\Delta_4)^{\frac{d-5}{2}}$$

$$\Rightarrow \int dPS_4 \frac{\Gamma(\epsilon)\Gamma(1-\epsilon)^2}{\Gamma(2-2\epsilon)} \frac{s_{13}s_{24}(s_{23}+s_{34}+s_{24})^{-\epsilon}}{s_{34}} (-\Delta_4)^{\frac{d-5}{2}}$$

use cyclicity in momenta of the light quarks $(3 \to 2 \to 1)$ and change of variables:

$$s_{13} = z - s_{23} - s_{12} \qquad s_{24} = \bar{z} - s_{14} - s_{34}$$

$$s_{12} = vwz \qquad s_{34} = \bar{z}\bar{v}$$

$$s_{14} = \bar{z}vx \qquad s_{23} = (a^+ - a^-)u + a^-$$

$$\Rightarrow \int_0^\delta dz (z\bar{z})^{d-3} \int_0^1 du\ dv\ dx\ dw\ (u\bar{u})^{\frac{d-5}{2}} v^{d-3} (\bar{v}w\bar{w}x\bar{x})^{\frac{d-4}{2}} \times$$

$$\Big[(a^+ - a^-)u + a^-\Big] x\bar{x}^{-1} \Big[v(wz+\bar{z})\Big]^{-\epsilon}$$

Sample Kernel

Evaluation of the integral leads to a sum of Hypergeometric functions:

$$c_1 \; \bar{z}^{1-2\epsilon} z^{2-2\epsilon} \; {}_2\mathrm{F}_1(2-\epsilon, \epsilon; 3-2\epsilon; z) + c_2 \; \bar{z}^{1-2\epsilon} z^{2-2\epsilon} \; {}_2\mathrm{F}_1(1-\epsilon, \epsilon; 2-2\epsilon; z)$$

where the c_i are functions of ϵ and we are still differential in the photon energy.

- Best case: fully analytic expression to all orders in ϵ, e.g. in terms of Hypergeometric and β-functions (the evaluation of the integral over z and the series in ϵ are interchangeable, if the former does not introduce new poles)

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- Best case: fully analytic expression to all orders in ϵ, e.g. in terms of Hypergeometric and β-functions (the evaluation of the integral over z and the series in ϵ are interchangeable, if the former does not introduce new poles)
- Second best case: obtain result in terms of e.g. Mellin-Barnes representation, which can then be expanded as a series in ϵ

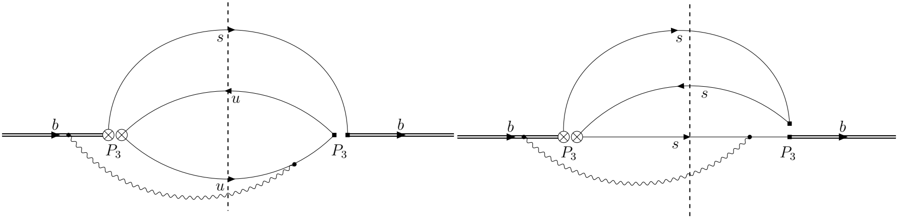
5-particle phase space

$$s_{1345}/q^2 = t_7$$
$$s_{134}/q^2 = t_6 t_7$$
$$s_{13}/q^2 = t_6 t_7 \bar{t}_2$$
$$s_{23}/q^2 = t_3 \bar{t}_7 (1 - t_2 t_4)(t_6 \bar{t}_9 + t_9)$$
$$s_{14}/q^2 = t_2 t_4 t_6 t_7$$
$$s_{24}/q^2 = y_5^- + (y_5^+ - y_5^-) t_5)$$
$$s_{34}/q^2 = t_2 t_6 t_7 \bar{t}_4$$
$$s_{15}/q^2 = t_7 \bar{t}_6 [1 - t_9(1 - t_2 t_2)] - y_{10}$$
$$s_{25}/q^2 = y_8^- + (y_8^+ - y_8^-) t_8$$
$$s_{35}/q^2 = t_7 t_9 \bar{t}_6 (1 - t_2 t_4)$$
$$s_{45}/q^2 = y_{10}^- + (y_{10}^+ - y_{10}^-) t_{10}$$

$$\int d\Phi^D_{1\to 5} = \mathcal{K}_\Gamma^{(5)} (q^2)^{2D-5} \int_0^1 \prod_{j=2}^{10} dt_j [t_5 \bar{t}_5]^{-1-\epsilon} [t_8 \bar{t}_8 t_{10} \bar{t} 10]^{-\frac{1}{2}-\epsilon}$$
$$\times [t_2 t_6 \bar{t}_6 \bar{t}_7]^{1-2\epsilon} [(\bar{t}_2 t_3 \bar{t}_3 t_4 \bar{t}_4 t_9 \bar{t}_9)]^{-\epsilon} t_7^{2-3\epsilon}$$

Renormalization

To cancel UV divergences, insertions of the bare operators $P_i^{(0)}$ into the tree-level diagrams have to be computed



With this, the renormalization constants δZ_{ij} can be used to cancel the UV-divergences via the relation

$$\sum_{i=1..6} \mathcal{C}_i P_i^{(0)} = \sum_{i=1..6} \mathcal{C}_i P_i + \frac{\alpha_s}{4\pi\epsilon} \sum_{i,j=1..6} \mathcal{C}_i \delta Z_{ij} P_j$$

IR regularization

- regions where photon is collinear to light quarks gives rise to collinear divergences
  $\rightarrow$ automatically regularized in DimReg
- divergences are artifact of massless limit
  $\rightarrow$ could more naturally be regulated by light quark masses, but massless case is already quite complicated
- fortunately, amplitudes in the quasi-collinear limit factorize:

$$b \rightarrow q_1 q_2 \bar{q}_3 \gamma \Rightarrow b \rightarrow \sum_i q_1 q_2 \bar{q}_3 \times f_i$$

with f_i a DGLAP splitting function describing emission of γ from q_i

Comparing the splitting functions in the two different schemes of mass regulators and DimReg leads to the relation

$$\frac{d\Gamma_m}{dz} = \frac{d\Gamma_\epsilon}{dz} + \frac{d\Gamma_{shift}}{dz}$$

Shifting part can be calculated from three-particle-cut diagrams:

$$\frac{\Gamma_{shift}}{dz} = \frac{1}{2m_b}\frac{1}{2N_c}\int dPS_3 \mathcal{K}_3(s_{ij})\frac{\alpha_e}{2\pi\bar{z}}\left\{ Q_1^2\left[1+\frac{(z-s_{23})^2}{(1-s_{23})^2}\right]\right\}\times$$
$$\times\left[\frac{1}{\epsilon}-1+2\log\frac{(1-s_{23})\mu}{m_{q_1}(1-z)}\Theta(z-s_{23})+(\text{cyclic})\right]$$

$\Rightarrow$ trade the surviving $1/\epsilon$ terms for $\log(\frac{m_q}{m_b})$ terms