Karlsruher Institut für Technologie

Residual flavour symmetries

From Bottom-up to top down strategies for flavourful leptoquarks

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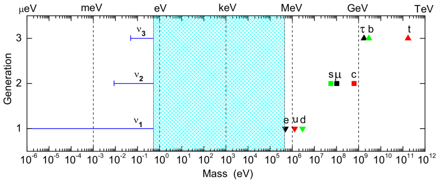
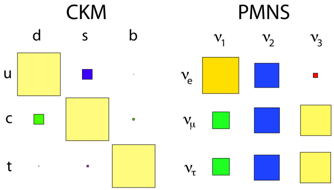
Outline

1. Flavour puzzle & discrete flavour symmetries
2. $R_{K^{(*)}}$ anomalies & flavourful leptoquark solutions
3. Bottom-up scan over symmetry groups
4. Conclusion

Motivations





The flavour puzzle

- **Why 3 families ?**
- **Why these specific mixing patterns ?**
- Why such mass hierarchies ?

$\Longrightarrow$ Discrete non-abelian flavour symmetries !

An A_4 example for leptons

[1002.0211] Rev. Mod. Phys. (2010) - G. Altarelli, F. Feruglio

Lepton mass terms

$$\mathcal{L}_{m_L} = \bar{E}_R m_\ell \ell_L + \frac{1}{2}\bar{\nu}_L^c m_\nu \nu_L + \text{h.c.}$$

Specific mixing patterns for leptons:

$$U_{PMNS} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.233 \to 0.495 & 0.448 \to 0.679 & 0.639 \to 0.783 \\ 0.287 \to 0.532 & 0.486 \to 0.706 & 0.604 \to 0.754 \end{pmatrix} \sim \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} = U_{TBM}$$

Is it possible to reproduce U_{TBM}?

In the basis where $\mathbf{m}_\ell$ **is diagonal**, m_ν^{TBM} is completely[1] specified by $\mathbf{S_{TBM}}$

$$m_\nu^{TBM} = S_{TBM}^T m_\nu^{TBM} S_{TBM}$$

$$\text{with} \quad S_{TBM} = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

1. Plus $\mu - \tau$ symmetry.

An A_4 example for leptons

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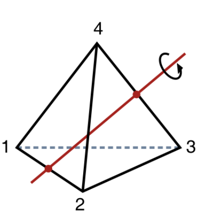
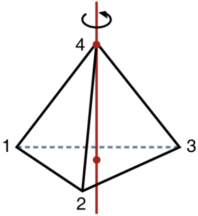
Group that leaves the tetrahedron invariant





$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Field	L	e^c	μ^c	τ^c	ν^c	ϕ_T	ϕ_S
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$

Yukawas from flavons ϕ_i charged under A_4 and other "driving symmetries".

$$\begin{cases} \phi_T \to (v_T, 0, 0) \\ \phi_S \to (v_S, v_S, v_S) \end{cases}$$

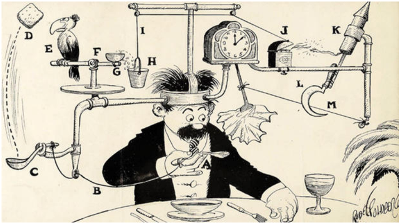
m_ℓ invariant under T, m_ν invariant under S.

$$\Longrightarrow U_{PMNS} = U_{TBM}$$

Reverse engineering A_4

Is it possible to recover A_4

from IR predictions ?



In the mass basis

$$T'_\nu = \text{diag}(1, -1, -1), \qquad T'_l = \text{diag}(1, \omega, \omega^2)\,.$$

Going back to the flavour basis by undoing the $U_{PMNS} = U_{TBM}$ rotation, you reach

$$T_\nu = U_{PMNS} T'_\nu U^T_{PMNS} = S_{TBM}, \qquad T_l = T'_l\,.$$

By closing the algebra composed of T_ν and T_l you recover A_4.

We will generalize this approach to scan over groups !

But still some subtleties ...

Reverse engineering A_4

The IR prediction of any flavour symmetry is hidden in the mass basis, i.e the residual flavour symmetry is hidden by larger group

$$\mathcal{L}_{m_L} = \bar{E}'_R m'_\ell \ell'_L + \frac{1}{2}\bar{\nu}'^{c}_L m'_\nu \nu'_L + \text{Non-Diagonal Charged Current}$$

The "accidental" symmetries of the mass terms are

$$T_l = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) = U(1)^3,$$
$$T_{\nu_1} \times T_{\nu_2} = \text{diag}(1,-1,-1) \times \text{diag}(-1,-1,1) = Z_2 \times Z_2$$

$\Longrightarrow$ need to perform a discrete scan over the phases and generators !

This can be automated using the `GAP` package

Flavour anomalies as a guideline

Recent hints of LFNU

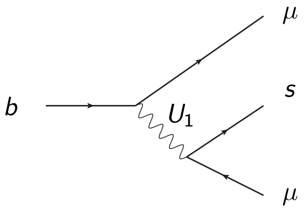
$$R_{K^{(\star)},[a,b]} = \frac{\int_a^b dq^2 \left[d\Gamma(B \to K^{(\star)}\mu^+\mu^-)/dq^2\right]}{\int_a^b dq^2 \left[d\Gamma(B \to K^{(\star)}e^+e^-)/dq^2\right]} .$$

and several others related to muons...

A popular explanation: Vector leptoquark

$$U_1 \sim (3, 1, 2/3)$$

$$\mathcal{L} \supset y\bar{Q}_L\gamma_\mu L_L U_1^\mu + ...$$



$\Rightarrow$ Structure the couplings ?

Residual flavour symmetries

Assume a parent flavour symmetry $\mathcal{G}_\mathcal{F}$, broken to the different sectors, for e.g.

$$\mathcal{G}_\mathcal{F} \to \begin{cases} \mathcal{G}_\mathcal{L} \to \begin{cases} \mathcal{G}_\nu \\ \mathcal{G}_\mathsf{l} \end{cases} \\ \mathcal{G}_\mathcal{Q} \to \begin{cases} \mathcal{G}_\mathsf{u} \\ \mathcal{G}_\mathsf{d} \end{cases} \\ \mathcal{G}_{\mathcal{L}\mathcal{Q}} \to \mathcal{G}'_{\mathcal{L}\mathcal{Q}} \end{cases} \qquad \Longrightarrow \text{Controls the LQ coupling!}$$

- The νSM fermion mass terms (in mass basis)

$$\mathcal{L}^{SM}_{mass} \supset \frac{1}{2}\bar{\nu}^c_L\, m_\nu\, \nu_L + \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{u}_R\, m_u\, u_L + \text{h.c.}$$

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is invariant under $U(1)^3_f$ for $f = l, q$ and $Z_2 \times Z_2$ for ν_L

$$\nu_L \to T_{\nu_i}\nu_L, \quad \text{with} \quad T_{\nu 1} = \mathrm{diag}\,(1,-1,-1) \quad \text{and} \quad T_{\nu 2} = \mathrm{diag}\,(-1,1,-1)\,,$$

$$f \to T_f f, \quad \text{with} \quad T_f = \mathrm{diag}\left(e^{i\alpha_f}, e^{i\beta_f}, e^{i\gamma_f}\right) \quad \text{for} \quad f \in \{E_R, l_L, d_R, d_L, u_R, u_L\}.$$

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- The LQ interaction terms

$$\mathcal{L} \subset \underbrace{\left(V^\dagger_{u_L}\, y\, V_\nu\right)}_{\lambda_{u\nu}} \bar{u}_L\gamma^\mu\nu U_{1\mu} + \underbrace{\left(V^\dagger_{d_L}\, y\, V_{e_L}\right)}_{\lambda_{dl}} \bar{d}_L\gamma^\mu e_L U_{1\mu} + \ldots$$

Note that the λ_{QL} can be all expressed in terms of λ_{dl}

Patterns from λ_{dl} symmetry constraints

[1901.10484] JHEP - Ivo de Medeiros, Jim Talbert

*Hypothesis: We enforce that the LQ terms are invariant under the **same** symmetries as the fermion mass terms*

$$T_d^{(T,\dagger)} \lambda_{dl} T_l \stackrel{!}{=} \lambda_{dl},$$

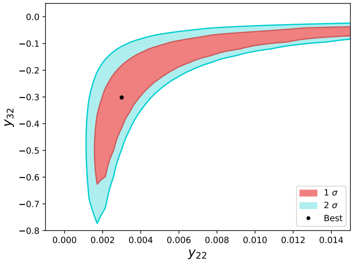
which imposes phase equalities

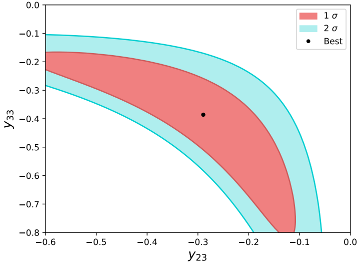
$$\begin{pmatrix} e^{i(\alpha_d+\alpha_l)}\lambda_{de} & e^{i(\alpha_d+\beta_l)}\lambda_{d\mu} & e^{i(\alpha_d+\gamma_l)}\lambda_{d\tau} \\ e^{i(\beta_d+\alpha_l)}\lambda_{se} & e^{i(\beta_d+\beta_l)}\lambda_{s\mu} & e^{i(\beta_d+\gamma_l)}\lambda_{s\tau} \\ e^{i(\gamma_d+\alpha_l)}\lambda_{be} & e^{i(\gamma_d+\beta_l)}\lambda_{b\mu} & e^{i(\gamma_d+\gamma_l)}\lambda_{b\tau} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}.$$

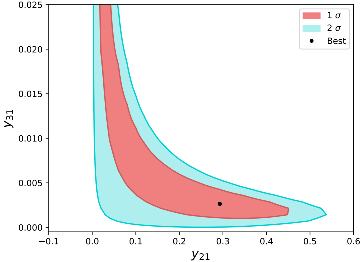
Different patterns:

- Isolation $\lambda_{dl}^{[e]} = \begin{pmatrix} \lambda_{de} & 0 & 0 \\ \lambda_{se} & 0 & 0 \\ \lambda_{be} & 0 & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\mu]} = \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ 0 & \lambda_{s\mu} & 0 \\ 0 & \lambda_{b\mu} & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\tau]} = \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ 0 & 0 & \lambda_{s\tau} \\ 0 & 0 & \lambda_{b\tau} \end{pmatrix}$
- Two-columned $\lambda_{dl}^{[e\mu]} = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix}, \quad \lambda_{dl}^{[e\tau]} = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix}, \quad \lambda_{dl}^{[\tau]} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$
- Three-columned ...

Flavourful U_1 as an anomaly solutions







Work ongoing

- Finish fits
- Pheno
- Collider studies

Strategy and objectives

- Assume that the same flavour symmetry controls fermionic mixing + LQ coupling
- Find groups that lead to aimed LQ patterns + fermionic mixing

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$$\underbrace{\{\mathcal{G}_u,\ \mathcal{G}_d,\ \mathcal{G}_\nu, \mathcal{G}_l\}}_{\text{mass basis}} \quad \underbrace{\longrightarrow}_{\text{LQ patterns \& SM mixing rotations}} \quad \underbrace{\{\mathcal{G}'_u,\ \mathcal{G}'_d,\ \mathcal{G}'_\nu, \mathcal{G}'_l\}}_{\text{"flavour" basis}} \quad \underbrace{\longrightarrow}_{\text{Close algebra}} \quad \mathcal{G}_{\mathcal{F}}$$

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The CKM and PMNS approximation

We wish to recover specific patterns for the leading order PMNS and CKM matrices.

We set

$$U_{PMNS} \simeq U_{\mu\tau} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta_{\mu\tau} & \sqrt{2}\sin\theta_{\mu\tau} & 0 \\ -\sin\theta_{\mu\tau} & \cos\theta_{\mu\tau} & 1 \\ \sin\theta_{\mu\tau} & -\cos\theta_{\mu\tau} & 1 \end{pmatrix} + \mathcal{O}\left(\theta^{l}_{\mathbf{13}}\right),$$

$$V_{CKM} \simeq U_C \equiv \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}\left(\theta_c^{\mathbf{2}}, \theta_c^{\mathbf{3}}\right).$$

$U_{\mu\tau}$ embed popular approximation for the PMNS matrix such as TBM

$$U_{\mu\tau} \supset U_{TBM}$$

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Choice of basis

In the flavour basis, the generators "know" about the mixing matrices !

Include as well the "knowledge" about specific chosen LQ pattern !

To do so, we further move to a basis where λ_{dl} is diagonal by

$$\lambda'_{dl} = \Lambda_d^* \lambda_{dl} \Lambda_l^\dagger$$

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To do so, we further move to a basis where λ_{dl} is diagonal by

$$\lambda'_{dl} = \Lambda_d^* \lambda_{dl} \Lambda_l^\dagger$$

Therefore, the residual generators in the leptoflavour basis are

$$T_l^{'} = \Lambda_l T_l \Lambda_l^\dagger, \quad T_\nu^{'} = \Lambda_l U_{PMNS} T_\nu U_{PMNS}^\dagger \Lambda_l^\dagger, \quad T_d^{'} = \Lambda_d T_d \Lambda_d^\dagger, \quad T_u^{'} = \Lambda_d V_{CKM}^\dagger T_u V_{CKM} \Lambda_d^\dagger,$$

Strategy and objectives

- Assume that the same flavour symmetry controls fermionic mixing + LQ coupling
- Derive authorized LQ patterns
- Find which groups are leading to aimed LQ patterns + fermionic mixing

How to do ?

$$\underbrace{\{\mathcal{G}_u,\ \mathcal{G}_d,\ \mathcal{G}_\nu,\mathcal{G}_l\}}_{\text{mass basis}} \underbrace{\longrightarrow}_{\text{LQ patterns \& SM mixing rotations}} \underbrace{\{\mathcal{G}'_u,\ \mathcal{G}'_d,\ \mathcal{G}'_\nu,\mathcal{G}'_l\}}_{\text{"flavour" basis}} \underbrace{\longrightarrow}_{\text{Close algebra}} \mathcal{G}_{\mathcal{F}}$$

Results

Lepton Isolation and Lepton Mixing					
$\lvert\tan\theta_{\mu\tau}\rvert$	T_l^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G_L}$	Electron/Muon
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[-1, 1, -1]$	$[12, 3]$	A_4	✓/✓
1	$[1, \omega_4, -\omega_4]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/X
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[1, -1, -1]$	$[24, 12]$	S_4	✓/X
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_4, -1, \omega_4]$	$[48, 3]$	$\Delta(48)$	✓/✓
1	$[\omega_4, 1, -1]$	$[1, -1, \omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	X /✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, -\omega_4, -\omega_4]$	$[48, 30]$	$A_4 \rtimes Z_4$	✓/X
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, -1, -1]$	$[72, 42]$	$Z_3 \times S_4$	X /✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[\omega_5, \omega_5^3, \omega_5]$	$[75, 2]$	$\Delta(75)$	✓/✓
$1/\sqrt{2}$	$[\omega_3, 1, \omega_3^2]$	$[1, \omega_3, 1]$	$[81, 7]$	$\Sigma(81)$	✓/✓
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/ X
1	$[\omega_4, 1, -1]$	$[1, -1, 1]$	$[96, 186]$	$Z_4 \times S_4$	✓/✓

The general recipe *[2005.12293] - JB, Ivo de Medeiros, Jim Talbert*

How to use the above results ?

- Move to the leptoflavour basis
- Furthermore, to the "model" basis
- Find preserved group elements
- Choose representations
- Find flavon vev
- Compute the wanted Yukawa
- Write down effective operators

$\rightarrow$ It works but subtleties when degenerate eigenvalues

Conclusion

Summary

- We investigated anomaly motivated patterns
- We built up a systematic approach to find flavour symmetries
- Building models, we closed the loop bottom-up - top-down
- Provide proof of principle of the methods

To go beyond

- Perform same analysis without degeneracy
- Extend CKM approximation ?
- Apply similar methods to other BSM scenarios ?

Thanks !

Ambiguity in the mixing prediction

The Altarelli-Feruglio model is supposed to predict $U_{PMNS} = U_{TBM}$...
But in fact, in the reverse engineering process, it is not guaranteed !

This is due to the degeneracy present in the mass basis generators

$$T'_\nu = \text{diag}(1,-1,-1) = R^\dagger T'_\nu R, \qquad T'_l = \text{diag}(1,\omega,\omega^2)\,.$$

where the unitary matrix R is

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\delta} \\ 0 & -\sin\theta e^{i\delta} & \cos\theta \end{pmatrix},$$

Therefore, ambiguity in $m'_\nu \to R^T m'_\nu R$

$$T'^T_\nu m'_\nu T'_\nu = R^T T'^T_\nu R^\star R^T m''_\nu R R^\dagger T'_\nu R$$

which translates into the PMNS ambiguity $U_{PMNS} \to U_{PMNS} R$

In Altarelli-Feruglio model, possible way out. But not always the case ...

Scan basic procedure

1) Discretize continuous parameters:

Assume that $\mathcal{G}_a \sim Z_{n_a}$ $\Longrightarrow$ Discretize phases in RFS

Discretize free parameters in mixing matrices

angles $\theta_i = 2\pi \left(\frac{n}{m}\right)_i$, phases $\Theta_i = 2\pi \left(\frac{n}{m}\right)_i$, LQ couplings $\theta_i^{LQ} = \left(\pm\sqrt{\frac{n}{m}}\right)_i$

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2) Move to the "leptoflavour" basis

3) Close algebra (GAP)

$$
\begin{aligned}
\text{Leptoquarks, PMNS \& CKM:} \quad & \mathcal{G}_F \sim \{T'_d, T'_l, T'_u, T'_\nu\} \\
\text{Leptoquarks, PMNS \& CKM:} \quad & \mathcal{G}_F \sim \{T'_d, T'_u\} \times \{T'_l, T'_\nu\} \\
\text{Leptoquarks \& PMNS:} \quad & \mathcal{G}_F \sim \{T'_l, T'_\nu\} \\
\text{Leptoquarks \& CKM:} \quad & \mathcal{G}_F \sim \{T'_u, T'_d\}
\end{aligned}
$$

Example 1: $\Delta(96)$ for leptonic mixing and electron isolation

One result of the scan is, for PMNS only and electron isolation pattern

$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/✗

N.B: There are no degeneracy in the $T's$ $\rightarrow$ No possible ambiguities in the mixing !

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Leptoflavour basis: $$T_l' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & \omega_3^2 \end{pmatrix}, \qquad T_\nu' = \frac{1}{3}\begin{pmatrix} 1+2i & -1+i & 1-i \\ -1+i & 1-i & -1-2i \\ 1-i & -1-2i & 1-i \end{pmatrix},$$

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$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/✗

N.B: There are no degeneracy in the $T's$ $\rightarrow$ No possible ambiguities in the mixing !

$$\text{Leptoflavour basis:} \quad T_l^{'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & \omega_3^2 \end{pmatrix}, \quad T_\nu^{'} = \frac{1}{3}\begin{pmatrix} 1+2i & -1+i & 1-i \\ -1+i & 1-i & -1-2i \\ 1-i & -1-2i & 1-i \end{pmatrix},$$

Now: "model" basis ($T'' = P^{-1}T'P$), to identify group elements

$$\text{Model basis:} \quad T_l^{''} = \begin{pmatrix} \omega_3 & 0 & 0 \\ 0 & \omega_3^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T_\nu^{''} \frac{1}{3}\begin{pmatrix} 1-i & 1+2i & 1-i \\ 1+2i & 1-i & 1-i \\ 1-i & 1-i & 1+2i \end{pmatrix}.$$

Example 1: $\Delta(96)$ for leptonic mixing and electron isolation

One result of the scan is, for PMNS only and electron isolation pattern

$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, 1, -\omega_4]$	$[96, 64]$	$\Delta(96)$	✓/✗

N.B: There are no degeneracy in the $T's$ $\rightarrow$ No possible ambiguities in the mixing !

$$\text{Leptoflavour basis:} \quad T_l^{'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & \omega_3^2 \end{pmatrix}, \quad T_\nu^{'} = \frac{1}{3}\begin{pmatrix} 1+2i & -1+i & 1-i \\ -1+i & 1-i & -1-2i \\ 1-i & -1-2i & 1-i \end{pmatrix},$$

Now: "model" basis ($T'' = P^{-1}T'P$), to identify group elements

$$\text{Model basis:} \quad T_l^{''} = \begin{pmatrix} \omega_3 & 0 & 0 \\ 0 & \omega_3^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T_\nu^{''} \frac{1}{3}\begin{pmatrix} 1-i & 1+2i & 1-i \\ 1+2i & 1-i & 1-i \\ 1-i & 1-i & 1+2i \end{pmatrix}.$$

There are 3 generators a, b and c in $\Delta(96)$. We find, for $\mathbf{\bar{3}_1}$, the following match

$$T_l'' = a^2 cd, \quad T_\nu'' = a^2 bc^2 d^3$$

$\Longrightarrow$ $\mathbf{\bar{3}_1}$ is a suitable irrep for $\bar{L}$.

Example 1: $\Delta(96)$ for leptonic mixing and electron isolation

Next step: build the Yukawa matrices
The Yukawa "combos" that we have to build are the following

$$m_\nu''^\dagger m_\nu'' = \frac{1}{3}\begin{pmatrix} \frac{1}{2}\left(m^2_{\nu_1}+2m^2_{\nu_2}+3m^2_{\nu_3}\right) & \frac{1}{2}\left(m^2_{\nu_1}+2m^2_{\nu_2}-3m^2_{\nu_3}\right) & \left(m^2_{\nu_2}-m^2_{\nu_1}\right) \\ \frac{1}{2}\left(m^2_{\nu_1}+2m^2_{\nu_2}-3m^2_{\nu_3}\right) & \frac{1}{2}\left(m^2_{\nu_1}+2m^2_{\nu_2}+3m^2_{\nu_3}\right) & \left(m^2_{\nu_2}-m^2_{\nu_1}\right) \\ \left(m^2_{\nu_2}-m^2_{\nu_1}\right) & \left(m^2_{\nu_2}-m^2_{\nu_1}\right) & \left(2m^2_{\nu_1}-m^2_{\nu_2}\right) \end{pmatrix},$$

$$m_l''^\dagger m_l'' = \begin{pmatrix} m^2_{l_2} & 0 & 0 \\ 0 & m^2_{l_3} & 0 \\ 0 & 0 & m^2_{l_1} \end{pmatrix},$$

This can be achieved by choosing

	$\bar{L}_L$	(e_R, μ_R)	τ_R	(ν_{R1}, ν_{R2})	ν_{R3}	ϕ_ν	ϕ_l	ξ
$\Delta(96)$	$\bar{\mathbf{3}}_1$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\bar{\mathbf{3}}_1$	$\bar{\mathbf{3}}_1$	$\bar{\mathbf{1}}'$

with Lagrangian

$$\mathcal{L} \supset \left[\bar{L}_L\phi_l\right]_1 E_{R3} + a_2\left[\bar{L}_L\phi_l\right]_2 E_{R12} + a_2'\left[\bar{L}_L\phi_l\right]_2 \xi E_{R12} + a_\nu\left[\bar{L}_L\phi_f\right]_1 \nu_{R3} + b_\nu\left[\bar{L}_L\phi_\nu\right]_2 \nu_{R12}$$