TRR 257 - Particle Physics Phenomenology
after the Higgs Discovery

Exact quark-mass dependence of the Higgs-gluon form factor at three loops in QCD

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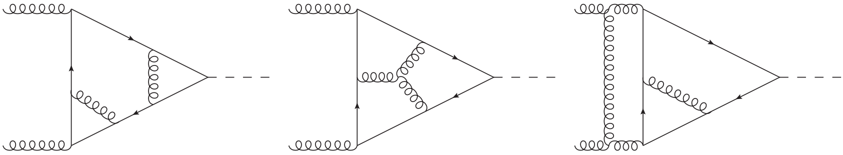
Based on arXiv:2001.03008

Motivation

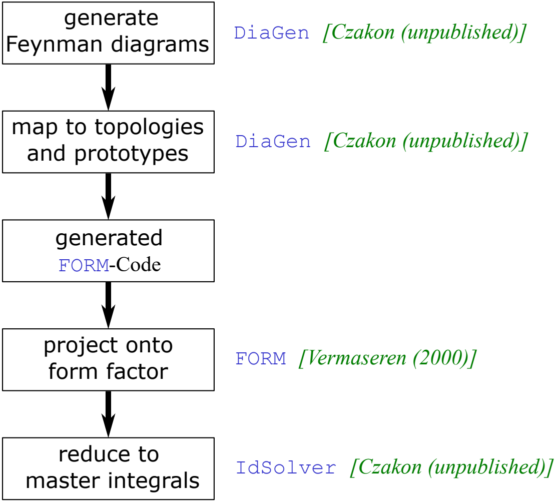


- Interest in the Higgs-gluon form factor due to studies on cross section predictions for hadron-collider processes involving an intermediate Higgs boson
- Amplitude $gg \to H$ contributes to single- and double-Higgs production
  $\Rightarrow$ Applications require knowledge of the Higgs-gluon form factor
- In this talk: Calculation of the three-loop Higgs-gluon form factor in QCD with a single massive quark

Recent development

- Top quark mass dependence of the Higgs-gluon form factor *[Davies,Gröber,Maier,Rauh,Steinhauser (2019)]*
- Analytic results for the light-fermion contributions to the Higgs-gluon form factor *[Harlander,Prausa,Usovitsch (2019)]*
- Exact quark-mass dependence of the Higgs-gluon form factor *[Czakon,MN (2020)]*

Setup


generate Feynman diagrams
DiaGen [Czakon (unpublished)]
map to topologies and prototypes
DiaGen [Czakon (unpublished)]
generated FORM-Code
project onto form factor
FORM [Vermaseren (2000)]
reduce to master integrals
IdSolver [Czakon (unpublished)]

From Feynman diagrams to scalar integrals

- Get rid of gluon wave functions and tensor structure
- Most general tensor structure of the amplitude:

$$\mathcal{M}^{\mu\nu} = (p_1^\mu p_1^\nu + p_2^\mu p_2^\nu)A + p_1^\mu p_2^\nu B + p_2^\mu p_1^\nu C + (p_1 \cdot p_2) g^{\mu\nu} D$$

  ($p_{1,2}$: momenta of the external gluons)
- Colour structure is trivial
- Physical quantities depend on coefficient C only
- C is projected out by

$$C = \frac{1}{(p_1 \cdot p_2)(d-2)} \left[g_{\mu\nu} - \frac{p_{2\mu} p_{1\nu}}{(p_1 \cdot p_2)} \right] \mathcal{M}^{\mu\nu}$$

From Feynman diagrams to scalar integrals

- Traces of γ-matrices in the amplitude are calculated
- Colour factors are computed with the FORM-package color *[van Ritbergen,Schellekens,Vermaseren (1999)]*
- For every Feynman diagram an automatically generated FORM-procedure is called depending on the topology
- Purpose of this code:
  - Matching scalar integrals in the projected form factor to prototypes of the form

$$PRID(n_1, \cdots, n_{12}) = \int \frac{d^d k_1}{i\pi^{d/2}} \int \frac{d^d k_2}{i\pi^{d/2}} \int \frac{d^d k_3}{i\pi^{d/2}} \frac{1}{D_1^{n_1} \cdots D_{12}^{n_{12}}},$$

- For $gg \to H$: 505 diagrams $\Rightarrow$ 10209 scalar integrals

Reduction to master integrals

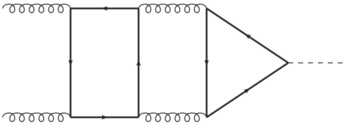
- Integration-by-parts (IBP) identities *[Chetyrkin, Tkachov (1981)]*:

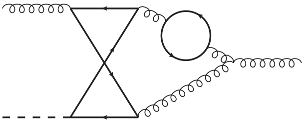
$$0 = \int \frac{d^d k_1}{i\pi^{d/2}} \cdots \int \frac{d^d k_L}{i\pi^{d/2}} \frac{\partial}{\partial k_i^\mu} \frac{p_j^\mu}{D_1^{n_1} \cdots D_N^{n_N}} ,$$

  where the p_j are loop momenta or external momenta.
- Automatic approach to solve IBP relations: Laporta's algorithm *[Laporta (2000)]*
- For $gg \to H$: 426 master integrals

Two-scale approximation

- The complexity of multi-loop calculations grows rapidly with the increasing number of scales
- At three-loop level different quark flavours could run in separated loops





- Introduce one heavy quark with mass M and a light quark with mass zero

Method of differential equations

- Introduce dimensionless variable $z \equiv \frac{s}{4M^2} + i0^+$
- Exploit IBP relations again to construct a system of first-order linear differential equations

$$\frac{\mathrm{d}M_i(z,\epsilon)}{\mathrm{d}z} \equiv \sum_j A_{ij}(z,\epsilon)\, M_j(z,\epsilon)\;,$$

  where the coefficients $A_{ij}(z,\epsilon)$ are rational functions in z and ϵ.
- Insert truncated ϵ-expansions for the master integrals

$$M_i(z,\epsilon) \equiv \sum_{l=0}^{\overline{n}_i-\underline{n}_i} \epsilon^{\underline{n}_i+l}\, I_{\underline{k}_i+l}(z)$$

- The functions I_k satisfy the following system of first-order linear differential equations

$$\frac{\mathrm{d}I_k(z)}{\mathrm{d}z} \equiv \sum_l B_{kl}(z)\, I_l(z)$$

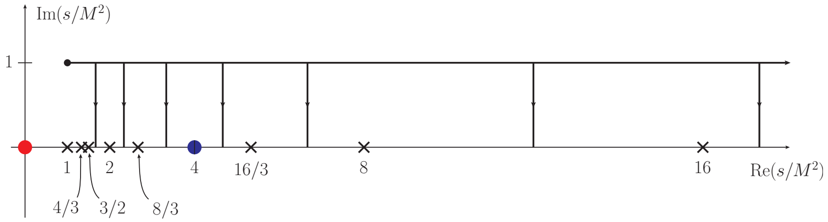
Method of differential equations

- Instead of seeking an analytic solution, we solve the system numerically.
- To provide proper boundaries for the numerical evolution, we solve the differential equations in the limit $z = 0$ via a power-log ansatz

$$I_k(z) \equiv \sum_{l=\underline{l}_k}^{\infty} \sum_{m=\underline{m}_k}^{\overline{m}_k} c_{klm}\, z^l \ln^m z \, .$$

The underlying algorithm to determine c_{klm} has been implemented in a private C++ software that was originally developed for *[Czakon,Fiedler,Huber,Misiak,Schutzmeier,Steinhauser (2015)]*.

Numerical solution



- Spurious and physical poles in the differential equations
- Numerical instabilities in their close vicinity
  $\Rightarrow$ Avoid instabilities by integrating along contours in the complex plane
- Utilise the algorithm by Bulirsch and Stoer implemented in the Boost library `odeint` *[Ahnert,Mulanski (2012)]* to collect numerical values for the master integrals over the whole kinematic range

Results

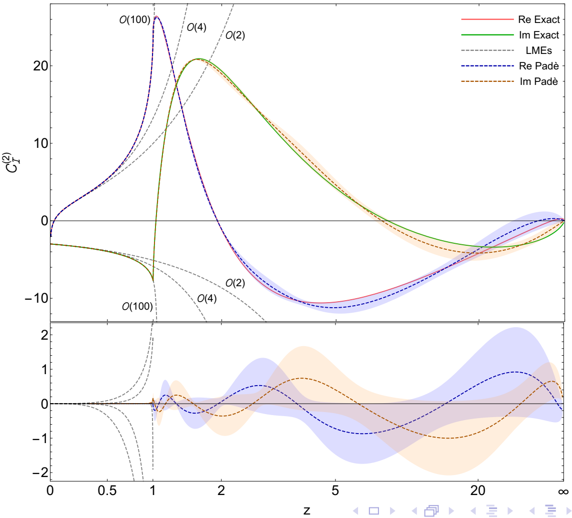
- The form factor $\mathcal{C}$ is expanded in the strong coupling constant, α_s, and the number of massless quark flavors, n_l:

$$\mathcal{C} = \mathcal{C}^{(0)} + \frac{\alpha_s}{\pi}\mathcal{C}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{C}^{(2)} + \mathcal{O}\left(\alpha_s^3\right) , \qquad \mathcal{C}^{(n)} = \sum_{k=0}^{n} \mathcal{C}^{(n,k)}\, n_l^k \; .$$

- M is defined in the on-shell scheme, the strong coupling in $\overline{\mathrm{MS}}$ scheme with massive-quark decoupling.
- Infrared divergencies may be removed according to *[Catani (1998)]* yielding the finite remainder $\mathcal{C}_I$.

Results

- Compare the exact form factor to Padé approximation estimated by *[Davies,Gröber,Maier,Rauh,Steinhauser (2019)]*



Summary

- The Higgs-gluon form factor is known exactly at three loops in QCD with a single massive quark.
- We provide expansions of the form factor in the kinematic limits to high orders.
- We have confirmed that an approach via Padé approximants is sufficient in the case, where the massive quark is the top.
- Our results remove any uncertainties on the value of the form factor obtained via Padé approximants.