
What is the Top Quark Mass?

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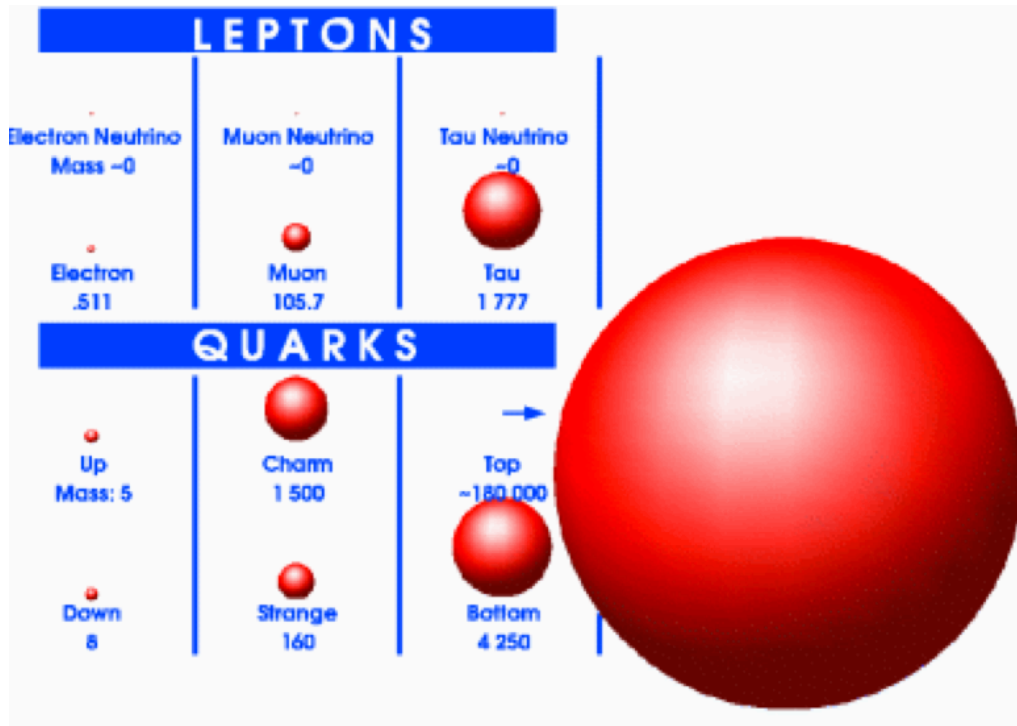
Based arXiv:2004.12915 prepared for
for Annual Reviews on Nuclear and Particle Science

fdk Π Doktoratskolleg
Particles and Interactions



FWF
Der Wissenschaftsfonds.

.. not just the heaviest SM particle



- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background in new physics searches
- One of crucial motivations for New Physics

- Very special physics laboratory: $\Gamma_t \gg \Lambda_{\text{QCD}}$
 - Top treated as a particle: p_T , spin, σ_{tot} , $\sigma(\text{single top})$, $\sigma(\text{tt}+X)$,... $\rightarrow q \gg \Gamma_t$
 - Quantum state sensitive to low-E QCD and unstable particle effects: m_t , endpoint regions $\rightarrow q \sim \Gamma_t$
 - **Multiscale problem:** $p_T, m_t \gg \Gamma_t \gg \Lambda_{\text{QCD}}, \dots$ (depends on resolution scale of observable \rightarrow **high top mass sensitivity \Leftrightarrow low resolution scale**)

Top Mass Measurements

Most precise method: Direct Reconstruction

kinematic mass determination

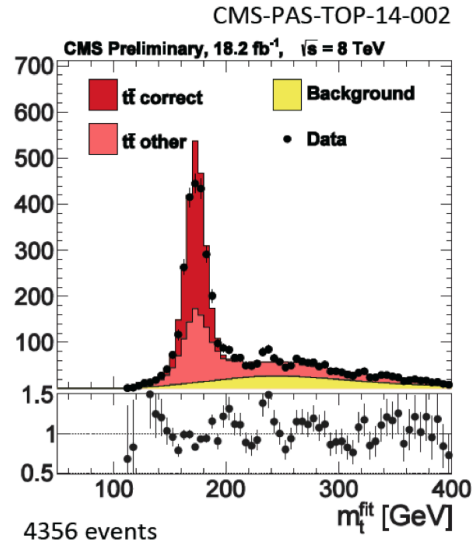
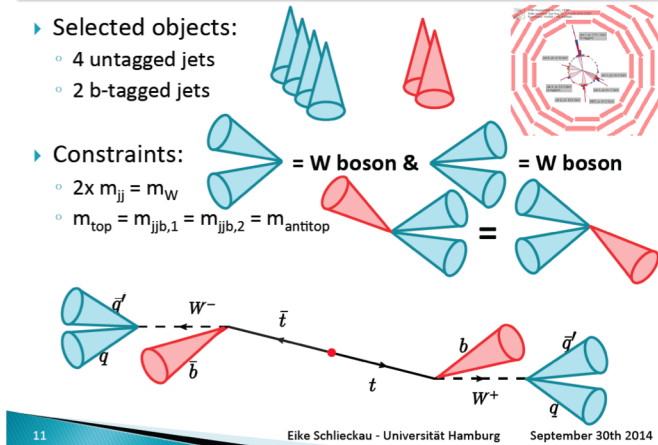
Kinematic Fit

Selected objects:

- 4 untagged jets
- 2 b-tagged jets

Constraints:

- $2 \times m_{jj} = m_W$
- $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$



Determination of the best-fit value of the Monte-Carlo top quark mass parameter

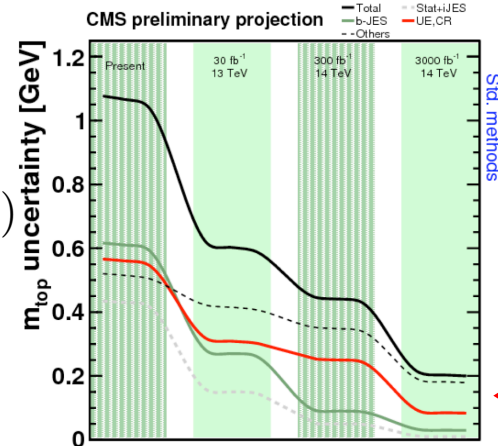
- ⊕ High top mass sensitivity
- ⊖ Precision of MC ?
- ⊖ Meaning of m_t^{MC} ?

$$m_t^{MC} = 172.9 \pm 0.4 \text{ GeV} \quad (\text{world average})$$

$$m_t^{MC} = 172.26 \pm 0.61 \text{ GeV} \quad (\text{CMS combined})$$

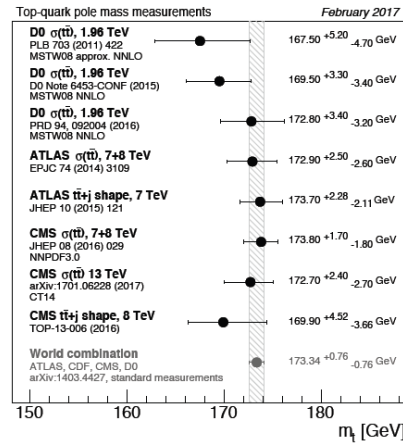
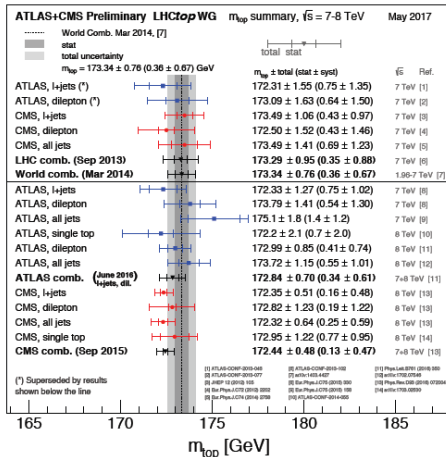
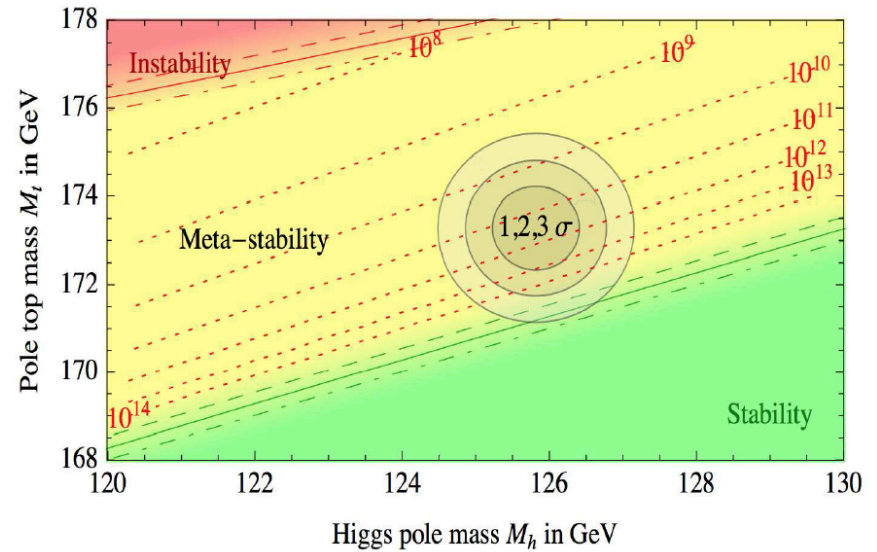
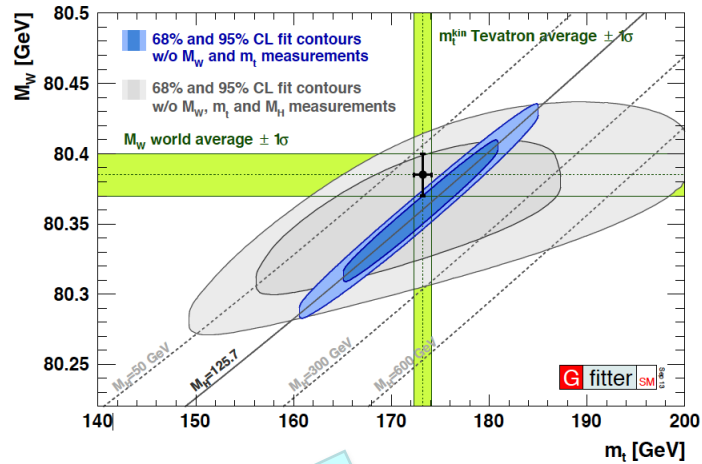
$$m_t^{MC} = 172.69 \pm 0.48 \text{ GeV} \quad (\text{ATLAS combined})$$

$$m_t^{MC} = 174.34 \pm 0.64 \text{ GeV} \quad (\text{Tevatron})$$



Δ $m_t \sim 200$ MeV (HL-LHC projection)

Why a Precision Top Mass is Important



Aims:

m_{top} wanted !

M_{top} is a renormalized QCD parameter !

- Reduce error in m_{top}^{MC}
- Improve / understand better MC
- Clarify mass scheme m_{top}^{MC} !

Content

Aims of future research on top mass measurements:

- Understand and resolve the physical and conceptual questions involved in the MC top mass interpretation problem (direct measurements).
- Better understanding of experimental and theoretical aspects of alternative top mass measurement methods.
 - based on theory predictions with well-defined mass scheme
 - uncertainties comparable to direct measurements difficult for HL-LHC

Content:

- Physics of mass renormalization schemes
- Status of top mass measurements
- The controversy
- Recent insights in the character of m_t^{MC}

Comment: Most the problems would be resolved, if we had an e^+e^- collider where we could do measurement of the total cross section at the top-antitop threshold. But this talk will concentrate on issues related to the LHC.

e^+e^- collider: $\delta m_t^{\text{well-defined scheme}} \sim 50\text{-}100 \text{ MeV}$ straightforward

Mass Extraction and Renormalization Schemes

The Principle of Top Mass Determinations

- Top quark is not a physical particle (“colored parton”)
- Top mass defined from theoretical prescriptions (renormalization schemes)
- Different schemes are related by a perturbative series.

$$m_t^A - m_t^B = \sum_{n=1} c_n \alpha_s^n(\mu)$$

Parton level cross section formally scheme-invariant,
but can be practically scheme-dependent due to truncation

$$\hat{\sigma}(Q, m_t^A, \alpha_s(\mu), \mu; \delta m^A) = \hat{\sigma}(Q, m_t^B, \alpha_s(\mu), \mu; \delta m^B)$$

- For comparison with exp. data one has to account for non-perturbative corrections

$$\sigma^{\text{exp}} = \hat{\sigma}(Q, m_t^X, \alpha_s(\mu), \mu; \delta m^X) + \sigma^{\text{NP}}(Q, \Lambda_{\text{QCD}})$$

Typically at LHC: $\sigma^{\text{NP}} \sim \left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^n, \quad n = 1$

Linear effects always arise from color neutralization processes.

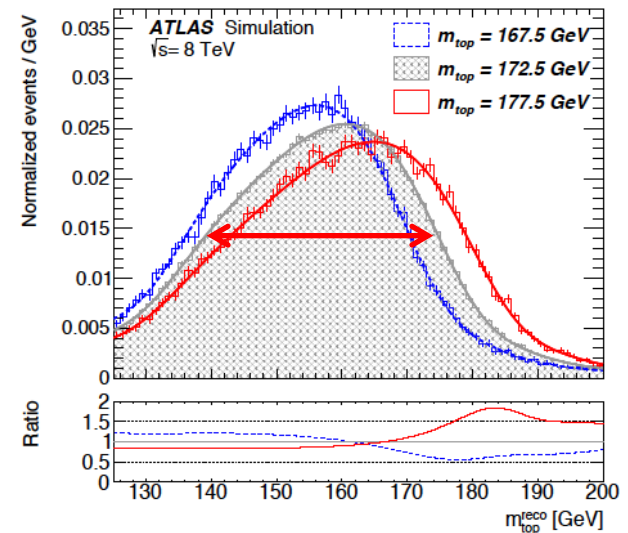
→ High precision control over soft partonic and NP effects needed when
mass sensitivity generated by small dynamical scales

Mass Extraction and Renormalization Schemes

- Parton level cross section and NP corrections MUST be separately consistent with QCD so that the top quark mass (as well as $\alpha_s(Q)$) can be determined reliably!
→ otherwise systematic bias: model instead of field theory parameters
- Which mass scheme is best? → Consider analogy to strong coupling α_s
 - Relevant dynamical scale $Q \Rightarrow \alpha_s(Q)$ frequently best choice (MSbar)
 - All quantum corrections to quark-gluon interactions from scales above Q are absorbed into $\alpha_s(Q)$ → IR-safe definition of strong coupling
 - Multiple scale problems: factorization allows to make adequate scale choices (but difficult in fixed-order p.th.)

We seek for a scale-dependent mass scheme $m_t(Q)$ with properties similar to the strong coupling $\alpha_s(Q)$.

- Multi-scale issue:
In general high mass sensitivity is associated with QCD dynamics at a low scale
→ typically: scale \sim width of distribution



Mass Extraction and Renormalization Schemes

Top Mass Renormalization Schemes

- Related to different treatments of the top self energy.

$$\overline{p} \rightarrow \overline{p} + \overline{p} \text{ (with gluon loop) } + \dots \sim \frac{i}{\not{p} - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

$$\Sigma(p, m_t^0, \mu) \sim m_t^0 \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu) \right] + \dots$$



Large linearly IR-sensitive contributions (soft gluons in top rest frame):
 $O(\Lambda_{\text{QCD}})$ renormalon behavior at higher orders

$\overline{\text{MS}}$ mass:

- Absorb $1/\epsilon$ term into the mass ($\overline{\text{MS}}$): $\overline{m}_t(\mu) = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) \right] \right\} + \dots$
- All self-energy corrections from scales $> \mu$ are absorbed into $m_t(\mu)$
 \rightarrow IR-safe short-distance mass definition (short-distance mass)
- RG-evolution similar to α_s : $\frac{d}{d \ln \mu} \overline{m}_t(\mu) = -\overline{m}_t(\mu) \left(\frac{\alpha_s(\mu)}{\pi} \right) + \dots$
- Large IR contributions in A^{fin} can cancel with other linearly sensitive corrections in the cross section coming from soft gluons in the top rest frame
- $\overline{\text{MS}}$ only well-defined for $\mu \gtrsim m_t \rightarrow$ e.g. total cross section at high energies

Mass Extraction and Renormalization Schemes

MSR mass:



- Absorb virtual top quark fluctuations into the mass as well:

Motivated by Foldy-Wouthuysen transformation:

$$m_t^{\text{MSR}}(R) = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(R)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/R) \right] \right\} - R \left(\frac{\alpha_s(R)}{\pi} \right) A^{\text{fin}}(1) + \dots$$

- All self-energy corrections from scales $> R$ are absorbed into $m_t^{\text{MSR}}(R)$

→ IR-safe short-distance mass definition

→ applies to $R \lesssim m_t$

Mateu, Lepenik, Preisser, AHH '17

Jain, Scimemi, Stewart, AHH '08

- RG-evolution is linear in R :

$$\frac{d}{d \ln R} m_t^{\text{MSR}}(R) = -\frac{4}{3} R \left(\frac{\alpha_s(R)}{\pi} \right) + \dots$$

- “Non-relativistic” mass scheme numerically close to low-scale threshold masses known from top pair threshold computations at a future lepton collider:

1S mass, PS mass, kinetic mass, etc.

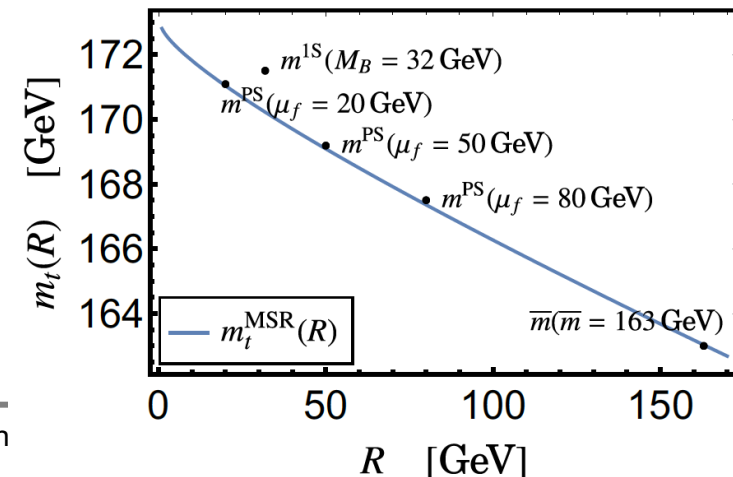
- By construction:

$$m_t^{\text{MSR}}(m_t) = \bar{m}_t(m_t) [1 + \mathcal{O}(\alpha_s^2)]$$

MSR scheme is the extension of the $\overline{\text{MS}}$ mass scheme for renormalization scales below m_t

→ kinematic mass scheme for

$R \sim (\text{dynamic scale}) \ll m_t$



Mass Extraction and Renormalization Schemes

Pole Mass: (canonical kinematic mass scheme)

- Absorb ALL self-energy corrections into the mass

$$m_t^{\text{pole}} = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu) \right] \right\} + \dots$$

- RG-invariant: $\frac{d}{d \ln \mu} m_t^{\text{pole}} = 0$

Realizes parton picture of a free top quark → radiation off top resolved at all scales

Mass of the LSZ state for on-shell top scattering amplitudes.

(Standard mass for most FO-NLO/NNLO calculations.)

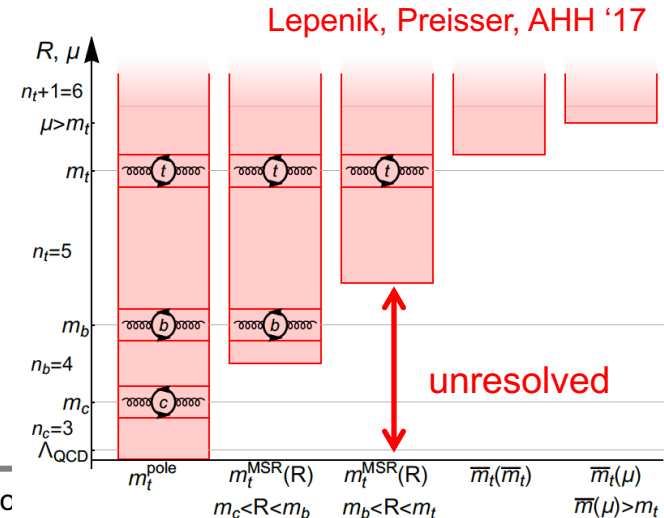
- Large contributions in A^{fin} absorbed into m_t^{pole} as well and cannot cancel with other linearly sensitive corrections in the cross section: $O(\Lambda_{\text{QCD}})$ renormalon problem !
- MSR and pole mass are numerically close for small R:

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = \frac{4}{3} \left(\frac{\alpha_s(R)}{\pi} \right) R + \dots$$

Limit $R \rightarrow 0$ impossible due to Landau pole.

MSbar and pole mass differ by ~ 10 GeV:

$$m_t^{\text{pole}} - \bar{m}_t(\mu) = \frac{4}{3} \left(\frac{\alpha_s(\mu)}{\pi} \right) \bar{m}_t(\mu) + \dots$$



Mass Extraction and Renormalization Schemes

Pole Mass Renormalon Ambiguity:

- $O(\Lambda_{\text{QCD}})$ renormalon problem related to diverging behavior of perturbative series

$$\sim \alpha_s^n n!$$

Strongest possible divergence behavior known.

Asymptotic pole mass

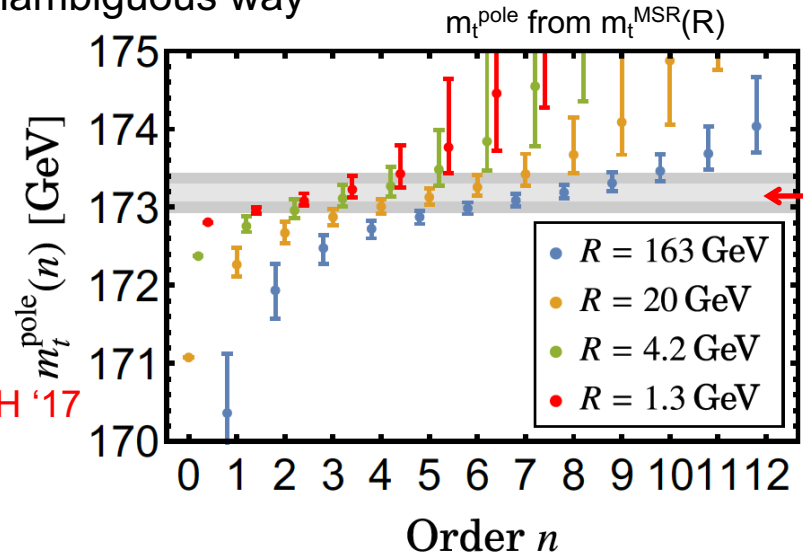
- Pole mass scheme cannot be defined in an unambiguous way

- Order-dependent value
- “Asymptotic” pole mass defined at truncation order where the corrections are minimal. Ambiguous to $O(\Lambda_{\text{QCD}})$

ambiguity: 110 MeV Beneke etal '17

250 MeV Lepenik, Preisser, AHH '17

- Truncation order depends the dynamical scale of the observable.



- Pert. series in QCD are always asymptotic.

$O(\Lambda_{\text{QCD}})$ renormalons arise also from physical NP corrections

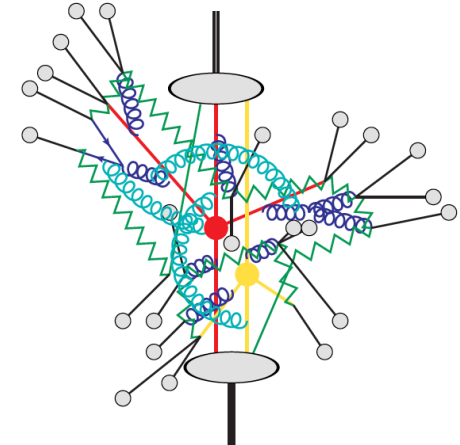
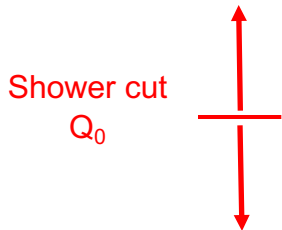
Correct treatment of linear NP corrections essential.

$$\sigma^{\text{NP}} \sim \left(\frac{\Lambda_{\text{QCD}}}{Q} \right)^n, \quad n = 1$$

Status of Top Mass Determinations at the LHC

Multi-Purpose Event Generators

- Backbone of all experimental analyses:
 - Pythia Sjöstrand, etal. '15
 - Herwig Bellm, etal. '16
 - Sherpa Gleisberg, etal. '09
 - Combines:
 - LO matrix elements
 - Parton shower (Markov chain):
 - p_T -ordered dipole (mom. recoil local)
 - coherent branching (non-global restricted)
 - Hadronization model: string, cluster
 - Employed for theoretical predictions for all other top mass sensitive cross sections
 - **Direct measurements:** templates (m_t^{reco} , $M_{b\text{-jet,lepton}}$), matrix-element/idiogram
 - b-jet and B-meson energy distribution, secondary vertices in B production
 - J/ ψ method, M_{T2} ,
- Measurements of MC top mass parameter m_t^{MC}

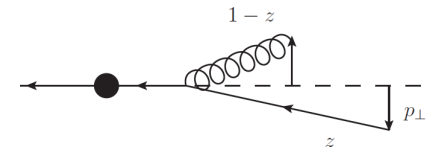


Status of Top Mass Determinations at the LHC

- Parton shower provides approximation to collinear and soft radiation
 - NLL precise for a few simple observables, but **LL or less in general**
 - Top quarks: theory input applies to **quasi-collinear** tops only!
 - No systematic treatment of **finite lifetime** effects
 - No systematic treatment of MPIs, **color-reconnection**

} Needs to be improved!

→ Self-energy corrections are not simulated: absorbed in m_t^{MC} !
⇒ m_t^{MC} is close to the pole mass



- Hadronization model provides description of σ^{NP} (tuning)
 - can compensate for deficiencies of the parton shower
- In such a case: σ^{parton} and σ^{NP} potentially separately incompatible with QCD
 - extraction of QCD parameters (top mass, α_s) affected by systematic errors that may not at all be captured by common MC uncertainty variations

THIS IS THE CORE OF THE INTERPRETATION PROBLEM OF m_t^{MC}
OBTAINED FROM THE DIRECT MEASUREMENTS

Status of Top Mass Determinations at the LHC

Experimental Analyses

Direct Measurements:

- Template method (ATLAS), matrix element/ideogram method (CMS)
- Based on highly top mass sensitive distributions ($M_{lb\text{-jet}}$, m_t^{reco} , etc) that are dominated by parton shower and hadronization model and cannot be improved by NLO matching.

$$m_t^{\text{MC}} = 172.9 \pm 0.4 \text{ GeV} \quad (\text{world average})$$

$$m_t^{\text{MC}} = 172.26 \pm 0.61 \text{ GeV} \quad (\text{CMS combined})$$

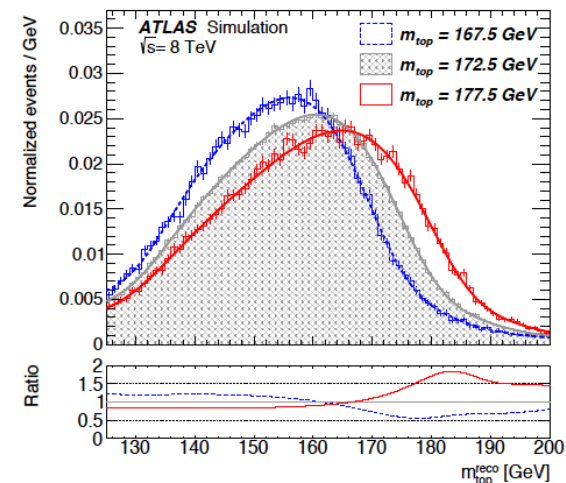
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- Mostly discussed in the context of the m_t^{MC} interpretation problem.

Nothing to improve in the experimental analysis.

Purely theoretical (Monte-Carlo-event generator) problem.



Status of Top Mass Determinations at the LHC

Pole Mass Measurements:

- Based on total and differential cross section for which the parton level calculation can be done reliably at NLO or NNLO/NNLL
- Called “pole mass measurements” only because theorists used pole mass scheme for their calculations. → misleading!

Better: Measurements of m_t in well-defined scheme

- Total inclusive cross section:

$$m_t^{\text{pole}} = 172.9^{+2.5}_{-2.6} \text{ GeV (ATLAS, 7 and 8 TeV data)}$$

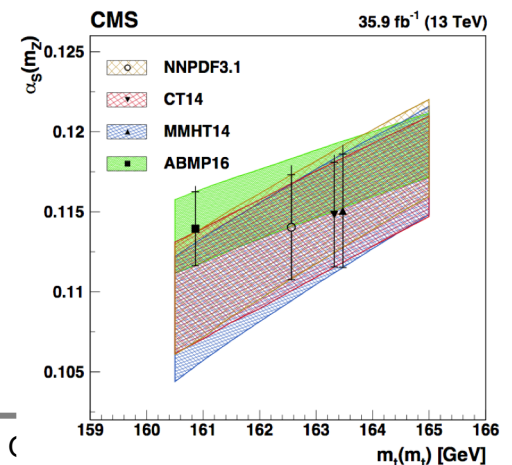
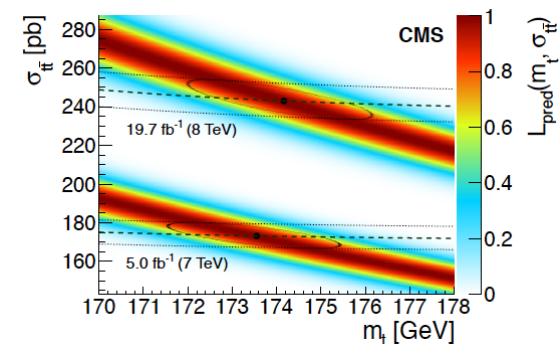
$$m_t^{\text{pole}} = 173.8^{+1.7}_{-1.8} \text{ GeV (CMS, 7 and 8 TeV data)}$$

$$m_t^{\text{pole}} = 169.9^{+2.0}_{-2.2} \text{ GeV (CMS, 13 TeV data)}$$

lower precision due to impact of norm uncertainties

(strong additional correlation to pdfs, α_S)

→ reliable mass interpretation, but imprecise



CMS arXiv:1812.10505

Status of Top Mass Determinations at the LHC

- Recently also differential cross sections: $M_{t\bar{t}+jet}$, $M_{t\bar{t}} + y(tt)$, lepton energies
→ distributions elevate top mass sensitivity due to structures

$$M_{t\bar{t}} + y(t\bar{t}) : m_t^{\text{pole}} = 170.5 \pm 0.8 \text{ GeV} \quad (\text{CMS})$$

$$M_{t\bar{t}+jet} : m_t^{\text{pole}} = 171.1_{-1.1}^{+1.2} \text{ GeV} \quad (\text{ATLAS})$$

$$\text{leptons} : m_t^{\text{pole}} = 173.2 \pm 1.6 \text{ GeV} \quad (\text{ATLAS})$$

Important questions to address:

- Reliability of FO parton level cross sections used for m_t determination
 - $M_{t\bar{t}} \sim 2m_{\text{top}}$: $t\bar{t}$ bar in color-octet configurations, Coulomb effects
- Recent suggestion: soft-dropped boosted top quark mass masses
AHH, Mantry, Pathak, Stewart '17
 - Reminder: The gold-plated platinum observable is the top threshold cross section at a future e^+e^- collider. An analogous observable does not exist for hadron colliders.
[$t\bar{t}$ bar always a color singlet at a lepton collider !]

The Controversy

- No general consensus on
 - how to formulate the m_t^{MC} interpretation problem
 - how to quantify the associated uncertainty
 - relevance of the problem

View 1:

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_{m_t}^{\text{MC}} \quad \text{Nason}$$

- Δ_m^{MC} an uncertainty in addition to uncertainties quoted in direct measurements but much smaller than these and negligible
- Pole mass renormalon ambiguity is 110 MeV \ll experimental uncertainty (HL-LHC)

View 2:

$$m_t^{\text{MC}, Q_0} = m_t^{\text{MSR}}(R_0) + \Delta_{m_t}^{\text{MC}}(R_0, Q_0) \quad \text{Hoang, Stewart}$$

- Shower cut Q_0 of the parton showers plays an essential role because radiation with scales $< Q_0$ is treated unresolved
 - m_t^{MC} depends on Q_0 and the parton shower type
 - m_t^{MC} close to $m_t^{\text{MSR}}(R_0 \approx Q_0)$
- Shower cut Q_0 acts like a IR factorization scale and should be chosen > 1 GeV
- Pole mass renormalon ambiguity is 250 MeV \sim experimental uncertainty (HL-LHC)

Recent Quantitative Results

- Combined analysis of direct and total cross section measurement

$$\rightarrow m_t^{\text{pole}} - m_t^{\text{MC}} < 2 \text{ GeV}$$

Kieseler, Lipka, Moch '16

- Calibration of m_t^{MC} using 2-jettiness for boosted tops in e^+e^- collisions

Butenschoen, Dehnadi, AHH, Preisser, Mateu, Stewart '16

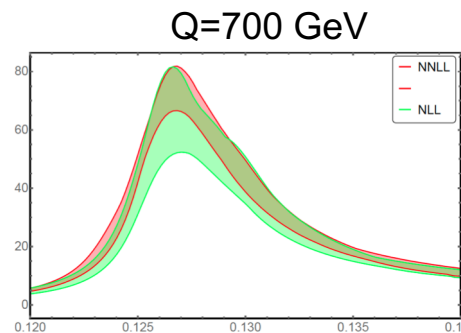
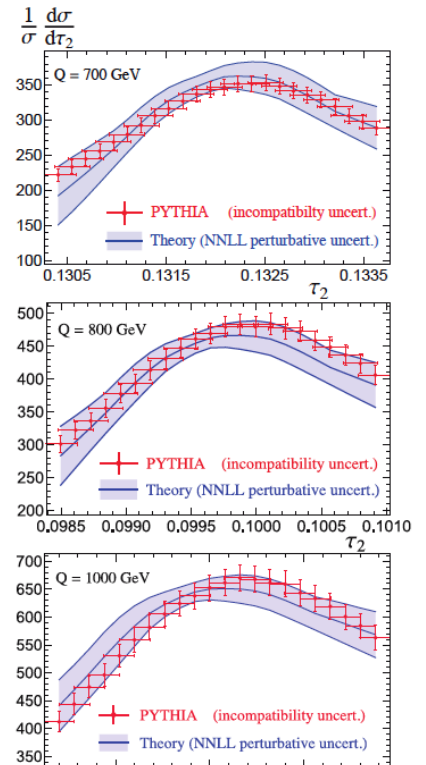
- Numerical relation between Pythia m_t^{MC} and MSR/pole mass using 2-jettiness in e^+e^- collisions
- Fits of NNLL+NLO+had.corr. theory predictions with Pythia 8.205 templates
- Good agreement between Pythia and analytic calculations

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(1 \text{ GeV}) + (0.18 \pm 0.23) \text{ GeV}$$

$$m_t^{\text{MC}} = m_t^{\text{pole}} + (0.57 \pm 0.29) \text{ GeV}$$

- Extended to soft-dropped groomed top jet mass distribution at LHC

Mantry, Pathak, Stewart, AHH '17



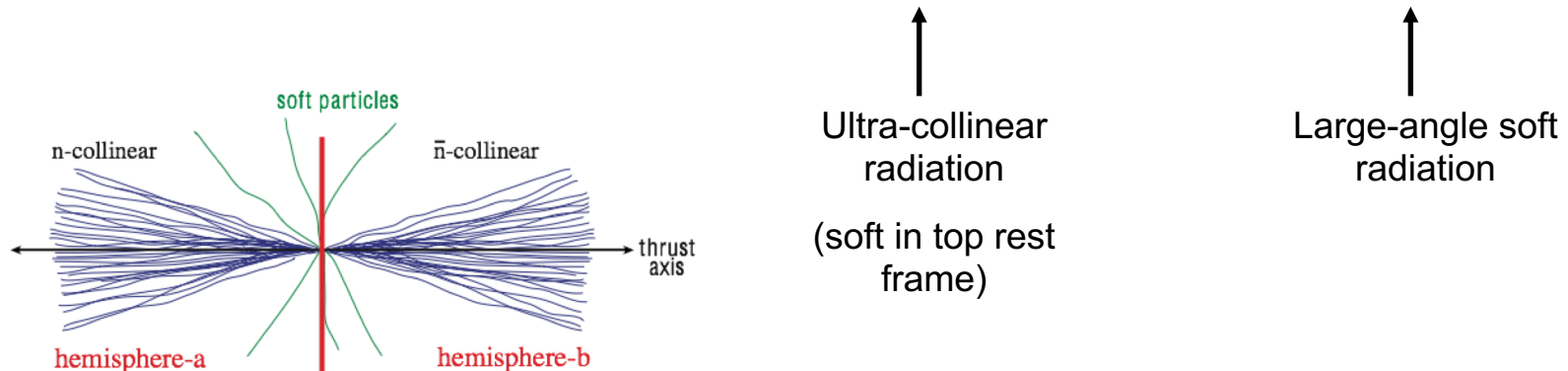
Recent Quantitative Results

Why boosted top quarks as theoretically clean :

- Fat top jet mass distribution can be computed in QCD factorization (SCET) because the top decays are well separated in phase space.
- Non-perturbative corrections enter through convolutions with a soft function.
- Top decay factorizes as well and can be added through methods known from B physics

$$\left(\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(l^+, l^-, \mu)$$

Fleming, Mantri, Stewart, AHH, 2007



Recent Quantitative Results

- Calibration results are indirect and do not allow to scrutinize the parton shower and hadronization models individually to test whether m_t^{MC} is a perturbative quark mass definition of a model parameter.

$$m_t^{\text{MC}, Q_0} = m_t^{\text{pole}} + \Delta^{\text{pert}}(Q_0) + \Delta^{\text{non-pert}}(Q_0) + \Delta^{\text{MC}}$$

pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup
- (Affected by finite width effects?)

Non-perturbative contribution:

- Effects of hadronization model
- May depend on parton shower setup

Monte Carlo shift:

- Contribution arising from systematic MC uncertainties
- E.g. color reconnection, b-jet modeling, ...
- Should be covered by 'MC uncertainty' or better negligible

Analysed for Herwig angular-ordered parton shower

- Boosted (quasi-collinear) top quarks
- Stable top quarks
- 2-jettiness (production stage QCD dynamics only)

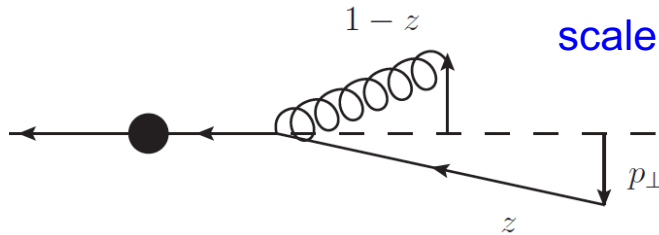
} Parton showers likely to work reliably in these limits

AHH, Plätzer, Samitz 1807.06617

Recent Quantitative Results

Catani, Marchesini, Webber 1991
Gieseke, Stephens, Webber, 2003

→ Coherent branching: (basis of the Herwig parton shower)



scale in α_s : $\mu^2 = p_{\perp}^2 + (1 - z)^2 m^2$ cutoff: $p_{\perp}^2 > Q_0^2$

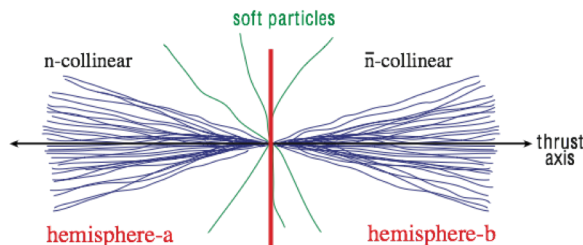
Usually not present in analytic QCD !

2-Jettiness τ_2 distribution In the peak region (for e^+e^- and boosted tops) can be analytically computed in QCD factorization (SCET) at NLL+NLO and coherent branching (CB) at NLL.

$$\left(\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$

Fleming, Mantri, Stewart, AHH, 2007

$$\times \int_{-\infty}^{\infty} dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(l^+, l^-, \mu)$$



↑
Ultra-collinear radiation

↑
Large-angle soft radiation

Plätzer, Samitz, AHH; arXiv:1807.06617

Recent Quantitative Results

- The following statements were strictly proven at parton level:
 - NLL precise parton shower mandatory and sufficient to control the scheme of m_t^{MC} with NLO (i.e. $O(\alpha_s)$) precision
 - Herwig 7 parton shower (coherent branching) is NLL precise for e^+e^- 2-jettiness in the resonance region (i.e. jet masses in the peak region)
 - For shower cut $Q_0=0$: $m_t^{\text{MC}} = m_t^{\text{pole}}$ (at NLO)
 - In realistic parton showers $Q_0 > 0$
 \Rightarrow the generator mass is $m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} \alpha_s(Q_0) Q_0 + \dots$
„coherent branching mass“
 - $m_t^{\text{CB}}(Q_0)$ is a short-distance mass and does not have the pole mass renormalon ambiguity
 - Numerical relations:
$$m_t^{\text{MSR}}(Q_0) - m_t^{\text{CB}}(Q_0) = 120 \pm 70 \text{ MeV}$$
$$m_t^{\text{pole}} - m_t^{\text{CB}}(Q_0) = 480 \pm 260 \text{ MeV}$$

Where do we stand?

- The **result is a first step** in a more general endeavor to understand the MC top mass m_t^{MC} and cannot yet be directly applied to the direct measurements due to the restrictions to boosted, stable tops and e^+e^- 2-jettiness.
- More work needed to generalize the analysis and to also get insights into the impact of the hadronization model $\rightarrow \Delta_m^{\text{non-pert}}, \Delta_m^{\text{MC}}$

What have we learned already?

- Only for a **NLL precise MC** we can calculate the parton level relation of m_t^{MC} to any renormalization scheme
- Certainly **m_t^{MC} is not the pole mass** due to $Q_0 \neq 0$, but more closely related to the MSR mass $m_t^{\text{MC}}(R \approx Q_0)$
- The **MC top mass interpretation problem is an essential issue** to be worked on when considering uncertainties at the level of 0.5 GeV (i.e relevant today!)

HL-LHC top mass measurements with 200 MeV precision possible,

but much work still needs to be done to get to the same level of theory precision.