The top mass at hadron colliders

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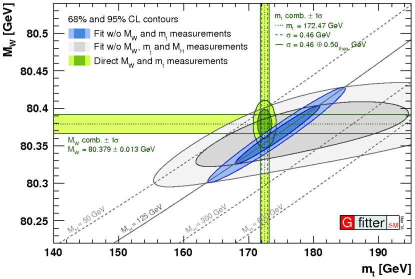
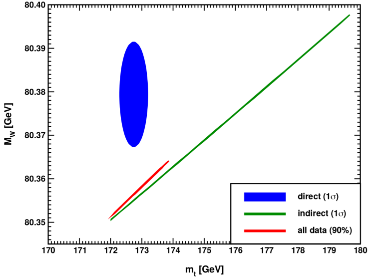
Top and precision physics



From PDG:



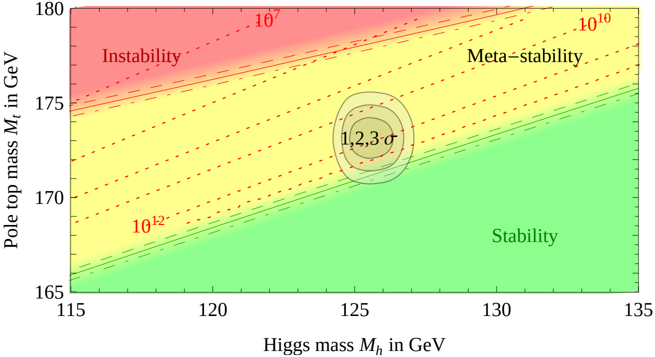
$$\Delta G_\mu / G_\mu = 5 \cdot 10^{-7}; \quad \Delta M_Z / M_Z = 2 \cdot 10^{-5};$$

$$\Delta\alpha(M_Z)/\alpha(M_Z) = \begin{cases} 1 \cdot 10^{-4} (\text{Davier et al.; PDG}) \\ 3.3 \cdot 10^{-4} (\text{Burkhardt, Pietrzyk}) \end{cases}$$

M_W can be predicted from the above with high precision, provided M_H and M_T (entering radiative corrections) are also known (and depending on how aggressive is the error on $\alpha(M_Z)$).

Top and vacuum stability

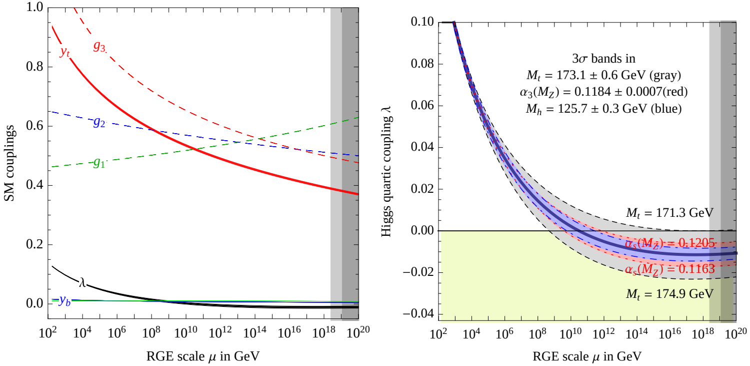
Degrassi et al. 2012



With current value of M_t and M_H the vacuum is metastable.
No indication of new physics up to the Plank scale from this.

Top and vacuum stability

Degrassi et al. 2012



The quartic coupling λ_H becomes tiny at very high field values, and may turn negative, leading to vacuum instability.
M_t as low as 171 GeV leads to $\lambda_H \to 0$ at the Plank scale.

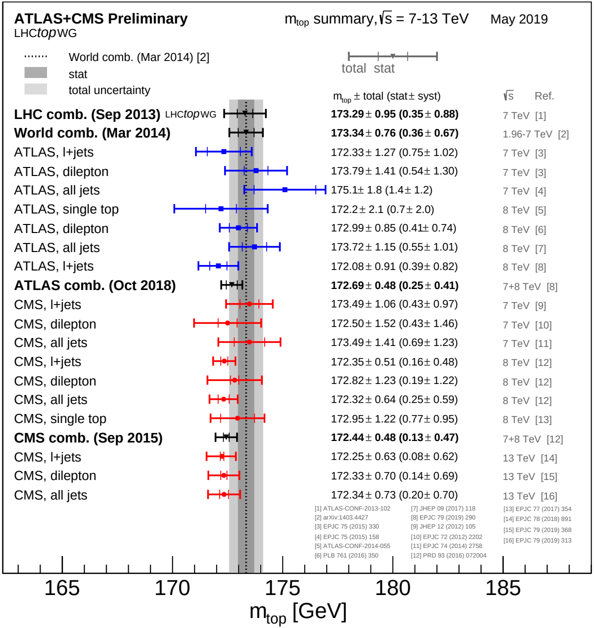
How to measure the top mass

- At the moment the top mass is measured at Hadron Colliders.
- An e^+e^- collider spanning the $t\bar{t}$ threshold can do much better, but we are far from it.

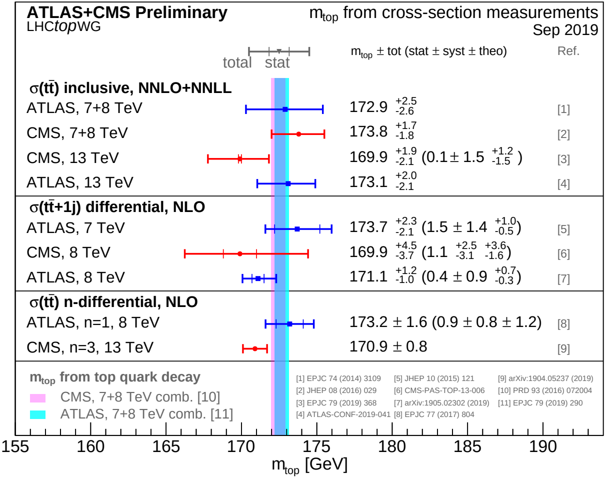
Measurements are classified as

- Direct measurements: measurements based upon the reconstruction of the full production event, that use as mass observable the reconstruction of its decay products.
- Measurements based upon cross section or differential distributions, decay product distributions insensitive to production dynamics, mass of boosted top jets, etc.

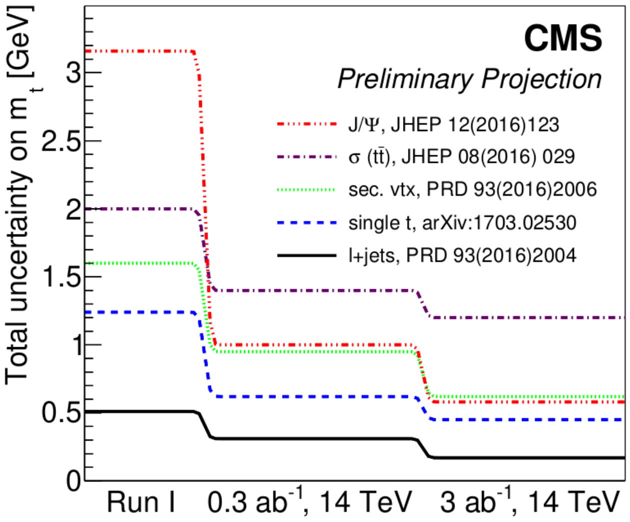
Measurements



Measurements



High Luminosity Projections



General problem

- In all measurements, the pole mass is extracted comparing measured top-mass sensitive observables to computed ones.
- As the required precision increases, one becomes sensitive to effects that are not computed with sufficient accuracy

Among the most sticky points:

- Transition from soft to hadronization stage in MC (only from models!).
- Soft radiation in MC (only OK in the collinear limit ...)

All these points easily lead to ambiguities that affect the extracted mass by an amount at least of order of a hadronic scale. When measurements approach the 500 MeV accuracy, they become worrysome.

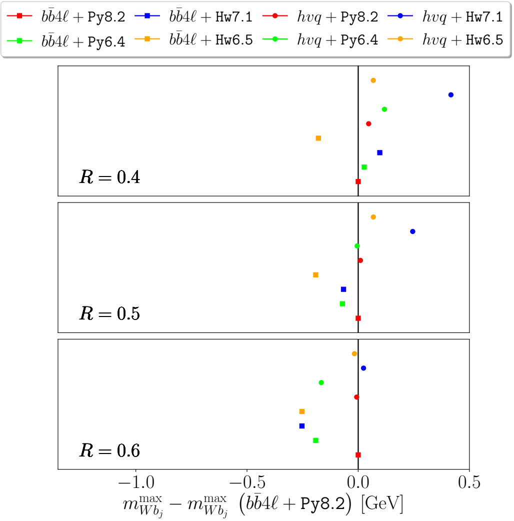
Theoretical proposals

- Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart,2016 Use boosted top jet mass + SCET.
- Agashe,Franceschini,Kim,Schulze,2016: peak of b-jet energy insensitive to production dynamics.
- Kawabata,Shimizu,Sumino,Yokoya,2014: shape of lepton spectrum. Insensitive to production dynamics and claimed to have reduced sensitivity to strong interaction effects.
- Frixione, Mitov: Selected lepton observables.
- Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ,2013; Bayu etal: M_t from $t\bar{t}j$ kinematics.
- $t\bar{t}$ threshold in $\gamma\gamma$ spectrum (needs very high luminosity), Kawabata,Yokoya,2015

A traditional way to assess uncertainties is to change parameters or/and implementations of the calculation.

As an example: Ferrario Ravasio,Jezo,Oleari,P.N,2017-2019 performed a theoretical study of top mass determinations using direct measurements:

- Focussed upon the invariant mass of the reconstructed $W-B-$jet system.
- Found large uncertainties when experimental uncertainties are applied to the invariant mass peak
- Found small uncertainties if the position of the peak was measured with a perfect detector.



Consider the groups of squares (our best generator) at fixed R. They span a range not larger than 250 MeV. This means that if you had a perfect detector, the intrinsic theoretical error estimate would be $\pm$125 MeV.

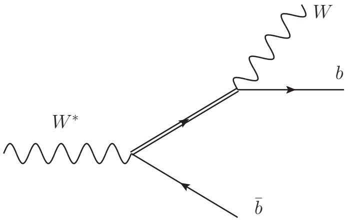
However, the doubt remains: what if our models are all biased, in a way that we fail to see?

In order to gain more insight into the form of the power suppressed corrections to top-mass sensitive observables, we have study the top mass measurement in a framework where power suppressed effects can be computed without ambiguity, i.e. the so called large-b_0 approximation.

Linear Power Corrections from Renormalons

Ferrario Ravasio, Oleari, P.N.2019

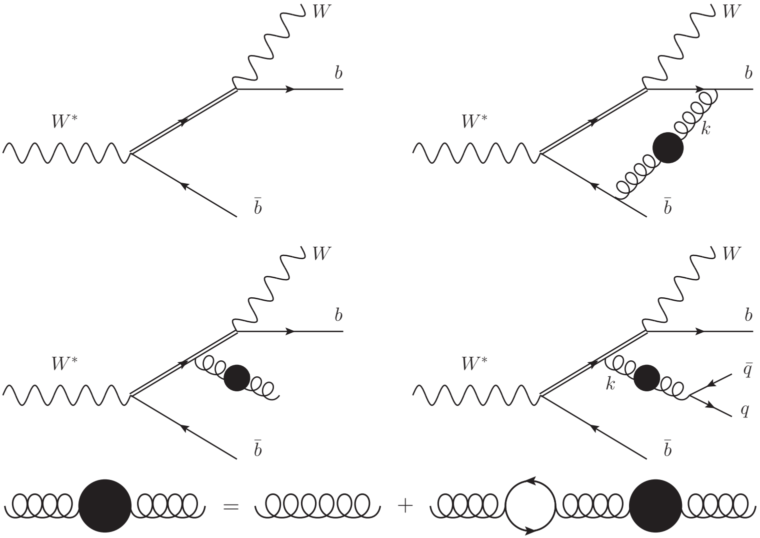
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.
- We can examine any infrared safe observable, no matter how complex.

Diagrams up to leading N_f one gluon correction:



All-order result

Introducing the notation

- Φ_b, phase space for $Wb\bar{b}$;
- Φ_{g^*}, phase space for $Wb\bar{b}g^*$, where g^* is a massive gluon with mass k^2,
- $\Phi_{q\bar{q}}$, phase space for $Wb\bar{b}q\bar{q}$, with $\mathrm{d}\Phi_{q\bar{q}} = \frac{\mathrm{d}k^2}{2\pi}\mathrm{d}\Phi_{g^*}\mathrm{d}\Phi_{\mathrm{dec}}$

the all-order result can be expressed in terms of

- $\sigma_b(\Phi_b)$, the differential cross section for the Born process;
- $\sigma_v(k^2, \Phi_b)$, the virtual correction to the Born process due to the exchange of a gluon of mass k;
- The real cross section $\sigma_{g^*}(k^2, \Phi_{g^*})$, obtained by adding one massive gluon to the Born final state;
- The real cross section $\sigma_{q\bar{q}}(\Phi_{q\bar{q}})$, obtained by adding a $q\bar{q}$ pair, produced by a massless gluon, to the Born final state;

All-order result

Consider a (IR safe) final state observable O. Define:

$$
\begin{aligned}
N^{(0)} &= \left[\int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{b}}\right]^{-1}, \quad \langle O\rangle_{\mathrm{b}} = N^{(0)} \int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{b}}(\Phi_{\mathrm{b}})\, O(\Phi_{\mathrm{b}}), \\
\widetilde{V}(k^2) &= N^{(0)} \int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{v}}^{(1)}(k^2, \Phi_{\mathrm{b}}) \left[O(\Phi_{\mathrm{b}}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{R}(k^2) &= N^{(0)} \int \mathrm{d}\Phi_{g^*}\, \sigma_{g^*}^{(1)}(k^2, \Phi_{g^*}) \left[O(\Phi_{g^*}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{\Delta}(k^2) &= \frac{1}{2}\frac{3}{\alpha_s T_{\mathrm{F}}}\, k^2\, N^{(0)} \int \mathrm{d}\Phi_{q\bar{q}})\, \sigma_{q\bar{q}}^{(2)}(\Phi_{q\bar{q}}) \times [O(\Phi_{q\bar{q}}) - O(\Phi_{g^*})]
\end{aligned}
$$

$\langle O\rangle_{\mathrm{b}} + \widetilde{V}(k^2) + \widetilde{R}(k^2)$ is the average value of O in a theory with a massive gluon with mass k^2, accurate to order α_s.

Notice: $\widetilde{V}(k^2) + \widetilde{R}(k^2)$ has a finite limit for $k^2 \to 0$, while each contribution is log divergent.

defining $\widetilde{T}(k^2) = \widetilde{V}(k^2) + \widetilde{R}(k^2) + \widetilde{\Delta}\,(k^2)$ our final result is

$$\langle O\rangle = \langle O\rangle_{\rm b} - \frac{3\pi}{\alpha_s T_{\rm F}} \int_0^\infty \frac{{\rm d}k}{\pi} \frac{\rm d}{{\rm d}k}\left[\widetilde{T}(k^2)\right] {\rm Im}\left\{\log\left[1+\Pi\left(k^2,\mu^2\right) - \Pi_{\rm ct}\right]\right\},$$

where

$$\Pi\left(k^2,\mu^2\right) - \Pi_{\rm ct} = \alpha_s b_0 \left(\log\frac{k^2}{\mu^2} = \frac{5}{3} - i\pi\right), \qquad b_0 = -\frac{4N_{\rm F}T_{\rm F}}{12\pi}$$

So

$${\rm Im}\left\{\log\left[1+\Pi\left(k^2,\tilde{\mu}^2\right) - \Pi_{\rm ct}\right]\right\} = -{\rm atan}\left(\frac{\alpha_s \pi b_0}{1+\alpha_s b_0 log\frac{k^2}{\tilde{\mu}^2}}\right)$$

that essentially exhibits the same Landau pole discussed earlier.
If we thus have:

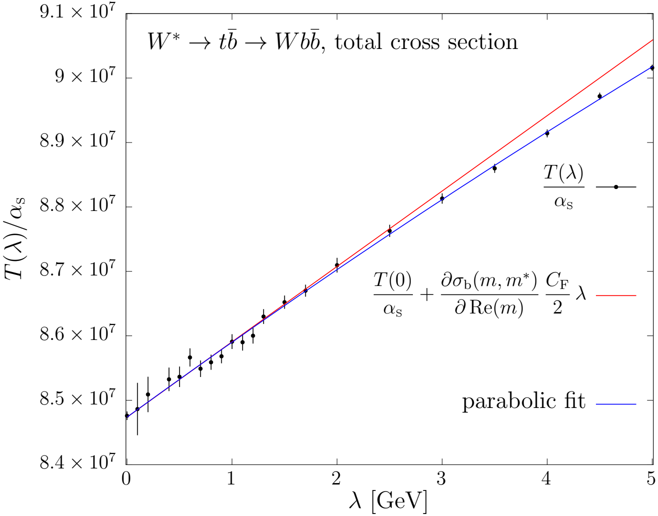
$$\widetilde{T}(k^2) = a + b\,k + \mathcal{O}(k^2) \tag{1}$$

we have a linear renormalon in our result.

Comments

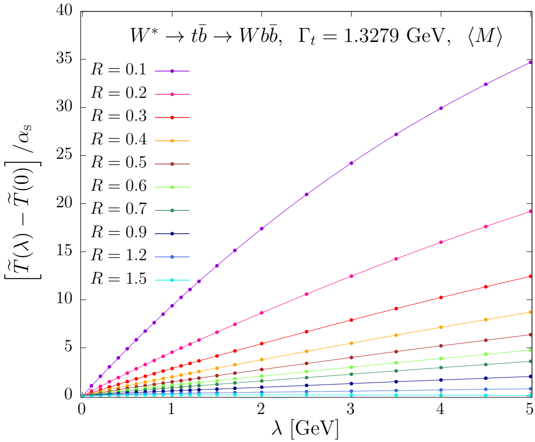
- In order to get our results, we need $\lim_{k^2 \to \infty} \widetilde{T}(k^2) = 0\,.$
  This happens if we use the Pole Mass Scheme for m_t.
- The need to include the Δ term has a long story:
  - Seymour,P.N. 1995, I.R. renormalons in e^+e^- event shapes.
  - Dokshitzer,Lucenti,Marchesini,Salam, 1997-1998 Milan factor
- We compute $T(k^2)$ numerically. The $k^2 \to 0$ limit implies the cancellation of two large logs in V and R. However, the precise value at $k^2 = 0$ can also be computed directly by standard means (which we do).

Total cross section

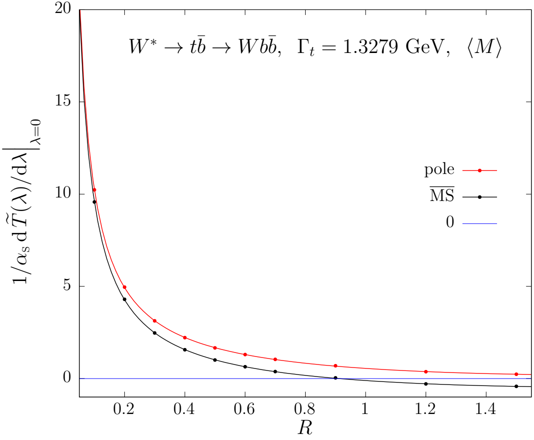


No linear renormalon in $\overline{\mathrm{MS}}$ scheme!

Reconstructed top mass



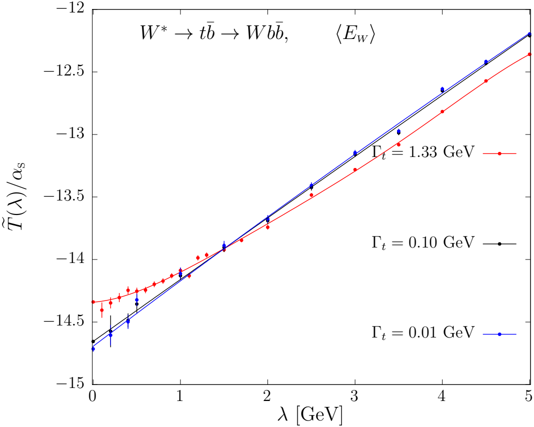
Reconstructed top mass



For large radii, m_{pole} is better!

Leptonic Observables

Choose as mass sensitive observable the average E_W.



For $k \gg \Gamma$, the slope is roughly 0.45. The $\overline{\mathrm{MS}}$ conversion would add -0.067.

Leptonic Observables

It seems that physical linear renormalons are present also in leptonic observables.

But, for $k \ll \Gamma$, the slope of $T(k)$ decreases, approaching 0.067!

So, the top finite width screens the linear renormalons!

This an exact statement!
It can be proven analytically.

Linear Power Corrections from Renormalons

We find:

- The total cross section (in the simplified model!) does not have them. However, if cuts are present (even if only on the lepton!) they are there.
- Jets have large linear power corrections with coefficients of order $1/R$. These have some sort of universality, and may be controlled by calibration. However, power corrections with no $1/R$ enhancement are also there, and are not universal.

Linear Power Corrections from Renormalons

- Leptonic observables have linear power corrections in the narrow width limit. These are seen to be absent for distributions defined in the top rest frame, consistently with the B decay example.
- In general, the top finite width screens the linear power corrections due to top emissions. Thus, for observables not involving jets, like the leptonic observables, we see that the linear corrections disappear for finite with.

In theory, one may exploited this fact to perform top mass measurements free of linear power corrections...

Putting things in perspective

- We have some certainty about the presence/absence of power corrections when we have an OPE:
  - $e^+e^- \to$ hadrons: power corrections like $1/Q^4$, from the dimensionality of G^2.
  - DIS: twist expansion, twist 4 versus twist 2, $1/Q^2$ corrections
  - B physics: absence of linear power corrections to semileptonic decays

In hadronic collisions: we have no OPE! This is a general fact, that affects all hadron collider physics (and thus also the top mass measurement).

Beneke and Braun, arXiv:hep-ph/9506452

Abstract:

The resummed Drell-Yan cross section in the double-logarithmic approximation suffers from infrared renormalons. Their presence was interpreted as an indication for non-perturbative corrections of order $\Lambda_{\text{QCD}}/(Q(1-z))$. We find that, once soft gluon emission is accurately taken into account, the leading renormalon divergence is cancelled by higher-order perturbative contributions in the exponent of the resummed cross section.

Their calculation: leading N_f one gluon correction:

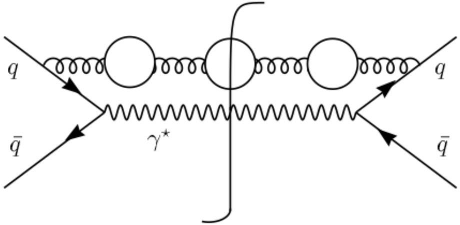


Figure 1: α_s^4-contribution to the partonic Drell-Yan cross section. $\gamma^\star$ represents a photon with invariant mass Q^2 that splits into a lepton pair.

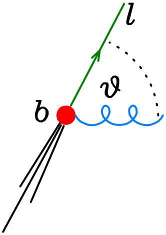
Putting things in perspective

Large n_f calculations provided some insight for non-perturbative corrections:

- Back in 1995 it was shown by Beneke and Braun that previous claims on the presence of linear power corrections in Drell-Yan production were not justified.
- We have found it useful to clarify some aspect of top mass measurement in hadronic collisions.
- But we need more. We need some insight to be applied also to MC models.
- Do we have intuitive arguments to support or dismiss the presence of linear renormalons, that we can verify using the large n_F calculations?

As an example, look at an intuitive argument to justify the absence of linear power corrections to the leptonic distribution in the decay of a heavy quark:

- A b quark in a B meson undergoes Fermi motion, i.e. it has momentum of order Λ. But its kinetic energy is of order Λ^2/m_b, because it is non-relativistic. So, no linear power corrections there.
- The decay can take place in a time fraction when the b is in a virtual state associated with the emission of a soft gluon.



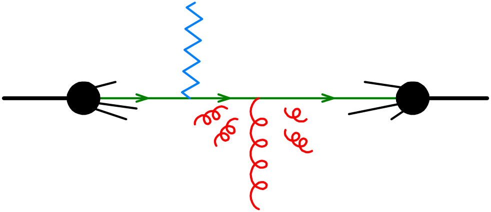
The decay product are boosted with velocity $v = k/m_b$, where k is the soft gluon momentum. The corresponding change in the lepton momentum is $\delta p_l \approx vp\cos\theta$. But this effect linear in v vanish under azimuthal average.

As a result, the semileptonic spectrum has no linear power corrections *if expressed in terms of a short distance mass*

Putting things in perspective

Another such argument, not related to quark mass measurement goes as follows:

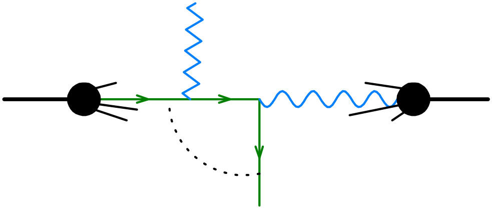
In Z production at large trasverse momentum the soft radiation pattern is not azimuthally symmetric. If a soft gluon effects can be mimiked by giving the gluon a small mass, we expect that the Z pt spectrum should be affected by linear power corrections.



Recently, together with Ferrario-Ravasio,Limatola, we have examined another such argument.

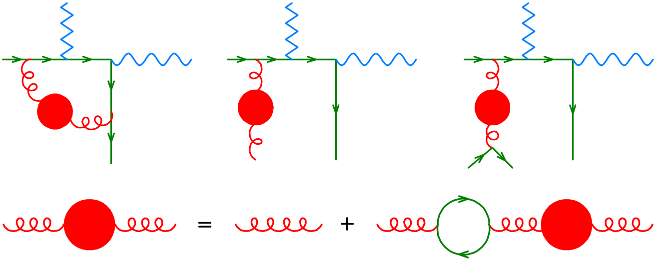
Power corrections to the Z p_t spectrum

We have considered the process $q\gamma \to Z + j$:



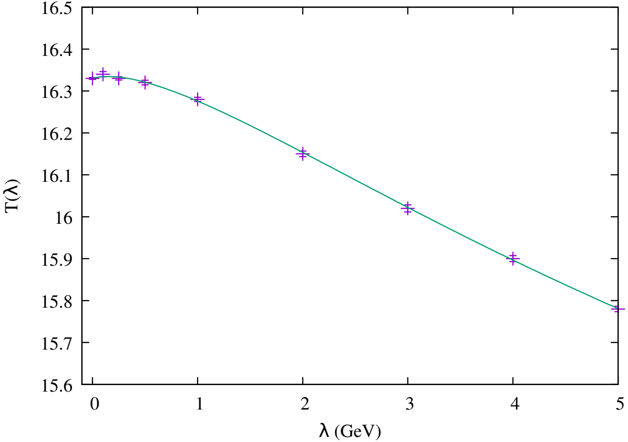
This process has also a soft radiation pattern that may hint to the presence of linear renormalons.

We compute all its large-n_f corrections:



Now the are complications with the handling of collinear factorization and the like, but it can be carried out to the end.

The resulting plot for the Z cross section with a transverse momentum cut of 20 GeV:



showing no clear evidence of linear effects.

Perspectives

- Mass measurements at hadron collider pose some challenging problems when one wants a precision comparable to a hadronic scale.
- These problems are related to a lack of understanding of the linear power corrections in hadronic collisions.
- There is one technique that can provide some hints, that is the large b_0 approximation.
- It seems that substantial theoretical progress is needed in this direction in order to meet the precision that can be achieved at the high-luminosity LHC.