The role of $m_{b,c}$ in semileptonic B decays

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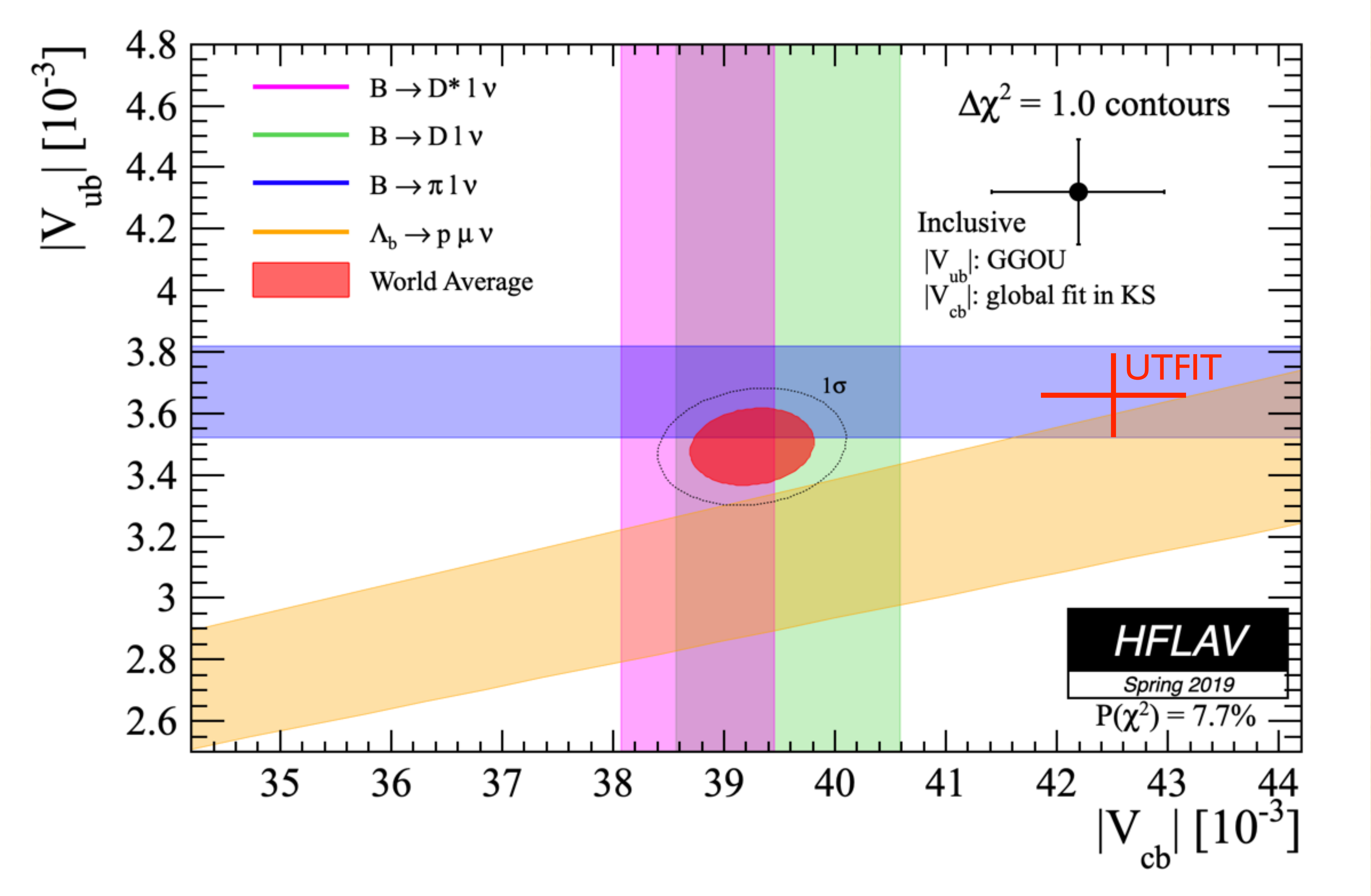
Mini Workshop on Quark Masses, KIT, 26 October 2020



Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement. This plot does **not** reflect all developments.

The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT)

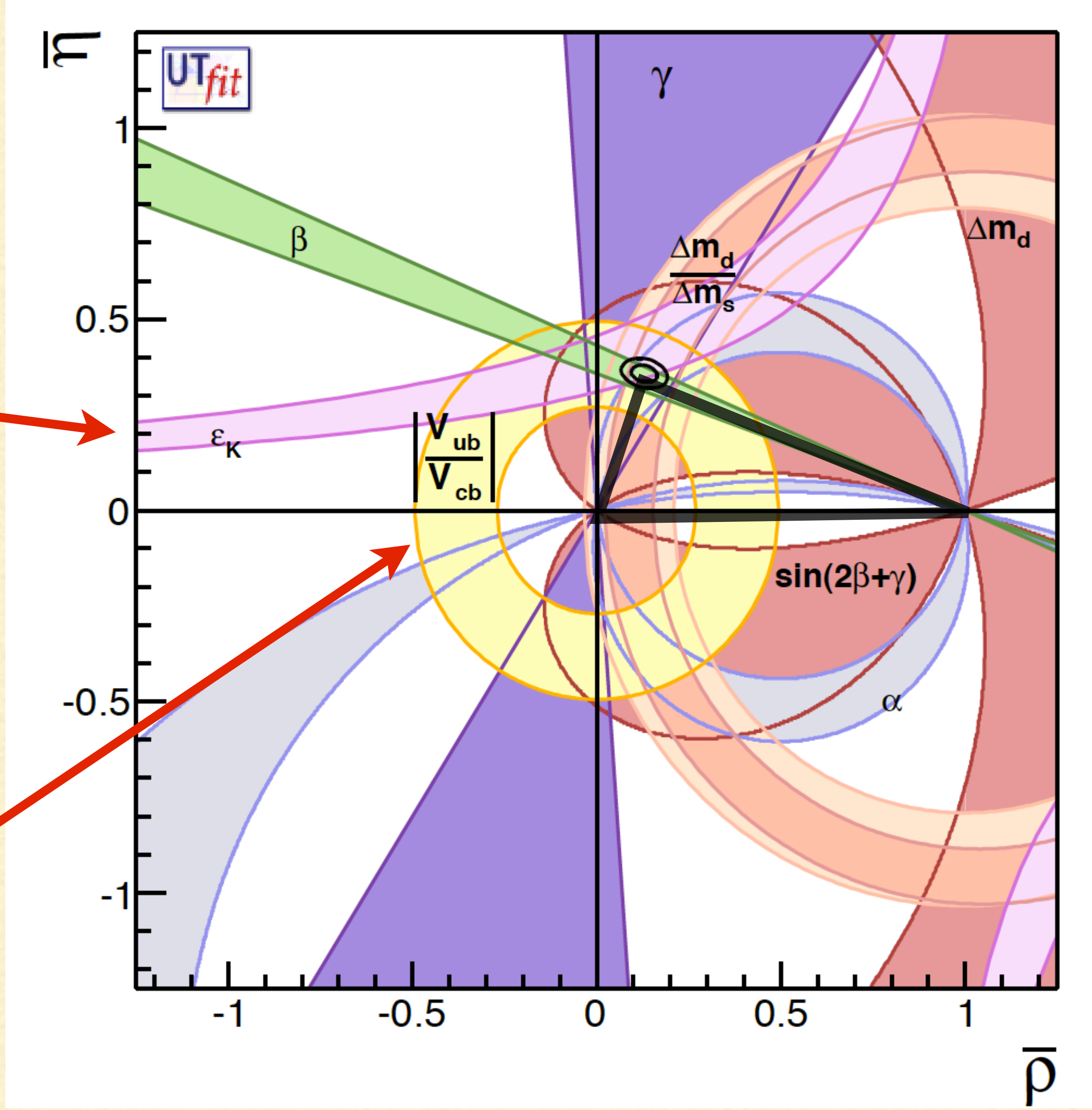
V_{cb} plays an important role in UT

$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

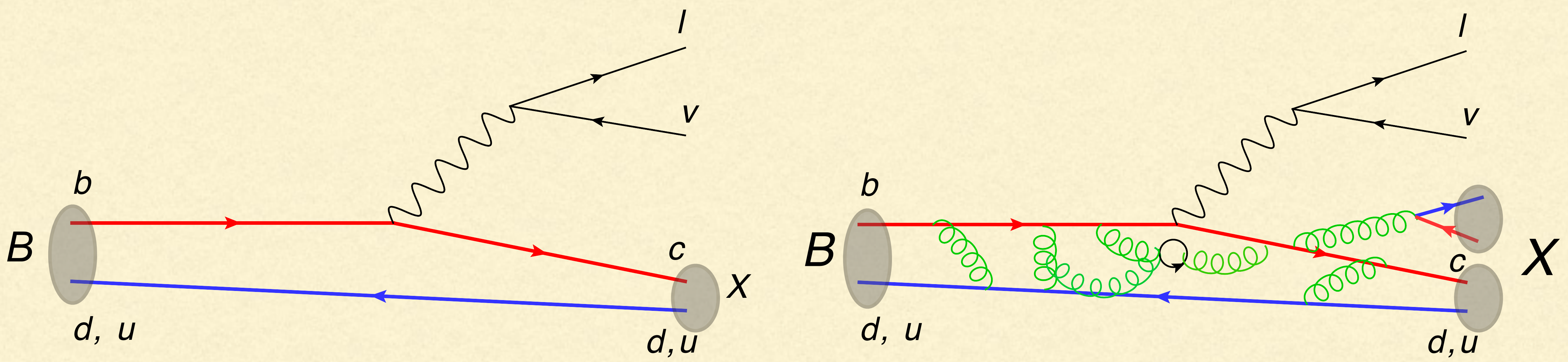
$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where it often dominates the theoretical uncertainty.
V_{ub}/V_{cb} constrains directly the UT



Our inability to determine precisely V_{cb} hampers significantly NP searches

INCLUSIVE DECAYS: BASICS



- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

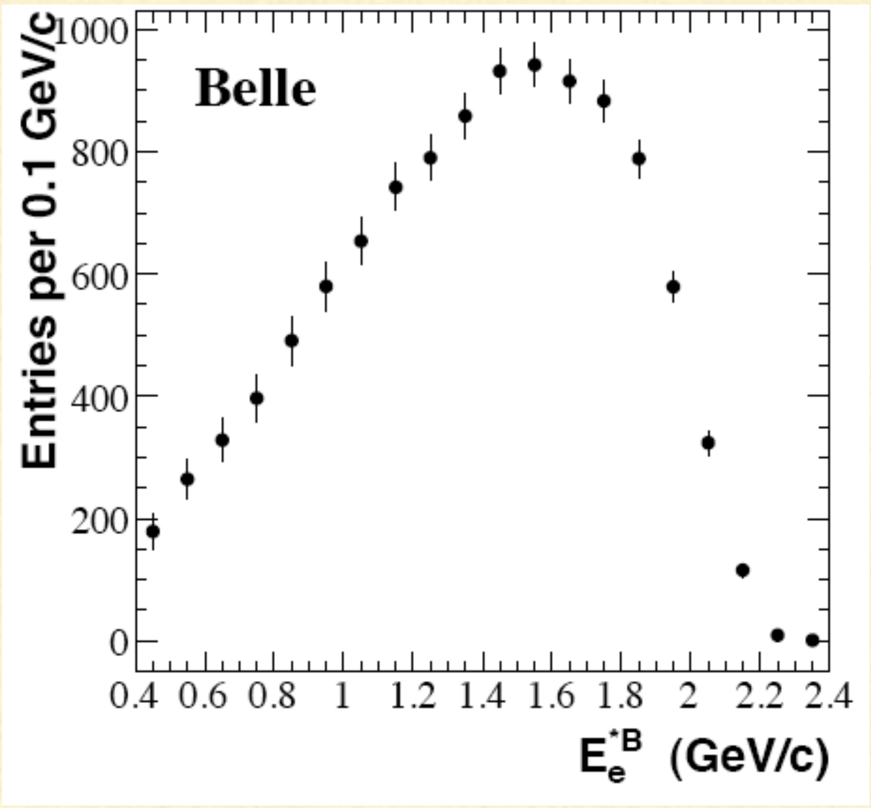
$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \middle| \bar{b}_v \left(i\vec{D}\right)^2 b_v \middle| B \right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B} \left\langle B \middle| \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v \middle| B \right\rangle_\mu$$

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.

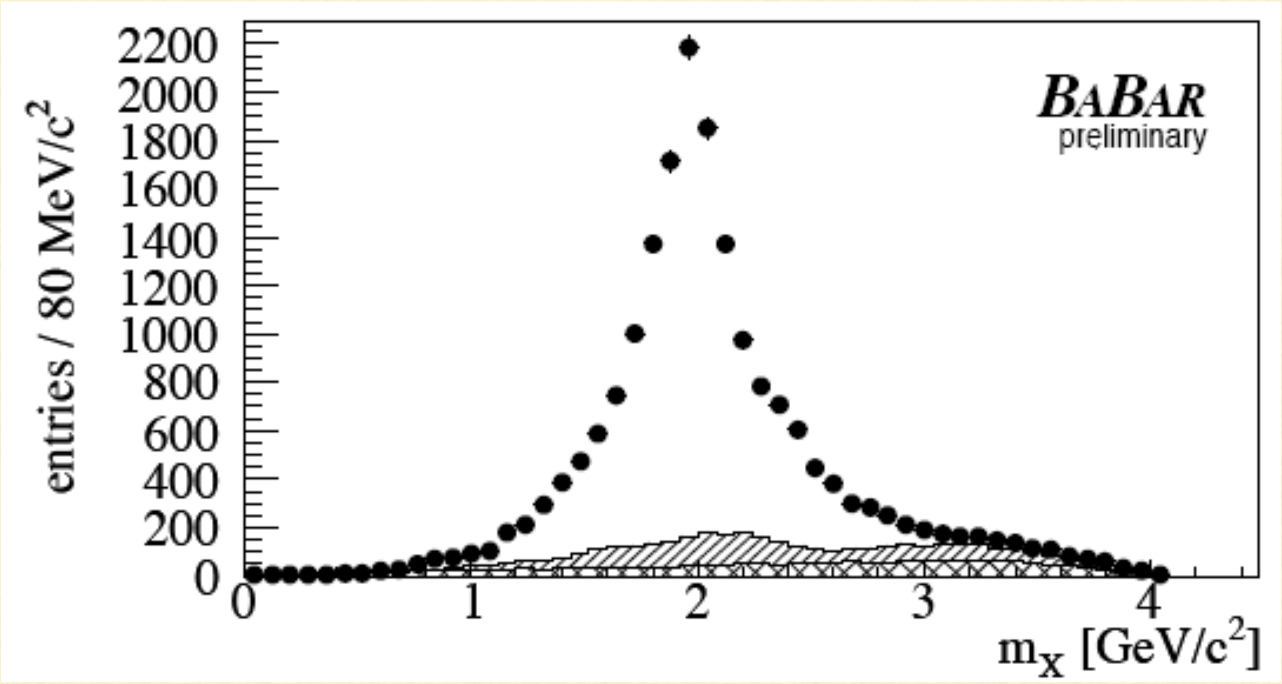
Current HFLAV **kinetic scheme** fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice

EXTRACTION OF THE OPE PARAMETERS

E_l spectrum



hadronic mass spectrum



Global shape parameters (first moments of the distributions, various lower cut on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, V_{ub},...)

THE KINETIC SCHEME

Bigi, Shifman, Uraltsev, Vainshtein

$$m_b^{kin}(\mu) = m_b^{pole} - \left[\bar{\Lambda}(\mu)\right]_{\rm pert} - \frac{\left[\mu_\pi^2(\mu)\right]_{\rm pert}}{2m_b^{kin}(\mu)}$$

$$\mu_\pi^2(0) = \mu_\pi^2(\mu) - [\mu_\pi^2(\mu)]_{pert}$$

$$\rho_D^3(0) = \rho_D^3(\mu) - [\rho_D^3(\mu)]_{pert}$$

$$\left[\bar{\Lambda}(\mu)\right]_{\rm pert} = \frac{4}{3} C_F \mu \frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(\frac{4}{3} - \frac{1}{2}\ln\frac{2\mu}{\mu_b}\right)\beta_0 - C_A\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right]\right\}$$

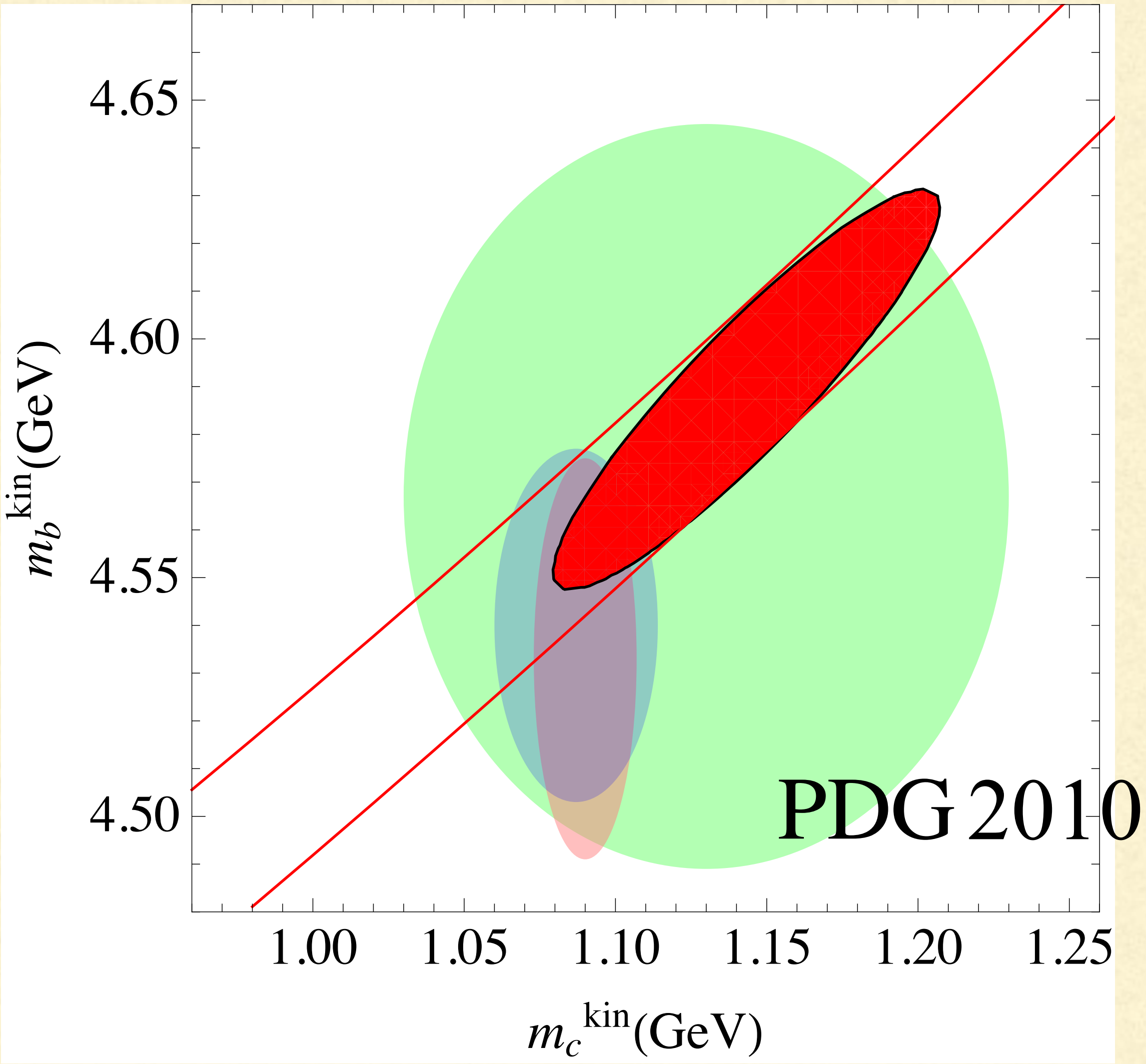
$$\left[\mu_\pi^2(\mu)\right]_{\rm pert} = C_F \mu^2 \frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(\frac{13}{12} - \frac{1}{2}\ln\frac{2\mu}{\mu_b}\right)\beta_0 - C_A\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right]\right\}$$

$$[\rho_D^3(\mu)]_{pert} = \frac{2}{3} C_F \mu^3 \frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(1 - \frac{1}{2}\ln\frac{2\mu}{\mu_b}\right)\beta_0 - C_A\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right]\right\}$$

Czarnecki, Melnikov, Uraltsev hep-ph/9708372

- Provides a short-distance, renormalon free definition of heavy quark mass *and* OPE parameters, by introducing a Wilson cutoff $\mu \approx 1\,GeV$ to factor out IR physics. This can be realised in different ways, beyond 1-loop SV sum rules are most practical (see Matteo's talk)
- The kinetic mass is one of a family of renormalon subtracted masses (PS, MRS…) but only in this case the definition is tailored on the HQE, although not Lorentz invariant
- 3-loop conversion now available 2005.06487
- The expansion for m_b is truncated at $O(1/m_b)$, however $O(\alpha_s/m_b^2)$ *vanish* according to hep-ph/0302262. We do not re-expand in μ/m_b

10 YEARS BACK…



Schwanda & PG, 1102.0210

The HFAG semileptonic fits seemed incompatible with values of $m_{b,c}$ from sum rules by the Karlsruhe and Hoang's groups. We found why…

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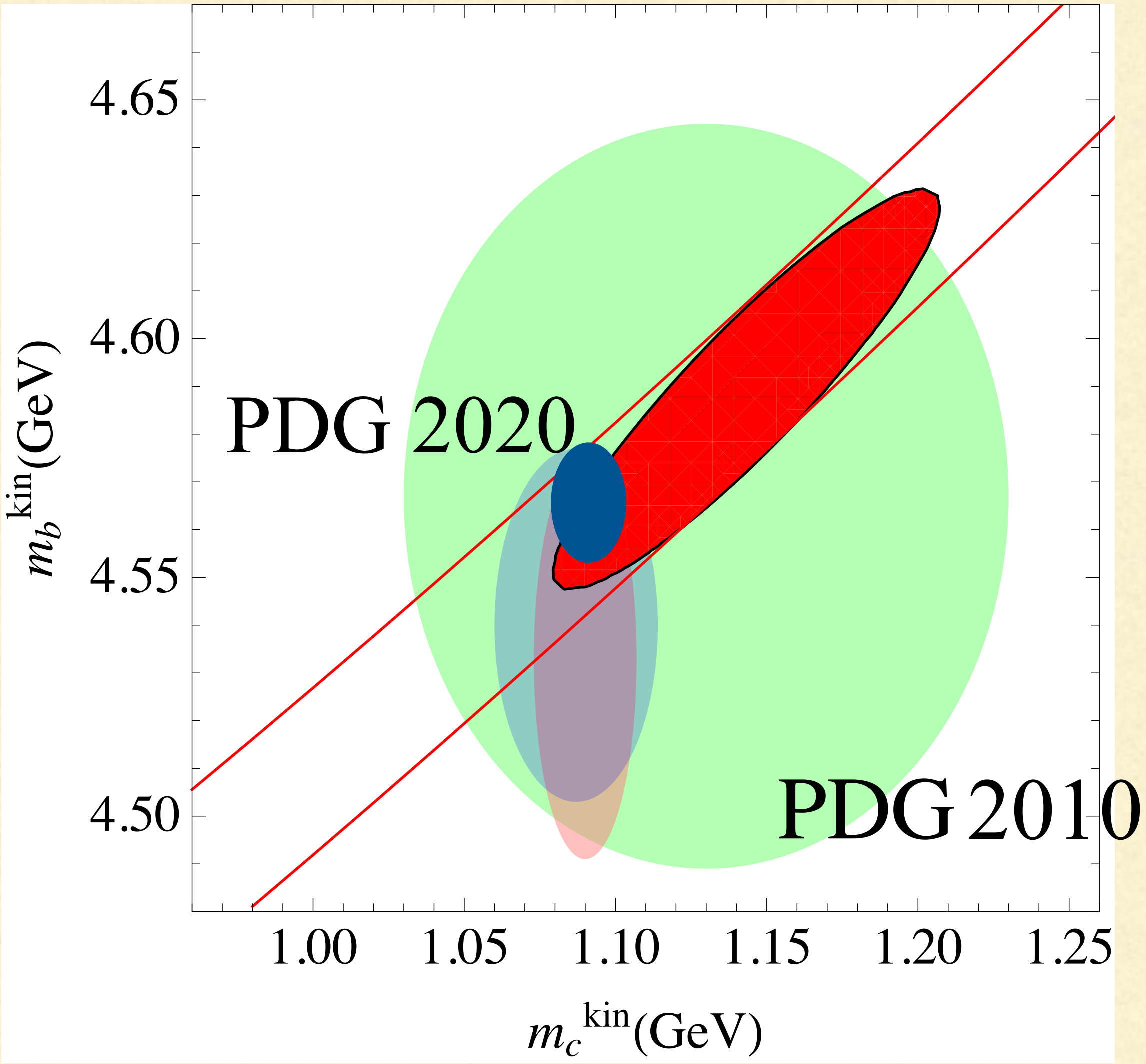

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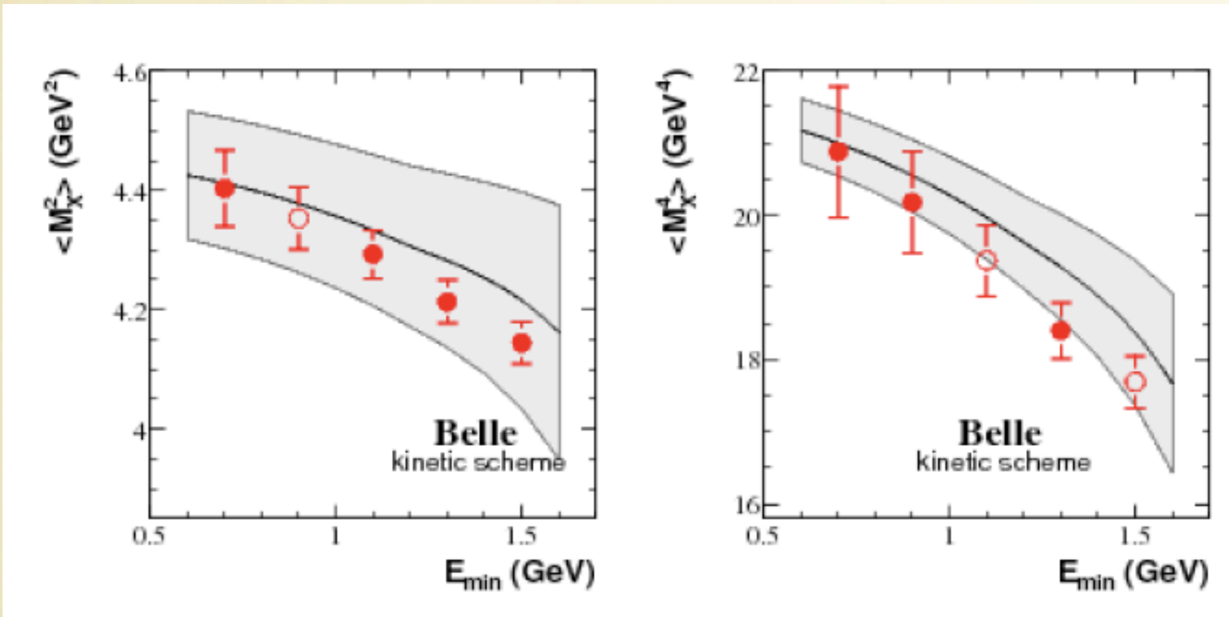
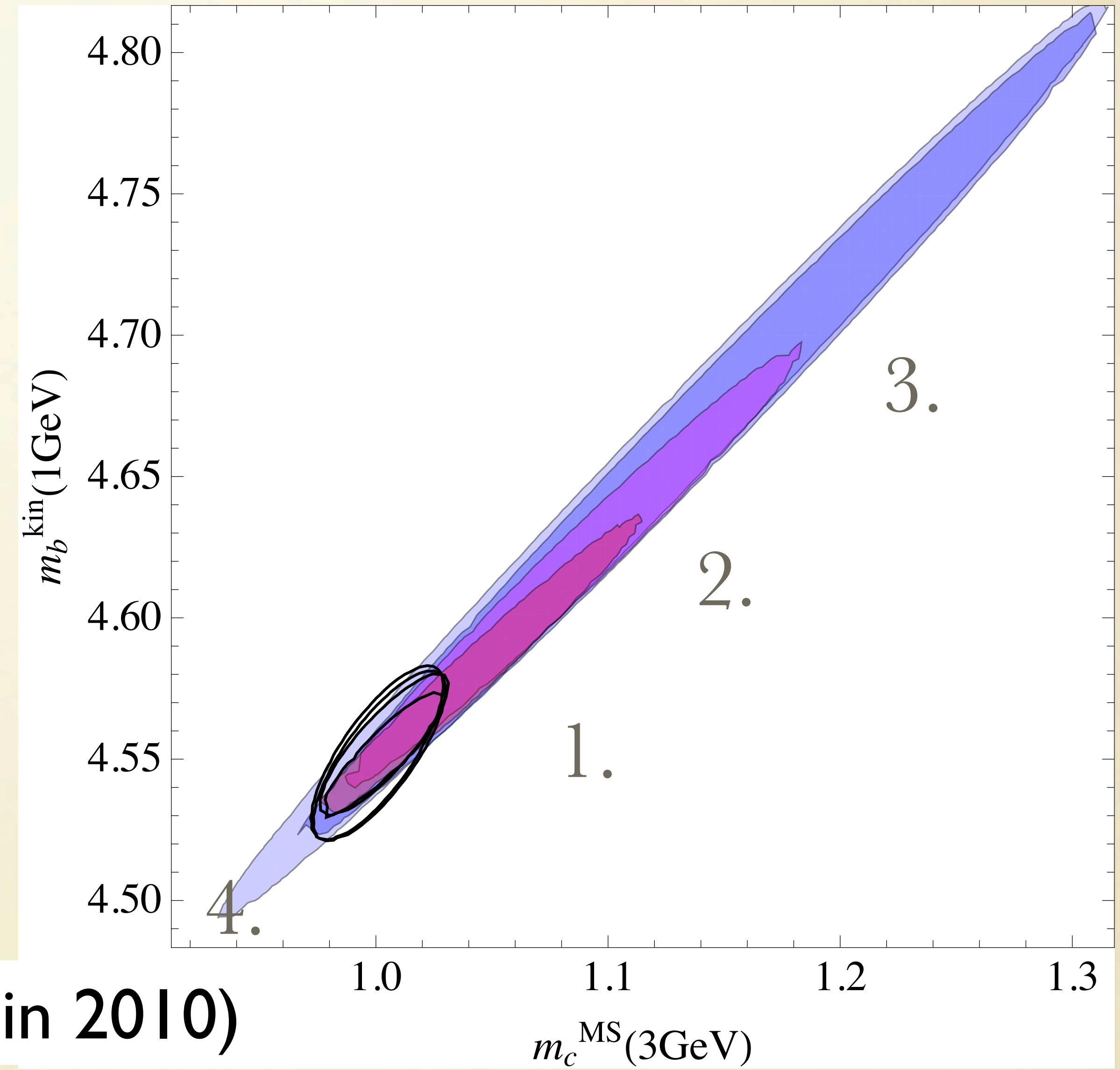
Theoretical correlations



Correlations between theory errors of moments with different cuts difficult to estimate

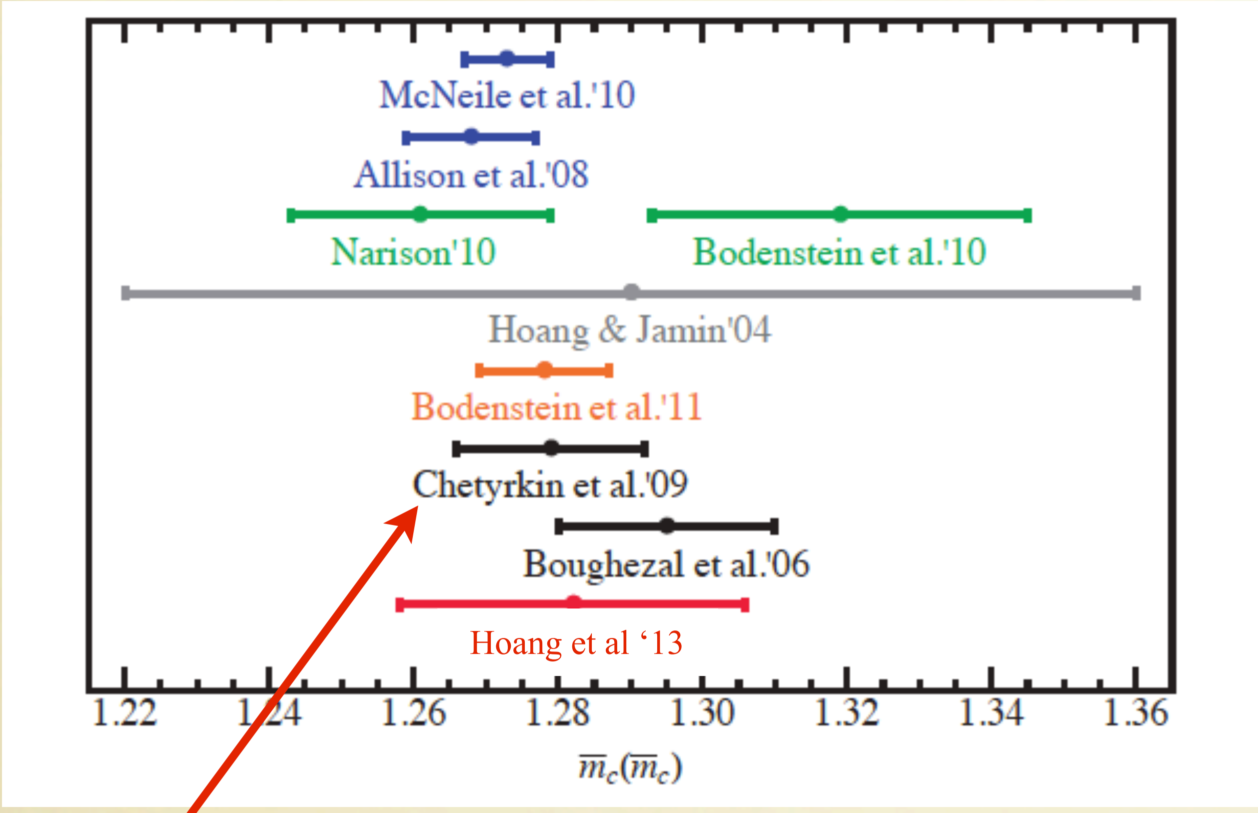


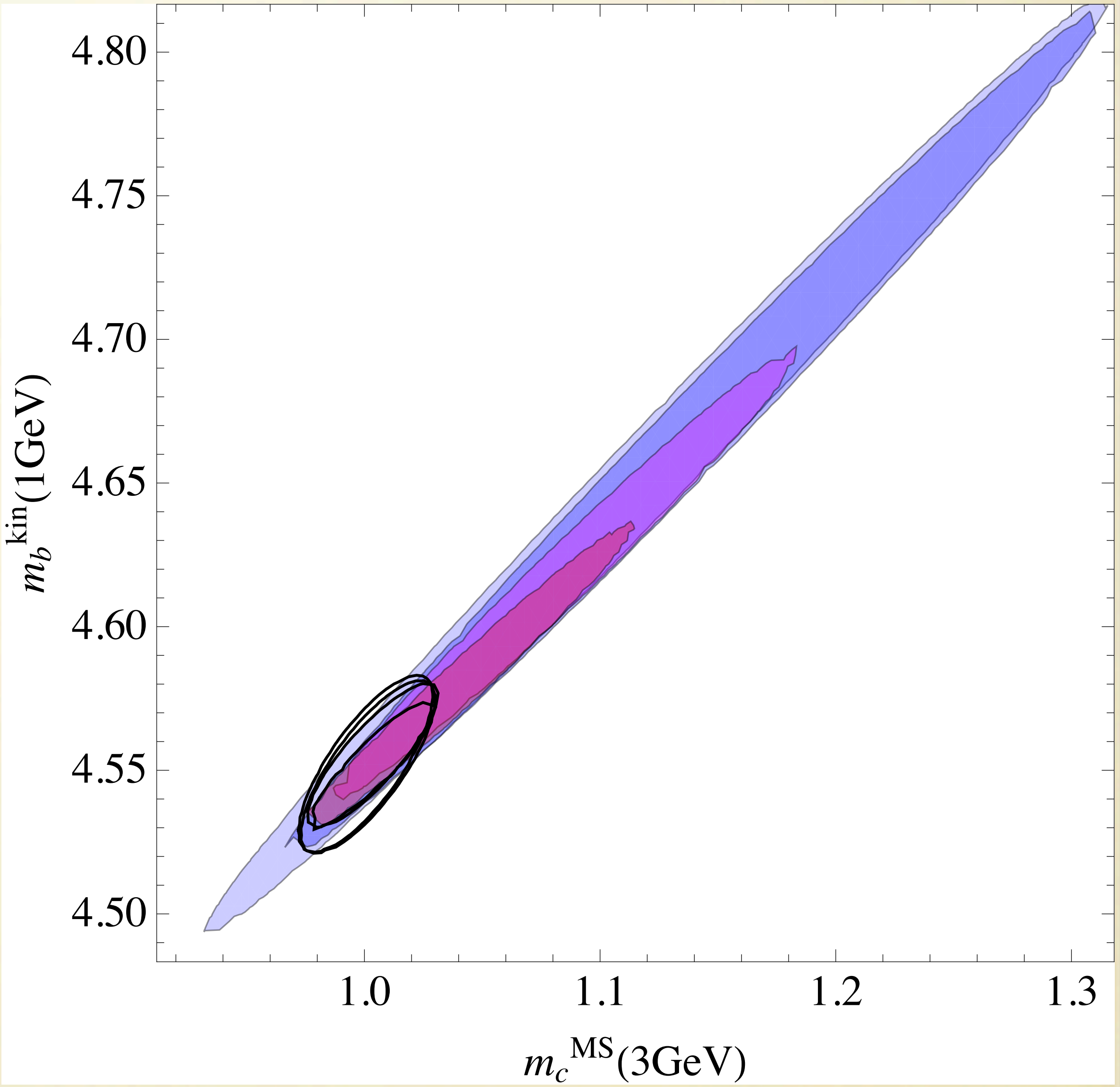
Schwanda, PG 1307.4551

1. 100% correlations (unrealistic but used in 2010)
2. corr. computed from low-order expressions
3. constant factor 0<ξ<1 for 100MeV step
4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated

CHARM MASS DETERMINATIONS





Schwanda, PG 1307.4551

our default choice

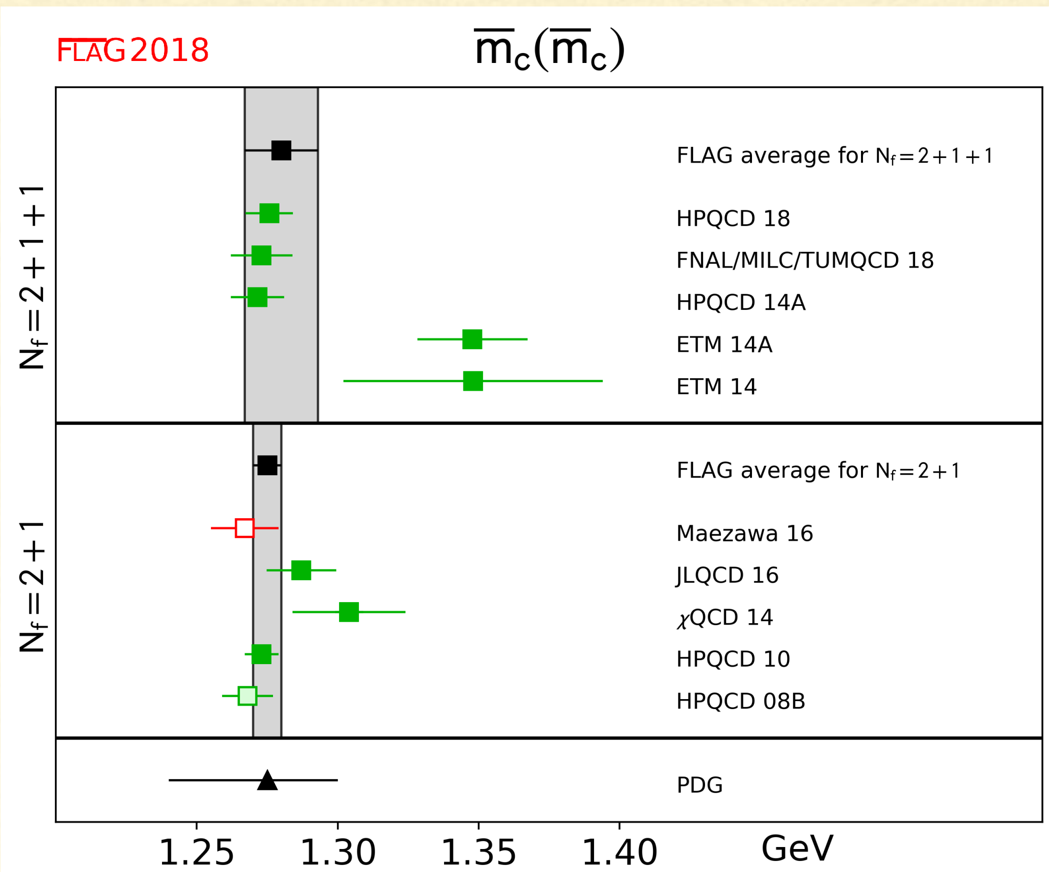
sum rules studies of $\sigma(e^+e^- \to \text{hadrons})$

almost all at NNNLO

Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b

CHARM MASS DETERMINATIONS TODAY



Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	$\overline{m}_c(\overline{m}_c)$	$\overline{m}_c(3\text{ GeV})$
HPQCD 18	[58]	2+1+1	A	★	★	★	★	1.2757(84)	0.9896(61)
FNAL/MILC/ TUMQCD 18	[59]	2+1+1	A	★	★	★	–	1.273(4)(1)(10)	0.9837(43)(14)(33)(5)
HPQCD 14A	[60]	2+1+1	A	★	★	★	–	1.2715(95)	0.9851(63)
ETM 14A	[86]	2+1+1	A	○	★	○	★	1.3478(27)(195)	1.0557(22)(153)
ETM 14	[61]	2+1+1	A	○	★	○	★	1.348(46)	1.058(35)
Maezawa 16	[33]	2+1	A	■	★	★	★	1.267(12)	
JLQCD 16	[87]	2+1	A	○	★	★	–	1.2871(123)	1.0033(96)
χQCD 14	[88]	2+1	A	○	○	○	★	1.304(5)(20)	1.006(5)(22)
HPQCD 10	[41]	2+1	A	○	★	○	–	1.273(6)	0.986(6)
HPQCD 08B	[56]	2+1	A	○	★	○	–	1.268(9)	0.986(10)
PDG	[7]							$1.275^{+0.025}_{-0.035}$	

All fits reported here use $m_c(3\text{GeV})=0.986(13)\text{GeV}$. No need to use the kinetic scheme for m_c. We use $m_c(2\text{GeV})$ and $m_c(3\text{GeV})$

FIT RESULTS

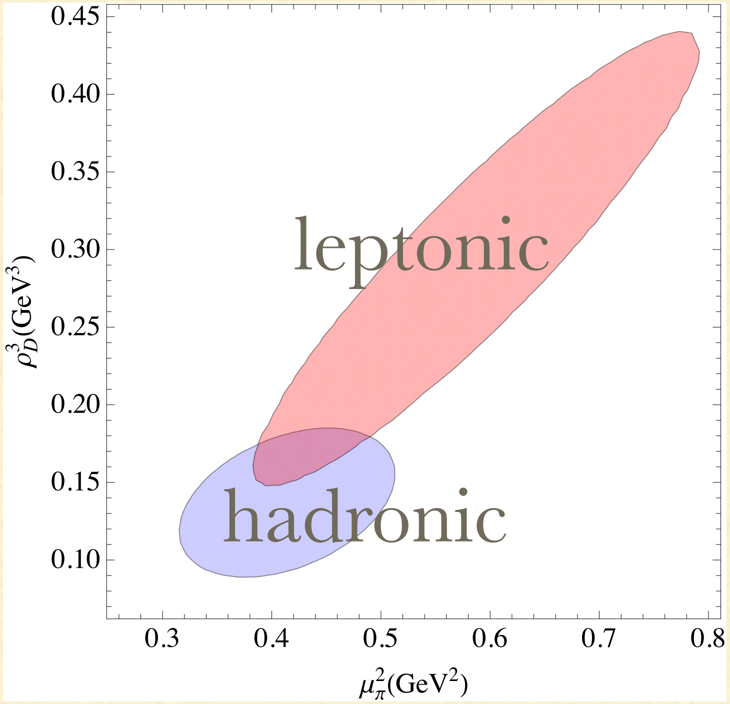
m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

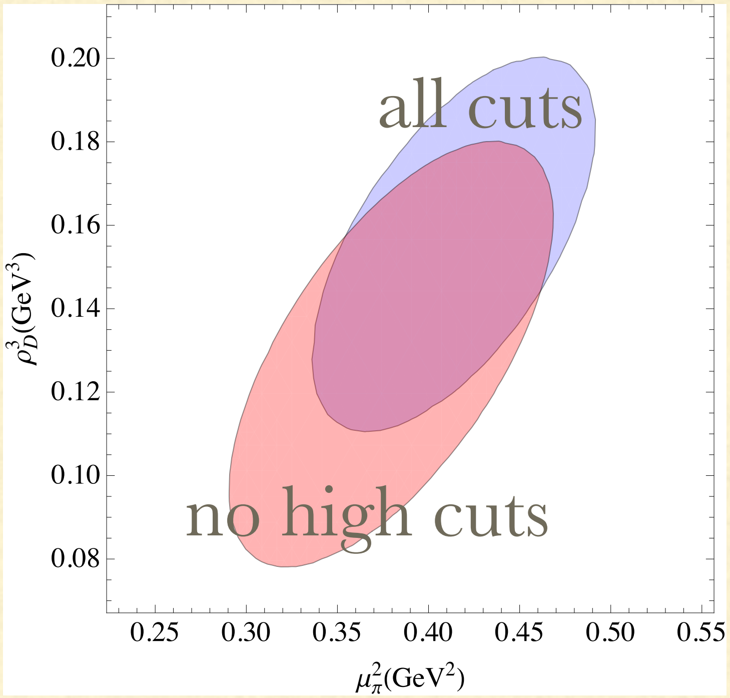
this is HFLAV fit

Alberti, Healey, Nandi, PG, 1411.6560

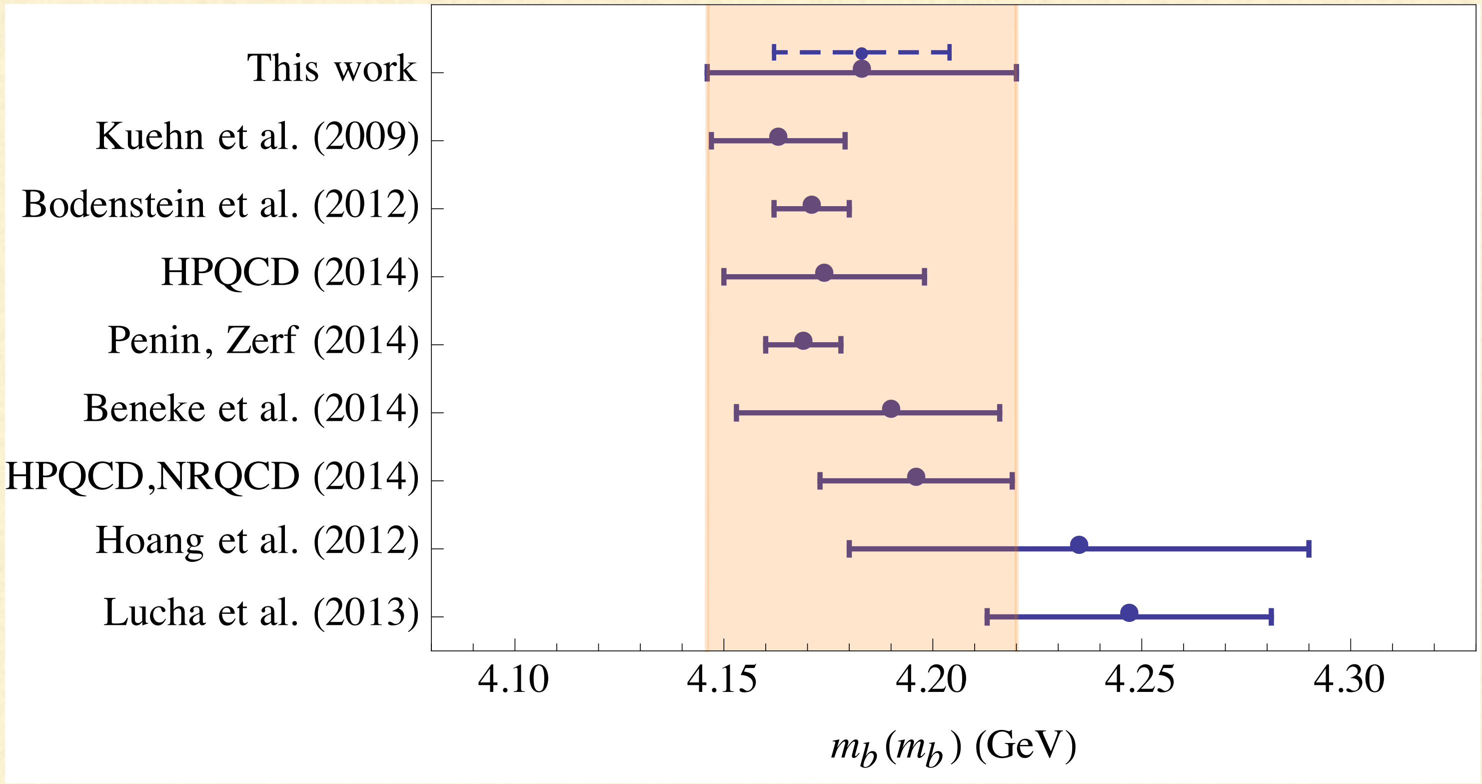
Without mass constraints $m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{GeV}$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results





INDIRECT m_b DETERMINATION



Alberti, Healey, Nandi, PG, 1411.6560

PDG 2020/FLAG:
$m_b(m_b)$=4.198(12)GeV

The fit gives $\boldsymbol{m_b^{kin}(1GeV)=4.553(20)GeV}$

scheme translation error (2loop) $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$

$\boldsymbol{m_b(m_b)=4.183(37)GeV}$

Conversely, we can use m_b as external input to improve precision on V_{cb}

HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

Mannel,Turczyk,Uraltsev **1009.4622**

can be estimated by ***Lowest Lying State Saturation*** (LLSA) approx by truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\, \mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\, \mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases, see1206.2296. In LLSA **good convergence of the HQE.**

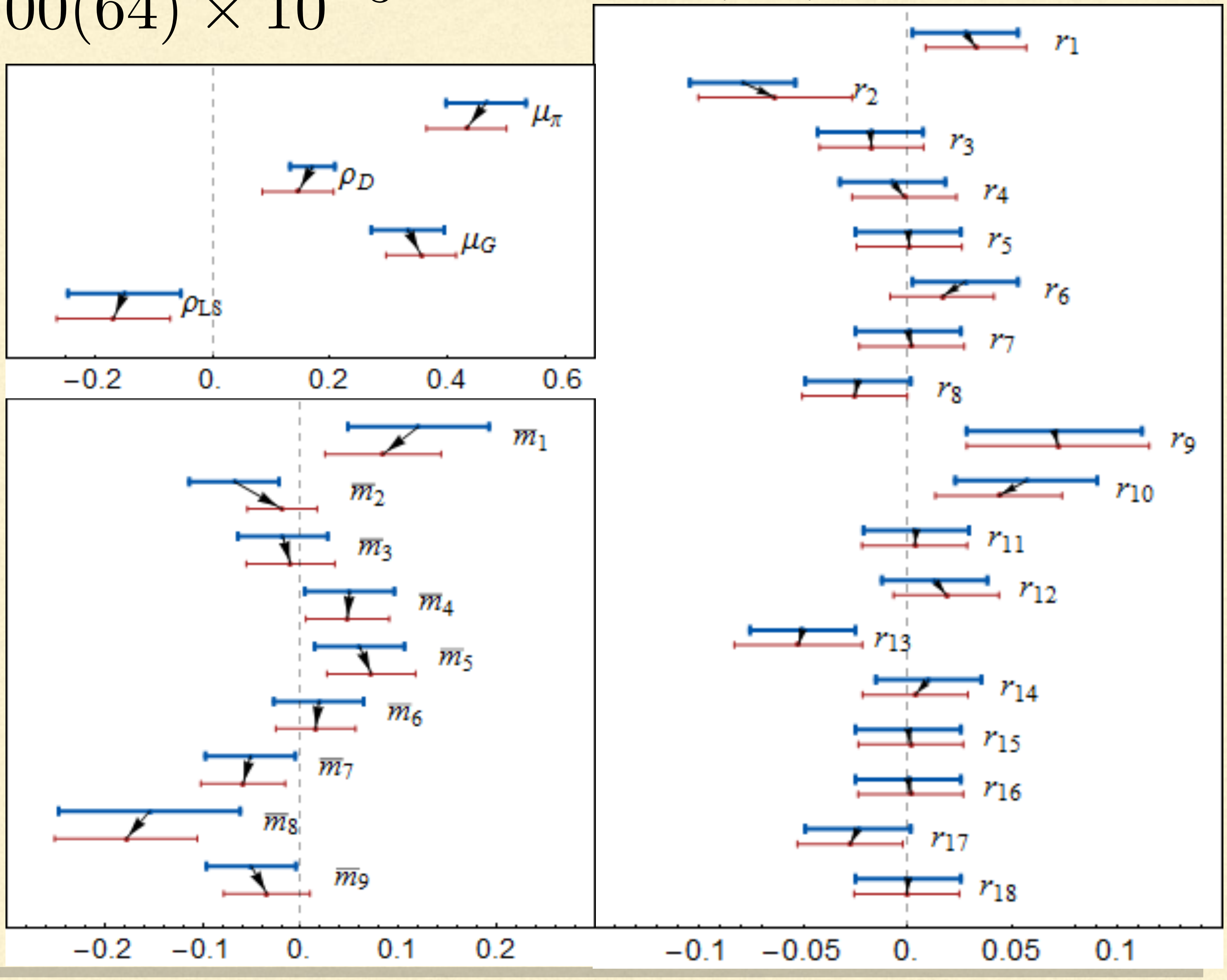
We used LLSA as loose constraint (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in the fit including higher powers

SENSITIVITY TO HIGHER POWER CORRECTIONS

$$|V_{cb}| = 42.00(64) \times 10^{-3}$$

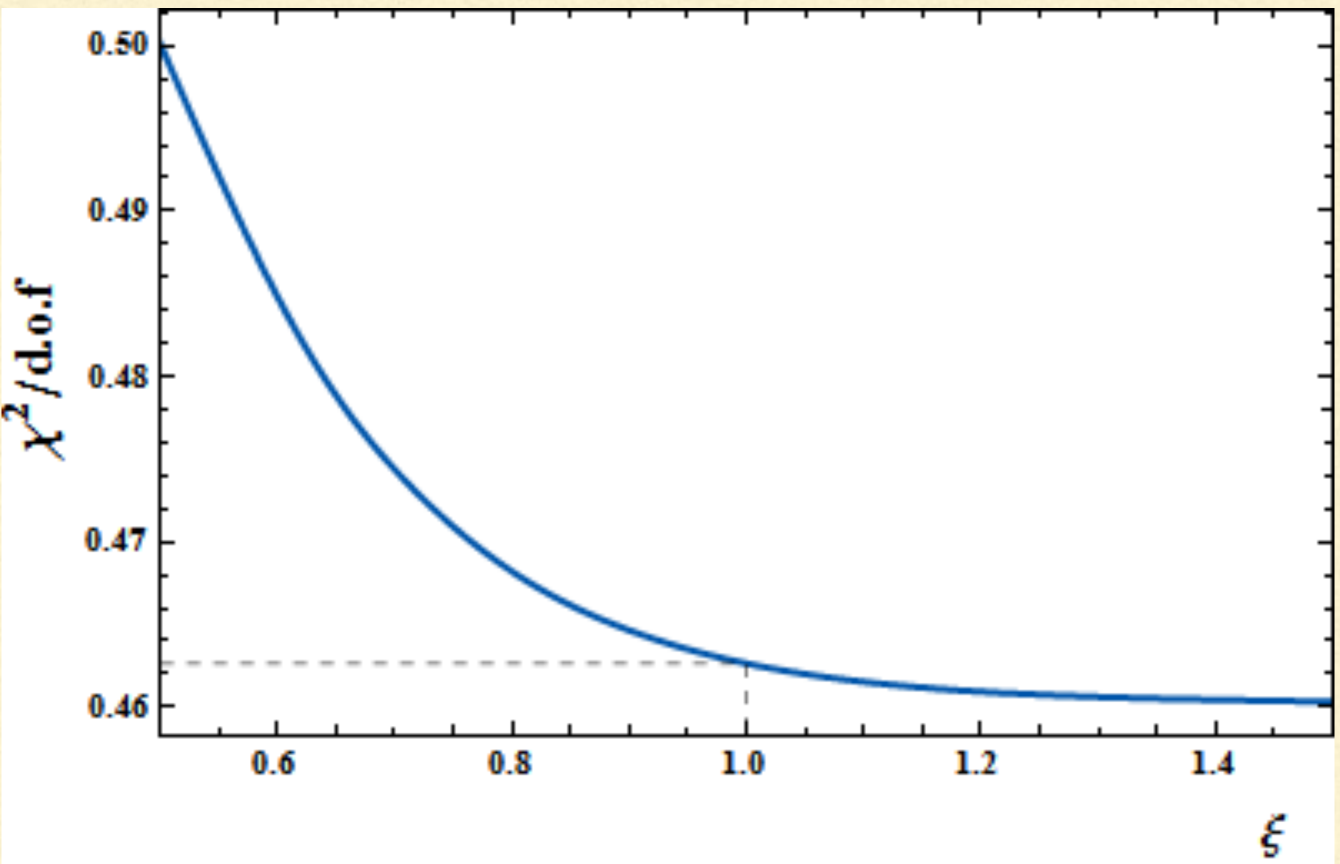
if one uses $m_c(2GeV)$ and includes PDG average for m_b
1.5% uncertainty

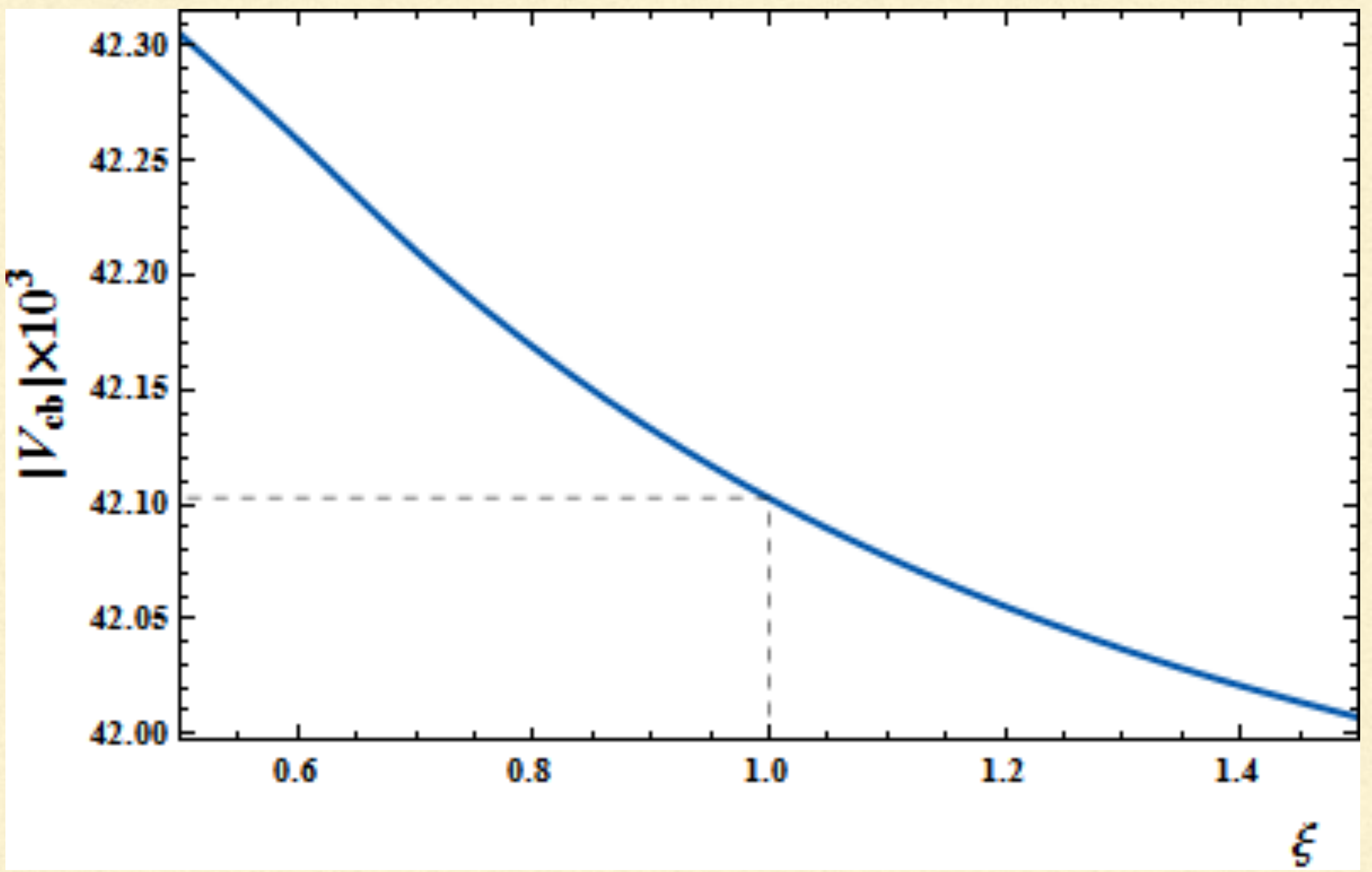
PG,Healey,Turczyk 1606.06174



Shifts in the OPE parameters from the LLSA using the 2014 fit (blue thick) to the fit including higher-order corrections (red thin). Error bars represent the error in the priors and the resulting fit error, respectively.

DEPENDENCE ON LLSA UNCERTAINTY





WE RESCALE ALL LLSA UNCERTAINTIES BY A FACTOR ξ

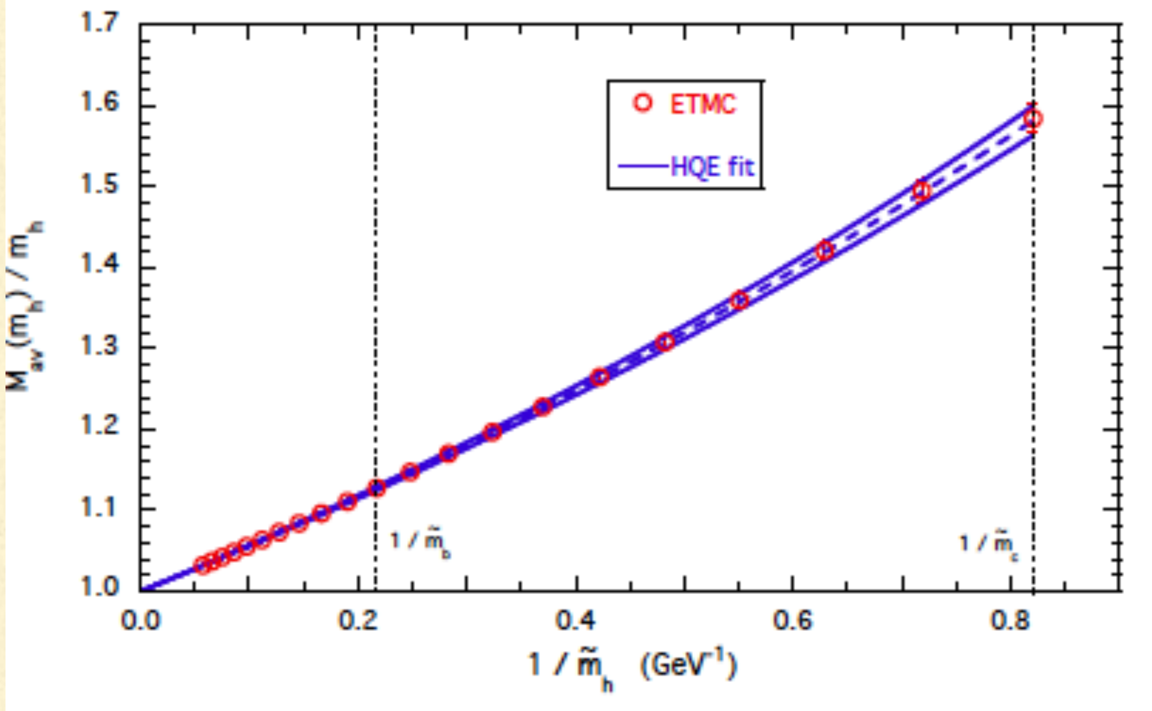
PROSPECTS for INCLUSIVE V_{cb}

- Theoretical uncertainties generally larger than experimental ones
- $O(\alpha_s/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- 3loop relation between MS and kin scheme just completed 2005.06487
It can be used to improve the precision of the m_b input
- $O(\alpha_s^3)$ corrections to total width feasible, needed for 1% uncertainty
- Electroweak (QED) corrections require attention
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now, q^2 moments (Fael, Mannel, Vos)…
- **Lattice QCD** is the next frontier

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \dots$$



- on the lattice one can compute mesons for arbitrary quark masses

  see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections
- Results consistent with s.l. fits, improvements under way, also following new 3loop calculation of pole-kinetic mass relation

INCLUSIVE SL DECAYS ON THE LATTICE

Hashimoto, PG 2005.13730

$$\frac{d\Gamma}{dq^2 dq^0 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}$$

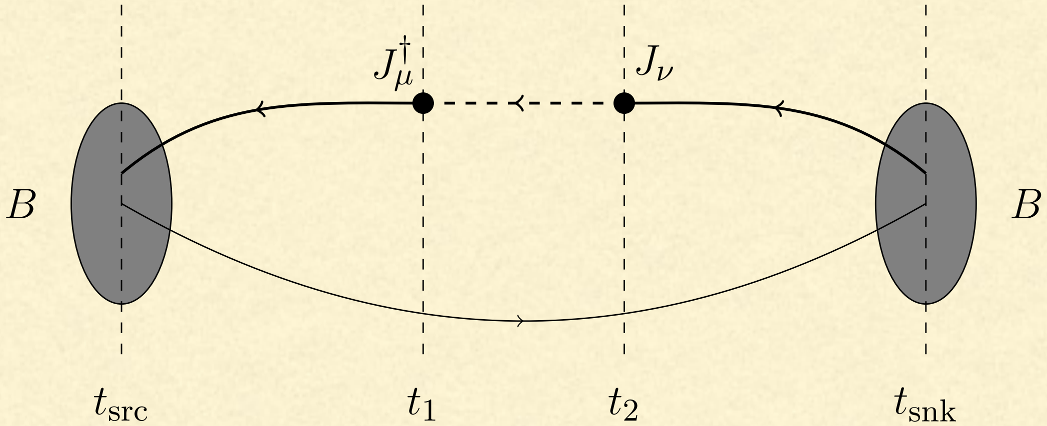
$$W^{\mu\nu}(p,q) = \sum_{X_c} (2\pi)^3 \delta^{(4)}(p-q-r) \times$$

$$\times \frac{1}{2E_{B_s}} \langle B_s(\boldsymbol{p}) | J^{\mu\dagger} | X_c(\boldsymbol{r}) \rangle \langle X_c(\boldsymbol{r}) | J^\nu | B_s(\boldsymbol{p}) \rangle$$

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q}^2 \sqrt{\boldsymbol{q}^2} \sum_{l=0}^{2} \bar{X}^{(l)}$$

$$\bar{X}^{(l)} \equiv \int_{\sqrt{m_{D_s}^2 + \boldsymbol{q}^2}}^{m_{B_s} - \sqrt{\boldsymbol{q}^2}} d\omega\, X^{(l)}$$ (ω hadr. energy)

4point functions on the lattice are related to the hadronic tensor in euclidean



$$\sum_{\boldsymbol{x}} e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \frac{1}{2m_{B_s}} \langle B_s(\boldsymbol{0}) | J_\mu^\dagger(\boldsymbol{x},t) J_\nu(\boldsymbol{0},0) | B_s(\boldsymbol{0}) \rangle$$

$$\sim \langle B_s(\boldsymbol{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) e^{-\hat{H}t} \tilde{J}_\nu(\boldsymbol{q}) | B(\boldsymbol{0}) \rangle$$

$$\int_0^\infty d\omega\, K(\omega,\boldsymbol{q}) \langle B_s(\boldsymbol{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) \delta(\hat{H}-\omega) \tilde{J}_\nu(\boldsymbol{q}) | B_s(\boldsymbol{0}) \rangle$$

$$= \langle B_s(\boldsymbol{0}) | \tilde{J}_\mu^\dagger(-\boldsymbol{q}) K(\hat{H},\boldsymbol{q}) \tilde{J}_\nu(\boldsymbol{q}) | B_s(\boldsymbol{0}) \rangle$$

K approximated by polynomial

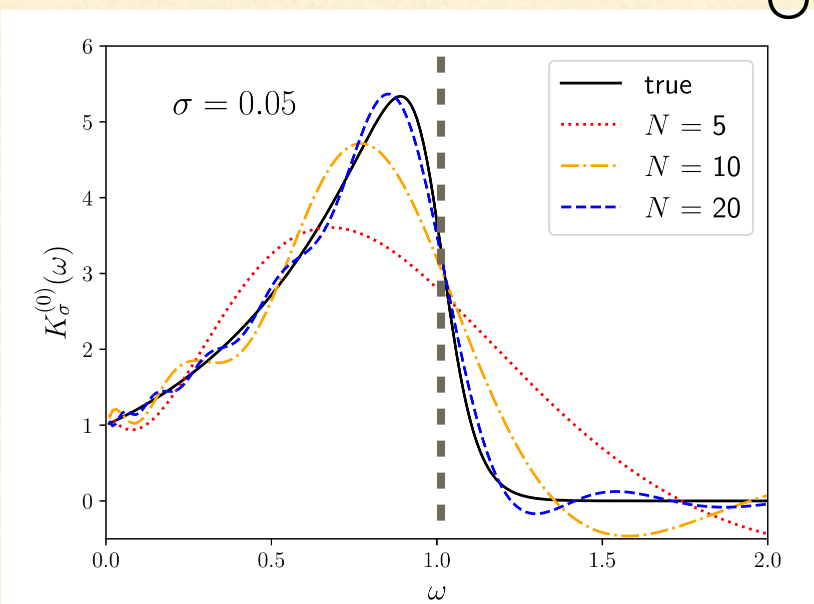
$$K(\hat{H},\boldsymbol{q}) = k_0(\boldsymbol{q}) + k_1(\boldsymbol{q}) e^{-\hat{H}} + \cdots + k_N(\boldsymbol{q}) e^{-N\hat{H}}$$

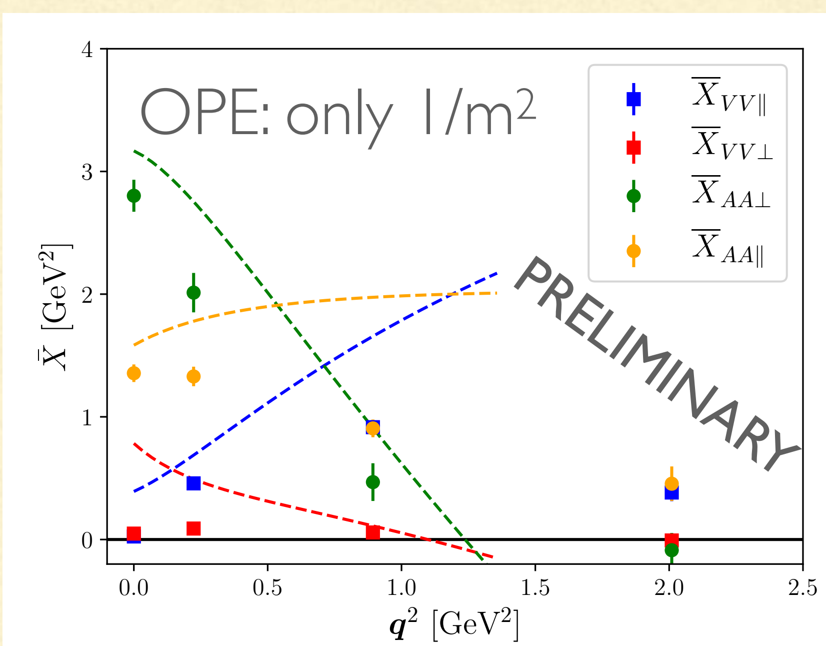
A PILOT NUMERICAL STUDY

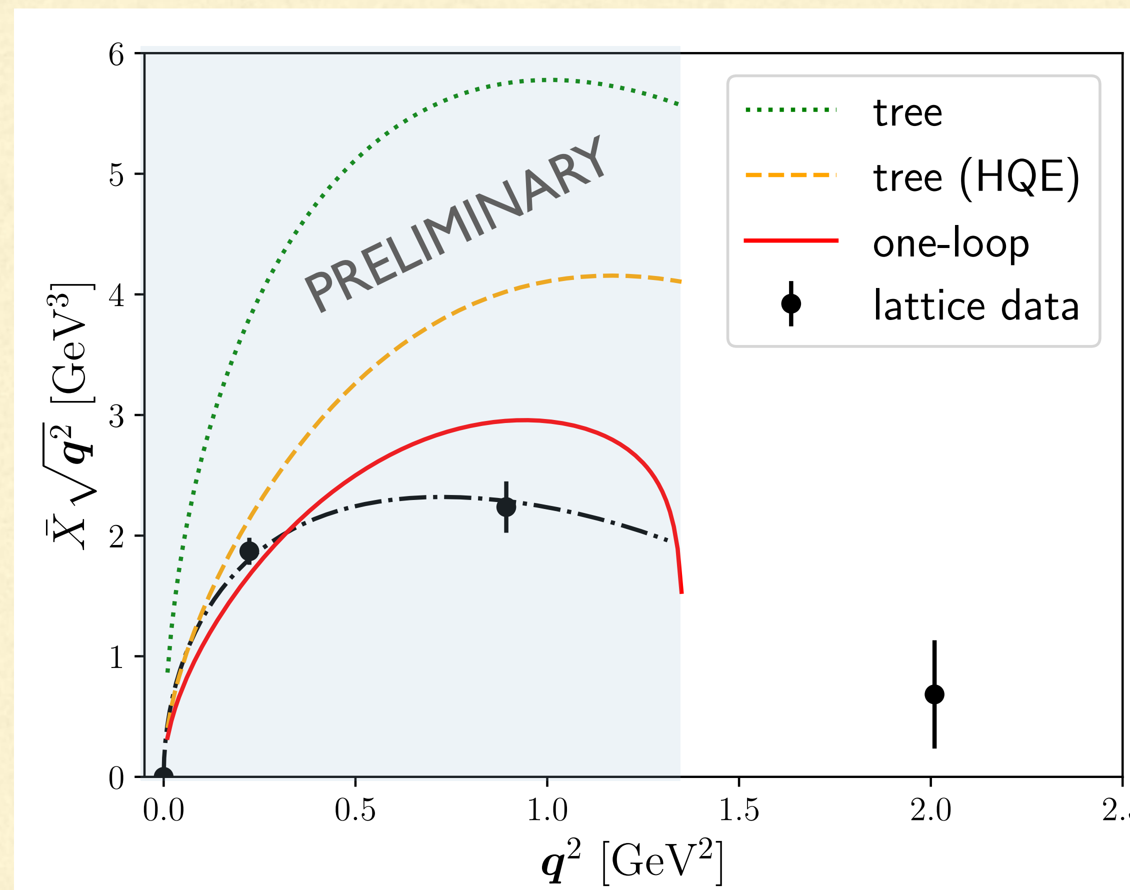
Hashimoto, PG 2005.13730

Smeared spectral functions can be computed on the lattice, see also 1704.08993

Using Chebychev polynomials to approx the kernel, 2+1 flavours of Moebius domain wall fermions with 1/a=3.6GeV, one gets M_{Bs}=3.45 GeV, i.e. $m_b \approx$2.70GeV $m_b - m_c \sim$1.7GeV only







$$\Gamma/|V_{cb}|^2 = 4.88(57) \times 10^{-13} \text{ GeV}$$ Lattice

$$\Gamma/|V_{cb}|^2 = 5.41(82) \times 10^{-13} \text{ GeV}$$ OPE

including all known corrections

CONCLUSIONS

- Inclusive s.l. B decays appear in a good shape: they can be improved by higher order calculations, lattice studies and new data from Belle II, as well as improved determinations of m_b and m_c
- The recent 3loop calculation makes it possible to employ precise determinations of m_b in the kinetic scheme, looking forward to 3loop corrections to the width
- Inclusive/Exclusive V_{cb} tension remains, but weaker. Hopefully, it will disappear. LQCD is likely to decide the fate of the V_{cb} puzzle