The role of *mb,c* in semileptonic B decays

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Last few years: new analyses of B-factories data, new calculations of FFs by several lattice collaborations and with light-cone sum rules, rising to the challenges of a precision measurement. This plot does **not** reflect all developments.

The importance of $|V_{cb}|$

 $\overline{\mathsf{r}}$ UT_{fit} The most important CKM unitarity test is the Unitarity Triangle (UT) $\Delta m_{\rm cl}$ V_{cb} plays an important role in UT Δm **Am** 0.5 $\varepsilon_K \approx x|V_{cb}|^4 + ...$ ub and in the prediction of FCNC: $\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2$ **i** $sin(2\beta+\gamma)$ $1 + O(\lambda^2)$ -0.5 where it often dominates the theoretical uncertainty. V_{ub}/V_{cb} constrains directly the UT -0.5 0.5 Ω $\boldsymbol{\rho}$

Our inability to determine precisely V_{cb} hampers significantly NP searches

INCLUSIVE DECAYS: BASICS

- **Simple idea:** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops \Box parameterize non-pert physics: *double series in αs,* **Λ***/mb*
- Lowest order: decay of a free *b*, linear **Λ**/m_b absent. Depends on m_{b,c}, 2 ш parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$
M_{i} = M_{i}^{(0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(1)} + \left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)} + \left(M_{i}^{(\pi,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(\pi,1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} + \left(M_{i}^{(G,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(G,1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}} + M_{i}^{(D,0)} \frac{\rho_{D}^{3}}{m_{b}^{3}} + M_{i}^{(LS,0)} \frac{\rho_{LS}^{3}}{m_{b}^{3}} + \dots
$$

$$
\mu_{\pi}^{2}(\mu) = \frac{1}{2M_{B}} \left\langle B \left| \overline{b} \left(i \overrightarrow{D} \right)^{2} b \right| B \right\rangle_{\mu} \qquad \mu_{G}^{2}(\mu) = \frac{1}{2M_{B}} \left\langle B \left| \overline{b} \right|_{2}^{i} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_{\mu}
$$

Reliability of the method depends on our control of higher order effects. Quarkhadron duality violation would manifest as inconsistency in the fit. €

Current HFLAV **kinetic scheme** fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice

EXTRACTION OF THE OPE PARAMETERS

 Global shape parameters (first moments of the distributions, various lower cut on E_l) tell us about m_b , m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the $quarks \rightarrow$ useful in many applications (rare decays, $V_{ub,...}$)

THE KINETIC SCHEME ³i⁷⁷ ⁼ ³ ↵*s*(*µb*) *C^F* ³⁶⁰⁰ ⁺ 7200 *m*² *b* ↵*^s* N In order to change to the kinetic scheme we use ⁼ *^C^F ^µ*² ↵*s*(*µb*) n ₁ ↵*s* h✓13 ¹² ¹ ln ²*^µ* ◆ *µb* ⁰ *C^A* \neg EM ^h(*E ^m^b* 2) ³i⁷⁷ ⁼ *^m*³ In the change to the change of the chang

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³ ¹

2

2809

⁰ *C^A*

*µ*2

⌘

1481

⁶ ¹³

*µ*2

^G(*µb*)

mkin

i

^b (*µ*) = *mpole*

^b ⇥

¹²◆io (48)

^b (*µ*) *,* (47)

pert

$$
m_b^{kin}(\mu) = m_b^{pole} - \left[\bar{\Lambda}(\mu)\right]_{\text{pert}} - \frac{\left[\mu_\pi^2(\mu)\right]_{\text{pert}}}{2m_b^{kin}(\mu)} \qquad \qquad [\bar{\Lambda}(\mu)]_{\text{pert}} = \frac{4}{3}C_F\mu\frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(\frac{4}{3} - \frac{1}{2}\ln\frac{2}{\mu}\right)\right]\right\}
$$
\n
$$
\mu_\pi^2(0) = \mu_\pi^2(\mu) - \left[\mu_\pi^2(\mu)\right]_{\text{pert}} \qquad \qquad [\mu_\pi^2(\mu)]_{\text{pert}} = C_F\mu^2\frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(\frac{13}{12} - \frac{1}{2}\ln\frac{2}{\mu}\right)\right]\right\}
$$
\n
$$
\rho_D^3(0) = \rho_D^3(\mu) - \left[\rho_D^3(\mu)\right]_{\text{pert}}
$$
\n
$$
\text{Laplace}
$$
\n
$$
[\rho_D^3(\mu)]_{\text{pert}} = \frac{2}{3}C_F\mu^3\frac{\alpha_s(\mu_b)}{\pi} \left\{1 + \frac{\alpha_s}{\pi}\left[\left(1 - \frac{1}{2}\ln\frac{2}{\mu}\right)\right]\right\}
$$
\n
$$
\text{Laplace}
$$

*m*³

⇡

C^F µ

⁰ (*µ*) + ↵*^s*

C^F

h

1

⇡

 $\frac{k}{b}$ $\frac{k}{\mu}$ $\left[\bar{\Lambda}(\mu)\right]_{\rm pert} = \frac{4}{3} C_F \mu \frac{\alpha_s(\mu_b)}{\pi} \Big\{1 + \frac{\alpha_s}{\pi} \Big[\Big(\frac{4}{3} - \frac{1}{2} \ln \frac{2\mu}{\mu_b}\Big) \beta_0 [\rho_D^3(\mu)]_{pert} = \frac{2}{3}C_F\mu^3\frac{\alpha_s(\mu_b)}{\pi}\Big\{1+\frac{\alpha_s}{\pi}\Big[\Big(1-\frac{1}{2}\ln\frac{2\mu}{\mu_b}\Big)\beta_0$ $\left[\mu_{\pi}^{2}(\mu)\right]_{\text{pert}} = C_{F}\mu^{2}\frac{\alpha_{s}(\mu_{b})}{\pi}\Big\{1 + \frac{\alpha_{s}}{\pi}\Big[\Big(\frac{13}{12} - \frac{1}{2}\ln\frac{3}{2}\Big)\Big\}$ $\frac{3}{D}(\mu)$]*pert*
 $\frac{1}{D}(\mu)$ ²
D(μ)² $\left[\bar{\Lambda}(\mu)\right]_{\text{pert}} = \frac{4}{3}C_F\mu$ $\alpha_s(\mu_b)$ π $\left\{1 + \frac{\alpha_s}{\alpha_s}\right\}$ π $\left[\left(\frac{4}{3}-\frac{1}{2}\right)\right]$ $\ln \frac{2\mu}{2}$ μ_b ◆ $\beta_0 - C_A$ $\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right\}$ $\left\{1 + \frac{\alpha_s}{\alpha_s}\right\}$ π $\left[\left(\frac{13}{12}-\frac{1}{2}\right)\right]$ $\ln \frac{2\mu}{2}$ μ_b ◆ $\beta_0 - C_A$ $\frac{b}{\pi} \left\{ 1 + \frac{\alpha_s}{\pi} \left[\left(\frac{13}{12} - \frac{1}{2} \ln \frac{2\mu}{\mu_b} \right) \beta_0 - C_A \left(\frac{\pi^2}{6} - \frac{13}{12} \right) \right] \right\}$ C_{α} $\alpha_s(\mu_b)$ C_{α} $\alpha_s(\mu_b)$ C_{α} α_s α $[\rho_D^3(\mu)]_{pert}=\frac{2}{3}C_F\mu^3\frac{\alpha_s(\mu_b)}{\pi}\Big\{1+\frac{\alpha_s}{\pi}\Big[\Big(1-\frac{1}{2}\ln\frac{2\mu}{\mu_b}\Big)\,\beta_0-\ C_A\Big\}$ $\left\{1 + \frac{\alpha_s}{\alpha_s}\right\}$ π $\left[\left(1-\frac{1}{2}\right)$ $\ln \frac{2\mu}{2}$ μ_b ◆ $\beta_0 - C_A$ $\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right\}$

⇤¯(*µ*) ⇤

 $\sqrt{2 \text{angle}}$ Melnikov Uraltsey hen-ph/9708372 Czarnecki, Melnikov, Uraltsev hep-ph/9708372

⁶ ln 2⌘ ⁺ *^L^µ*

2 ln 2⌘ + *L^µ*

- **Provides a shot** ers, by introducing a Wilsor י-ג
. listance, renormalon free definition of heavy quark mass *and (*
roducing a Wilson cutoff µ≈1GeV to factor out IR physics. This ⁰ *C^A ^C^F ^µ*³ ↵*s*(*µb*) n *C*_{**F**} = 2. International context states in the MS scheme and the MS scheme at the Matthews (30. International scheme at α **perturbative Matteo's talk**) ⇡ and ⇢³ ⇡ ⇡ 2 *µb* S, C v introducing a W **Son cute** of μ \approx 1 GeV to factor out IR physics Provides a short-distance, renormalon free definition of heavy quark mass and OPE *kin,*⁰ ⁷⁷ ⁼ *^G*² *^F* ↵*mb*(*µ*)³*m*MS *^b* (*µb*)² parameters, by introducing a Wilson cutoff $\mu \approx$ *I GeV* to factor out IR physics. This can ² *,* (53) be realised in different ways, beyond 1-loop SV sum rules are most practical (see Matteo's talk)
The kinetic mass is one of a family of renormalon subtracted masses (PS, MRS…) but
- *i*₂ The kinetic mass is one of a family of renormalon subtracted masses (PS, MRS. with the definition of the kinetic scheme parameters used in the semileptonic fits we will decouple the charm a
The semileptonic fits we will decouple the charm and used in the charm and used in the charm and used in the c only in this case the definition is tailored on the HQE, although not Lorentz invariant ↵*s* $\frac{1}{2}$ _{otracted mi} ⌘² + 1 2 *L^µ* ◆ *µ*² ⇡ *m*² *b* ↵*^s* ⇡ \overline{D} $\mathsf{S}.$..) b \mathcal{L} H (*N^L* = 3*, N^V* = 1*,* ⁰ = 9)
	- *kin,*⁰ 77 \overline{p} conversion now 3-loop conversion now available 2005.06487 *,* (53)

³ *^L^µ* + 4⌘ + 2⌘² ⁺

 \overline{a} The expansion for mb is truncated at *O(1/mb)*, however *O(αs/mb2) vanish* according to П σ (*usitip*) vanisita (13 ²⇡²) \overline{a} hep-ph/0302262*.* We do not re-expand in *µ/mb* ✓4 ◆ 3

+

⇣↵*^s*

⌘2

C^F Akin

³ 2 ln 2⌘ + 2*L^µ*

11

9 ⌘3

10 YEARS BACK…

The HFAG semileptonic fits seemed incompatible with values of mb,c to the rules by the Karlsruhe and Hoang's groups (N/e found why and the ethnical sum-rule of the sum-rule determination of the sumfrom sum rules by the Karlsruhe and Hoang's groups. We found why…

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THEORETICAL CORRELATIONS

Correlations between theory errors of moments with different cuts difficult to estimate

- 1. 100% correlations (unrealistic but used in 2010)
- 2. corr. computed from low-order expressions
- 3. constant factor 0<ξ<1 for 100MeV step
- 4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated

charm mass determinations

Remarkable improvement in recent years. *mc* can be used as precise input to fix *mb*

CHARM MASS DETERMINATIONS TODAY

 $am(3C_0)(1-0.986/13)$ Col/No need to use All fits reported here use mc(3GeV)=0.986(13)GeV. No need to use the kinetic scheme for m_c . We use $m_c(2GeV)$ and $m_c(3GeV)$

and 0.11 fm) with unitary pion masses in the range from 200 to 420 MeV. For the valence from 200 MeV. For the v

FIT RESULTS

Without mass constraints $\lim_{n \to \infty}$ (10.57) 0.02 - (0.015) 0.51.1 100 V 0.00 m 0.00 m $m_b^{kin} (1 {\rm GeV}) - 0.85 \, \overline{m}_c (3 {\rm GeV}) = 3.714 \pm 0.018 \, {\rm GeV}$

and second rows give central values and uncertainties, the

- **results depend little on 12.0916 assumption for correlations and** choice of inputs, 1.8% determination of V_{cb}
	- 20-30% determination of the OPE parameters *^X .*
- and sum rules results squared mass, and sum

lepton energy applied by some of the experiments. Since

2

INDIRECT mb DETERMINATION

PDG 2020/FLAG: 0.020 0.014 0.068 0.039 0.061 0.096 0.16 0.68 *mb(mb)*=4.198(12)GeV

1.02

Alberti, Healey, Nandi, PG, 1411.6560

The fit gives m_b ^{kin}(1GeV)=4.553(20)GeV tion between di↵erent *central* moments and a correlation briversely, we can use m_b as external input to improve pre \mathbf{V} ditional constraint *mkin* 1.04 scheme translation error (2loop) *mbkin*(1GeV)=*mb(mb)*+0.37(3)GeV *mb(mb)=4.183(37)GeV* Conversely, we can use m_b as external input to improve precision on Vcb

HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m⁵$. At $1/m_b⁴$

 $2M_Bm_1 = \langle ((\vec{\rho})^2)^2 \rangle$ $2M_Bm_2 = g^2\langle \vec{E}^2 \rangle$ $2M_Bm_3 = g^2 \langle \vec{B}^2 \rangle$ $2M_Bm_4 = g\langle \vec{p} \cdot \text{rot } \vec{B} \rangle$ $2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$ $2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$ $2M_B m_7 = g\langle (\vec{S}\cdot\vec{p})(\vec{p}\cdot\vec{B})\rangle$ $2M_B m_8 = g\langle (\vec{S}\cdot\vec{B})(\vec{p})^2\rangle$ $2M_B m_9 = g \langle \Delta (\vec{\sigma} \cdot \vec{B}) \rangle$

Mannel,Turczyk,Uraltsev **1009.4622**

can be estimated by *Lowest Lying State* **Saturation** (LLSA) approx by truncating

$$
\langle B|O_1O_2|B\rangle = \sum_n \langle B|O_1|n\rangle \langle n|O_2|B\rangle
$$

see also Heinonen, Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$
\rho_D^3 = \epsilon \mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon \mu_G^2 \qquad \epsilon \sim 0.4 \text{GeV}
$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases, see1206.2296. In LLSA good convergence of the HQE.

 We used LLSA as loose constraint (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in the fit including higher powers

SENSITIVITY TO HIGHER POWER CORRECTIONS

DEPENDENCE ON LLSA UNCERTAINTY

we rescale all LLSA uncertainties by a factor *ξ*

PROSPECTS for INCLUSIVE *Vcb*

- **Theoretical uncertainties generally larger than experimental ones**
- \Box $O(\alpha_s/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- Burn 3loop relation between MS and kin scheme just completed 2005.06487 It can be used to improve the precision of the m_b input
- $O(\alpha_s^3)$ corrections to total width feasible, needed for 1% uncertainty
- Electroweak (QED) corrections require attention
- **New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk** could be measured already by Babar and Belle now, q² moments (Fael, Mannel, Vos)…
- **Lattice QCD** is the next frontier

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

- on the lattice one can compute mesons for arbitrary quark masses see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct $2+1+1$ simulation, a=0.62-0.89 fm, m_{π} =210-450 MeV, heavy masses from m_c to 3 m_c , ETM ratio method with extrapolation to static point. о
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to 1/m³ corrections п
- Results consistent with s.l. fits, improvements under way, also following new ш 3loop calculation of pole-kinetic mass relation

INCLUSIVE SL DECAYS ON THE LATTICE Hashimoto, PG 2005.13730 ` and *^p^µ* \ddotsc for the leftons \ddotsc and \ddotsc and \ddotsc S (*pq*)*^µ* with *^q^µ* = (*p*` ⁺*p*⌫¯)*^µ*. The di↵erential decay rate is written as [15, 16] $\overline{2}$ from four-point functions that contain interpolating operators to create and and and annihilate the *B*
B*asimuoto*, it is *zoositype* evaluated in the rest frame of the initial *B^s* meson. TAYS ON THE LATTICE $\frac{1}{2}$ Hashimoto, PG 2005.13730 trias. The hadronic tensor $\frac{1}{2}$ is the second tensor $\frac{1}{2}$ $\frac{1}{2}$ this decay mode is *^J^µ* = (*^V ^A*)*^µ* = ¯*c^µ*(1 5)*b*. One can perform an integral over the lepton energy *E*` where *K*(!*, q*) represents an integral kernel determined $\begin{array}{|c|c|c|c|c|c|}\n\hline\n\textbf{I} & \textbf{I} &$ $\frac{1}{1}$ is implicity, the zood, is $\frac{1}{2}$ \overline{m} frame of the initial *B^s* meson, respectively. Thus, the total decay rate may be calculated as H $F = \frac{1}{2}$ $P(X \cup S \cup F)$ ⌫¯ for the leptons ` and ¯⌫ in the final state, re- AY ⁵ C *x* 2*m^B^s* h*Bs*(0)*|J† ^µ*(*x, t*)*J*⌫(0*,* 0)*|Bs*(0)i (8) where *G^F* is the Fermi constant and *|Vcb|* is one of the Cabibbo-Kobayashi-Maskawa matrix elements. The transfer momentum *q^µ* and the lepton energy *E*` are $r \sim 11$ 1 1 *V* 2*m^B^s* ^h*Bs*(0)*|J*˜*†* INCLUSIVE SL DECAIS UN INE LAI IICE decay process *b* ! *c*`⌫¯. After describing the kinematics

$$
\frac{d\Gamma}{dq^2 dq^0 dE_{\ell}} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}
$$
\n
$$
W^{\mu\nu}(p,q) = \sum_{X_c} (2\pi)^3 \delta^{(4)}(p-q-r) \times
$$
\n
$$
\times \frac{1}{2E_{B_s}} \langle B_s(\mathbf{p}) | J^{\mu\dagger} | X_c(\mathbf{r}) \rangle \langle X_c(\mathbf{r}) | J^{\nu} | B_s(\mathbf{p}) \rangle
$$
\n
$$
\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{q_{\text{max}}^2} dq^2 \sqrt{q^2} \sum_{l=0}^2 \bar{X}^{(l)} \qquad \bar{X}^{(l)} \equiv \int_{\sqrt{m_{D_s}^2 + q^2}}^{m_{B_s} - \sqrt{q^2}} d\omega \, X^{(l)} \quad (\omega \text{ hadr energy})
$$
\n4point functions on the lattice are related to the hadronic tensor in euclidean.

4point functions on the lattice are related to the hadronic tensor in euclidean *POINT TUNCTIONS ON THE IATTICE Are relate J* the nadronic tensor in euclidean Fig. 10 and the letter of the left of the lead of the letter in equipped the letter of the letter of $\frac{1}{2}$ where *q*² **E** are related to the riadibility to decay *4 point functio c* related evaluated in the rest frame of the initial *B^s* meson. ated to $\frac{1}{2}$ the hadronic tensor in euclidean \mathbb{R}^2 $\overline{\text{Oint}}$ fun e related to the hadronic tensor in euclidean

V

2*m^B^s*

the momentum *p^µ* for the initial *B* meson, the momenta

⁸⇡³ *^Lµ*⌫*W^µ*⌫

It is summed over all possible final states *X^c* to represent

$$
\sum_{\mathbf{B}} e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2m_{B_s}} \langle B_s(\mathbf{0}) | J^{\dagger}_{\mu}(\mathbf{x},t) J_{\nu}(\mathbf{0},0) | B_s(\mathbf{0}) \rangle
$$

$$
\sim \langle B_s(\mathbf{0}) | \tilde{J}^{\dagger}_{\mu}(-\mathbf{q}) e^{-\hat{H}t} \tilde{J}_{\nu}(\mathbf{q}) | B(\mathbf{0}) \rangle
$$

$$
= \langle B_s(\mathbf{0}) | \tilde{J}^{\dagger}_{\mu}(-\mathbf{q}) \delta(\hat{H} - \omega) \tilde{J}_{\nu}(\mathbf{q}) | B_s(\mathbf{0}) \rangle
$$

$$
= \langle B_s(\mathbf{0}) | \tilde{J}^{\dagger}_{\mu}(-\mathbf{q}) K(\hat{H}, \mathbf{q}) \tilde{J}_{\nu}(\mathbf{q}) | B_s(\mathbf{0}) \rangle
$$

 $\frac{1}{2}$ approximated by polynomial $\frac{1}{2}$ K approximated by polynomial $K(H,q) =$ *^X*(2) = (*m^B^s* !) *K* approximated by polynomia the Cappins Annual Cabibbo - Cappins and It is summed over all possible final states *X^c* to represent K approximated by polynomial p*q*² smearing is introduced for the initial B meson interpretation in the interpolating operator λ K approximated by polynomial $K(\hat{H}, q)$

transfer momentum *q^µ* and the lepton energy *E*` are

*dq*²

this decay mode is *^J^µ* = (*^V ^A*)*^µ* = ¯*c^µ*(1 5)*b*.

and J^ν are inserted at t¹ and t2, respectively.

X^c

decay rate, we present a pilot lattice study.

spectively. Then, the hadronic state *X^c* has momentum

in (1), and the remaining integrals over *q*² and *r*² can

X¯(*l*)

while the *i*-th direction is supposed to be perpendicular

, (3)

 $\hat H ({\bm q}) = k_0 ({\bm q}) + k_1 ({\bm q}) e^{-\hat H} + \cdots + k_N ({\bm q}) e^{-N\hat H}$ that the integral is implicit on the right hand side; all integrals in $K(\hat{H}, a) = k_0(a) + k_1(a)e^{-\hat{H}} + \cdots$ Here, we take the momentum *q* in the *k*-th direction, *p*² + ... + $c_{N}(\boldsymbol{q})e^{-N}$ \mathbf{r} $V(\hat{H} \mid \hat{e} \mid \hat{e}) + k(\hat{e})e^{-\hat{H}} + k(\hat{e})e^{-N\hat{H}}$ $n_1(1, q) = n_0(q) + n_1(q)$ ^h*Bs*(0)*|J*˜*† ^µ*(*q*)*eHt* ^ˆ *^J*˜ where *K*(!*, q*) represents an integral kernel determined $K(\hat{H}, \mathbf{q}) = k_0(\mathbf{q}) + k_1(\mathbf{q})e^{-\hat{H}} + \cdots + k_N(\mathbf{q})e^{-N\hat{H}}$ λ in the form $\overline{}$

^d! *^K*(!*, ^q*)h*Bs*(0)*|J*˜*†*

 $\frac{1}{2}$

be rewritten in terms of ! and *q*², energy and spatial

time dependence of the matrix elements of the matrix elements of the matrix elements of the matrix elements of
The matrix element in (8) may be dependent in (8) may be dependent in (8) may be dependent in (8) may be depen

^eⁱq·^x ¹

transfer momentum *q^µ* and the lepton energy *E*` are

h*Bs*(0)*|J†*

the intermediate states may exist between the currents.

j=1

⌫(*q*)*|B*(0)i*,* (9)

⌫(*q*)*|Bs*(0)i

p*q*²

^µ(*q*)(*H*^ˆ !)*J*˜

^µ(*x, t*)*J*⌫(0*,* 0)*|Bs*(0)i (8)

^h *^µ[|]* ⌫ⁱ *,* (12)

A PILOT NUMERICAL STUDY

Hashimoto, PG 2005.13730

Smeared spectral functions can be computed on the lattice, see also 1704.08993

100, and the measurement is performed with four diagnosis performance with four diagnosis performance with four

^j (*eH*^ˆ)*[|]* ⌫i*/*^h *^µ[|]* ⌫ⁱ at *^j* ' 10 or

Longitudinal (k) and perpendicular (?) polarizations are plot-

ted for vector (*V V*) and Axial-vector (*AA*) channels.

matrix elements h *^µ|T* ⇤

Using Chebychev polynomials to approx the kernel, 2+1 flavours of Moebius domain wall fermions with 1/a=3.6GeV, one gets $M_{Bs} = 3.45$ GeV, i.e. $m_b \approx 2.70$ GeV $m_b-m_c \sim 1.7$ GeV only ted for vector (*V V*) and Axial-vector (*AA*) channels.

 $\sum_{a,b}$ $\Gamma/|V_{cb}|^2 = 4.88(57) \times 10^{-13}$ GeV. Lattice $\Gamma / |V_{cb}|^2 = 5.41(82) \times 10^{-13} \text{ GeV}$ OPE including all known corrections the smaller phase space for the artificially small *b* quark $\frac{1}{\Gamma_1} \sum_{2.5}^{1.5}$ $\frac{1}{2.5}$ $\frac{1}{2.5}$ $\frac{1}{2}$ $\$ $\frac{1}{\sqrt{2}}$

spectator and to the physical *b* mass. We show the lowest

mass, but missing higher order corrections and uncer-

CONCLUSIONS

- Inclusive s.l. B decays appear in a good shape: they can be improved by higher order calculations, lattice studies and new data from Belle II, as well as improved determinations of mb and m_c
- The recent 3loop calculation makes it possible to employ precise determinations of m_b in the kinetic scheme, looking forward to 3loop corrections to the width
- Inclusive/Exclusive *V_{cb}* tension remains, but weaker. Hopefully, it will disappear. $LQCD$ is likely to decide the fate of the V_{cb} puzzle